Flexoelectro-optic effect and two-beam energy exchange in a hybrid photorefractive cholesteric cell with a short-pitch horizontal helix

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We develop a theoretical model to describe two-beam energy exchange in a hybrid photorefractive cholesteric cell with a short-pitch helix oriented parallel to the cell substrates (so-called uniformly lying helix configuration). Weak and strong light beams incident on the hybrid cell interfere and induce a periodic space-charge field in the photorefractive substrate of the cell, which penetrates into the cholesteric liquid crystal (LC). Due to the flexoelectro-optic effect an interaction of the photorefractive field with the LC flexopolarization causes the spatially periodic modulation of the helix axis in the plane parallel to the cell substrates. Coupling of a weak signal beam with a strong pump beam at the LC permittivity grating, induced by the periodically tilted helix axis, leads to the energy gain of the weak signal beam. Dependence of the signal beam gain coefficient on the parameters of the short-pitch cholesteric LC is studied.

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I. INTRODUCTION

Energy transfer between two light beams interacting in solid inorganic photorefractive crystals is a well-known effect [1]. In hybrid organic-inorganic photorefractives a liquid crystal (LC) sample is placed adjacent to a solid photorefractive layer or between two solid photorefractive layers. Incident intersecting coherent light beams interfere and generate space charges in the inorganic photorefractive layers. These charges create a spatially modulated space-charge field, which penetrates into the adjacent LC layer causing a director-modulation-induced grating of the LC permittivity. Both incident light beams propagate across the LC layer and interact on the grating. Due to the beams coupling on the grating, one of the beams is amplified. Very strong two-beam energy transfer between coupled beams has been observed in these hybrid systems with the gain coefficient of two orders of magnitude larger than in solid inorganic photorefractive crystals [2–8].

The first theoretical models for hybrid organic-inorganic photorefractive systems [9,10] supposed the beam-coupling mechanisms to be similar to those in conventional LC cells. Coupling between the director and the space-charge electric field would then be caused by the LC static dielectric anisotropy. This hypothesis predicts the maximal energy transfer to occur when the grating spacing and the LC cell thickness are of comparable dimensions. However, in the experiments [5–8] this maximum occurs when the ratio of grating spacing to cell thickness is rather small. In our previous papers [11–14] discussing the formation of a director grating in hybrid systems, we supposed that the space-charge electric field penetrating from photorefractive substrates into the LC couples with the director through an interaction with the LC flexoelectric polarization, rather than through the LC static dielectric anisotropy. An additional assumption was made that the magnitude of the director grating is a nonlinear function of the space-charge electric field. Such a nonlinearity might be the consequence of the dipole molecule segregation under a strongly inhomogeneous electric field in LC and following the director-concentration coupling. These assumptions allowed for the description of the experimental results obtained for both nematic [11] and cholesteric LC cells [12,13].

In recent years, there has been much interest in the short-pitch cholesteric LCs confined in a sandwich cell with the helix axis aligned parallel to the substrate planes. If the pitch is much less than the wavelength of light, the cholesteric LC (CLC) layer acts as a uniaxial wave plate with the optic axis along the helix axis. When the electric field is applied perpendicular to the helix axis, the CLC may exhibit a flexoelectro-optic response: The helix axis rotates around an axis parallel to the field. The rotation magnitude depends on the flexoelectric coefficients and is linear with respect to the applied field [15–17]. In the LC materials developed for optimization of this effect, the rotation angles can reach values of 45° [18–20] with switching times on the order of microseconds [21–25]. Together with other specific features, it makes the flexoelectro-optic effect in the short-pitch CLCs a potential mechanism for developing the high-speed optoelectronic devices [19,26–29].

In this paper, we extend the study of the energy transfer between light beams in hybrid photorefractive cholesteric cells presented in our papers [12–14] to the case of the short-pitch CLC with the helix axis aligned parallel to the cell substrates. We show that a spatially periodic space-charge field arising in the photorefractive substrate under incident light beams produces a spatially periodic flexoelectro-optic response in the short-pitch CLC: The CLC helix axis is periodically tilted in the direction parallel to the cell substrates. This results in the appearance of the CLC permittivity diffraction grating. Interaction of the light beams at the grating leads to amplification of the weak signal beam. We calculate the signal beam gain and study its dependence on the parameters of the short-pitch CLC.

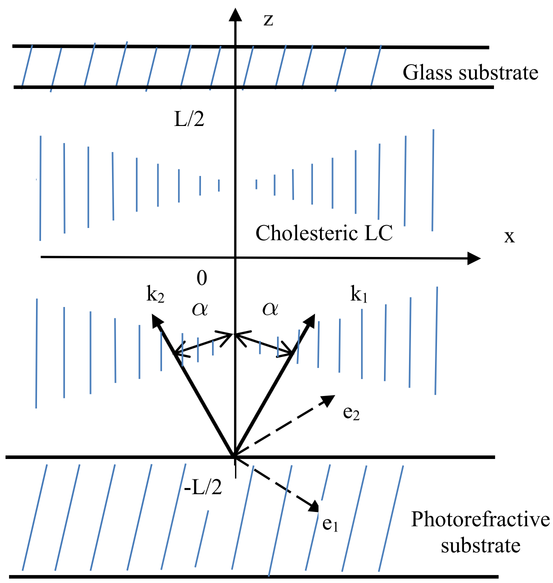

FIG. 1. Cholesteric LC cell with the initial helix axis directed along the x axis, showing light beams incident from the photorefractive medium, together with associated wave and polarization vectors; α is an angle of refraction.

The paper is organized as follows. In Sec. II we introduce the model of a hybrid cholesteric cell placed in the interference pattern of two incident light beams, and obtain expressions for the electric field in the short-pitch CLC cell. In Sec. III we obtain an expression for the spatially periodic rotation angle of the CLC helix axis under the photorefractive field. In Sec. IV we consider propagation of light beams in the short-pitch CLC and obtain an expression for the exponential gain coefficient. Results of numerical calculations of the gain coefficient and their discussion are presented in Sec. V. In Sec. VI we present some brief conclusions.

II. PHOTOREFRACTIVE ELECTRIC FIELD IN CLC

Let the z axis be directed perpendicular to the planes of a hybrid photorefractive cell, which contains the CLC layer bounded by the planes $z = -L/2$ and $z = L/2$. We suppose that the cholesteric helix axis is directed parallel to the planes $z = \pm L/2$ along the x axis. The cell bottom substrate at $z = -L/2$ is photorefractive; the cell top substrate at $z = L/2$ is nonphotorefractive (glass). The hybrid cell is illuminated by two intersecting coherent light beams $\mathbf{E}_1 = A_1\mathbf{e}_1 \exp(i\mathbf{k}_1\boldsymbol{r} - i\omega t)$ and $\mathbf{E}_2 = A_2\mathbf{e}_2 \exp(i\mathbf{k}_2\boldsymbol{r} - i\omega t)$. The bisector of the beams is directed along the z axis, and the wave vectors $\mathbf{k}_1, \mathbf{k}_2$ lie in the xz plane. At the entrance plane $z = -L/2$ the polarization vectors $\mathbf{e}_1, \mathbf{e}_2$ of the beams also lie in the xz plane (Fig. 1).

The beams produce a light intensity interference pattern in the photorefractive substrate,

$$I(z) = (I_1 + I_2)\left\{1 + \tfrac{1}{2}[m \exp(iqx) + \text{c.c.}]\right\}, \tag{1}$$

where $m = 2\cos(2\delta)A_1A_2^*/(I_1 + I_2)$ is the modulation parameter; 2δ is the angle between incident beams in the photorefractive medium; $I_1 = A_1A_1^*$, $I_2 = A_2A_2^*$ are the intensities of incident beams; and $q = k_{1x} - k_{2x} = 2k_1 \sin\delta$ is the wave number of the intensity pattern.

Inside the photorefractive substrate, the light intensity pattern given by Eq. (1) induces a space charge. The space-charge density is modulated along the x axis with period equal to $\Lambda = 2\pi/q$ and gives rise to an electric potential $\tilde{\Phi}(x) = \tilde{\Phi}_0 + [\tilde{\Phi}\exp(iqx) + \text{c.c.}]$ at the cell boundary $z = -L/2$. Here $\tilde{\Phi}_0$ is an arbitrary constant (which may be taken to be zero), and $\tilde{\Phi} = iE_{sc}(q)m/2q$, where $E_{sc}(q)$ is the space-charge field. In particular, in an infinite photorefractive medium and for a diffusion-dominated space-charge field, $E_{sc}(q)$ takes the following form [30]:

$$E_{sc}(q) = \frac{iE_d}{1 + \frac{E_d}{E_q}}, \quad E_d = q\frac{k_bT}{e}, \quad E_q = \left(1 - \frac{N_a}{N_d}\right)\frac{eN_a}{\varepsilon_0\varepsilon_{Ph}q}, \tag{2}$$

where E_d is the diffusion field, E_q is the so-called saturation field, ε_{Ph} is the dielectric permittivity of photorefractive material, e is the electron charge, and N_a and N_d are the acceptor and donor impurity densities, respectively.

The electric field in the CLC layer can be found from the Poisson equation,

$$\nabla \cdot (\varepsilon_0\hat{\tilde{\varepsilon}} \cdot \mathbf{E} + \mathbf{P}_f) = 0, \tag{3}$$

where $\mathbf{P}_f = e_{11}\mathbf{n}\nabla \cdot \mathbf{n} + e_{33}(\nabla \times \mathbf{n} \times \mathbf{n})$ is the flexopolarization [31], $\tilde{\varepsilon}_{ij} = \tilde{\varepsilon}_\perp\delta_{ij} + \tilde{\varepsilon}_a n_i n_j$ is the low-frequency dielectric permittivity of the CLC, n_i are the components of the director $\mathbf{n}$, e_{11} and e_{33} are the flexoelectric coefficients, $\tilde{\varepsilon}_a = \tilde{\varepsilon}_\parallel - \tilde{\varepsilon}_\perp$ is the static dielectric anisotropy, and $\tilde{\varepsilon}_\parallel$ and $\tilde{\varepsilon}_\perp$ are the components of the dielectric tensor along and perpendicular to the director.

To solve Eq. (3) inside the CLC, we use the relation $\mathbf{E}(x,z) = -\nabla\Phi(x,z)$, and seek a solution for the electric potential $\Phi(x,z)$ in the form $\Phi(x,z) = \Phi_0(z) + [\Phi(z)\exp(iqx) + \text{c.c.}]$ with boundary conditions

$$\Phi_0(\mp L/2) = 0, \quad \Phi(-L/2) = \frac{iE_{sc}(q)}{2q}m, \quad \Phi(L/2) = 0. \tag{4}$$

We will consider only small deviations of the director in response to the electric field. In this case, we can neglect the feedback of the director response on the electric field and derive the following equations for $\Phi_0(z)$ and $\Phi(z)$:

$$\frac{\partial^2}{\partial z^2}\Phi_0(z) = 0, \tag{5}$$

$$\left[\frac{\tilde{\varepsilon}_\perp + \tilde{\varepsilon}_\parallel}{2} - \frac{1}{2}\tilde{\varepsilon}_a \cos 2\varphi_0(x)\right]\frac{\partial^2}{\partial z^2}\Phi(z) - \tilde{\varepsilon}_\perp q^2\Phi(z) = 0, \tag{6}$$

where $\varphi_0(x) = \frac{2\pi}{p}x$ and p is the pitch of the cholesteric helix.

We note that a characteristic length of the electric potential variation along the x axis is an order of magnitude of the grating spacing Λ; therefore, for the short-pitch CLC with $p \ll \Lambda$ one can average Eq. (6) over the cholesteric pitch yielding

$$\frac{\tilde{\varepsilon}_\perp + \tilde{\varepsilon}_\parallel}{2}\frac{\partial^2}{\partial z^2}\Phi(z) - \tilde{\varepsilon}_\perp q^2\Phi(z) = 0. \tag{7}$$

Solving Eqs. (5) and (7) with boundary conditions (4) we get the following expressions for the electric field in the CLC cell:

$$E_x = E_{0x}\exp(iqx) + \text{c.c.},\quad E_z = E_{0z}\exp(iqx) + \text{c.c.},$$
$$E_{0x} = -\frac{E_{sc}(q)m}{2\sinh(\tilde{q}L)}\sinh\tilde{q}(z-L/2),$$
$$E_{0z} = \frac{i\tilde{q}E_{sc}(q)m}{2q\sinh(\tilde{q}L)}\cosh\tilde{q}(z-L/2), \tag{8}$$

where $\tilde{q} = \sqrt{2\tilde{\varepsilon}_\perp/(\tilde{\varepsilon}_\perp + \tilde{\varepsilon}_\parallel)}\,q$ and m is the modulation parameter at the LC cell boundary $z = -L/2$.

III. FLEXOELECTRO-OPTIC RESPONSE

The equilibrium director profile can be found by minimizing the total free energy functional of the CLC cell defined by

$$F = F_{el} + F_l + F_E + F_{fl}, \tag{9}$$

where

$$F_{el} = \frac{1}{2}\int [K_{11}(\nabla\cdot\mathbf{n})^2 + K_{22}(\mathbf{n}\cdot\nabla\times\mathbf{n} + 2\pi/p)^2 + K_{33}(\mathbf{n}\times\nabla\times\mathbf{n})^2]dV,$$
$$F_l = -\frac{\varepsilon_0\varepsilon_a}{4}\int(\mathbf{n}\cdot\mathbf{E}_{h\nu})^2 dV,$$
$$F_E = -\frac{1}{2}\int(\mathbf{D}\cdot\mathbf{E})\,dV,\quad F_{fl} = -\int(\mathbf{P}_f\cdot\mathbf{E})dV. \tag{10}$$

Here F_{el} is the bulk elastic energy of a distorted CLC layer, F_l is the contribution of the light field to the total free energy functional, F_E is the contribution from the dc electric field created in the CLC cell by the photorefractive substrate, and F_{fl} is the contribution from the interaction of the dc electric field with the CLC flexoelectric polarization.

Some terms in Eq. (9) can now be neglected. The LC dielectric anisotropy at optical frequencies $\varepsilon_a \ll 1$, implies the light field contribution F_l can be ignored. In addition, the LC dielectric anisotropy term F_E can also be neglected compared to the contribution from the LC flexopolarization, which was shown in [11]. For simplicity, we assume $K_{11} = K_{22} = K_{33} = K$ and following Ref. [15] we set the flexoelectric coefficients $e_{11} \approx -e_{33} \equiv \tilde{e}$.

Presenting the CLC director in the form $\mathbf{n} = 0,\ \cos\varphi(x,y,z),\ \sin\varphi(x,y,z)$ the total free energy functional (9) reduces to

$$F = \frac{K}{2}\int\left[\left(\frac{\partial\varphi}{\partial y}\right)^2 + \left(\frac{\partial\varphi}{\partial z}\right)^2 + \left(\frac{\partial\varphi}{\partial x} - \frac{2\pi}{p}\right)^2 + \frac{2\tilde{e}}{K}\frac{\partial\varphi}{\partial y}E_z\right]dV. \tag{11}$$

From this point on we consider only a finite box-shaped region, with the y side of size L_1, a quantity on the order of the beam spot size, but not necessarily coinciding exactly with it. In this region, we assume φ to be given by the equation

$$\varphi = \frac{2\pi}{p}x + \beta y E_z, \tag{12}$$

and substituting Eqs. (8) and (12) into Eq. (11) we arrive at

$$F = \frac{K\beta}{2}\left|\frac{\tilde{q}}{q}E_{sc}(q)me^{-\tilde{q}L/2}\right|^2\int[\beta(\cos^2qx + y^2q^2\sin^2qx)\times\cosh^2\tilde{q}(z-L/2) + \beta y^2\tilde{q}^2\cos^2qx\sinh^2\tilde{q}(z-L/2) + 2(\tilde{e}/K)\cos^2qx\cosh^2\tilde{q}(z-L/2)]dV. \tag{13}$$

Evaluating the integral in the above expression, we find that the CLC free energy reaches its minimal value at

$$\beta = -\frac{2r_0(1+\sinh 2\tilde{q}L/2\tilde{q}L)}{1+\frac{1}{3}{L_1}^2(q^2-\tilde{q}^2) + \left[1+\frac{1}{3}{L_1}^2(q^2+\tilde{q}^2)\right]\frac{\sinh 2\tilde{q}L}{2\tilde{q}L}}, \tag{14}$$

where $r_0 = \tilde{e}/K$.

As in [15], introducing the vector $\tilde{\boldsymbol{k}}$ along the helix axis we rewrite Eq. (12) in the form $\varphi = \tilde{\boldsymbol{k}}\cdot\boldsymbol{r}$ with $\tilde{k}_x = \tilde{k}\cos\phi = 2\pi/p$ and $\tilde{k}_y = \tilde{k}\sin\phi = \beta E_z$. Therefore, the rotation angle of the helix axis is given by the relationship

$$\tan\phi = \frac{p}{2\pi}\beta E_z. \tag{15}$$

Considering only small values of angle ϕ and recalling that $E_z = E_{0z}\exp(iqx) + \text{c.c.}$ [see Eq. (8)] we can rewrite Eq. (15) as

$$\phi(x,z) = \phi(z)\exp(iqx) + \text{c.c.}, \tag{16}$$

where

$$\phi(z) = \frac{p}{2\pi}\beta E_{0z}. \tag{17}$$

Therefore, the tilt of the CLC helix axis under the photorefractive electric field defined by Eq. (16) is spatially periodically modulated along the x axis with period $2\pi/q$ and magnitude $\phi(z)$.

IV. SIGNAL BEAM GAIN COEFFICIENT

Let us introduce the local Cartesian system (x',y',z') rotated by the angle $\phi(x,z)$ around the z axis with the x' axis directed along the tilted helix axis of the CLC. In this Cartesian system, the CLC dielectric tensor at optical frequencies takes the form [31]

$$\hat{\varepsilon} = \begin{vmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \bar{\varepsilon} + \frac{1}{2}\varepsilon_a\cos\frac{4\pi}{p}x' & \frac{1}{2}\varepsilon_a\sin\frac{4\pi}{p}x' \\ 0 & \frac{1}{2}\varepsilon_a\sin\frac{4\pi}{p}x' & \bar{\varepsilon} - \frac{1}{2}\varepsilon_a\frac{4\pi}{p}x' \end{vmatrix}, \tag{18}$$

where $\bar{\varepsilon} = (\varepsilon_\perp + \varepsilon_\parallel)/2$, $\varepsilon_a = \varepsilon_\parallel - \varepsilon_\perp$ is the dielectric anisotropy at optical frequencies.

Supposing the cholesteric pitch to be much less than the wavelength of the light we can average the dielectric tensor (18) over the cholesteric pitch and rewrite the averaged tensor $\hat{\varepsilon}$ in the initial Cartesian system (x,y,z). It yields the following expression for the dielectric tensor of the short-pitch CLC:

$$\hat{\varepsilon} = \hat{\varepsilon}_1 + [\hat{\varepsilon}_2(z)\exp(iqx) + \text{c.c.}], \tag{19}$$

where

$$\hat{\varepsilon}_1 = \begin{vmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \bar{\varepsilon} & 0 \\ 0 & 0 & \bar{\varepsilon} \end{vmatrix}, \quad \hat{\varepsilon}_2 = -\frac{1}{2}\varepsilon_a\phi(z)\begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}. \tag{20}$$

The wave equation for the electric vector $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$ of both light beams in the LC with a dielectric tensor $\hat{\varepsilon}_1$ reads

$$\nabla \times \nabla \times \mathbf{E} - \frac{\omega^2}{c^2}\hat{\varepsilon}_1\mathbf{E} = 0. \tag{21}$$

Equation (21) can be decomposed into equations for the electric vector of each beam:

$$\nabla \times \nabla \times \mathbf{E_1} - \frac{\omega^2}{c^2}\hat{\varepsilon}_1\mathbf{E_1} = 0, \quad \nabla \times \nabla \times \mathbf{E_2} - \frac{\omega^2}{c^2}\hat{\varepsilon}_1\mathbf{E_2} = 0. \tag{22}$$

Taking into account the form of the dielectric tensor $\hat{\varepsilon}_1$, Eqs. (22) are the same as in a nematic LC and their solutions are the plane waves $\mathbf{E}_1 = A_1\mathbf{e}_1\exp(i\mathbf{k}_1\boldsymbol{r} - i\omega t)$, $\mathbf{E}_2 = A_2\mathbf{e}_2\exp(i\mathbf{k}_2\boldsymbol{r} - i\omega t)$ with the wave vectors $\mathbf{k}_1$, $\mathbf{k}_2$ and the polarization vectors $\mathbf{e}_1$, $\mathbf{e}_2$ sited in the xz plane as is shown in Fig. 1.

In the LC characterized by a dielectric tensor with a spatially periodic modulation, $\hat{\varepsilon}_1 + [\hat{\varepsilon}_2(z)\exp(iqx) + \text{c.c.}]$, the light beam electric vector $\tilde{\mathbf{E}}$ satisfies the modified equation:

$$\nabla \times \nabla \times \tilde{\mathbf{E}} - \frac{\omega^2}{c^2}\{\hat{\varepsilon}_1 + [\hat{\varepsilon}_2\exp(iqx) + \text{c.c.}]\}\tilde{\mathbf{E}} = 0. \tag{23}$$

Neglecting small terms of the second order with respect to the small angle $\phi(z)$, the x and z components of Eq. (23) coincide with the x and z components of Eq. (21) and do not contain the electric field component $\tilde{E}_y$. It implies that $\tilde{E}_x = E_x = E_{1x} + E_{2x}$, $\tilde{E}_z = E_z = E_{1z} + E_{2z}$. The y component of Eq. (23) gives us an equation for $\tilde{E}_y$:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\tilde{E}_y + \frac{\omega^2}{c^2}\left\{\bar{\varepsilon}\tilde{E}_y + \left[-\frac{1}{2}\varepsilon_a\phi(z)\exp(iqx) + \text{c.c.}\right]E_x\right\} = 0. \tag{24}$$

Therefore, we can seek a solution to Eq. (23) in the form

$$\tilde{\mathbf{E}} = \mathbf{E}_1 + \mathbf{E}_2 + \boldsymbol{j}\,\tilde{E}_y, \tag{25}$$

where $\boldsymbol{j}$ is a unit vector along the y axis and $\tilde{E}_y$ satisfies Eq. (24).

We define beam 1 to be the signal and beam 2 to be the pump, with the consequence that the pump amplitude $|A_2| \gg |A_1|$. Then, in Eq. (24) we can set $E_x = E_{1x} + E_{2x} \approx E_{2x}$ and seek the solution to this equation in the form

$$\tilde{E}_y = A(z)\exp(i\mathbf{k}\boldsymbol{r} - i\omega t). \tag{26}$$

Further, we follow a procedure analogous to that first outlined by Kogelnik [32], which we have used in our previous related papers [11–13]. According to this, we suppose the magnitude $A(z)$ to vary slowly across the cell. Note that electric fields $\mathbf{E}_1$, $\mathbf{E}_2$ satisfy Eq. (22) and their magnitudes A_1, A_2 are the constants. Then, substituting Eq. (26) into Eq. (24), after some algebra we obtain an equation for $A(z)$:

$$2ik_z\frac{\partial A(z)}{\partial z} - \frac{1}{2}\varepsilon_a\phi(z)\frac{\omega^2}{c^2}\{\exp[i(q + k_{2x} - k_x)x] + \exp[-i(q - k_{2x} + k_x)x]\}A_2\mathbf{e}_{2x}\exp[i(k_{2z} - k_z)z] = 0. \tag{27}$$

Recalling that $q = k_{1x} - k_{2x}$, Eq. (27) averaged over the grating spacing has nonzero solutions only if $k_x \approx k_{1x}$. In this case it reduces to

$$\frac{\partial A(z)}{\partial z} + i\frac{\varepsilon_a}{4k_z}\frac{\omega^2}{c^2}\phi(z)A_2\cos\alpha\exp[i(k_{2z} - k_z)z] = 0. \tag{28}$$

Now we substitute $\phi(z)$ from Eq. (17) and E_{0z} from Eq. (8) into Eq. (28) and take into account that for the pump amplitude $|A_2| \gg |A_1|$ the expression for the modulation parameter m reduces to $m \approx 2\cos(2\delta)A_1/A_2$. Then the solution to Eq. (28), which satisfies the boundary condition $A(-L/2) = 0$ is as follows:

$$A(z) = \frac{\varepsilon_a}{4k_z}\frac{\omega^2}{c^2}\frac{p}{2\pi}\beta\frac{\tilde{q}E_{sc}(q)\cos(2\delta)\cos\alpha}{2q\sinh(\tilde{q}L)}A_1 \times\left\{\frac{\exp[\tilde{q}(z - L/2) + i(k_{2z} - k_z)z] - \exp[-\tilde{q}L - i(k_{2z} - k_z)L/2]}{\tilde{q} + i(k_{2z} - k_z)} - \frac{\exp[-\tilde{q}(z - L/2) + i(k_{2z} - k_z)z] - \exp[\tilde{q}L - i(k_{2z} - k_z)L/2]}{\tilde{q} - i(k_{2z} - k_z)}\right\}. \tag{29}$$

If the optical dielectric anisotropy is small, $|\varepsilon_a| = |n_e^2 - n_o^2| \ll 1$, then the wave vector $\mathbf{k}$ of the wave (26) is close to the initial signal beam wave vector $\mathbf{k}_1$. In this case, we can define the signal beam gain at the exit of the CLC cell as

$$G = \left.\frac{|\mathbf{E}_1 + \tilde{E}_y\boldsymbol{j}|^2}{|\mathbf{E}_1|^2}\right|_{z=L/2} = \frac{|A_1|^2 + |A(L/2)|^2}{|A_1|^2}. \tag{30}$$

Using Eq. (29) we can calculate $A(L/2)$ and substitute it into Eq. (30). The result is expressed in terms of the exponential gain coefficient:

$$\Gamma = \frac{1}{L}\ln|G| = \frac{1}{L}\ln\left\{1+\left|\frac{n_e^2-n_o^2}{4k_1}\frac{\omega^2}{c^2}\frac{p}{2\pi}\beta\frac{E_{sc}(q)\cos(2\delta)}{q}\right|^2\right\}, \tag{31}$$

where the parameter β is defined by Eq. (14).

V. NUMERICAL CALCULATIONS

For numerical calculations we use the cholesteric mixture BC038/CB15, supposing that its helix axis is directed parallel to the cell substrates. Theory developed in our paper [13] provides a good description for the experimental data for the signal beam gain coefficient in the hybrid cell with this mixture, when the helix axis is perpendicular to the cell substrates. Therefore, it will allow us to compare results obtained for the gain coefficient in the same CLC, but having different orientations of the helix axis, horizontal and vertical.

The ordinary and extraordinary refractive indices of the mixture BL038/CB15 are $n_o = 1.527$ and $n_e = 1.799$, respectively [5]. As in our previous papers [11–13] accounting for a change of the dipole concentration spatial distribution in the LC cell under an inhomogeneous photorefractive field, we replace in Eq. (31) the flexoelectric parameter $r_0 = \tilde{e}/K$ by its effective value $r = r_0(1+\mu q^2|E_{sc}|^2)$, where μ is the fitting parameter. Based on the experimental data, the parameter μ was estimated in Ref. [11] for a nematic LC TL208 and in Ref. [13] for a cholesteric LC mixture BL038/CB15. In Ref. [13] the cholesteric LC had its helical axis oriented perpendicular to the cell substrates (uniformly standing helix) in contrast to the current study where the helical axis is parallel to the substrates (uniformly lying helix); moreover, in this current work the cholesteric pitch is short compared to the wavelength. A short-pitch cholesteric is optically equivalent to a nematic LC with the effective (averaged) dielectric tensor $\hat{\varepsilon}_1$. Therefore, for a definiteness at numerical calculations we used the value for a parameter obtained for a nematic LC [11]: $\mu = 2\times 10^{-21}\,\mathrm{J^{-2}\,C^2\,m^4}$. The incident beams have wavelength $\lambda = 532$ nm; the LC cell thickness $L = 5\,\mu$m. To evaluate $E_{sc}(q)$ we set the ratio of the acceptor to donor impurity densities to be very small, i.e., $N_d \gg N_a$, with $N_a \approx 3.8\times 10^{21}\,\mathrm{m^{-3}}$; the dielectric permittivity of the photorefractive substrate is given by $\varepsilon_{Ph} = 200$ [5,13].

Parameter L_1 characterizing the transverse CLC cell dimension influences the gain coefficient via the function $\beta(L_1)$ [see Eqs. (31) and (14)]. In Fig. 2 we show the function $|\beta(L_1)|/2r_0$ versus the parameter L_1 for several values of the grating spacing Λ. It is seen from this figure that the function $|\beta(L_1)|/2r_0$ changes appreciably depending on the value of the grating spacing. However, influence of the parameter L_1 is minimized when its value is rather small. As an example, we choose for further calculations $L_1 = 0.1\,\mu$m.

The magnitude of the helix axis rotation angle, $\phi(z)$, determines the magnitude of the refractive index grating, which is responsible for the coupling of the beams. It is proportional to the magnitude of the photorefractive field component E_{0z} and therefore depends on the distance from the photorefractive substrate and the grating spacing. In Fig. 3 we present the function $\phi(z)$ versus grating spacing for several distances from the photorefractive substrate setting, for example, the flexoelectric parameter $r_0 = 1\,\mathrm{C\,m^{-1}\,N^{-1}}$ and cholesteric pitch $p = 0.3\,\mu$m.

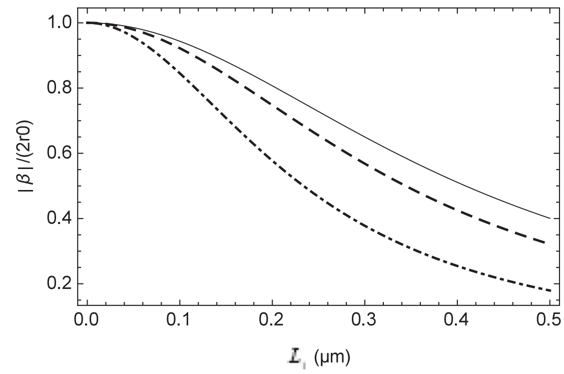

FIG. 2. Dependence of the function $|\beta(L_1)|/2r_0$ on the transverse CLC cell dimension L_1 for grating spacing $\Lambda = 1\,\mu$m (solid line), $2\,\mu$m (dashed line), and $4\,\mu$m (dot-dashed line). The cell thickness is $L = 5\,\mu$m.

Presented in Fig. 3, the dependence of the helix axis rotation angle magnitude on the grating spacing Λ reflects, as follows from Eq. (17), the corresponding dependence on Λ of the photorefractive field component E_{0z} [see Eq. (8)]. The magnitude of the director angle ϕ is proportional to the electric field E_{0z}; therefore, $\phi(z)$ decreases with the distance from the photorefractive substrate ($z = -L/2$) in a similar fashion as the photorefractive electric field component that penetrates into the CLC cell from the photorefractive substrate.

In Fig. 4 we compare the gain coefficient $\Gamma(\Lambda)$ for the short-pitch cholesteric mixture BL038/CB15 when its helix axis is oriented perpendicular (vertical helix) and parallel (horizontal helix) to the cell substrates. The dashed line is obtained using results of Ref. [13] and corresponds to the vertical helix axis.

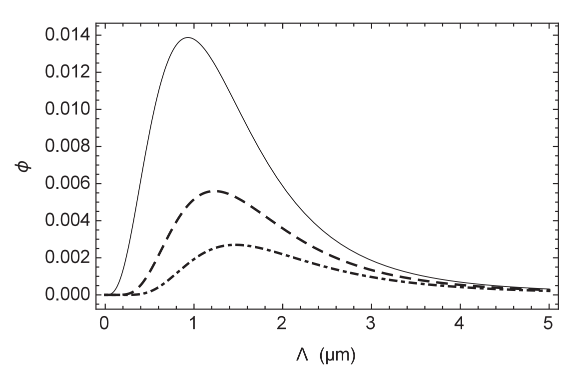

FIG. 3. Magnitude $\phi(z)$ of the helix axis rotation angle versus grating spacing. $z/L = -0.5$ (solid curve), -0.45 (dashed curve), and -0.4 (dash-dotted curve); ratio of flexoelectric coefficient to elastic constant $r_0 = 1\,\mathrm{C\,m^{-1}\,N^{-1}}$, cholesteric pitch $p = 0.3\,\mu$m, and the transverse CLC cell dimension $L_1 = 0.1\,\mu$m.

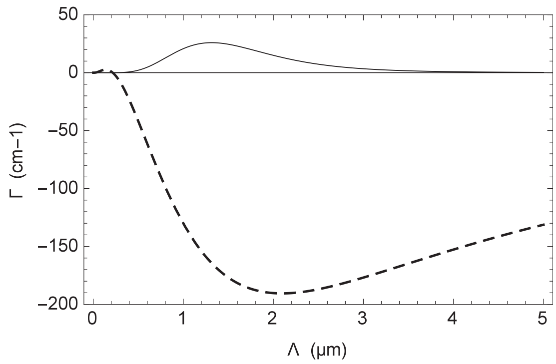

FIG. 4. Gain coefficient Γ versus grating spacing Λ in a hybrid photorefractive cell containing cholesteric LC mixture BL038/CB15. Cholesteric pitch $p = 0.25\,\mu$m; ratio of flexoelectric coefficient to elastic constant in the case of horizontal helix axis (solid line) $r_0 = 1\,\mathrm{C\,m^{-1}\,N^{-1}}$; the ratios of flexoelectric to elastic moduli in the case of vertical helix axis (dashed line) are those describing the experimental results in [13] and have an order of magnitude $\sim 1\,\mathrm{C\,m^{-1}\,N^{-1}}$1.

The solid line is calculated using Eq. (31) and corresponds to the horizontal helix axis.

It is seen that the dependence of the gain coefficient on the grating spacing is qualitatively different: In the short-pitch CLC with the horizontal helix axis the function $\Gamma(\Lambda)$ is similar to those observed in a hybrid photorefractive cell filled with nematic LC [11]. This is consistent with the fact that the dielectric tensor $\hat{\varepsilon}_1$ obtained for the short-pitch CLC [see Eq. (20)] has the same form as for an ordinary nematic LC.

From Eqs. (31) and (14) we can see that the gain coefficient strongly depends on the ratio of the flexoelectric coefficient to the CLC elastic constant. The bent-core LCs are reported to possess a large flexoelectricity [33–35] and their twist or bend elastic constants can be very small [36–38]. Therefore, the ratios of flexoelectric to elastic moduli for the bent-core LCs can considerably exceed those for conventional LCs. In Fig. 5(a) the gain coefficient Γ versus the grating spacing Λ is plotted for different values of the ratio of flexoelectric coefficient to elastic constant, when the CLC helix axis is horizontal. We keep other parameters the same as for BL038/CB15. It is seen that the gain coefficient can reach high values for reasonable values of the ratio $\tilde{e}/K$.

In Fig. 5(b) we illustrate the influence of the cholesteric helix pitch on the gain coefficient dependence on the grating spacing described by Eq. (31). It is seen that even at small pitches, $p \ll \lambda$, the gain coefficient can reach rather high values.

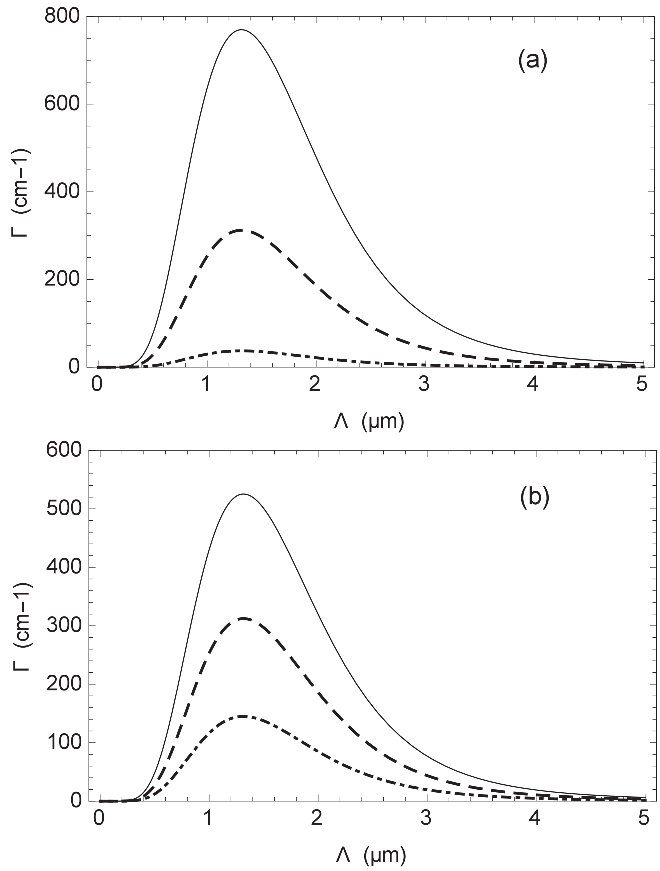

FIG. 5. Gain coefficient Γ versus grating spacing Λ for CLC with horizontal helix axis; (a) different values of the ratio of flexoelectric coefficient to elastic constant: $r_0 = 1$ (dash-dotted line), 3 (dashed line), 5 (solid line), $p = 0.3\,\mu$m; (b) different values of the cholesteric pitch: $p = 0.2\,\mu$m (dash-dotted line), $p = 0.3\,\mu$m (dashed line), $p = 0.4\,\mu$m (solid line); $r_0 = 3$.

VI. CONCLUSIONS

We have developed a theoretical model to describe the two-beam energy exchange in a hybrid photorefractive cholesteric cell with a short-pitch helix oriented parallel to the cell substrates. It is shown that under the space-charge field penetrating in the CLC from the photorefractive substrate and interacting with the CLC flexopolarization, the CLC helix axis can be spatially modulated along the x axis with period $\Lambda = 2\pi/q$. We show that coupling of a weak signal beam with a strong pump beam at the CLC permittivity grating induced by the periodically tilted helix axis leads to the energy gain of the signal beam.

It is found that the dependence of the signal beam gain coefficient on the grating spacing in the short-pitch CLC with the helix axis parallel to the cell substrates substantially differs from that in the CLC with the helix axis perpendicular to the cell substrates. In the case of the helix axis parallel to the cell substrates, this dependence is similar to that observed in the hybrid photorefractive cells filled with the nematic LCs.

We show that in hybrid cholesteric cells with the short-pitch helix parallel to the cell substrates the gain coefficient can reach rather high values, which can be further increased by selecting CLC with a high ratio of flexoelectric to elastic moduli. We pose the question of application of this theory to an experimental test in the hybrid photorefractive cells filled with the bent-core short-pitch CLCs, which possess increased values of the ratio of flexoelectric to elastic moduli.

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