Learning-based game theoretical framework for modeling pedestrian motion

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Pedestrians are active agents that undergo a repeated decision-making process while walking. These anisotropic, interactive, and feedback-oriented agents observe their surroundings, anticipate the future state of the network, and decide on their next movements accordingly, while ensuring a collision-free path toward their destination. Aiming at capturing these behavioral characteristics of human agents while walking, the present study puts forward a learning-based game theoretical approach for modeling pedestrian motion in dynamic environments. The proposed game structure provides a technical foundation to analyze optimal decision-making by pedestrians where the outcome of the game for each player's choice depends primarily on the strategies played by other players. This, in turn, ensures the frequently observed collision avoidance behavior of pedestrians while walking. The influence of nearby pedestrians on one's decision-making process and the feedback-oriented behavior of human agents are also captured via incorporating a learning structure. Optimum moving strategies are selected based on Nash equilibria calculations, where everyone is playing optimally given what all other players are playing. The validation results using real-world trajectories of pedestrians provide evidence of the model's capability in describing pedestrian motion and walking behaviors at microscopic as well as macroscopic levels.

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I. INTRODUCTION

Despite numerous studies in vehicular traffic, research on crowd dynamics is still young. Compared to vehicular traffic, crowd dynamics is more complex in many regards, justifying dedicated rationales and modeling approaches. Its complexity pertains to multidirectional movements, interactions between individuals, and effect of human psychology and cultural aspects on pedestrians' decision-making and behavior. Various analytical as well as experimental studies have been proposed to describe the underlying decision-making and model various crowd movement scenarios. Social force models [1–3], cellular automata [4–6], and analogy to fluid dynamics [7–9] are among the broadly used approaches in modeling pedestrians' behavior. The review papers [10–12] and books [13,14] present a general introduction and comparison of the existing models on pedestrian motion. Despite the considerable number of efforts on predicting and modeling crowd dynamics, practical applications of pedestrian motion models (e.g., navigating autonomous robots in dynamic environments) still show significant discrepancies with real-world pedestrian trajectories [15,16]. One of the major challenges in this regard is the lack of system perspective, i.e. focusing on predicting individual pedestrian's movement and making decisions accordingly afterward; therefore, ignoring the interactive nature of pedestrian behavior [17]. Indeed, many of these models are based on pure mechanics without a detailed incorporation of behavioral interactions of human agents [18]. Pedestrians in such frameworks are usually considered as merely reactive agents and their walking strategies are modeled as a response to the observed conditions in the environment [19]. Several research works, however, indicated that pedestrians anticipate the future motion of other pedestrians and dynamic objects [20–23]. In other words, rather than being passively subjected to external forces and solely reacting to them, they take into account these predictions/beliefs and decide on their own walking strategies accordingly [17,24]. This is indeed a crucial behavioral component of pedestrian motion that ensures successful human navigations without major collisions.

Generally, pedestrians are active agents that undergo a repeated decision-making process while walking. Beyond simply being automata, they are anisotropic, interactive, and feedback-oriented agents who: (1) observe their surroundings, predict the future state of the network, and decide on their future movements taking into account the potential movements of other pedestrians; and (2) expect the same behavior from other agents [18]. Neglecting these behavioral factors and particularly the interactive nature of human agents, models will fail to capture some crucial elements of pedestrian motion, such as the commonly observed behavior of pedestrians when they predict the future movement of nearby pedestrians and decide to give way to each other in order to avoid potential collisions [15,16]. It is thus important to put forward models that can represent the logical joint decision-making behavior of human agents, as well as the associated mutual effects and collision avoidance behavior in modeling pedestrian motion in dynamic environments.

A review of the previous studies on modeling interdependencies in various disciplines (e.g., economics, political science, biology, computers science, etc.) reveals the possibility of describing such interactive behaviors via the well-established game theoretical methodologies. In fact, game theory is one of the most notably used mathematical foundations to study a wide range of real-life situations where multiple

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rational players interact and take into account their knowledge, experience, or predictions of other players' behavior. Therefore, capturing the mutual anticipation of other players' potential moves and incorporating it into one's decision-making is one of the main contributions of game-theory-based frameworks over the merely reactive models in describing pedestrian motions [17]. Moreover, game theory can reflect individual preferences and decision-making by defining individual payoffs for each of the utility-oriented players in the game [17]. The methodology is also generalizable and can be extended to cover a variety of behavioral concepts, particularly the learning-based payoff functions.

Based on these considerations, the present study puts forward a game-theory-based framework to model pedestrians' motion in bidirectional flow scenarios. The game theory structure proposed here ensures the aforementioned anticipation behavior of pedestrians while walking. The influence of all other pedestrians into one's decision-making process and the feedback-oriented behavior of human agents are also captured via incorporating a learning structure into the model. Moving strategies are then selected based on Nash equilibria calculations. The proposed framework is validated using the real-world trajectories of pedestrians to ensure realistic human behaviors while walking. Results are presented based on microscopic level behavior of individual pedestrians as well as macroscopic characteristics of crowd dynamics.

The remainder of this paper is organized as follows: Sec. II presents a review of the game-theory-based studies on modeling pedestrian motions. Model formulation, including the game structure, the learning process, and the payoff formulations are discussed in Sec. III. Section IV introduces the Nash-based solution of the proposed model and the calibration results, followed by a discussion on the model validation at both microscopic and macroscopic levels in Sec. V. A comparison of the proposed game theoretical model and discrete choice-based models is also presented in this section. The paper is concluded with summary remarks and future research directions in Sec. VI.

II. LITERATURE REVIEW

Many research studies in various scientific disciplines have shown the capability of game theory in modeling the cooperative behavior of rational decision-makers in different scenarios. Economics, psychology, computer science, and biology are among the major disciplines that benefit most from game theory. Testifying its increasing importance in transportation engineering, there is also a growing body of literature that use game-theory-based approaches to analyze, describe, and model various behavioral scenarios [25–29]. Most of these studies have focused on the macroscopic level analysis of road/parking tolls policy [28,30–32], vehicle routing problems [33,34], transportation network reliability [35], urban traffic demand [27], and transport modes competition [36].

However, despite its proven capability to model the interactions and different aspects of human behavior, efforts toward building game theoretical frameworks for describing and modeling pedestrian motion are quite limited. One of the first game-theory based studies on developing a generic and consistent theoretical foundation for modeling pedestrian behavior is the study by Hoogendoorn and Bovy [37]. Incorporating optimal control and differential games, each individual pedestrian in their work is defined as an autonomous predictive controller who tries to minimize the subjective predicted cost of walking. The framework they introduced is a finite deterministic model with a focus on simulating crowd dynamics where the interactive behavior of pedestrians is modeled using the distance between them. The proposed game is also solved as an optimal control problem, rather than using common game theory solutions (e.g., Nash equilibria). In another study, Turnwald *et al.* [16] showed the potential capability of game theory in modeling humans' walking behavior at a microscopic level. Later, they extended their model by examining additional cost functions, including the length of the path, execution time, the distance between current location and destination, and the associated cost to maintain a desired speed [17]. Roy *et al.* [38] also proposed a Fokker-Plank Nash game for pedestrian motion assuming that individuals try to minimize the associated collision costs while walking. They proved the existence of Nash equilibria by transforming the game structure into an optimal control problem. All of these studies, however, restrict the existing field interactions among all pedestrians to the interactive behaviors between only two players and model the mutual interdependencies between two pedestrians that want to pass each other. Thereby, the existence and potential influence of other players on the game outcomes are mostly neglected.

One of the main issues with incorporating field interactions between all individual pedestrians in a crowd is indeed the extreme complexity in the game structure and the resulting computational costs. To overcome these challenges, several studies have proposed models which incorporate the concept of mean field games (MFGs) in modeling general crowd dynamics. The idea was introduced by Lasry and Lions [39]. They assumed that in large enough systems, individual agents are not influenced by the individual actions of other players, but only by the average properties. Accordingly, each agent makes his/her decisions relying on some statistics regarding the overall community of agents [39]. Motivated by this, a few studies have focused on the concept of MFGs to restate the game theory approach in modeling crowd dynamics as an interaction of one individual with the mass of others [24,40–44]. Many of these studies have dealt with numerical methods to solve the MFG and evaluate the existence and uniqueness of the solutions. Nevertheless, the real-world behavior of the pedestrians indicates that humans are more influenced by those in their visibility zone [45]. In other words, pedestrians are anisotropic agents with different reactions at different walking directions and would rarely react to the pedestrians/objects behind or far from them (except maybe for routing related decisions). Thus, incorporating the distribution of the characteristics of the overall community to reflect the individual interactions in pedestrian motion (as in MFG-based approaches) may lead to unrealistic outcomes.

In addition to the above approaches, there are also several studies that have used discrete choice models to explicitly incorporate humans' decision-making process into pedestrian motion modeling. More promising in this regard is the study by Antonini *et al.* [46] who have proposed a discrete choice framework for pedestrians' short-term behavior. A choice set consisting of a combination of different moving directions and speeds is defined for each individual, and movement of each agent is then determined by maximizing the utilities associated with each choice alternative. Robin *et al.* [47] also adopted a similar discrete choice framework taking into account two different types of pedestrian behavior, referred to as constrained and unconstrained behaviors. Discrete choice models can be categorized as merely reactive frameworks in which each pedestrian's movement is modeled as a response to the presence of other pedestrians. As alluded to in the previous section, this assumption neglects pedestrians' mutual anticipation behavior and thus, may lead to failure in capturing the commonly observed collision-avoidance behavior of human agents (e.g., when they predict the potential movements of surrounding pedestrians and decide to give way in order to avoid possible collisions). Note that a more detailed discussion will be presented in Sec. V D to compare the performance of the proposed game-theory-based model and the corresponding discrete choice framework in describing pedestrians' motion in dynamic environments.

Motivated by the aforementioned challenges in modeling pedestrian behavior, the present study puts forward a learning-based game theoretical approach for modeling pedestrian motion in dynamic environments. The main contributions of the introduced framework can be summarized as follows: (1) The proposed game-theory-based model ensures the interactive decision-making mechanism of pedestrians by formulating a joint payoff function for the players in the game structure, which also incorporates the anisotropic characteristics of human agents while walking. The payoff functions are also defined in a way that ensures the stochastic nature of human behavior; (2) Rather than being merely reactive, the model provides pedestrians with the ability to anticipate the future behavior of surrounding pedestrians and decide accordingly. This, in turn, can capture the frequently observed collision avoidance behavior of the pedestrians in real-world walking scenarios; and (3) Adopting a learning structure, the proposed game-theory-based model incorporates the influence of previous experience and observations into pedestrians' decision-making process and thereby, addresses the feedback-oriented behavior of human agents. Through extensive validation efforts, this study shows that the proposed game theoretical model can describe pedestrians' motion and walking behaviors at microscopic, as well as, macroscopic levels.

III. METHODOLOGY

Game theory is a mathematical framework to model the interactions and decision-making processes of two or more players in strategic settings (games). It provides a technical foundation to analyze optimal decision-making by the rational players where the outcome of the game for each player's choice depends primarily on strategies played by other players. It has been shown that the approach can provide a unique tool to capture the underlying interactions among various players with different preferences. This study presents a game theoretical approach to model pedestrian motions in dynamic environments. In the proposed game structure, players are rational pedestrians who try to find the fastest route to their desired destination. The strategy set for each player is then defined as the possible choices of movement at each time step. As discussed, the key advantage of this framework is its ability to capture individual pedestrian's preferences in dealing with various situations and choices by other pedestrians in the crowd.

While walking, players continuously make decisions on their moving directions and walking speed to optimize their walking utility [48]. These decisions are based on two main information sets: (1) the observations in the current time step and anticipating the future movements of other players in the visibility zone, and (2) estimating the general walking behavior of all surrounding pedestrians. Therefore, it is assumed that each pedestrian plays two games simultaneously: one with the pedestrian that has potentially the most influence on his/her behavior (usually the nearest neighbor in his/her visibility zone), and another learning-based game with all nearby pedestrians to construct an estimate about their general walking behavior. Maximizing the subjective joint payoff function according to the Nash equilibrium solution for the game theory results in the optimum strategy for the target pedestrian. This strategy, in turn, determines the optimum speed and direction to move at each time step. In the following, the logic and structure of the aforementioned games played by each pedestrian at each time step, and the mathematical formulation of the subjective payoff functions will be discussed in more details.

A. First game

In a congested environment (e.g., a narrow corridor), pedestrians interact with each other, while considering the obstacles on their way. One of the most determinant factors in modeling crowd dynamics is the anisotropic pedestrians' interactive behavior with those walking in their visibility zone. It is assumed that each individual plays a game with the closest pedestrian in his/her visibility zone. This assumption is based on the real-world observations where it can be seen that pedestrians' behaviors are not affected equally by the nearby pedestrians. In other words, although each individual interacts with nearby pedestrians, his/her decisions are mostly affected by those who have the shortest distance from him/her, and also located in his/her visibility zone. Accordingly, the first game in the proposed framework is defined to capture the effect of the nearest pedestrian in the visibility zone. Note that the effect of other nearby pedestrians will be captured in the second game introduced in the next section.

Before deciding on the appropriate form for the first game, let us first introduce two main categories of games in general:

(1) *Cooperative or noncooperative games*: Cooperative games involve those strategic settings where players form a kind of negotiation/coalition in order to play joint strategies. In noncooperative games, however, each player acts individually and tries to optimize his/her own payoff.

(2) *Zero-sum and non-zero-sum games*: In zero-sum games, the gain for one player will necessarily result in a loss for other players. Thereby, the sum of all gains and losses for all players is equal to zero. Other games, for which this condition is not enforced, are referred to as non-zero-sum games.

Back to pedestrians' motions and walking decisions, it can be concluded that pedestrians usually act independently

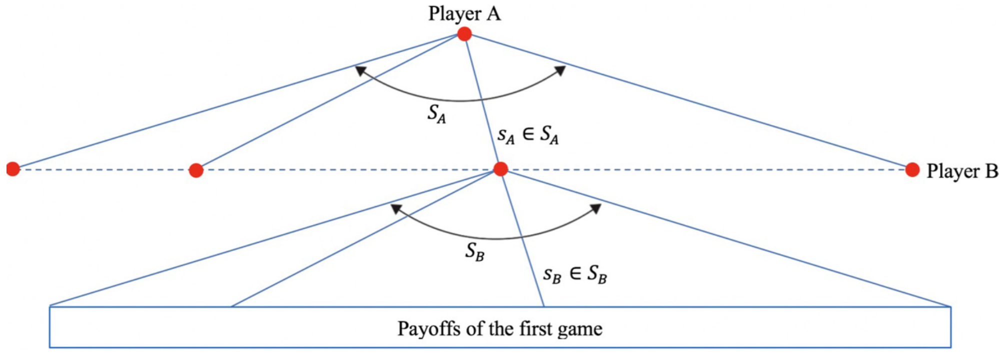

FIG. 1. Extensive form of the first game (only one subgame in P_B's information set is illustrated).

and try to maximize their own walking utilities in a self-interested manner. Thereby, the game played between players can be defined as a noncooperative game where each player is modeled individually [16]. A Nash-equilibrium solution for such games tries to predict the strategies that each individual will choose considering the potential moves by other players. On the other hand, the game played between pedestrians is not necessarily competitive; rather, it can be beneficial for both players. Hence, a non-zero-sum game is selected to describe a situation in which players have both complimentary, as well as conflicting interests. Finally, the first game in the proposed framework is defined as a two-person non-zero-sum noncooperative game between two pedestrians: (1) player A (P_A), which is the target player whose behavior is modeled individually under the noncooperative game structure; and (2) player B (P_B), which is the nearest pedestrian to P_A in his/her visibility zone. Note that there is no need for P_A to be located within P_B's visibility zone. Indeed, each individual may play this game with a pedestrian who is walking in his/her visibility but not necessarily at the same heading direction. A general m-player form of the first game can be given by

(1) A set of m players: $P = \{P_1, \ldots, P_i, \ldots, P_m\}$.

(2) A finite set of possible strategies for player i, denoted by S_i. Note that $s_i \in S_i$ indicates a possible strategy for player i. Let $S = \{s_1, \ldots, s_m\}$ denote a possible strategy profile for the game, i.e., a strategy for each player.

(3) A set of payoff functions illustrated by Π, where $\pi_i(s_i, s_{-i}) \in \Pi$ denotes the payoff of player i if he/she plays $s_i \in S_i$, while all other players play s_{-i}. Note that $s_{-i} = \{s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_m\}$ represents a strategy for every player except i.

The strategies in the corresponding pedestrian motion scenario are defined as the possible moving directions for each player at each time step.

To apply the game structure, the 360-degree zone around each individual is divided into n identical zones with the angle of θ degrees each, where $\theta = 360/n$. A pedestrian's movement decision at each time step falls into one of these n zones. Therefore, each zone represents a possible strategy for the player at each time step, defined based on the deviation from pedestrian's desired direction. The area consisting of the strategies (zones) located in a human's peripheral vision zone is referred to as pedestrians' visibility zone. Note that for a large enough n, this system can capture all the potential directions that a pedestrian can choose.

Figure 1 shows the extensive form of the first game played between P_A and P_B. Nodes and branches denote players and their strategies in the first game, respectively. Each individual has a set of pure strategies (possible moving directions) to choose at his/her first game with the nearest pedestrian in his/her visibility zone (S_i). These strategies can be defined based on the deviation from pedestrian's desired direction. Notice that P_B does not know which of his/her nodes is actually reached since he does not observe P_A's choice before his decision. In other words, the game is played simultaneously, rather than sequentially, and hence P_B's nodes are in the same information set, represented via a dotted line in Fig. 1.

As illustrated, each game results in some payoffs, or utilities, for the players. For simplicity, only one of the n possible subgames that can be played by P_B is illustrated in Fig. 1. The same subgame structure will be repeated for all other nodes in P_B's information set. Payoffs, however, can be different for each subgame as the outcome of the game for each player depends on the strategies chosen by both players. Therefore, playing the same strategy by one player may lead to different payoffs, depending on the strategy selected by other players. This, in turn, reflects the ability of the proposed game-based model to capture pedestrians' anticipation behavior, which is one of the main contributions of this approach over others. Indeed, rather than being a merely reactive agent, game theory allows pedestrians to anticipate the potential strategies that can be played by other pedestrians, estimate the associated payoffs, and decide on the best strategy to choose accordingly.

It should be noted that the proposed structure for pedestrians' first game can also capture the effects of obstacles on choosing the movement direction. When necessary, the game will substitute the role of P_B with the nearest obstacle. Indeed, if there exists an obstacle in P_A's visibility zone that is closer than any other pedestrian, P_A can coordinate a game with it and decide on his/her best strategy to move taking into account the location of the obstacle. The formulation of the payoff functions associated with each strategy profile and the Nash-based solution of the proposed game will be discussed in more detail in Secs. III and IV.

B. Second game (learning)

1. A note on "learning" in games

In addition to the game played with the nearest pedestrian in the visibility zone, each pedestrian considers another factor in deciding on which strategy to choose. Consider a normal case where a pedestrian is moving toward his/her desired direction along with another pedestrian who is walking at the right-hand side (no pedestrian on the left-hand side). Now assume that a blockage has happened in the target pedestrian's desired direction and therefore, he/she should choose another route to the destination by changing his/her direction at the current time step. Recalling that the right-hand side was occupied in the past time steps, the probability of choosing the right direction will be much lower for the target pedestrian without even looking. Overall, by repeatedly observing the environment while walking, pedestrians make inferences about the general walking behavior of nearby pedestrians with which they gradually adjust their expectations of the future surrounding environment. These expectations, in turn, affect their choice of walking strategies at each time step. This behavior stems from the feedback-oriented characteristics of human agents, which also justifies several self-organizing pedestrian motion phenomena, namely the lane formation in high densities [49], where individuals tend to segregate and follow each other in specific lanes. This phenomenon can be described considering the collision avoidance behavior and the feedback-oriented feature of human agents. From their experiences in the previous time steps, pedestrians learn that the immediate place in front of them becomes vacant at each time step, while other locations are usually occupied increasing the probability of potential conflicts. This feedback, in turn, leads the crowd to form lanes in order to minimize the need for direction and/or speed change to prevent potential collisions [49].

The described feedback-oriented characteristic of pedestrians can be captured through the concept of ”learning” in the field of game theory. "Learning" allows pedestrians to adapt their walking behavior in response to surrounding pedestrians' walking strategies in an interactive decision-making setting [50]. Note that a similar concept is applied in cellular automata models for crowd dynamics. These models define probabilities to reflect pedestrians' tendency to move to each of the adjacent cells. Predefining higher probabilities for the immediate front cell can then result in lane formation phenomena in crowd dynamics simulations. Such predefined rules, however, do not reflect the decision-making processes of individual pedestrians nor the dynamic interactions between them. Game theory, on the other hand, can potentially model the underlying decision-making processes while accurately capturing pedestrians' walking behavior, including the self-organized lane formation phenomenon. In a nutshell, pedestrians usually try to find the fastest route to their destination (not the shortest path necessarily), which may lead them to select strategies with higher probabilities of being available. These probabilities are constructed based on a belief about the nearby pedestrians' choices and get updated at each time step according to the previous observations. In order to capture this essential feature of pedestrian motion, the present study incorporates a learning game to model the feedback-oriented behavior of pedestrians while walking.

2. Mathematical representation of the proposed learning game

In a general learning structure, each player is assumed to construct an empirical estimate of other players' actions. Applying this structure to the proposed learning game for modeling pedestrians' motions, it is assumed that pedestrians keep in mind the number of times that each of their strategies was available in the past. Let $N(s_i)^{t-1}$ denote the number of times that strategy s_i was available to pedestrian i until time $t-1$, where $N(s_i)^0$ represents player i's initial belief about opponents' play before starting the game. Therefore, Pedestrian i believes that his/her strategy s_i will be available at time step t with probability of $P(s_i)^t$:

$$P(s_i)^t = \frac{N(s_i)^{t-1}}{t-1}. \tag{1}$$

After observing the new condition of the network at each time step, players update their estimates based on the observed actions taken by other pedestrians. Assume that the game has been played by all of the pedestrians at time step t, and only the strategy j for player i has gotten occupied by nearby pedestrians (while all other possible zones for him/her remained available). Observing the game outcomes at time step t, player i updates his/her beliefs for time step $t+1$ as follows [51]:

$$\begin{aligned} P(s_i)^{t+1} &= \frac{N(s_i)^{t-1} + 1}{t} \quad \text{if } s_i \neq j \\ P(s_i)^{t+1} &= \frac{N(s_i)^{t-1}}{t} \quad \text{if } s_i = j. \end{aligned} \tag{2}$$

Note that the above formulation is the standard structure of the *fictitious play* in game theory, introduced by Brown [52].

Considering the feedback-oriented walking behavior of humans, these estimates will affect pedestrians' choices at each time step and therefore, should be considered when modeling pedestrians' motions in dynamic environments.

C. Payoff functions

As alluded to in the previous sections, each pedestrian plays two simultaneous games at each time step: one with the nearest pedestrian in his/her visibility zone and another learning game with all of the nearby pedestrians to construct an estimate about the availability of the strategies in next time step. The final payoffs for each individual will then be a joint function which incorporates the effects of both games. In the following section, first, the payoff functions of each game will be defined and discussed. Then, a joint payoff formulation will be introduced for each of the possible strategy profiles of the game.

1. Payoffs of the first game

In the proposed game structure, each player has n pure strategies to choose in the first game with the nearest pedestrian in his/her visibility zone. Payoffs for the first game are defined as a function of certain factors that characterize the walking behavior of the pedestrians. The approach in this study is based on the assumption that underlying decision-making logic of humans is based on the "rational" theory [53,54]. According to this theory, humans have a "value" associated with each alternative (each strategy in the context of game theory), and they are trying to maximize such value. In modeling pedestrian' motion, the alternatives can be the direction of the movement and the walking speed. Therefore, the decision-making process of the pedestrians consists of two major components, which must be reflected in the payoff functions: the directional component and the movement (speed) component.

Starting with the directional component of the payoff functions, pedestrians should choose their direction of movement at each time step. It is assumed that pedestrians prefer to follow their desired direction, and deviate (if necessary) as late as possible [55]. Therefore, the directions with smaller deviation from pedestrian's desired direction must offer higher values.

The movement component of crowd dynamics has its roots in desired walking speed. Observations on crowd dynamics show that pedestrians tend to walk at the desired speed, which corresponds to the most convenient or least energy-consuming walking speed [55]. This preference usually leads to choosing the fastest rather than the shortest route to their destination. Thus, it is expected that the speeds closer to the desired walking speed can provide higher payoff values for pedestrians. Combining the above components into a single term in the payoff functions of the first game, the following equation can be formulated:

$$\pi_A(j,q)^{\mathrm{DM}} = \cos(\theta_j)\frac{V_{(j,q)}/V_d}{(1 + V_{(j,q)}/V_d)^{\frac{\beta_j - 1}{2}}}, \qquad (3)$$

where $\pi_A(j,q)^{\mathrm{DM}}$ is the directional-movement component of P_A's payoff function where P_A and P_B play $s_A = j$ and $s_B = q$, respectively [i.e., strategy profile $S = (j,q)$]; θ_j is the associated average deviation from the desired direction for strategy j; $V_{(j,q)}$ denotes the maximum achievable speed of P_A if strategy profile $S = (j,q)$ is played by the players (further explanation is provided in the following paragraph); V_d is the desired walking speed; and β_j is a parameter to be estimated.

It is assumed that pedestrians can keep walking at their current speed if: (1) they choose to move toward their desired direction, and (2) there is no potential for collision considering the future movement of the nearest pedestrian. On the other hand, walking speed towards the destination would be decreased by deviation from the desired walking direction. Also, the real-world observations of pedestrians' behavior indicate that movement of nearby pedestrians can affect the walking speed of each individual as they may reduce their speed to avoid colliding with a pedestrian walking toward them. Thus, lower payoffs are expected for those strategies that can lead to potential collisions and speed reductions. Accordingly, the maximum achievable speed for P_A under strategy profile $S = (j,q)$ can be defined by

$$V_{(j,q)} = V_A \cos(m\theta_j)e^{n(-R_{AB})},$$
$$R_{AB} = d_{AB} - W, \qquad (4)$$

where V_A is P_A's speed at current time step; m is equal to 0 if strategy j is toward P_A's desired direction, and 1 otherwise. n is equal to 0 if there is no potential for collision under strategy profile $S = (j,q)$, and 1 otherwise; R_{AB} is the effective distance between P_A and P_B, d_{AB} is the Euclidean distance between the center points of P_A and P_B; W is the average shoulder width of a human, set to 0.4 meter in the present study.

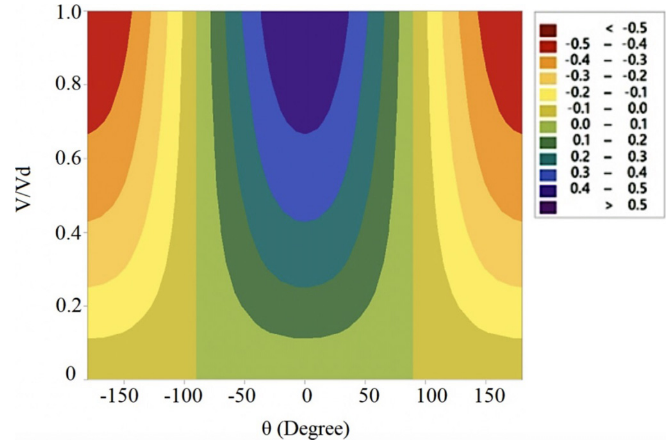

FIG. 2. The directional-movement component of crowd dynamics for different values of θ and V/V_d.

Figure 2 illustrates $\pi_A(j,q)^{\mathrm{DM}}$ for different values of walking direction and speed. The proposed structure ensures that pedestrians are at their maximum directional-movement utility when traveling toward their destination at desired speed.

Beside the directional-movement component in players' payoff function for the first game, another component should be considered to represent the motivation of each player in choosing a walking strategy. The real-world observations show that pedestrians usually prefer not to walk too close to each other [55]. They might even deviate slightly from their desired direction in order to increase their distance with another pedestrian walking in front of them. Moreover, in case of observing an obstacle on the way to the destination, the relative location of the obstacle to the pedestrian also turns to be a determinant factor in his/her choice of walking strategy. Figure 3 illustrates the schematic of the first game played between P_A and P_B. In this figure, θ_j is the average deviation from the desired direction for strategy j in P_A's strategy set, and θ_{AB} is the angle between the line connecting P_A and P_B, and the desired direction of P_A. The shaded area in the figure illustrates the relative location of P_A to P_B.

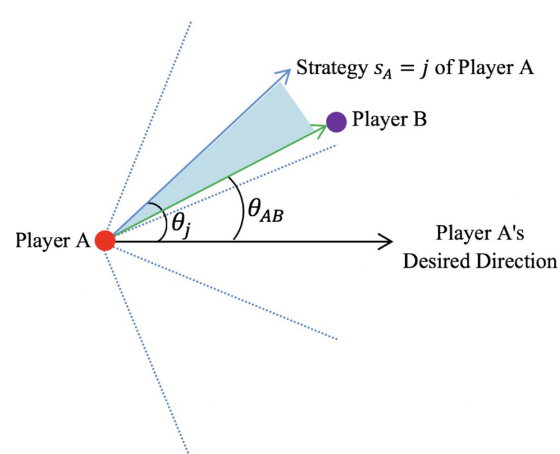

FIG. 3. The relative location of P_A to P_B. θ_{AB} is the angle between the line connecting P_A and P_B and the desired direction of P_A.

Apparently, the strategies associated with smaller shaded areas are expected to have lower payoffs for each player. This preference is reflected in the proposed payoff functions to represent a more realistic behavior in modeling pedestrians' motions. Notice that since pedestrians adjust their walking based on what they observe, the cost caused by the presence of other pedestrians or obstacles in the walking area only appears in the payoffs associated with those strategies that cover pedestrian's visibility zone. Accordingly, the corresponding term in the payoff functions of the first game can be formulated as follows:

$$\pi_A(j,q)^{\text{int}} = \left|\theta_{AB} - \theta_j\right|,$$
$$\text{If strategy } s_A = j \text{ is in } P_A\text{'s visibility zone;}$$
$$\pi_A(j,q)^{\text{int}} = 0, \quad \text{Otherwise}, \tag{5}$$

where $\pi_A(j,q)^{\text{int}}$ is the interaction term in P_A's payoff function for the first game if P_A and P_B play their $s_A = j$ and $s_B = q$ strategies, respectively. This term captures players' tendency to stay away from other pedestrians. θ_{AB} is the angle between the line connecting P_A and P_B and the desired direction of P_A; $|\theta_{AB} - \theta_j|$ is the relative location of P_A to P_B, if P_A decides to choose strategy j.

The next component to consider when formulating pedestrians' payoff functions for the first game, is the collision avoidance feature of human agents while walking. As alluded to in the previous sections, one of the key contributions of game-theory-based frameworks over other pedestrian motion models is their ability to capture human agents' anticipation behavior while walking. By introducing different payoffs for different strategy profiles, game theory allows players to decide on their best strategies considering the potential choices by other opponents. In other words, $\pi_i(s_i,\ s_{-i})$ can result in different payoff values than $\pi_i(s_i,\ s'_{-i})$, i.e., player i may not get the same payoff by playing strategy s_i, if the strategies selected by other players changes from s_{-i} to s'_{-i}.

In the Nash-equilibrium solution for games, players compute payoffs for each of their strategies under different strategy profiles and choose their optimum strategies accordingly. This behavior is in line with the real-world behavior of pedestrians, where each individual predicts the future motions of nearby pedestrians and considers them in his/her own motion planning process. This feature in pedestrian motion, in turn, leads to a mutual collision avoidance behavior between pedestrians that prevents potential conflicts while walking [16]. In the proposed game structure, a *collision avoidance* term is therefore added to players' payoff function whenever there is a potential collision situation:

$$\pi_A(j,q)^{\text{collision}} = e^{(-R_{AB})} \tag{6}$$

where $\pi_A(j,q)^{\text{collision}}$ is a component in P_A's payoff function, added if there is a potential collision when P_A and P_B play their $s_A = j$ and $s_B = q$ strategies, respectively; and R_{AB} is the effective distance between P_A and P_B, indicating that the collision avoidance behavior can be more influential when players are walking close to each other.

2. *Payoffs of the second game (learning)*

By playing the learning game introduced in Sec. III B 2, each pedestrian will construct an estimate/belief about the availability of each of his/her strategies for the next time step. Strategies with higher probabilities of being available (directions not occupied with other pedestrians) will lead to greater payoffs for the player. For example, if P_A recognizes that direction j was usually occupied and hence $s_A = j$ was unavailable during previous time steps, he/she will probably consider a lower weight for the payoffs provided by this strategy in the first game. In other words, the proposed learning structure enables P_A to calculate $P(s_i)^{t-1}$, the probability of strategy $s_A = j$ being available for P_A at time step t. Refer to Sec. III B 2 for more details.

3. *Formulating a joint payoff function*

Considering the effects of both games in players' final payoffs, a joint formulation for the payoff functions is defined by

$$\pi_A(j,\ q) = P(s_A)^t\big[\alpha_j^0 + \alpha_j^{\text{DM}}\pi_A(j,\ q)^{\text{DM}} + 1(VZ)\alpha_j^{\text{int}}\pi_A(j,\ q)^{\text{int}} + 1(col)\,\alpha_j^{\text{collision}}\pi_A(j,\ q)^{\text{collision}}\big] + \varepsilon_j \tag{7}$$

where $\pi_A(j,q)$ represents the payoff that P_A will get under the strategy profile of $S = (j,q)$; $1(VZ)$ is equal to 1 if the strategy $s_A = j$ falls into P_A's visibility zone, and 0 otherwise; $1(col)$ is equal to 1 if the strategy profile of $S = (j,q)$ leads to a potential collision, and 0 otherwise; ε_j is the error term to capture the stochastic behavior of human agents. α_j^0, α_j^{DM}, α_j^{int}, and $\alpha_j^{\text{collision}}$ are parameters to be estimated. It should be noted that the walking behavior of human agents is not a deterministic process, meaning that pedestrians may choose different strategies under similar situations. Thus, to capture the stochastic behavior of the pedestrians and reflect the effect of unobserved factors in their decision-making process, an error term is considered in the payoff formulations of the proposed game structure.

IV. NASH EQUILIBRIA AND GAME CALIBRATION RESULTS

A. Nash equilibrium solution of the game

The main goal of game theory is to predict the strategy profile that is most likely to be played by players. This solution needs to be stable, i.e., a strategy profile where everyone is playing optimally given what all other players are playing. The general mathematical description of such a profile for a general m-player game can be given by $S^* = \{s_1^*,\ \ldots,\ s_m^*\}$, where for all players $s_i \in S_i$

$$\pi_i(s_i^*,\ s_{-i}^*) \geqslant \pi_i(s_i,\ s_{-i}^*). \tag{8}$$

In this equation, $\pi_i(s_i^*,\ s_{-i}^*)$ denotes the payoff for player i if he/she chooses to play s_i^*, while all other players play s_{-i}^*. Equation (8) ensures that player i cannot make any profitable deviation from strategy s_i^* if all other players choose to s_{-i}^*. Since each player selects his/her best response to what everyone else is doing, the strategy profile S^* is called a Nash equilibrium.

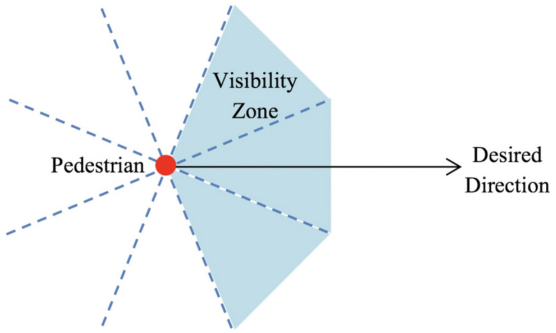

FIG. 4. Pedestrians' choices to move and visibility zone.

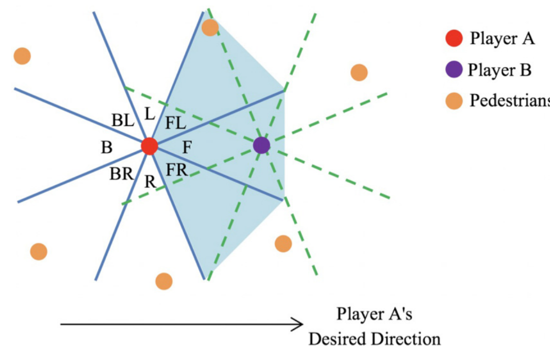

FIG. 5. Schematic of the first game for P_A for $\theta = 45^\circ$.

The present study adopts a Nash-based solution for the proposed game structure to predict and model pedestrians' motion and walking behavior. Note that Nash equilibrium predicts the most likely outcome of the game considering that each player maximizes his/her own utility while anticipating the strategies played by other players. This is indeed in line with the utility maximization/cost minimization principal used by numerous crowd dynamic models in the literature [16,17,37,38,40,43,46–48]. Moreover, the Nash-based solution offers another key dimension by capturing the corresponding anticipation behavior of human agents discussed in the previous sections. In the proposed game structure, each player computes his/her corresponding payoff for each possible strategy profile and then chooses the strategy that maximizes his/her payoff given the strategies selected by other players.

B. Model calibration results

This section presents the results for the numeric parameter estimation of the proposed game structure. For simplicity and without any loss of generality, we chose the strategy range of $\theta = 45^\circ$. With this selection, the 360° zone around each individual is divided into eight identical zones, as illustrated in Fig. 4. Note that considering an average shoulder width of 0.4 meter, each individual is modeled as a circle with a radius of 0.2 meter.

The visibility zone for each pedestrian is defined as the 135° zone in front of them, corresponding to $[-67.5^\circ, 67.5^\circ]$ deviation from their desired direction (the shaded area in Fig. 4). According to this structure, the possible moving directions for each pedestrian are categorized into eight independent zones, resulting in eight pure strategies for him/her to choose from at each time step. As discussed, these strategies can be defined based on the deviation from the desired direction. Table I indicates the strategy set for each pedestrian that is used to calibrate and validate the proposed game theoretical framework. The following acronyms are hereafter used to refer to each of these strategies: Moving Forward (F), Forward Left (FL), Forward Right (FR), Left (L), Right (R), Backward Left (BL), Backward Right (BR), and Backward (B).

Figure 5 illustrates a schematic of the first game played between the target pedestrian (P_A) and the nearest pedestrian/obstacle in his/her visibility zone (P_B). As illustrated in Fig. 5, the visibility zone for each pedestrian involves the three strategies of F, FL, and FR in the proposed structure.

Figure 6 represents the extensive form of the first game, where players simultaneously choose a strategy from their available strategy set. Notice that this figure is indeed the corresponding form of Fig. 1 for the proposed setting with 8 pure strategies.

To evaluate the proposed model, this study uses real-world pedestrian trajectories associated with a bidirectional flow passing through a corridor and exiting from an instructed direction [56]. Figure 7 illustrates a schematic of this experiment, where the corridor has the following geometries: width of the corridor $(b_{\text{cor}}) = 3.60\,\text{m}$ and width of the entrances $(b_{\text{ent}}) = 0.90\,\text{m}$.

At the beginning of the experiment, each participant is assigned a random number and located within the designated sites behind the entrances on both sides of the corridor. They were then asked to cross and exit the corridor from instructed directions determined by their assigned personnel numbers during a total period of 2.5 min. Pedestrians with an even personnel number are instructed to exit the corridor on the

TABLE I. Strategy set for each pedestrian at each time step for $\theta = 45^\circ$.

Strategy	Moving direction	Deviation from desired direction (deg)	Average deviation from desired direction (deg)
F	Forward	[−22.5, 22.5]	0
FL	Forward Left	[22.5, 67.5)	45
FR	Forward Right	[−67.5, −22.5)	−45
L	Left	[67.5, 112.5)	90
R	Right	[−112.5, −67.5)	−90
BL	Backward Left	[112.5, 157.5)	135
BR	Backward Right	[−157.5, −112.5)	−135
B	Backward	[−157.5, −180] or [157.5, 180]	180

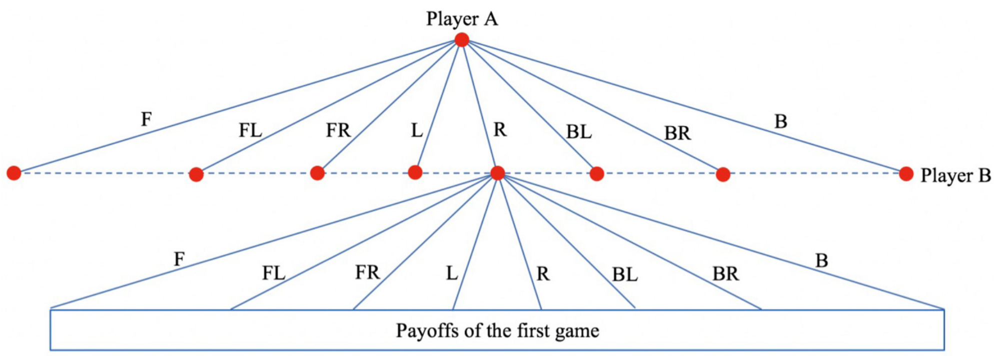

FIG. 6. The extensive form of the first game.

right, and those with odd personnel numbers are instructed to exit the corridor on the left. It is worth noting that forcing specific exit directions creates more crossing movements while walking, and thus, more complex dynamics. Therefore, such an experiment can better evaluate the model's capability in describing pedestrians' walking behaviors in dynamic environments.

The dataset used for calibrating the game parameters includes the trajectories of 170 pedestrians during a 1-min period, corresponding to a 6.1-m-longsegment on the corridor. Each pedestrian is assumed to play the proposed game every 0.1 sec. The speed of each individual is calculated at every 0.1 sec, and the desired speed is assumed to be 1.3 m/s for all of the pedestrians. The desired direction is also defined as the direction of the line connecting the pedestrian to his/her destination. The selected strategies for each pedestrian are determined based on the deviation from the desired direction at each time step. The initial estimates for the learning game are set based on the best fits provided by the model: $P(F)_A^0 = 0.98$, $P(FL)_A^0 = P(FR)_A^0 = 0.93$, $P(L)_A^0 = P(R)_A^0 = 0.65$, $P(BL)_A^0 = P(BR)_A^0 = 0.30$, $P(B)_A^0 = 0.10$. Table II shows the parameter estimation results and the standard deviations for the calibrated parameters. These values provided the best fit for the introduced game structure, leading to a log-likelihood ratio of 0.979.

Note that according to the proposed game framework, P_A will play the first game with P_B only if he/she is located in P_A's visibility zone. Thus, for the structure illustrated in Fig. 5, the interaction term in the payoff functions will only appear for those strategy profiles that include the strategies of F, FL, or FR in P_A's strategy set. In addition, the collision avoidance term in P_A's payoff functions will only appear for the strategies of F, FL, FR, L, or R, since P_B is in P_A's visibility zone and assumed to move through, up to one zone, at each time step. Therefore, there is no chance for a potential collision between the two players if P_A chooses to move BL, BR, or B.

V. MODEL VALIDATION

In this section, the proposed game theoretical framework is validated to assess its capability in capturing and modeling real-world pedestrian behavior. New trajectories of 178 pedestrians during another 1-min period are used to validate the model's capability in predicting the actual strategies played by each player. Considering 0.1 sec time step, 27 761 strategies are tested and validated.

A. Microscopic analysis of crowd dynamics

To validate the proposed framework at microscopic level, the individual strategies played by each pedestrian in the game are compared to the corresponding real-world choices of the pedestrians. Accordingly, the values of the joint payoff functions are calculated for each player at each time step, using the parameter estimation results presented in Table II. The optimum strategy for each pedestrian is then identified based on the Nash-based solution of the game. Comparing the predicated and corresponding actual choices indicates an 86.89% accuracy in predicting the moving strategies chosen by each pedestrian at each time step. To further evaluate the calibration results, root mean square error (RMSE) is also

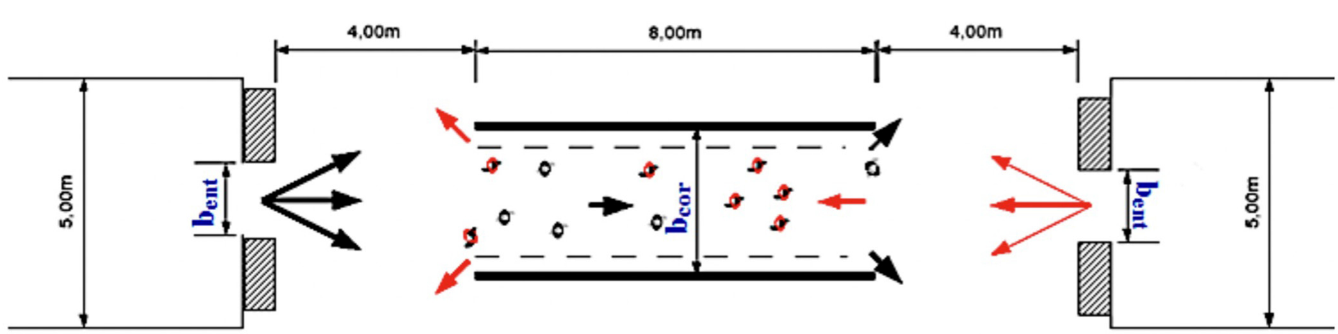

FIG. 7. The schematic of the corridor experiment: bidirectional flow, free choice of destination [56].

TABLE II. Model calibration results.

Parameter	Calibrated value	STD
α_{F}^{0}	0.000	0.000
$\alpha_{\mathrm{F}}^{\mathrm{DM}}$	13.160	0.451
$\alpha_{\mathrm{F}}^{\mathrm{int}}$	− 0.873	0.062
β_{F}	12.531	0.056
$\alpha_{\mathrm{F}}^{\mathrm{collision}}$	0.977	0.011
α_{FL}^{0}	− 0.159	0.020
$\alpha_{\mathrm{FL}}^{\mathrm{DM}}$	1.079	0.404
$\alpha_{\mathrm{FL}}^{\mathrm{int}}$	− 13.691	0.504
$\alpha_{\mathrm{FL}}^{\mathrm{collision}}$	1.004	0.066
α_{FR}^{0}	− 6.449	0.815
$\alpha_{\mathrm{FR}}^{\mathrm{DM}}$	3.330	0.058
$\alpha_{\mathrm{FR}}^{\mathrm{int}}$	1.154	0.037
$\alpha_{\mathrm{FR}}^{\mathrm{collision}}$	1.015	0.018
α_{L}^{0}	− 8.853	0.064
$\alpha_{\mathrm{L}}^{\mathrm{DM}}$	− 0.484	0.094
β_{L}	20.114	0.133
$\alpha_{\mathrm{L}}^{\mathrm{collision}}$	0.978	0.022
α_{R}^{0}	0.203	0.029
$\alpha_{\mathrm{R}}^{\mathrm{DM}}$	− 1.381	0.054
β_{R}	0.174	0.034
$\alpha_{\mathrm{R}}^{\mathrm{collision}}$	0.981	0.044
α_{BL}^{0}	− 6.039	0.082
$\alpha_{\mathrm{BL}}^{\mathrm{DM}}$	3.783	0.153
β_{BL}	− 2.471	0.011
α_{BR}^{0}	− 1.314	0.001
$\alpha_{\mathrm{BR}}^{\mathrm{DM}}$	− 16.979	0.047
β_{BR}	6.462	0.101
α_{B}^{0}	2.334	0.231
$\alpha_{\mathrm{B}}^{\mathrm{DM}}$	1.748	0.008
β_{B}	− 1.377	0.321

calculated as follows [26]:

$$\mathrm{RMSE} = \sqrt{\frac{1}{n} f(x_i, x'_i)}, \tag{9}$$

where $f(x_i, x_i') = \sum_i 1(x_i' - x_i)^2$ and n is the number of observations. x_i' and x_i denote the predicted and actual values, respectively. $1(x_i' - x_i)$ is equal to zero if the predicted and actual values are identical (i.e., $x_i' = x_i$) and 1 otherwise. For the proposed game theoretical model, the actual strategies played by each pedestrian at each time step are evaluated and compared to the corresponding optimum strategies predicted by the model. The discrete RMSE value for 27 761 strategies chosen by all of the pedestrians is then computed as 0.29, indicating the relatively high prediction power of the model in predicting the moving decisions made by individual pedestrians at each time step.

These findings from model validation at microscopic level indicate the capability of the proposed game theoretical model in describing the real-world behavior of pedestrian dynamic environments.

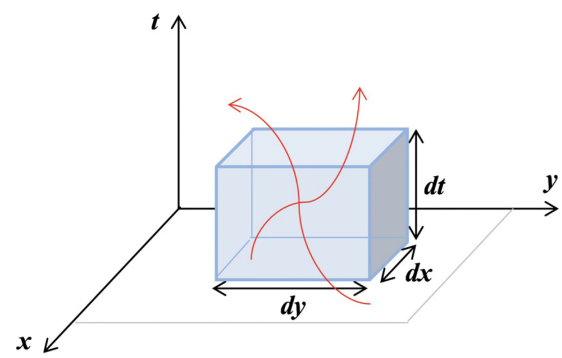

FIG. 8. Three-dimensional pedestrian trajectories [57].

B. Macroscopic analysis of crowd dynamics

This section presents a macroscopic level analysis of crowd dynamics to provide a more accurate evaluation of the model performance and a better understanding of the model's capability in capturing the complex dynamics in the crowd. As discussed to in the previous sections, pedestrians are assumed to play two simultaneous games at each time step and an optimum strategy is then determined for each player based on maximizing the subjective joint payoff functions. Each pedestrian then moves in that direction and at the speed associated with his/her optimal strategy at the current time step. The game is played again in the next time step based on the new locations of the pedestrians and the updated beliefs from the previous time steps. The repeated cycles of the game generate the predicted trajectories of a single pedestrian, used to evaluate the macroscopic features of crowd dynamics.

In order to analyze the macroscopic characteristics, pedestrian flow, speed, and density are calculated based on the method introduced by Saberi and Mahmassani [57]. This method is an extension of Edie's definition [58] to a three-dimensional time-space diagram for pedestrians, where the x and y axes are, respectively, the width and length of the walking facility, and the z axis represents time. In their work, Saberi and Mahmassani calculated the time spent and the distance traveled inside a hypothetical volume, V, with dimensions of dx, dy, and dt (see Fig. 8). The corresponding crowd density (k) and flow (q) are then defined as follows:

$$k = \frac{\sum_{i\in N} t_i}{|V|} = \frac{\sum_{i\in N} t_i}{dxdydt}, \tag{10}$$

$$q = \frac{\sum_{i\in N} d_i}{|V|} = \frac{\sum_{i\in N} d_i}{dxdydt}, \tag{11}$$

where t_i and d_i denote the total time spent and distance traveled by pedestrian i inside shape V, respectively. Saberi and Mahmassani [57] showed that their method results in a more accurate estimation of flow and density compared to traditional methods.

Applying Eqs. (10) and (11), the actual and simulated density and flow values are calculated for the proposed game structure. Figure 9 indicates the density profiles based on actual (a) and simulated (b) pedestrian trajectories during a 1-min period over a 1.5-m-long segment in the middle of the test corridor. As illustrated, both profiles create approximately identical dynamics. Lower density values can be identified

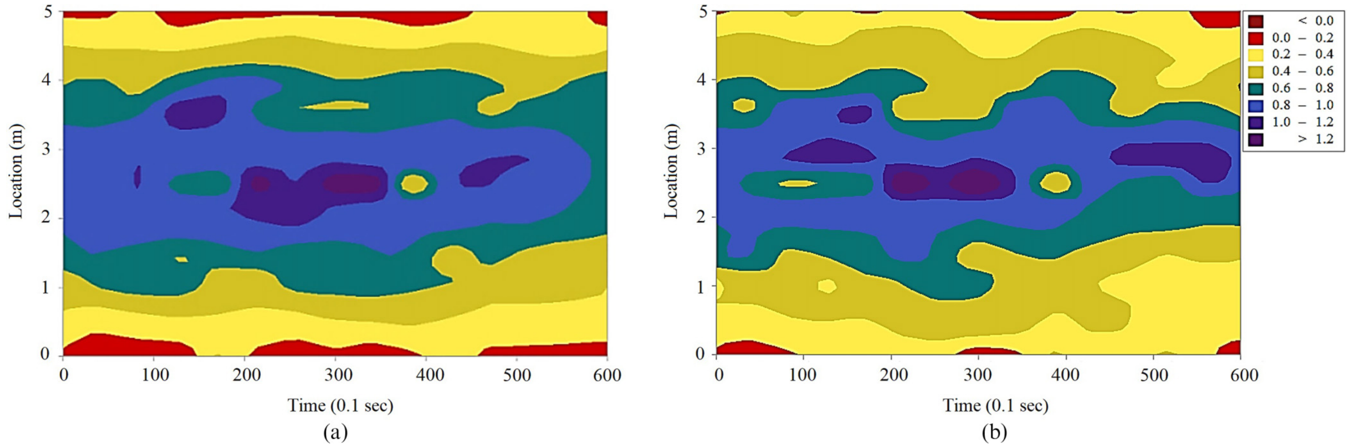

FIG. 9. Density profiles (ped/m^2) based on (a) actual and (b) simulated pedestrian trajectories.

near the entrance-exit points (meter markers 0 and 5) in both graphs. An increased crowd density is also identified around the center of the corridor between meter markers 2 and 3. Figure 10 also illustrates the similar patterns in flow profiles for actual and simulated crowd dynamics. It can be seen that the proposed game theoretical framework can capture the flow fluctuations in pedestrians' motion, and more interestingly, the increased flow values around the meter markers 2 and 3.

Figure 11 illustrates the changes in density over time, based on actual and simulated pedestrian trajectories during a one-minute period over the selected 1.5-m segment in the middle of the corridor. As shown, the simulated density-time curve fits well to the corresponding curve based on the real-world pedestrian dynamics. Simulated and actual trajectories generate almost similar fluctuations of the density over time, revealing the model's capability in capturing macroscopic characteristics of crowd dynamics. It can be seen that simulated dynamics result in relatively higher values for crowd density in comparison to the actual values over the same segment of the corridor. However, the continuous RMSE evaluation of the simulated and actual density values yields to a considerably low average error of 0.150 (ped/m^2).

Figure 12 illustrates the flow-density relationship based on the actual and simulated trajectories during a 1-min period over the same 1.5-m-long segment in the middle of the corridor. As indicated by the best-fit curves through the data points, a very similar trend can be verified in both curves. However, the maximum density and flow rates predicted by the model are less than the observed values. The difference between actual and simulated values in both cases is mainly due to dividing the strategy space into only eight possible strategies. Defining more strategies, each with smaller deviation angle from the desired direction can further smooth the movements of pedestrians and potentially address this inconsistency.

Generally, validation results of the proposed game theoretical framework provide evidence on the model's capability in predicting pedestrians' microscopic choice behavior and capturing the macroscopic characteristics of crowd dynamics.

C. Analysis of collisions

As discussed, one of the crucial behavioral factors in pedestrians' motion is the anticipation and mutually interactive features of human agents, based on which pedestrians develop an estimate about the future state of the surrounding network and decide on their own walking strategies accordingly. This, in turn, results in an almost collision-free movement of pedestrians even in dense crowds. It is thus important

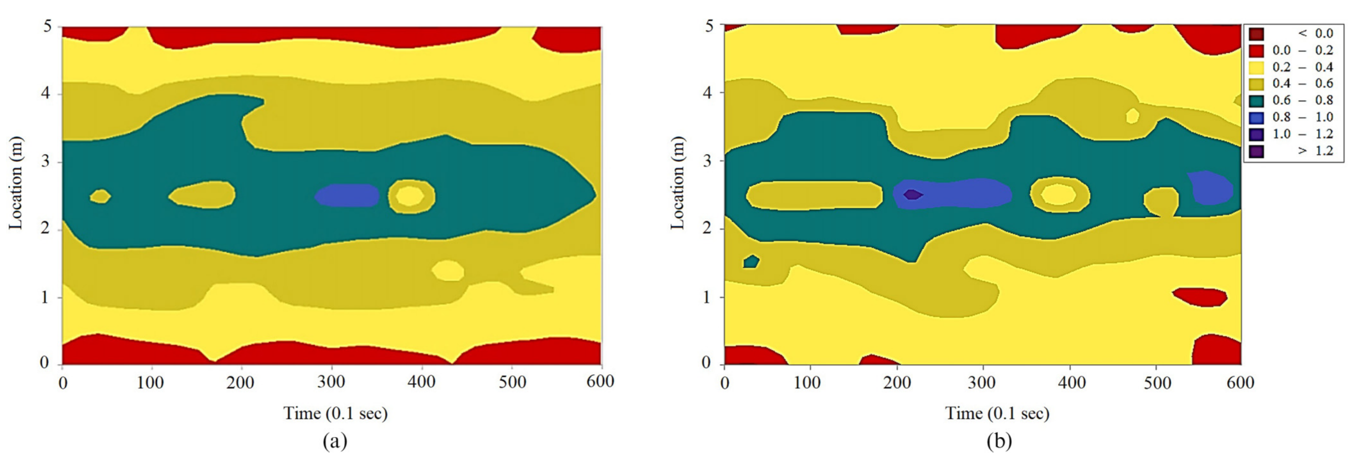

FIG. 10. Flow profiles (ped/sec/m) based on (a) actual and (b) simulated pedestrian trajectories.

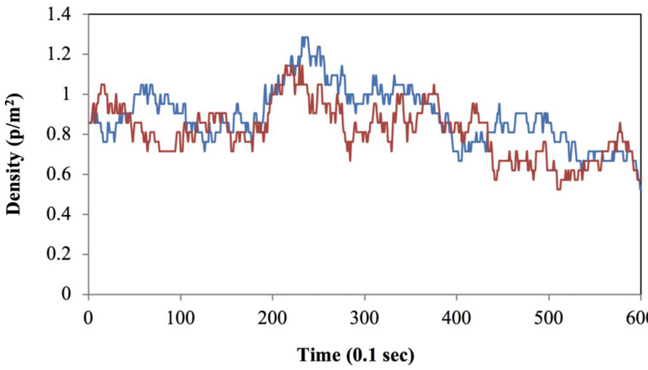

FIG. 11. The density-time curve for bidirectional flow based on the actual (blue line) and simulated (red line) trajectories.

to incorporate these behavioral characteristics when modeling pedestrians' motion in dynamic environments.

The proposed game-theory-based model accounts for these features by: (1) incorporating a learning structure to address human agents' feedback-oriented behavior while walking, (2) defining a collision avoidance component in the payoff functions of those strategy profiles that can potentially result in a collision between the pedestrians, and (3) developing a Nash equilibrium-based solution for the proposed game to capture pedestrians' mutually interactive decision making. In this section, the performance of the proposed game theoretical framework in modeling pedestrians' collision avoidance behavior is investigated.

To analyze potential collisions in the real-world scenario, the effective distance to the nearest neighbor (D_{Act}) is calculated based on the real-world data for each individual pedestrian at three randomly selected time steps, namely $t_1 =$ 27 (sec), $t_2 =$ 35 (sec), and $t_3 =$ 49 (sec). Then, using the simulated trajectories, provided by the model, the corresponding distances between P_A and P_B at each game is calculated at the same time steps (D_{Sim}). Table III presents the statistical analysis of the distance to the nearest neighbor for actual and simulated trajectories.

Analyzing the actual and simulated trajectories at all three time steps indicates that pedestrians have well managed their movements such that a minimum distance of 30 cm is maintained between all of the pedestrians along the corridor. This indeed represents the collision avoidance behavior of pedestrians that is also captured by the proposed model.

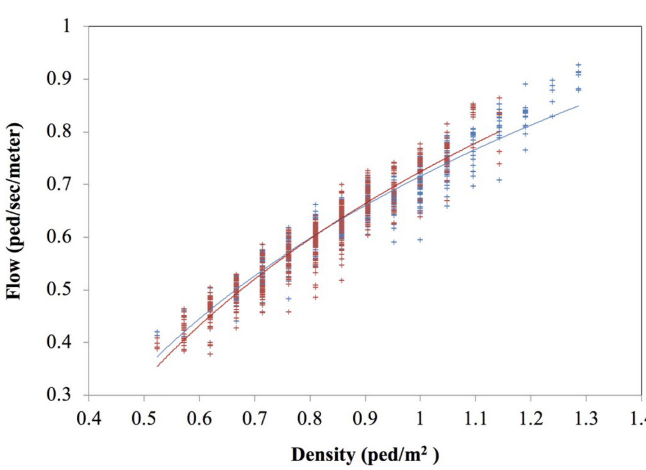

FIG. 12. Flow-density diagram based on the actual (blue) and simulated (red) trajectories.

TABLE III. Statistical analysis of the actual and simulated distances to nearest pedestrian.

	$t_1 =$ 27 (sec)		$t_2 =$ 35 (sec)		$t_3 =$ 49 (sec)	
	D_{Act}	D_{Sim}	D_{Act}	D_{Sim}	D_{Act}	D_{Sim}
Mean (m)	1.36	1.35	1.35	1.26	1.26	1.32
Std Dev (m)	1.64	1.55	1.59	1.35	0.88	1.01
Minimum (m)	0.49	0.38	0.39	0.31	0.35	0.42
Median (m)	0.91	0.86	0.87	0.96	1.12	0.81
Maximum (m)	7.17	6.97	7.11	7.19	5.79	6.32
p-value	0.9822		0.7562		0.7515	

Table III also presents the results of the analysis of variance (ANOVA) for D_{Act} and D_{Sim}. The large p values in the associated t tests for all three cases show that being 99% confident, there is no evidence of a significant difference between the actual and simulated distance between a pedestrian and his/her nearest neighbor. In other words, the distance distribution among the nearest pedestrians simulated by the proposed game-theory-based model shows no significant difference from the corresponding real-world behavior. Figure 13 illustrates the distribution of the actual and simulated distances to the nearest neighbor for $t_1 =$ 27 (sec). These findings provide evidence of the ability of the proposed game theoretical framework in modeling the collision avoidance behavior of human agents while walking.

D. Comparison of the game theoretical and discrete choice models

Discrete choice models (DCMs) are behavioral frameworks developed to model choices made by decision makers between discrete alternatives. Some researchers have also used DCMs to predict the walking behavior of human agents and describe pedestrian motion in dynamic environments [35,36].

Despite similarities in defining basic concepts, there are fundamental differences between game-theoretical and discrete choice modeling approaches. As indicated, payoffs in the game-theory-based model are defined for each possible state of the network in the future (i.e., for each potential strategy profile) to capture human agents' anticipation and collision avoidance behavior while walking. DCMs, on the other hand, are based on pure utility maximization behavior where utilities (payoffs) are defined for each possible option of the decision-maker, based on the current state of the network. In other words, decision-makers' utility gain from one option is independent of the decisions made by other decision makers in a discrete choice modeling framework. The individual decisions in DCMs are therefore modeled as the best response to the observed surrounding environment, leading to a merely reactive model without considering the estimation ability of humans in anticipating probable future movements of other decision makers.

In this section, a quantitative analysis is conducted to compare the performance of the proposed game theoretical

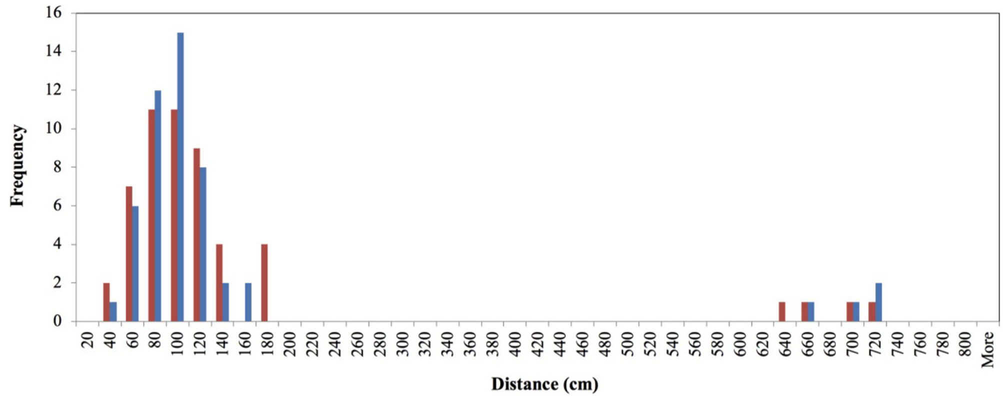

FIG. 13. Distribution of the actual (blue) and simulated (red) distances to nearest pedestrian in visibility zone at $t_1 = 27$ (sec).

model and the corresponding discrete choice framework in describing pedestrians' motion. The structure of the DCM is selected based on the work of Antonini *et al.* [46]. For the comparison to be valid, both models use the same features when computing the utilities/payoff functions for each option/strategy profile. The corresponding DCM for the proposed game structure is then developed considering the same moving alternatives (strategies) as well as the same variables in defining the utility functions, while neglecting: (1) the learning game structure in the proposed game-theory-based model, and (2) the collision avoidance component of the payoff functions for different strategy profiles of the game. Therefore, the associated utility gain for decision-maker A if he/she decides to choose his/her j^{th} option can be formulated as (note that for simplicity, the notations of the corresponding variables in the two models are kept identical where possible)

$$\pi_A(j) = \alpha_j^0 + \alpha_j^{\text{DM}}\pi_A(j)^{\text{DM}} + 1(VZ)\alpha_j^{\text{int}}\pi_A(j)^{\text{int}} + \varepsilon_j, \tag{12}$$

where $\pi_A(j)$ represents the utility that decision-maker A will get if he/she chooses his/her option $s_A = j$; $\pi_A(j)^{\text{DM}}$ is the directional-movement component of decision-maker A's utility gain from choosing option $s_A = j$ [refer to Eq. (3)]; $\pi_A(j)^{\text{int}}$ represents the interaction between decision-maker A and the nearest pedestrian to him/her [refer to Eq. (5)]; $1(VZ)$ is equal to 1 if option $s_A = j$ falls into decision-maker A's visibility zone, and 0 otherwise; ε_j is the error term to capture the stochastic behavior of human agents; and α_j^0, α_j^{DM}, and α_j^{int} are parameters to be estimated.

Identical to the proposed game-theory-based framework, the corresponding DCM is calibrated and also validated using the same dataset of pedestrian trajectories introduced in Sec. IV B. The model parameters of DCM are calibrated using the maximum likelihood estimators (MLE), and the utility value for each moving option is calculated based on the calibrated parameters. Each pedestrian is then assumed to maximize his/her utility gain from decisions made at each time step. Table IV presents the parameter estimation results for the corresponding DCM.

Table V shows the model evaluation results and the performance of both models in describing pedestrians' motion. It can be seen that the proposed game theoretical model has improved the prediction performance of the corresponding DCM by approximately 8%. Even larger improvements can be expected for pedestrian motion modeling in more crowded scenarios, as there is a higher chance of collision between individuals and thereby, the mutual interaction and anticipation of future movements of the nearby pedestrians play a more critical role. The comparison results highlight the importance of incorporating the anticipation as well as feedback-oriented characteristics of human agents into pedestrian motion modeling. In fact, merely reactive models, such as DCMs, fail to

TABLE IV. DCM calibration results.

Parameter	Calibrated value	STD
α_{F}^0	0.000	0.000
$\alpha_{\text{F}}^{\text{DM}}$	10.672	0.061
$\alpha_{\text{F}}^{\text{int}}$	− 0.203	0.765
β_{F}	9.001	0.871
α_{FL}^0	5. 976	0.002
$\alpha_{\text{FL}}^{\text{DM}}$	6.223	0.723
$\alpha_{\text{FL}}^{\text{int}}$	− 3.519	0.910
α_{FR}^0	− 6.449	0.003
$\alpha_{\text{FR}}^{\text{DM}}$	7.878	0.303
$\alpha_{\text{FR}}^{\text{int}}$	0.056	0.143
α_{L}^0	4.030	0.405
$\alpha_{\text{L}}^{\text{DM}}$	− 0.484	0.133
β_{L}	7.108	0.023
α_{R}^0	− 0.323	0.066
$\alpha_{\text{R}}^{\text{DM}}$	− 0.921	0.026
β_{R}	0.984	0.079
α_{BL}^0	− 2.872	0.032
$\alpha_{\text{BL}}^{\text{DM}}$	0.903	0.347
β_{BL}	− 3.007	0.108
α_{BR}^0	− 2.107	0.333
$\alpha_{\text{BR}}^{\text{DM}}$	− 10.049	0.549
β_{BR}	2.443	0.009
α_{B}^0	− 0.937	0.555
$\alpha_{\text{B}}^{\text{DM}}$	0.038	0.503
β_{B}	− 1.666	0.775

TABLE V. Comparison of the game theoretical and corresponding discrete choice model.

Model	No. of decisions evaluated	No. of correctly predicted decisions	Percent correct (%)	Improvement (%)
DCM	27761	21081	79.04	(Base)
Game theory	27761	24121	86.89	7.85

account for these behavioral factors in describing pedestrians' motion and may lead to less accurate models and predictions.

VI. SUMMARY AND CONCLUSION

In spite of the chaotic appearance of individual pedestrian's behavior, particular patterns can be observed in pedestrian motion. The characteristics of pedestrian movements are affected by individual decisions and their set of actions and strategies. Thus, any behavioral based model needs to be directly related to the decision-making process of individual pedestrians. On the other hand, previous studies show that human agents are anisotropic, interactive, and feedback-oriented agents who anticipate the possible movements of the nearby pedestrians and decide accordingly to ensure a collision-free movement. However, a major part of the studies on modeling pedestrian motion have incorporated merely reactive models where pedestrians' movements are described as a reaction to the conditions they face, without considering the anticipation behavior of human agents. Considering these limitations, the present study has put forward a model to describe pedestrians' motion and behavior based on a game theoretical framework, while incorporating a learning process that can capture pedestrians' feedback-oriented walking behavior. The proposed game structure provides a technical foundation to analyze optimal decision-making of pedestrians where the outcome of the game for each player's choice also depends on the strategies played by other players. This, in turn, ensures the frequently observed collision avoidance behavior of pedestrians while walking.

In the proposed game structure, players are rational pedestrians who are assumed to play two games simultaneously: one with the nearest pedestrian in their visibility zone, and the other a learning game with all of the nearby pedestrians. In the first game, a finite strategy set consisting of possible moving directions are defined for each player and the payoffs for each strategy profile are calculated based on directional movements, interactions, and collision avoidance considerations. In the second game, each pedestrian plays a learning game considering the number of times each of his/her strategies in the first game was available during the previous time steps. A joint payoff function is then formulated for possible strategy profiles in the game. The moving strategies at each time step are selected based on the Nash equilibria calculations, where everyone is playing optimally given what all other players are playing. The game parameters are calibrated using the real-world walking trajectories of a bidirectional flow passing through a corridor. Validation results indicate the model's capability in capturing and modeling the microscopic decision-making behavior of individual pedestrians as well as its ability in predicting the empirically observed macroscopic characteristics of pedestrian flow dynamics. Statistical analysis of the actual and simulated pedestrian trajectories also provides evidence on the model's performance in capturing the collision avoidance behavior of human agents while walking.

Note that all the calibration and validation results are based on the roughly discretized version of the model. Smaller deviation ranges may better differentiate pedestrians' available strategies to play at each game, and hence lead to more accurate simulation results. Considering a continuous strategy profile for each player has been left for future research. Such an approach can result in a game with infinite strategies. The accuracy gain in the model's prediction should then be evaluated against the additional computational cost.

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