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Talent and experience shape competitive social hierarchies

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The hierarchy of social organization is a ubiquitous property of animal and human groups, linked to resource allocation, collective decisions, individual health, and even to social instability. Experimental evidence shows that both the intrinsic abilities of individuals and social reinforcement processes impact hierarchies; existing mathematical models, however, focus on the latter. Here, we develop a rigorous model that incorporates both features and explore their synergistic effect on stability and the structure of hierarchy. For pairwise interactions, we show that there is a trade-off between relationship stability and having the most talented individuals in the highest ranks. Extending this to open societies, where individuals enter and leave the population, we show that important societal effects arise from the interaction between talent and social processes: (i) Despite a positive global correlation between talent and rank, paradoxically, local correlation is negative, and (ii) the removal of an individual can induce a series of rank reversals. We show that the mechanism underlying the latter is the removal of an older individual of limited talent, who nonetheless was able to suppress the rise of younger, more talented individuals.

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Introduction. Hierarchy is a central organizing principle of complex systems, manifesting itself in various forms in biological, social, and technological systems [1]. Therefore, to understand complex systems, it is crucial to quantitatively describe hierarchies [2–5] and to identify their origins and benefits [6,7]. Among the various forms of hierarchy, here we are concerned with social hierarchies emerging through competition, including dominance and status hierarchies or socioeconomic stratification [8,9]. Ultimately, such a hierarchy represents a ranking of individuals based on social consensus: A high ranking individual is expected to win a conflict against a low ranking one. This type of organization is present in societies ranging from insects to primates and humans [3,10–12], and has been linked to resource allocation, individual health, collective decisions, and social stability [7,13–15].

The prevalence of social hierarchies has motivated a long history of theoretical research in statistical physics and mathematical biology [6,16–19]. The unifying theme in explaining the emergence of hierarchies is the positive reinforcement of differences known as the winner effect: Initially, equally ranked individuals repeatedly participate in pairwise competitions, and after an individual wins, the probability that they win later competitions increases. Conditions for hierarchies to emerge under this mechanism and their structure have been thoroughly investigated [9,11,18–20].

Yet, from experiments focusing on animal groups, we known that in addition to social reinforcement, intrinsic attributes also play a critical role in hierarchy formation [9,11]. The relative strength of the two effects depends on context; however, it was observed that they both affect hierarchies ranging from species with relatively simple social interactions, such as cichlid fish [21], to species that form highly complex societies, such as primates [13,22].

Despite the clear indication from experiments that both talent and reinforcement matter, we are lacking a general theoretical understanding of their synergistic impact [23,24]. Here, we develop a rigorous model incorporating both and show that this captures a much richer landscape. For pairwise interactions, we show a trade-off between relationship stability and having more talented individuals be the high-ranked leaders. We then extend the model to open populations, where individuals enter and leave the group, and we characterize both the global and the local structure of hierarchies.

Another pressing issue is to understand the response of hierarchical structure to perturbation, e.g., the effect of removing an individual. In particular, animal behavior experts must often make strategic decisions to remove individuals from captive societies due to health issues or in an attempt to promote social stability, which sometimes lead to an unanticipated reorganization of hierarchy and even societal collapse [14,25]. We show herein that if either talent or social reinforcement dominates hierarchy formation, the associated models predict a smooth response and no rearrangement. It is only if their effects are equally important, that the removal of an individual can lead to a nontrivial series of rank reversals.

Model. Our starting point is a classic model by Bonabeau *et al.* that considers only social reinforcement [6]. It describes a

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group with N members, where the rank of each member is determined by its ability to defeat others in pairwise competitions. This ability is quantified by a score $x_i(t)$, where the subscript indexes the individuals. The scores are initially identical $[x_i(t=0) \equiv 0]$ and they change through two discrete-time processes. The first is through positive feedback: In each time step, participants are randomly paired to compete with each other, and the winner increases its score by δ. Individual i wins against j with probability

$$Q_{ij}(t) = \frac{1}{1+\exp[-\beta(x_i(t)-x_j(t))]}, \qquad (1)$$

where β is an inverse temperaturelike parameter: For large β the outcome of the fight is deterministic, and for $\beta = 0$ both parties have an equal chance to win. The second process is forgetting: The effect of a fight wears off exponentially, i.e., $x_i(t)$ is reduced by $\mu x_i(t)$ $(0 \leqslant \mu \leqslant 1)$ in each time step. Describing the full process with the deterministic equation

$$x_i(t+1) = (1-\mu)x_i(t) + \frac{\delta}{N-1}\sum_{j\neq i} Q_{ij}(t), \qquad (2)$$

it was shown that, depending on the relative strength of reinforcement and decay, the model supports either egalitarian $(x_i \equiv 0)$ or hierarchical $(x_i \not\equiv 0)$ steady state solutions [6,26].

To introduce intrinsic attributes, we offset the score of each participant in Eq. (1) by base abilities b_i and b_j,

$$Q_{ij}(t) = \frac{1}{1+\exp[-\beta(x_i(t)+b_i-x_j(t)-b_j)]}. \qquad (3)$$

Parameter b quantifies talents that are independent of social processes, yet are relevant to conflict outcomes, such as strength or intelligence. This modification, although formally simple, requires a different mathematical description and leads to a series of nontrivial behaviors and unanticipated emergent properties.

Two individuals. To understand the consequences of intrinsic differences, it is insightful to first investigate a population of $N = 2$. The deterministic equation describing the steady state is

$$0 = -\mu\Delta x + \delta\left(\frac{2}{1+\exp\left[-\beta(\Delta x+\Delta b)\right]} - 1\right), \qquad (4)$$

where $\Delta x = x_1 - x_2$ and $\Delta b = b_1 - b_2 \geqslant 0$. Introducing dimensionless quantities $\Delta\bar{x} = \beta\Delta x$, $\Delta\bar{b} = \beta\Delta b$, and $\epsilon = \mu/(\delta\beta)$ leads to

$$0 = -\epsilon\Delta\bar{x} + \frac{2}{1+\exp[-\Delta\bar{x}-\Delta\bar{b}]} - 1, \qquad (5)$$

meaning that the steady state is determined by the talent difference and a single parameter ϵ measuring the relative strength of decay to social reinforcement [27].

Systematically changing ϵ, we observe a transition at $\epsilon_c(\Delta\bar{b})$ separating regimes with one and two stable solutions; the nature of the transition depends on the presence of intrinsic differences. If $\Delta\bar{b} = 0$ [Fig. 1(a), black line], we recover the original model: For $\epsilon > \epsilon_c(0)$ we find one solution, representing the egalitarian state $\Delta\bar{x} = 0$, and at $\epsilon_c(0)$ two symmetric hierarchical solutions $(\Delta\bar{x}_1 = -\Delta\bar{x}_2 \neq 0)$ emerge through a pitchfork bifurcation. If $\Delta\bar{b} > 0$ [Fig. 1(a), red line]:

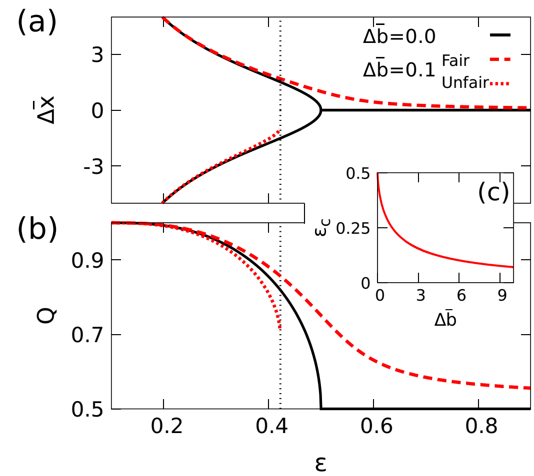

FIG. 1. Fairness and stability ($N = 2$). (a) Score difference as a function of ϵ, without (solid black) and with a difference in talent (dashed and dotted red). If $\Delta b > 0$, for large ϵ only one hierarchical solution exists corresponding to the fair ranking, i.e., rank is determined by talent (dashed); and through a discontinuous transition at ϵ_c (vertical line) a new solution emerges corresponding to the opposite, unfair ordering (dotted). (b) The probability Q that the dominant defeats the subordinate quantifies the stability of a hierarchical relationship; as ϵ decreases, social stability increases. Shown for the fair (dashed) and unfair (dotted) states. (c) Critical point ϵ_c as a function of $\Delta\bar{b}$.

For $\epsilon > \epsilon_c(\Delta\bar{b})$ we again find just one solution; this solution, however, is not egalitarian $(\Delta\bar{x} > 0)$, but it is "fair" in that the more talented individual outranks the less talented. At $\epsilon_c(\Delta\bar{b})$ a new stable solution appears through a discontinuous transition supporting the opposite order, which is "unfair," meaning that the less talented outrank the more talented. In other words, social reinforcement can outpace intrinsic abilities. We call the $\Delta\bar{x} > 0$ solution "fair" and the $\Delta\bar{x} < 0$ one "unfair," since high-ranked individuals tend to have better access to resources, more impact on collective decisions, and a higher chance to foster offspring.

Figure 1(c) shows the dependence of ϵ_c on $\Delta\bar{b}$. In general, no closed-form solution is available; limiting cases, however, can be worked out analytically: For small differences we find $(\epsilon_c - 1/2) \sim \Delta\bar{b}^{2/3}$ and for large differences $\epsilon_c = \Delta\bar{b}^{-1}$. The latter indicates that increasing talent difference or decreasing reinforcement pushes the system to a regime where only the fair solution exists. Since the fair solution intuitively benefits society, this prompts the following question: What is the role of social reinforcement?

To answer this question, we quantify the stability of a dominant-subordinate relationship with Q, the probability that the dominant wins a conflict, $Q \approx 1/2$ indicates an unstable relationship, and $Q \approx 1$ a well-defined relationship. Stable relationships reduce overall aggression and are positively associated with individual health [15]. Figure 1(b) shows that strong social reinforcement (high δ and thus low ϵ) increases Q, revealing a fundamental trade-off between stability and fairness: Stable relationships require a strong social reinforcement; however, strong reinforcement allows for unfair hierarchical states. A similar trade-off was experimentally observed in rankings of products in a marketplace competing for the attention of consumers: Strong social reinforcement led to less accuracy in selecting the highest-quality product, and to larger differences in market share [28].

Open populations. So far, we have focused on the relationship of two individuals, but now we turn our attention to larger, changing populations. We study groups of N individuals where the talent of each individual is drawn randomly from a distribution $p(b)$. We initially allow the population to reach a stable ranking. Then, in each step, we remove a random individual and add a new member i to the bottom of the society, i.e., $x_i = 0$, and again allow the population to reach a stable ranking.

For simplicity we restrict our investigation to the $\beta \to \infty$ limit, in which case Q_{ij} becomes a step function. This allows us to explicitly formulate the condition for two consecutively ordered individuals to reverse ranks during the evolution of the hierarchy [29],

$$b(k+1) - b(k) > \Delta x \equiv \frac{\delta}{\mu(N-1)}, \qquad (6)$$

where $b(k)$ is the talent of the individual ranked kth (note that $k = 1$ is the top and $k = N$ is the bottom rank); and Δx is the score difference of two consecutively ranked individuals $x(k) - x(k+1)$ which turns out to be independent of their ranks [29]. Therefore, Δx is the additional talent needed to overcome the advantage of having higher rank. Parameters δ and μ only effect the system through Δx; therefore, treating Δx as a parameter completely specifies the dynamics. The $\beta \to \infty$ limit allows us to study a simplified representation of the dynamics in Eqs. (2) and (3): We check each consecutively ranked individual and if Eq. (6) is satisfied, we reverse their order; we repeat this until no more pairs are reversed. In the Supplemental Material, we derive various properties of the hierarchy through exact combinatorics and mean-field-like approximations [29].

The talent b of an individual represents an intrinsic ability or a combination of abilities that influence the outcome of a fight. In our analysis we derive a number of properties of social hierarchies for general continuous talent distribution $p(b)$, including heavy-tailed distributions. Whenever specific $p(b)$ is necessary for calculations or simulations, we focus on the standard normal distribution. Indeed, body size, intelligence, and other relevant abilities are often normally distributed.

We now systematically investigate the structure of the emergent hierarchy as a function of Δx, the additional talent difference needed to overcome rank difference. We measure the correlation between rank and talent ($\tau_{\rm tal}$) and between rank and experience ($\tau_{\rm exp}$) using Kendall's tau coefficient, where experience is the amount of time an individual has spent in the population. For example, $\tau_{\rm tal} = 1$ indicates talent completely determines rank and $\tau_{\rm tal} = 0$ indicates no correlation. Analytical calculations and simulations show that for large Δx, rank is dominated by experience, meaning that the only way to advance in the hierarchy is if a higher ranking individual is removed; and for small Δx rank is dominated by talent [Fig. 2(a)]. These two limiting cases are separated by a regime where both talent and experience matter, and theory predicts that the crossover point, where $\tau_{\rm tal} = \tau_{\rm exp} = 1/2$, is $\Delta x_c \approx 0.36$ for $N = 100$.

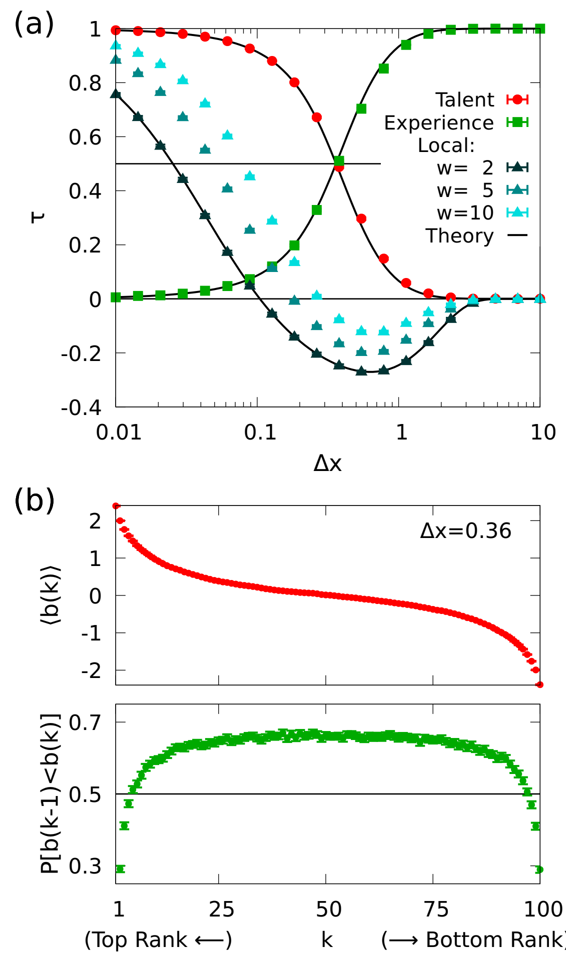

FIG. 2. Talent-rank correlation. (a) Kendall's tau as a function of Δx. Global talent-rank (red circles) and experience-rank (green squares) correlation shows a crossover between talent and experience dominated limiting cases. Counterintuitively, we find that locally talent and rank are anticorrelated (blue triangles) as shown for local windows of increasing size w. (b) Local rank-talent anticorrelation. In the crossover regime, the expected talent increases with rank (top panel), yet the probability that an individual's immediate superior is less talented is greater than 1/2 (bottom panel). In (a) and (b), results are shown for populations of $N = 100$, where continuous lines are analytical solutions [29]. Data points are simulations of the dynamics defined in Eq. (6), representing an average of 10 000 independent samples and error bars provide the 95% confidence interval (CI).

The experimental measurement of $\tau_{\rm tal}$ is challenging since it requires exact identification of the relevant talents; determining $\tau_{\rm exp}$, however, is straightforward. Indeed, Tung *et al.* established small captive groups of macaques by introducing animals one by one into an enclosure and found that the Spearman's correlation between rank and experience is $\rho_{\rm exp} = 0.61$, demonstrating that some real systems are in fact near the crossover point [13].

In addition to global correlations, we also quantify local orderedness by calculating $\tau_{\rm tal}(w)$, the talent-rank correlation averaged over a sliding window of length w. Counterintuitively, Fig. 2(a) shows that in the crossover regime $\tau_{\rm tal}(w)$ is negative, meaning that locally rank and talent are anticorrelated. Figure 2(b) provides an additional aspect of this

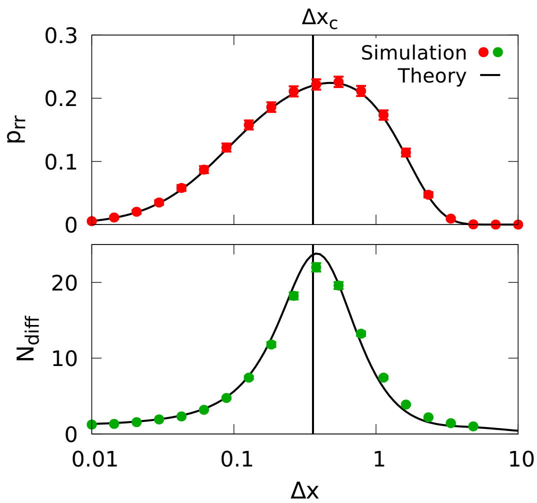

FIG. 3. Removal of a group member. In the limiting cases of talent and experience dominated societies, removal has a trivial effect, while in the crossover regime the removal of an individual causes rank reversals with finite probability (top panel). The average number of rank reversals $N_{\rm diff}$ peaks near $\Delta x_{\rm c}$ (bottom panel). Results are shown for populations of $N = 100$, where continuous lines are analytical solutions [29]. Data points are simulations of the dynamics defined in Eq. (6), representing an average of 50 000 time steps and error bars provide the 95% CI.

paradox situation: The expected talent $\langle b(k)\rangle$ of an individual ranked kth at a random time step monotonically increases with rank, yet the probability that the $(k-1)$th individual, the one immediately outranking the kth, is less talented than the kth is greater than $1/2$.

To understand the mechanism producing the local anticorrelation, first consider two consecutive individuals forming an ordered pair with respect to talent, i.e., $b(k) < b(k-1)$. If a new individual arrives with talent b such that $b(k) + \Delta x < b$ and $b(k-1) < b < b(k-1) + \Delta x$, it can pass the kth individual, but cannot pass the $(k-1)$th, lodging itself between the two and creating an unordered pair. Once an unordered pair exists, i.e., $b(k) > b(k-1)$, any individual passing the kth will necessarily pass the $(k-1)$th, too. Therefore, an unordered pair will remain unordered until one of the pair is removed. This asymmetry in creating ordered and unordered pairs is responsible for the local anticorrelation.

Finally, we also investigate the effect of removing an individual. We find that in the talent or experience dominated limiting cases the system's response is trivial and no reorganization happens. However, Fig. 3 shows that $p_{\rm rr}$, the probability that removal of an individual induces rank reversals, is nonzero in the crossover regime. For $N = 100$, both $p_{\rm rr}$ and the average number of these rank reversals $N_{\rm diff}$ peak near, but not exactly at, the crossover point $\Delta x_{\rm c}$. For removal-induced rank reversals to happen, at least three consecutively ranked individuals are needed in opposite order with respect to talent, i.e., $b(k+1) > b(k) > b(k-1)$. If the condition $b(k+1) - b(k-1) > \Delta x$ is satisfied, the removal of the kth individual allows the $(k+1)$th to pass the $(k-1)$th, which can lead to a series of rank reversals. In other words, the kth individual is not talented enough to further advance in society, but is capable of holding back a younger, more talented contender.

Understanding the response of hierarchies to external perturbation is an important issue. Particularly, the removal of animals from primate groups can sometimes lead to large shifts in hierarchy and instabilities endangering the group [14,25]. Here, we demonstrated that traditional models of hierarchy formation, those only considering either intrinsic abilities or social feedback, predict a trivial response to removal, and that both effects have to be present simultaneously to observe rank reversals.

So far we have focused on populations of $N = 100$ individuals. In the Supplemental Material, we extract the scaling behavior of various properties for large N [29]. Local quantities, such as $\tau_{\rm tal}(2)$ and $p_{\rm rr}$, scale as $\tau_{\rm tal}(2) = \tau_{\rm tal}^{(1)}(2, N\Delta x)$ and $p_{\rm rr} = p_{\rm rr}^{(1)}(N\Delta x)$ for small Δx, and are independent of N for large Δx, i.e., $\tau_{\rm tal}(2) = \tau_{\rm tal}^{(2)}(2, \Delta x)$ and $p_{\rm rr} = p_{\rm rr}^{(2)}(\Delta x)$. The location of their extreme value is at the crossover of these two regimes and in case of normal talent distribution scales as $\sim(\ln N)^{1/6}/N^{1/3}$. We find universal bounds

$$\begin{aligned} \tau_{\rm tal}(2) &\geqslant -2\ln 2 + 1, \\ p_{\rm rr} &\leqslant 0.294\ldots, \end{aligned} \tag{7}$$

for any continuous unbounded talent distribution, and these bounds are reached in the large population limit. For global talent correlation, on the other hand, we find that $\tau_{\rm tal} \to 1$ if $\sqrt{N}\Delta x \to 0$ and $\tau_{\rm tal} \to 0$ if $\sqrt{N}\Delta x \to \infty$. Therefore, the crossover point where $\tau_{\rm tal} = \tau_{\rm exp} = 1/2$ scales as $\Delta x_{\rm c} \sim 1/\sqrt{N}$. The average number of rank reversals $N_{\rm diff}$ depends on global correlations, and peaks near the crossover point $\Delta x_{\rm c}$. Note that in the parametrization of the model, provided in Eq. (6), $\Delta x = \delta/[\mu(N-1)]$, meaning that for $N \to \infty$, global correlation becomes talent dominated and local correlation may become negative depending on the value of $N\Delta x$. Other scalings of Δx are also possible through an adjustment of δ or μ, or if individuals do not randomly select opponents, but selectively compete with similarly ranked ones. The properties we observed for finite hierarchies may become more pronounced in the large population limit, for example, if

TABLE I. Structure of hierarchy in the large population limit assuming $\Delta x N^{\alpha} = C$. The numerical values are valid for any continuous unbounded talent distribution, while the scaling functions are specific to the talent distribution and are calculated in the Supplemental Material [29].

	In the limit of $N \to \infty$				
	$\tau_{\rm tal}$	$\tau_{\rm exp}$	$\tau_{\rm tal}(2)$	$p_{\rm rr}$	$N_{\rm diff}/N$
$1 < \alpha$	1	0	1	0	0
$\alpha = 1$	1	0	$\tau_{\rm tal}^{(1)}(2, C)$	$p_{\rm rr}^{(1)}(C)$	0
$1/2 < \alpha < 1$	1	0	$-2\ln 2 + 1$	$0.294\ldots$	0
$\alpha = 1/2$	$\tau_{\rm tal}(C)$	$1 - \tau_{\rm tal}(C)$	$-2\ln 2 + 1$	$0.294\ldots$	$f(C)$
$0 < \alpha < 1/2$	0	1	$-2\ln 2 + 1$	$0.294\ldots$	0
$0 = \alpha$	0	1	$\tau_{\rm tal}^{(2)}(2, C)$	$p_{\rm rr}^{(2)}(C)$	0
$\alpha < 0$	0	1	0	0	0

$N\Delta x \to \infty$ but $\sqrt{N}\Delta x \to 0$, global correlation τ_{tal} converges to one, while local correlation approaches its theoretical minimum. In Table I, we provide a detailed enumeration of possible behavior in the large population limit assuming $\Delta x N^{\alpha} = C$, where $C > 0$ is constant.

Discussion. We studied the synergistic effect of talent and social reinforcement on the structure of competitive social hierarchies, and we identified behaviors that cannot be observed if either effect dominates. Although we derived our model assuming pairwise conflicts and a winner effect, we believe that the results can be interpreted more generally: (i) The mechanism behind both local talent-rank anticorrelation and removal-induced rank reversals is that to pass someone in rank it is not enough to be more talented, but the talent difference has to be sufficient to compensate for the advantage of being higher ranked—a process relevant to many systems, where examples might include rankings of scientists, bestseller lists, or sports rankings. (ii) We introduced parameter b to capture individual talents; however, it can be thought of as a proxy for support of kin or as a simplified model of reputation received in exchange for nonadversarial social interactions.

Finally, our results prompt many research questions, both experimental and theoretical. For example, local anticorrelation and removal-induced rank reversals are predictions that are testable through experiments. Future theoretical work may investigate sources of complexity not captured by our model, for example, the role of aging or slow deterioration of talent, or nonlinear hierarchies, where social tiers are occupied by multiple individuals.

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