Stochastic modeling of nonstationary earthquake time series with long-term clustering effects

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Earthquake time series are widely used to characterize the main features of regional seismicity and to provide useful insights into earthquake dynamics. Properties such as intermittency and nonstationary clustering are common in earthquake time series, highlighting the complex nature of the earthquake generation process. In the present work we introduce a stochastic model with memory effects that reproduces the temporal scaling behavior observed in regional seismicity. For nonstationary earthquake activity, where the average seismic rate fluctuates, the solution of the stochastic model is the q-generalized gamma function that presents two power-law regimes for short and long waiting times, respectively, while for stationary activity it reduces to the standard gamma function. To validate the derived model, we study nonstationary earthquake time series from Southern California and Japan. The analysis shows that for various threshold magnitudes and spatial areas and after rescaling with the mean waiting time, the normalized probability density functions fall onto a unique curve, which is characterized by two power-law regimes for short and long waiting times, respectively, a scaling behavior that can exactly be recovered by the derived q-generalized gamma function. The results show the validity of the stochastic model and the derived scaling function, further signifying both short- and long-term clustering effects and memory in the evolution of seismicity.

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I. INTRODUCTION

Earthquakes are a typical example of complex dynamic phenomena, exhibiting a high variability in the time and space of their occurrence. Complexity in the temporal occurrence of earthquakes, in particular, is manifested in the strong variations between periods of stationary activity, where few regional earthquakes occur and the seismicity rate remains relatively constant, and periods of increased activity associated with sudden seismic bursts, where a significant increase of the seismicity rate may occur. As such, earthquake occurrence is an intermittent phenomenon, characterized by fluctuating behavior, nonstationary clustering, and (multi)fractal structures [1,2].

Regarding the temporal occurrence of seismicity, a long-standing question concerns whether earthquakes occur randomly in time, or if some kind of memory is present in the seismogenic process. A short-term clustering effect is evident in aftershock sequences, where the aftershock production rate decays as a power law with time according to the modified Omori formula [3]. However, the background activity in a seismic region is frequently considered uncorrelated and statistically independent in time, such that models that express randomness, like the general Poisson model, have been used to describe its temporal properties [4,5]. The latter properties are manifested in the waiting time (or interevent time, recurrence time) distribution between consecutive earthquakes, where a mixture distribution between triggered aftershocks that scale according to the modified Omori formula and a Poissonian background activity has been proposed [6,7]. Such distribution approximately takes the form of a gamma distribution [7,8]:

$$f(\tau) = C\left(\frac{\tau}{\beta}\right)^{\gamma-1} \exp\left(-\frac{\tau}{\beta}\right), \tag{1}$$

where C is a normalization constant, γ is the exponent that characterizes the power-law decay of short and intermediate waiting times, and β is a scaling parameter that marks the crossover to the exponential long waiting times decay.

This type of scaling has been found in stationary earthquake time series [7,8] and is approximately the one predicted by the epidemic-type aftershock sequence (ETAS) model [7,9,10]. However, intermittency and clustering in the temporal occurrence of seismicity contradicts a Poissonian presumption on the constant seismicity rates and random temporal occurrence. Furthermore, correlations and long-term clustering effects, associated with the background activity, have also been found in earthquake time series [11–16], which implies memory in the earthquake generation process [17–19]. In support to the latter, Ref. [15] studied nonstationary earthquake time series in the Corinth Rift (Greece) and found a bimodal waiting time distribution of two power-law regimes that characterize both short and long waiting times. Similar scaling behavior has previously been found by [12] and [20] for nonstationary earthquake time series in Southern California and Japan, respectively. To describe the observed scaling behavior, Ref. [15] proposed a unified scaling function that incorporates the two power-law regimes, namely the q-generalized gamma function [21], that takes the form

$$f(\tau) = C\left(\frac{\tau}{\tau_0}\right)^{\gamma-1}\exp_q\left(-\frac{\tau}{\tau_0}\right), \quad (2)$$

where C is the normalization constant and γ is the scaling exponent, similar as in Eq. (1), while the last term on the right–hand side of Eq. (2) is the q-exponential function defined as

$$\exp_q(x) = [1 + (1-q)x]^{1/(1-q)}. \quad (3)$$

The q-exponential function is associated with nonextensive statistical mechanics as it maximizes the nonadditive entropy S_q [2,22] and its applicability to earthquake dynamics has been demonstrated in numerous studies [2,14–16,19,23]. In Eq. (2) q is the nonextensive parameter that marks how far or close to exponential and thus to random behavior the system is and τ_0 marks the time of crossover to the second power-law regime. In the limit $q \to 1$ the q-generalized gamma function exactly recovers the ordinary gamma function, Eq. (1). Equation (2) is similar to the F distribution [24] known from statistics and is a particular case of the pathway model of Ref. [25]. It has also empirically been proposed to describe stock traded volume distributions in financial markets [26].

In the present work we extend previous results on the scaling properties of nonstationary earthquake time series and use statistical physics to propose an underlying mechanism for the temporal occurrence of earthquakes. Within this framework we develop a stochastic mechanism with memory effects whose exact solution for nonstationary series is the q-generalized gamma function given in Eq. (2). To test the validity of the derived model, we further study the scaling behavior of earthquake time series in Southern California and Japan. The analysis demonstrates results consistent with the stochastic model, indicating clustering effects at all time scales and both short- and long-term memory in the seismogenic process.

II. STOCHASTIC MECHANISM WITH MEMORY EFFECTS

Let's consider the following stochastic differential equation for the evolution of seismicity:

$$d\tau = -k(\tau - \bar{\tau})dt + \varphi\sqrt{\tau}W_t, \quad (4)$$

where the temporal occurrence of earthquakes is represented by the waiting time series τ after some time t. The latter stochastic equation manifests two parts controlling the evolution of seismicity. The first deterministic part aims to keep the seismic rate R stable to the typical value of $R = 1/\bar{\tau}$ according to a restoring constant k that represents the rate of relaxation to the mean waiting time $\bar{\tau}$. The second stochastic part represents memory effects in the evolution of seismicity. The stochastic term W_t is the standard Wiener process following a Gaussian distribution with zero mean and unitary variance that mimics the microscopic effects in the evolution of τ. Due to its random sign, W_t leads to an increase ($W_t > 0$) or decrease ($W_t < 0$) of τ. The term φ adds some noise to the process and can be expressed as a function of the mean waiting time $\bar{\tau}$ and the restoring constant k as $\varphi = \sqrt{(2/\gamma)k\bar{\tau}}$, where γ is a characteristic constant of the system.

The stochastic differential equation given in Eq. (4) is a classic example of multiplicative noise, further known in statistics as the Feller process [27]. It has previously been introduced by Heston to derive stochastic volatility in trading price returns [28,29] and by [21] to describe stock traded volume sequences in financial markets. Interestingly, financial time series present similar characteristics to earthquake time series, such as high fluctuations, power-law scaling, and multifractal behavior, among others [30–32].

To determine the evolution of the waiting time series τ after some time t, given by the probability distribution $f(\tau, t)$, we can write the corresponding Fokker-Planck equation for Eq. (4) [33,34]:

$$\frac{\partial f(\tau,t)}{\partial t} = \frac{\partial}{\partial \tau}[k(\tau-\bar{\tau})f(\tau,t)] + \frac{\partial^2}{\partial \tau^2}\left[\tau\bar{\tau}\frac{k}{\zeta}f(\tau,t)\right]. \quad (5)$$

The stationary solution of the latter Fokker-Planck equation, Eq. (5), is the gamma distribution [35]:

$$f(\tau) = \frac{\gamma^\gamma}{\Gamma[\gamma]\bar{\tau}}\left(\frac{\tau}{\bar{\tau}}\right)^{\gamma-1}\exp\left[-\frac{\gamma}{\bar{\tau}}\tau\right]. \quad (6)$$

Let's now consider local fluctuations in the seismic rate R associated with nonstationarities in the evolution of the earthquake activity over time scales much larger than k^{-1}, which is necessary for Eq. (4) to reach stationarity. In this case local fluctuations of the mean waiting time $\bar{\tau}$ appear and we assume that these fluctuations follow the stationary gamma distribution:

$$P(\bar{\tau}) = \frac{(\gamma/\lambda)^\delta}{\Gamma[\delta]}\bar{\tau}^{-(\delta+1)}\exp\left[-\frac{\gamma}{\lambda\bar{\tau}}\right]. \quad (7)$$

In this case Eq. (6) provides the conditional probability of τ given $\bar{\tau}$, hence

$$f(\tau) \to p(\tau|\bar{\tau}) = \frac{\gamma^\gamma}{\Gamma[\gamma]\bar{\tau}}\left(\frac{\tau}{\bar{\tau}}\right)^{\gamma-1}\exp\left[-\frac{\gamma}{\bar{\tau}}\tau\right]. \quad (8)$$

Thus, the joint probability for obtaining certain values of τ and $\bar{\tau}$ is $P(\tau, \bar{\tau}) = p(\tau|\bar{\tau})P(\bar{\tau})$. The marginal probability of τ, independent of $\bar{\tau}$, is now given by

$$P(\tau) = \int_0^\infty P(\tau|\bar{\tau})d\bar{\tau} = \int_0^\infty p(\tau|\bar{\tau})P(\bar{\tau})d\bar{\tau}. \quad (9)$$

From Eqs. (7)–(9) and by performing the integration, we get the solution for varying $\bar{\tau}$:

$$P(\tau) = \frac{\lambda\Gamma[\gamma+\delta]}{\Gamma[\gamma]\Gamma[\delta]}(\lambda\tau)^{\gamma-1}(1+\lambda\tau)^{-(\gamma+\delta)}. \quad (10)$$

By further carrying out the changes in the variables,

$$\lambda = \frac{q-1}{\tau_0}, \quad \delta = \frac{1}{q-1} - \gamma, \quad (11)$$

and considering the q-exponential function given in Eq. (3), Eq. (10) can be written as [21]

$$P(\tau) = \frac{(q-1)^{\gamma+1}\Gamma[1/(q-1)]}{\tau_0\Gamma\{[1/(q-1)]-\gamma\}\Gamma[\gamma]}\left(\frac{\tau}{\tau_0}\right)^{\gamma-1}\exp_q\left[-\frac{\tau}{\tau_0}\right]. \quad (12)$$

The last equation, Eq. (12), has the exact form of the q-generalized gamma function given in Eq. (2). Equation

(12) has been derived by the stochastic model, Eq. (4), for varying mean waiting time $\bar{\tau}$, i.e., nonstationary earthquake activity. If $\bar{\tau}$ does not fluctuate, i.e., if the earthquake activity is stationary, then the gamma function, Eqs. (1) and (6), is dynamically recovered.

III. SCALING PROPERTIES OF NONSTATIONARY EARTHQUAKE TIME SERIES

To validate the derived stochastic model, we study earthquake time series in Southern California and Japan [36] (Fig. 1). Earthquake activity in the studied regions is typically characterized by nonstationarities and fluctuating behavior, where stationary periods of low to moderate earthquake activity are interspersed by sudden seismic bursts, which are related to the occurrence of earthquake sequences and earthquake swarms, such as the 2016 Kumamoto earthquake sequence (Japan), or more frequently to the occurrence of stronger events followed by subsequent aftershock sequences, observed for instance after the 2011 M_W9.1 Tohoku mega-earthquake (Japan), or following the 1992 M_W7.3 Landers earthquake and the 2010 M_W7.2 Baja California earthquake (Southern California) (Fig. 2). During such earthquake sequences a significant increase of the seismicity rate is observed, represented by large spikes in Fig. 2. Such earthquake sequences indicate a short-term clustering effect, which is evident in almost every earthquake catalog [37].

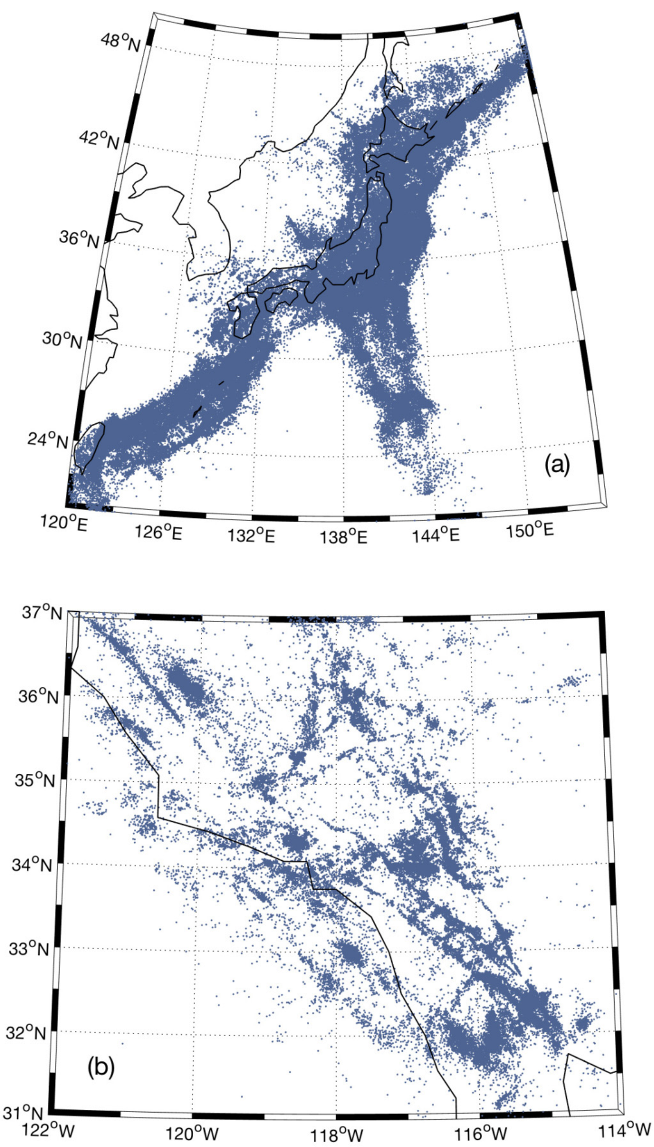

FIG. 1. Epicentral distribution of the 03/06/2002–30/06/2016 regional seismicity in Japan (a) and of the 01/01/1981–30/06/2011 seismicity in Southern California (b) for earthquakes with magnitude $M \geqslant 2$.

To extract further information regarding the temporal structure of seismicity and to validate the stochastic model, we define the distribution of waiting times τ, i.e., the time intervals between successive earthquakes, defined as $\tau_i = t_{i+1} - t_i$, where t_i is the time of occurrence of the ith event, $i = 1, 2, \ldots, N-1$, and N is the total number of events. The waiting time series for Southern California and Japan are shown in Fig. 3, presenting high fluctuations over almost eight orders of magnitude, indicating once more the intermittent character of the earthquake occurrence.

Similar to the analysis of [6,38], we cover the areas under study with a grid of cell sizes $L \times L$ and estimate the waiting time series in each cell. We estimate the probability density $P_{m,L}(\tau)$ of waiting times τ for all earthquakes with magnitude equal to or greater than a threshold magnitude m occurring within range L. The probability density $P_{m,L}(\tau)$ is then constructed by counting the number of τ that fall into logarithmically spaced bins and then normalized by dividing this number by the bin width and by the total number of counts, so that the probabilities of occupation sum to 1. We don't consider waiting times shorter than 1 min due to overlapping of the successive events in the seismograms and the possible incompleteness of the catalogs at very short-time scales [38,39].

The resulting $P_{m,L}(\tau)$, for various cell sizes L and threshold magnitudes m, are shown in Fig. 4. In each case, we rescale the observed variables with the mean waiting time $\bar{\tau}$, which is equivalent to rescaling with the mean seismicity rate ($R = 1/\bar{\tau}$). As Ref. [8] has explicitly showed, after rescaling with $\bar{\tau}$(or R), the observed $P_{m,L}(\tau)$ fall onto a unique curve. As evident from Fig. 4, the rescaled $P_{m,L}(\tau)$, for both datasets and for the various L and m, approximately fall onto a unique curve, which consists of two parts: for short and intermediate waiting times the observed $P_{m,L}(\tau)$ decay as a power law until some characteristic waiting time τ_c, while for long waiting times ($\tau > \tau_c$) $P_{m,L}(\tau)$ decay faster according to another power law. The two power-law regimes are evident from the linear decay of $P_{m,L}(\tau)$ with τ in the double logarithmic axes representation of Fig. 4, which span for almost 11 orders of magnitude in the y axis. Hence, the scaling behavior of nonstationary earthquake time series in Southern California and Japan is characterized by a bimodal distribution between two power laws in short and long waiting times, respectively, indicating both short- and long-term clustering effects.

Such scaling behavior can well be approximated by the q-generalized gamma function, Eq. (2), for the values of $C = 0.147 \pm 0.011$, $\tau_0 = 2.02 \pm 0.14$, $\gamma = 0.216 \pm 0.005$, and $q = 1.39 \pm 0.11$ for Japan and the values of $C = 0.052 \pm 0.009$, $\tau_0 = 3.55 \pm 0.45$, $\gamma = 0.1 \pm 0.008$ and

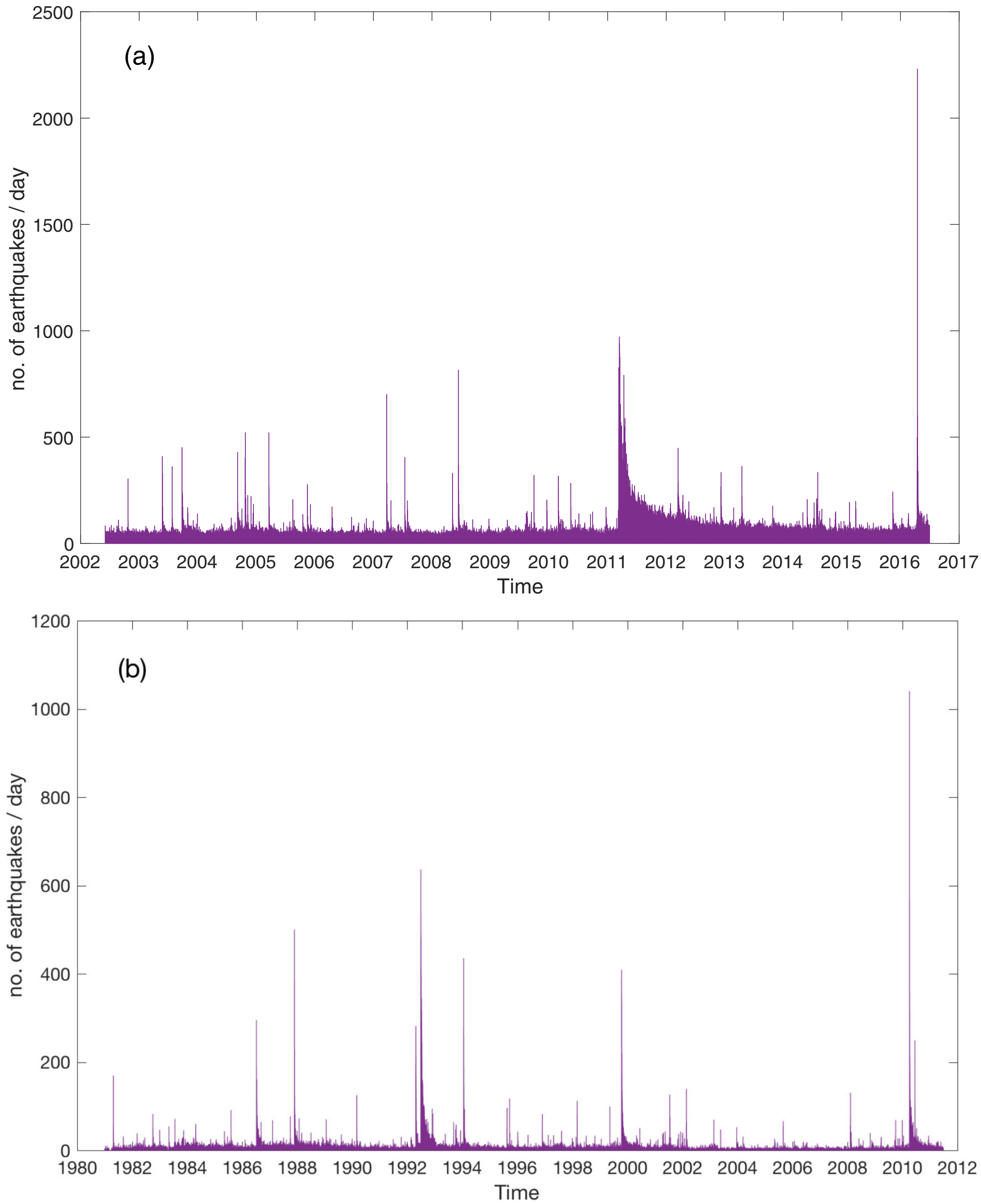

FIG. 2. Seismicity rate per day for Japan (a) and Southern California (b), for earthquakes with magnitude $M \geqslant 2$.

$q = 1.43 \pm 0.08$ for Southern California (Fig. 4). Thus, the q-generalized gamma function offers a unified function to describe the observed double power-law behavior in nonstationary earthquake time series, in accordance with the stochastic model that has been introduced in the previous section. According to this scaling behavior, for short waiting times, $P_{m,L}(\tau)$ scales as a power law $\sim\tau^{\gamma-1}$ up to a characteristic waiting time $\tau_c = \tau_0/\bar{\tau}$, indicating short-term clustering. For waiting times greater than τ_c, $P_{m,L}(\tau)$ scales as another power law $\sim\tau^{(1-\gamma)/(1-q)}$, showing clustering effects at the long term.

IV. DISCUSSION

In view of the statistical physics of earthquakes, the most important task is to elucidate general physical mechanisms that produce the scale invariant properties that are evident in the sizes of earthquakes and faults, as well as in the spatiotemporal occurrence of earthquakes [2,40]. In this line, in the present work we introduced a stochastic model with memory effects to describe the scaling properties of earthquake time series and consequently the temporal evolution of seismicity. During periods of stationary earthquake activity,

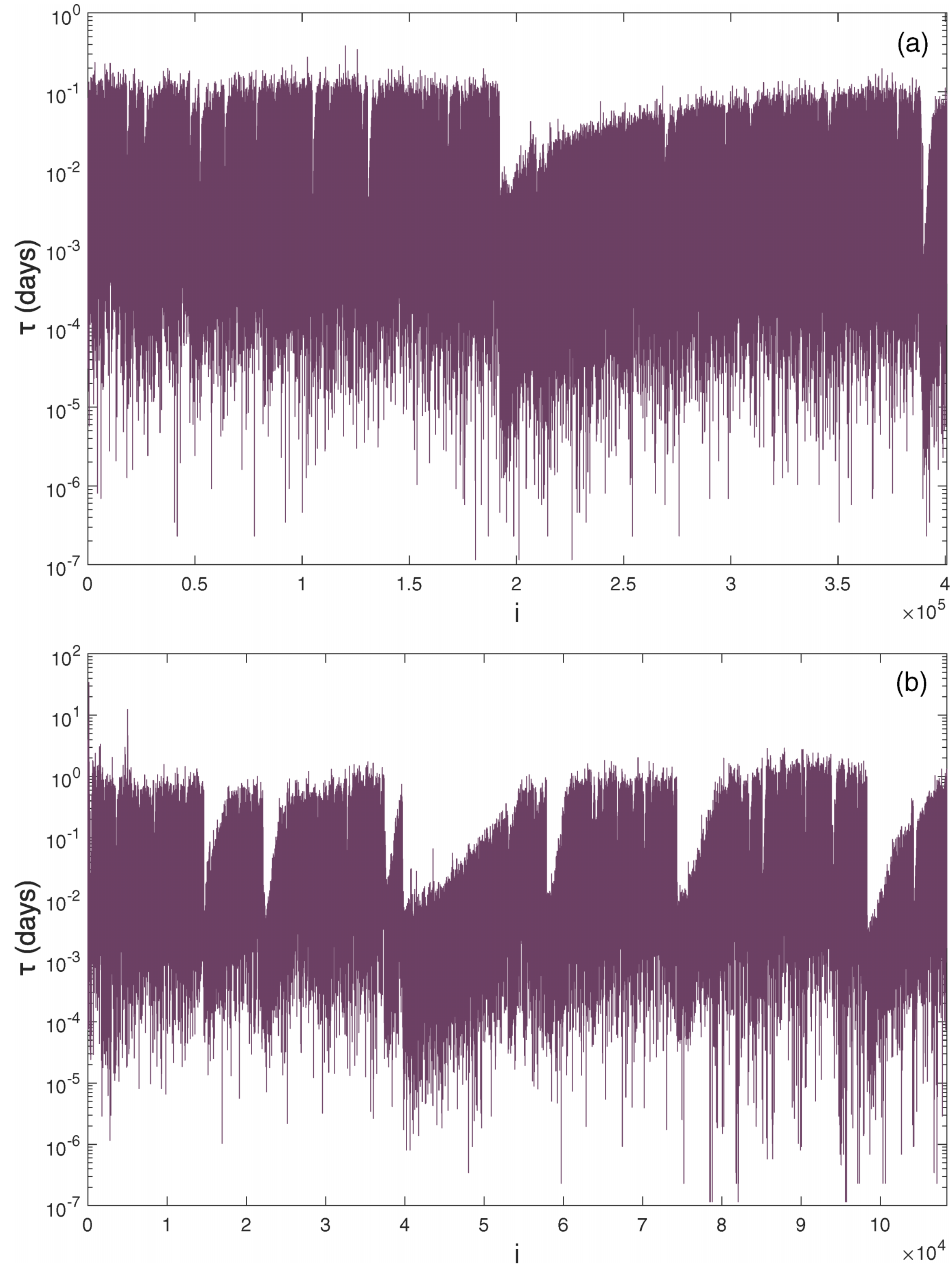

FIG. 3. The waiting time series τ (in days) for Japan (a) and Southern California (b), for earthquakes with magnitude $M \geqslant 2$.

where the average seismic rate is relatively constant, the stochastic model produces the gamma function as the scaling function of the waiting time series of earthquakes. However, during periods of nonstationary earthquake activity, where the average seismic rate fluctuates, the stochastic model provides the q-generalized gamma function as the scaling function of the waiting time distribution.

To test and validate the stochastic model, we studied nonstationary earthquake time series in Japan and Southern California. For both regions and for various threshold

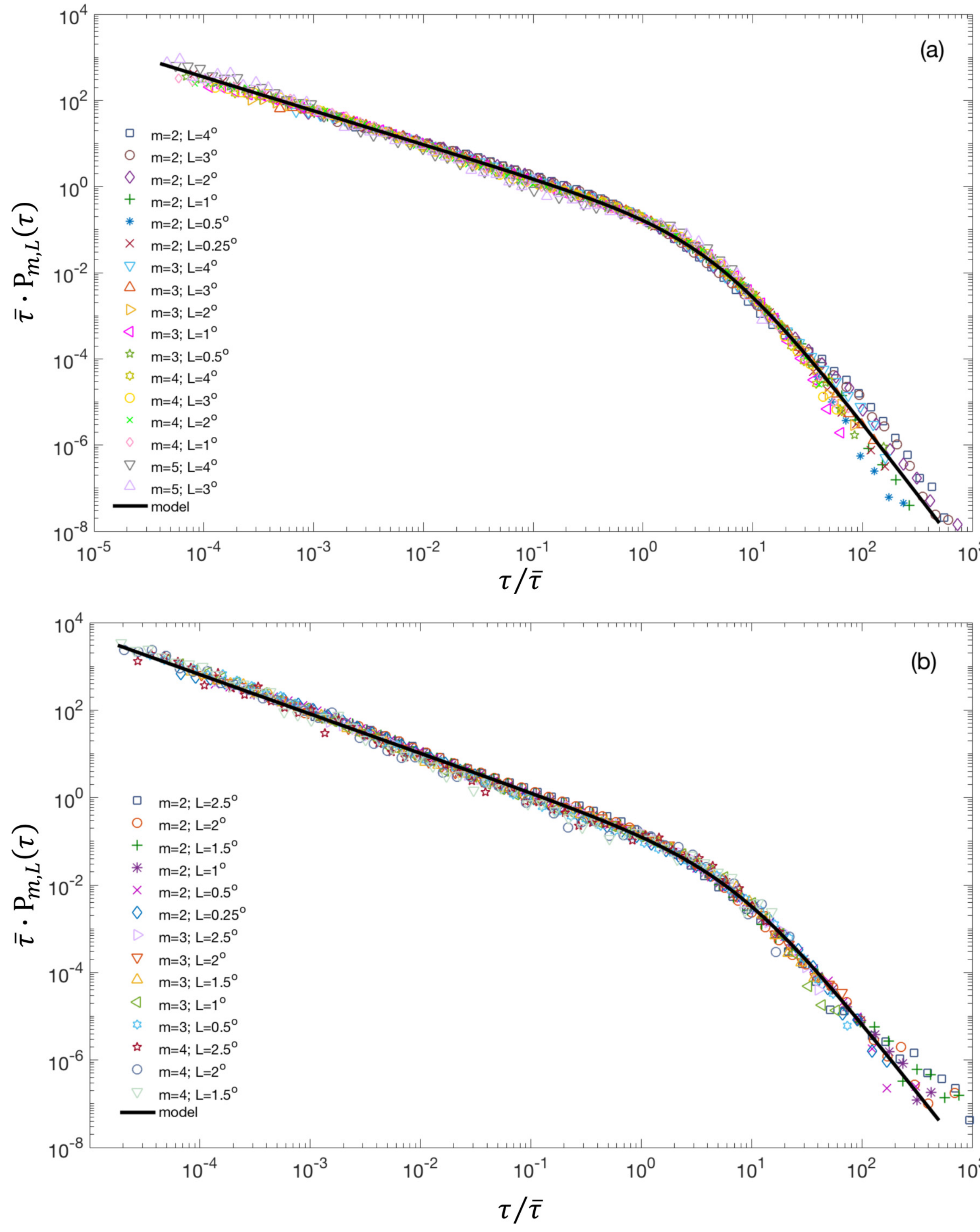

FIG. 4. Normalized probability densities $P_{m,L}(\tau)$ of the rescaled waiting times τ for Japan (a) and Southern California (b), for various cell sizes L and threshold magnitudes m. The model (solid line) represents the q-generalized gamma function, Eq. (2) for the values of $C = 0.147$, $\tau_0 = 2.02$, $\gamma = 0.216$, and $q = 1.39$ for Japan (top) and $C = 0.052$, $\tau_0 = 3.55$, $\gamma = 0.1$, and $q = 1.43$ for Southern California (bottom).

magnitudes m and spatial areas of size L, the analysis showed that the scaling structure of the waiting time series is characterized by a bimodal distribution and a crossover behavior between slow power-law decay for short waiting times and faster power-law decay for long waiting times. Such scaling behavior can exactly be reproduced by the stochastic model and the derived q-generalized gamma function that unifies the observed double power-law behavior for short and long waiting times, respectively. According to this scaling behavior, the normalized probability density $P_{m,L}(\tau)$ for short waiting times scales as a power law with τ as $\sim\tau^{\gamma-1}$, while for long waiting times $P_{m,L}(\tau)$ scales as another power law with τ as $\sim\tau^{(1-\gamma)/(1-q)}$, indicating both short- and long-term clustering effects in the evolution of seismicity. Such results lay further support to the double power-law behavior in the waiting time distribution found previously by [12,15,20] and the long-term clustering effects and memory in the earthquake activity shown by [11,17,18], respectively.

The multiplicative process used in the stochastic model can be understood as a cascade of seismic bursts that interposes the background earthquake activity, giving rise to intermittency and nonstationary earthquake rates in the evolution of seismicity. The bimodality in the observed $P_{m,L}(\tau)$ and the gradual crossover between the two power-law regimes for short and long waiting times, respectively, can then express these two processes. The first one is related to short-term clustering effects induced by aftershock sequences and earthquake swarms, and the second one to long-term clustering effects, related to the background activity. Short waiting times decay as $\sim\tau^{\gamma-1}$, providing a decay exponent of -0.784 for Japan and -0.9 for Southern California. The short-term clustering effect can further be expressed by the Omori formula that for a single aftershock sequence provides a power-law waiting time distribution with exponent $2-p^{-1}$, where p is the exponent of the modified Omori formula [3]. Combining the latter with the decay exponents for short waiting times, we find a p value of 0.82 for Japan and 0.91 for Southern California. These values do not represent the p value of the Omori formula in a strict sense, but rather an average value that expresses the average deacay rate of aftershocks in the studied regions. The latter turns out to be faster for Southern California than Japan, although short-term clustering effects do not only appear due to aftershock sequences, but also due to earthquake swarms. In addition, long waiting times decay as another power law $\sim\tau^{(1-\gamma)/(1-q)}$, providing the almost similar decay exponents of 2.01 for Japan and 2.09 for Southern California, which are almost identical to the exponent 2.2 found by [41] for various earthquake catalogs.

The stochastic model and the proposed scaling for the waiting time distribution introduced in the present study present the advantage of considering all earthquakes in the seismic catalog, regardless of their classification as foreshocks, mainshocks, and aftershocks and without the need to transform the time series into stationary ones, as it was previously performed by [8]. Nonetheless, the stochastic model predicts that during periods of stationary earthquake activity, the normalized probability densities $P_{m,L}(\tau)$ scale according to the gamma function, in accordance to previous results [7,8]. The latter implies that the scaling behavior in the temporal evolution of seismicity is controlled by the seismic rate, where nonstationarities in the earthquake activity, giving rise to a varying seismic rate, determine the emergence of the second power-law regime for long waiting times.

The results obtained in the present study seem robust for nonstationary earthquake time series, despite the possible incompleteness of the earthquake catalogs, the selected threshold magnitude, or the spatial size of the chosen area, further signifying self-similarity in the temporal structure of seismicity. The latter, as well as the almost perfect collapse of the rescaled $P_{m,L}(\tau)$ onto a unique curve, further implies that earthquake hazard assessments in the given regions $P_{m,L}(\tau)$ can be estimated by just knowing the average seismic rate R as $P_{m,L}(\tau) = Rf(\tau)$, where the scaling function $f(\tau)$ can well be approximated by the q-generalized gamma function.

The scaling properties found from the analysis can further be viewed in terms of probabilities of subsequent earthquakes. The observed scaling indicates that the probability of a subsequent earthquake is high immediately after the occurrence of the previous one and decreases slowly up to a characteristic waiting time τ_c, where a crossover to faster decaying probabilities is observed. In addition, the observed short- and long-term clustering effects further suggest that short waiting times are more likely to be followed by short ones and long waiting times by long ones. Such type of behavior implies that the longer it had been since the last earthquake, the longer it will be until the next one, referred to as "the paradox of the expected time until the next earthquake" [42,43].

To conclude, the stochastic model with memory effects that has been introduced in the present work to describe the temporal evolution of seismicity can well reproduce the observed scaling properties of nonstationary earthquake time series. These scaling properties, for various threshold magnitudes and spatial areas, are characterized by double power-law behavior for short and long waiting times, respectively, which can well be reproduced by the derived q-generalized gamma function that unifies the two power-law regimes, further indicating both short- and long-term clustering effects and memory in the seismogenic process.

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