Stabilizing effect of volatility in financial markets

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In financial markets, greater volatility is usually considered to be synonymous with greater risk and instability. However, large market downturns and upturns are often preceded by long periods where price returns exhibit only small fluctuations. To investigate this surprising feature, here we propose using the mean first hitting time, i.e., the average time a stock return takes to undergo for the first time a large negative (crashes) or positive variation (rallies), as an indicator of price stability, and relate this to a standard measure of volatility. In an empirical analysis of daily returns for 1071 stocks traded in the New York Stock Exchange, we find that this measure of stability displays nonmonotonic behavior, with a maximum, as a function of volatility. Also, we show that the statistical properties of the empirical data can be reproduced by a nonlinear Heston model. This analysis implies that, contrary to conventional wisdom, not only high, but also low volatility values can be associated with higher instability in financial markets. This proposed measure of stability can be extremely useful in risk control.

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I. INTRODUCTION

Volatility is typically considered a monotonic indicator of a financial market's risk and instability. Recently, however, such conventional wisdom has been questioned by the observation that sizable market downturns or upturns can be anticipated by periods of low volatility. Notable examples of this phenomenon include the 2008 financial crisis, preceded by the so-called "great moderation," and the Chinese crash in 2015. These episodes have received a lot of attention in the specialized press and have popularized the so-called Minsky's financial instability hypothesis [1] that periods of calm can project a false sense of security and lure agents into taking a riskier investment, preparing for a crisis [2]. Therefore, a better characterization of the relationship between volatility and market stability seems particularly important.

Searching for empirical regularities and modeling complex market dynamics have typically been the objective of financial times series analysis, econophysics, and complex systems [3–14]. Specifically, very recently, a new method to find the critical transition points within a financial market has been introduced in Ref. [14]. In this literature, the importance of the statistical properties of volatility for portfolio optimization strategies, risk management, and financial stability have been underlined in Refs. [15–18]. Along these lines, investigations have looked at the statistical properties of large volatilities [19], cross correlations between volume change and price change [20], the interplay between past market correlation structures and future volatility outbursts [21], and temporal sequences of financial market fluctuations around abrupt switching points [22]. There, the authors argue that the end of microscopic or macroscopic trends in financial markets have a parallel with metastable physical systems. Moreover, in a very recent paper [23], the authors provide a comprehensive analysis of a structural model for the dynamics of the prices of assets, namely, the interacting generalization of the geometric Brownian motion model, introduced in Ref. [24] and exhibiting a large number of metastable states. Specifically, the authors elucidate in detail the relation between switching dynamics between *metastable states* and the phenomenon of *volatility clustering*. In other words, the market states would indeed emerge naturally as attractors of collective nonlinear dynamics of interacting prices. In the presence of noise, many of these attractors will survive as *long-lived states* and volatility clustering is expected to arise naturally through the interplay of the dynamics within long-lived states and the dynamics of occasional noise-induced transitions between them [23]. In this respect, it is worthwhile to note that in Ref. [25] a set of distinct *quasistationary market states* in historical data has been identified and their dynamics and stability discussed.

Indeed, financial market stability is often associated with moderate levels of perceived uncertainty and measured looking at the intensity of price return fluctuations [26–28] or stochastic volatility estimators based on first passage time statistics [29]. However, both approaches cannot be reconciled with the observed evidence discussed above.

In this direction, a fundamental, and yet overlooked, question has to be addressed: What is the typical time scale before a

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large negative or positive stock return variation? To answer this question, we propose exploiting the notions of "level crossing" and "hitting time" to monitor the stability of price returns and observing its relationship with volatility [30–32]. In particular, the mean first hitting time (MFHT) or mean first passage time (MFPT), earlier introduced in Refs. [33–37], is the time it takes, on average, for a variable to cross for the first time a certain level, and it can provide the above-mentioned timescale to observe modifications in market scenarios.

Here, we propose this quantity (MFHT) as an indicator to measure the stability of price returns, defined as the resilience to large negative price variations: The longer this time, the more stable the series of price returns. Observing the daily closing prices of a large number of stocks traded in the New York Stock Exchange (NYSE), we find that this measure of stability has a nonmonotonic behavior, with a maximum, as a function of volatility, both for crashes and rallies. This result seems in line with the view discussed above that a higher price return instability, corresponding here to lower hitting times, is not only associated with high values but also with low values of volatility. As such, this measure can be considered as an important indicator of market stability and a potential tool for *risk control*.

Further, we are able to reproduce all the main statistical features of the price return dynamics of the considered stock market by using a nonlinear generalization of the Heston model proposed in Ref. [30].

The paper is organized as follows. In the next section, the definitions of the first hitting time (FHT) and the MFHT are given, together with their relevance in the scientific literature. In Sec. III, the MFHT as an indicator of financial stability is proposed and the main results of the paper, obtained by an empirical analysis, are shown. Specifically, the nonmonotonic behaviors of the MFHT for crashes and rallies are presented. A nonlinear generalization of the Heston model is proposed in Sec. IV, and some of the well-established statistical characteristics of the financial time series are shown in Sec.V. Conclusions are drawn in the last section.

II. FIRST HITTING TIME

The *first hitting time* (FHT), or first passage time (FPT), is defined as the random time it takes for a stochastic variable to cross for the first time a certain level. In Fig. 1, we show the schematic representation of the FHT calculated starting from the time series of returns. Specifically, the FHT is the time it takes for a stock price return to cross for the first time a large negative or positive threshold Θ_f starting from an initial position Θ_i. By ensemble averaging on all the time series of the market, we obtain the proposed indicator of price return stability, that is, the MFHT. The MFHT, or MFPT, was earlier introduced in scientific literature. Indeed, the study of the first passage time and exit problem has a long-standing tradition in physics, mathematics, engineering, and natural sciences [33–37]. The first pioneering papers in this subject are those by Smoluchowski, who first considered the problem of a random walk with reflecting and absorbing barriers [33]; Pontryagin *et al.*, who first derived the differential equation for the mean first passage time [34]; Kramers, with his celebrated paper on "Brownian motion in a field of force and the diffusion model of

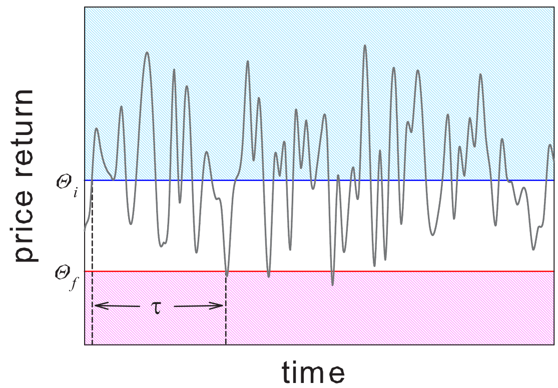

FIG. 1. Time series of returns and the corresponding first hitting time, that is, the time it takes for a stock price return to cross for the first time a large negative threshold Θ_f starting from an initial position Θ_i. By ensemble averaging on all the time series of the market, we obtain the proposed indicator of price return stability.

chemical reactions" [35], who understood well the mechanism of the escape process as a noise-assisted reaction; Chandrasekhar, who considered the importance of the occurrence of the escape problem in astronomical phenomena [36]; and Feller, who gave his fundamental contribution in mathematical literature, with his proposal of two singular diffusion problems, called Feller processes, and the related boundary problem appearing in these diffusion processes [37]. Indeed, the fact that the Feller process never attains negative values has made it an ideal candidate for modeling many natural and social science phenomena.

In finance, the concept of FPT appears in several domains: the valuation of barrier options, credit risk modeling, and optimal exercise time of American options. Even more, the study of the MFPT for two well-known mean-reverting processes, that is, the square root process of Feller and the generalized autoregressive conditional heteroskedasticity (GARCH) diffusion process, was recently given in Refs. [38,39].

III. MFHT AS AN INDICATOR OF FINANCIAL STABILITY

In this paper, in order to perform our empirical analysis and calculate the MFHT from the time series of returns, we have relied on a well-tested data set already employed in previous investigations by Refs. [40–44]. This includes 1071 stocks traded at the NYSE and continuously recorded for 12 years from 1987 to 1998 (3030 trading days). This period of trading is sufficiently long and inclusive of a large number of stocks, so that it can be considered representative of overall market behavior. In order to investigate episodes of instability associated with large negative or positive return variations, following the literature on speculative pressure in the exchange market, we identify price changes as "sizable" if they are larger than a certain threshold, typically defined starting from the standard deviation [45,46]. In line with this literature, the robustness of the identification mechanism is also assessed by considering different thresholds in the range of 1.5–3 times the standard deviation.

In detail, we first transform the series of stock prices into daily returns, $r(t) = [p(t) - p(t-1)]/p(t-1)$, and calculate the standard deviation σ_i^r of each series over the entire period. By averaging this quantity across all stocks in the sample, we obtain the overall volatility of the market over the observed period as $\bar{\sigma}^r = \sum_{i=1}^{N} \sigma_i^r / N = 0.022\,54$, with $N = 1071$. We then proceed to compute the MFHT by considering the FHT, and ensemble averaging over all FHTs measured in the price return series. This random quantity has been calculated by fixing the initial and final threshold as $\Theta_i = \theta_i \bar{\sigma}^r$ and $\Theta_f = \theta_f \bar{\sigma}^r$, where $\bar{\sigma}^r$ is the overall volatility of the market over the observed period and the two thresholds are defined, in line with Refs. [45,46]. The parameters θ_i and θ_f define the "stability" window and, therefore, how large a variation has to be in order to determine an escape from a metastable state. To assess the robustness of the results, shown in Figs. 2 and 3, we consider a wide range of realistic numerical parameters for θ_i and θ_f in the intervals $[+0.9, -1.6]$ and $[-0.5, -3.0]$, respectively, in order to identify an episode of instability (see Refs. [45,46]).

This allows us to obtain several subseries, each corresponding to one first hitting time. The standard deviation of each subseries gives the value of volatility v, corresponding to each FHT. Averaging all the FHTs corresponding to the same volatility value yields the nonmonotonic behavior of the MFHT versus the volatility shown in Fig. 2(a) (blue circles), where the values of the threshold parameters are $\theta_i = -0.1$ and $\theta_f = -1.5$. In particular, we note that the MFHT takes smaller values for lower levels of volatility, i.e., the series of returns exhibits negative jumps equal to or less than $-1.5\bar{\sigma}^r$, after short time intervals. This corresponds to a fast exit of the stock return from the fixed region $[\Theta_i, \Theta_f]$. As volatility increases, the time spent within this region also increases, which is a signal that the market is becoming more stable. A further increase of volatility, however, shortens the MFHT, and stability decreases. This implies that in the intermediate region we observe a *stabilizing effect of volatility*. Figure 2(a) is a clear representation of the relation between MFHT, i.e., the time returns stay within the fixed region, and the size of volatility v. Considering the MFHT as a measure of market stability, it is possible to argue that volatility plays a stabilizing effect when its values are within the range [0.004,0.01].

In order to cross validate the robustness of this result, we have also investigated whether this effect persists for (i) different thresholds with fixed interval size, $\Theta_f - \Theta_i = -1.4\bar{\sigma}^r$ [Fig. 2(b)], and (ii) fixed starting threshold Θ_i but different final thresholds Θ_f [Fig. 2(c)]. The results indicate that the nonmonotonic behavior of the MFHT as a function of volatility is "robust" to sizable variations of the two thresholds.

We perform a similar analysis for stock price upturns or rallies. We find that the nonmonotonic behavior, with a maximum, of the MFHT versus volatility occurs both in real market data and in simulations based on the proposed nonlinear Heston model [Eqs. (5) and (6)]. This means that we can extend our proposed measure of price return stability not only to negative variations of price returns but also to positive variations. The results are shown in Fig. 3. Again, we cross validate the results of Fig. 3(a) by considering different thresholds with a fixed interval size [Fig. 3(b)], and a fixed starting threshold with different final thresholds [Fig. 3(c)]. We note that, differently from the case of crashes, for a fixed difference between thresholds the curves of the MFHT versus volatility coalesce [see Fig. 3(b)]. This could be ascribed to

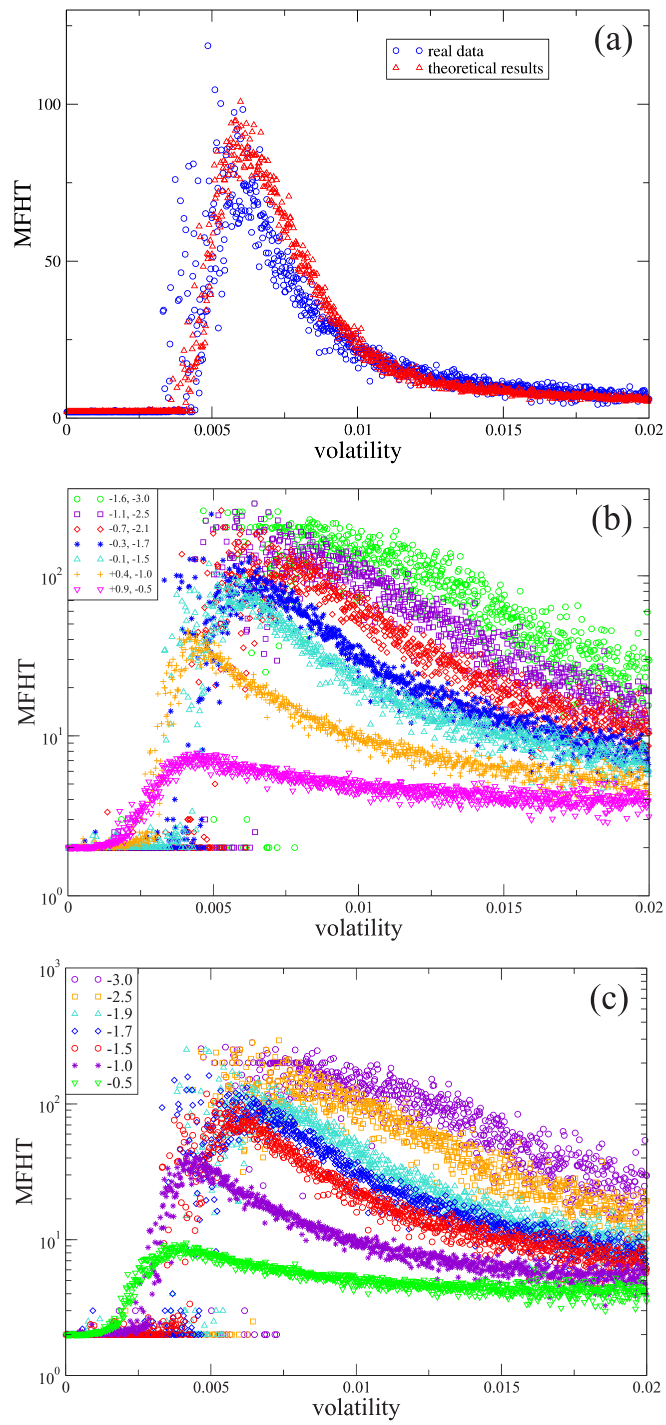

FIG. 2. Crashes: (a) MFHT as a function of volatility, with the thresholds $\Theta_i = -0.1\bar{\sigma}^r$ and $\Theta_f = -1.5\bar{\sigma}^r$, from empirical time series (blue circles) and theoretical results (red triangles), obtained from a nonlinear Heston model [Eqs. (5) and (6)]. (b) MFHT vs volatility (real data) for a fixed difference between thresholds, $\Theta_f - \Theta_i = -1.4\bar{\sigma}^r$, with $[\Theta_i,\Theta_f]$ ranging from $[+0.9\,\bar{\sigma}^r, -0.5\,\bar{\sigma}^r]$ to $[-1.6\,\bar{\sigma}^r, -3.0\,\bar{\sigma}^r]$. (c) MFHT vs volatility (real data) for a fixed starting threshold $\Theta_i = -0.1\bar{\sigma}^r$ and a different final threshold Θ_f, ranging from $-0.5\,\bar{\sigma}^r$ to $-3.0\,\bar{\sigma}^r$.

the asymmetry of the returns distribution, characterized by a negative skewness (see Fig. 5).

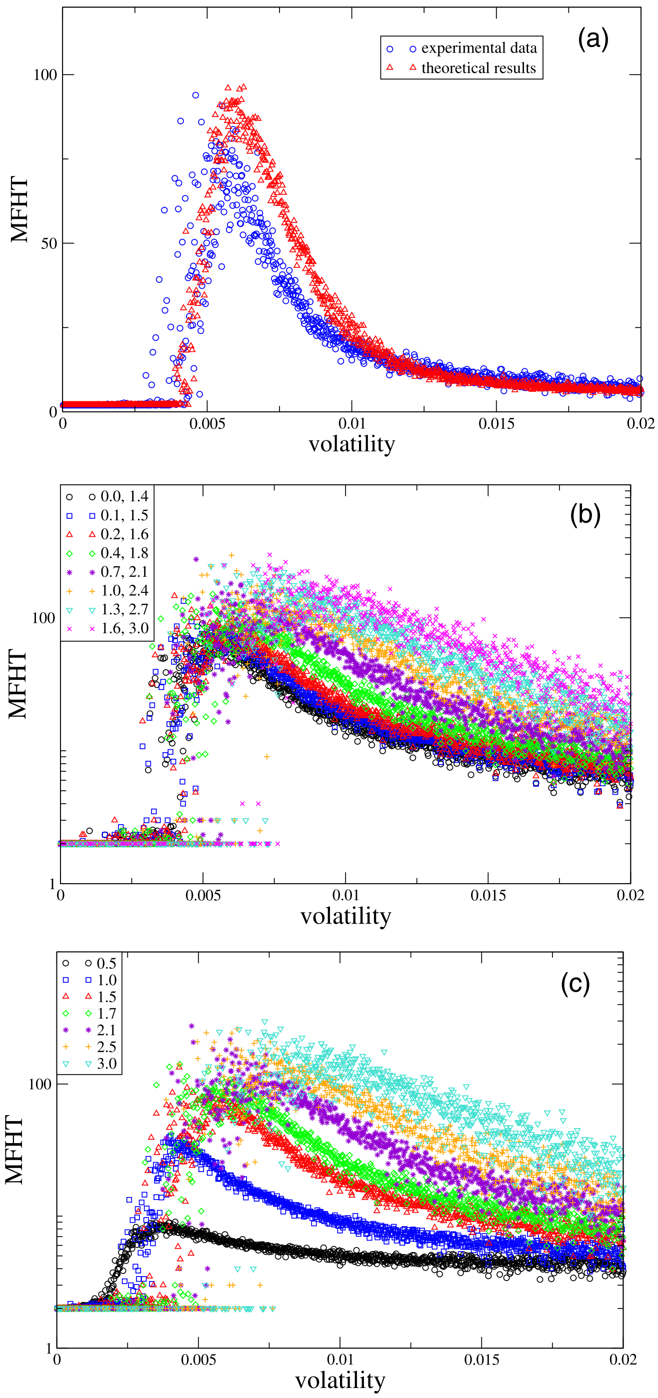

FIG. 3. Rallies: (a) MFHT as a function of volatility, with the thresholds $\Theta_i = +0.1\bar{\sigma}^r$ and $\Theta_f = +1.5\bar{\sigma}^r$, from empirical time series (blue circles) and theoretical results (red triangles), obtained from a nonlinear Heston model. (b) MFHT vs volatility (real data) for a fixed difference between thresholds, $\Theta_f - \Theta_i = +1.4\bar{\sigma}^r$, with $[\Theta_i, \Theta_f]$ ranging from $[-0.9\,\bar{\sigma}^r, +0.5\,\bar{\sigma}^r]$ to $[+1.6\,\bar{\sigma}^r, +3.0\,\bar{\sigma}^r]$. (c) MFHT vs volatility (real data) for a fixed starting threshold $\Theta_i = +0.1\bar{\sigma}^r$ and different final thresholds Θ_f, ranging from $+0.5\,\bar{\sigma}^r$ to $+3.0\,\bar{\sigma}^r$.

The nonmonotonic behavior observed in Figs. 2 and 3 is similar to what is known to occur also in all physical systems with metastable states. Indeed, the behavior of the MFHT as a function of volatility shows the typical signature of noise-enhanced stability (NES) observed in a variety of physical (classical and quantum), biological, chemical, and ecological systems [47–75]: The stability of a metastable state can be enhanced by the noise and its average lifetime is a measure of this stability. This noise-enhanced metastability is a consequence of the interplay between the thermal fluctuations and nonlinearity of the complex system investigated. This effect is observed by increasing the temperature as, by analogy, the stabilization of the price returns occurs for increasing volatility. The empirical evidence of a NES effect in the behavior of the MFHT (Figs. 2 and 3) suggests that price return dynamics could be depicted by considering the value of the return as the position of a fictitious *Brownian particle*, subjected to noise and moving in an effective potential with a metastable state.

IV. MODEL

Models reproducing most of the *stylized facts* of financial markets and their dynamics by nonlinear stochastic differential equations have been presented in the literature [5,23,24,76–79]. Moreover, financial markets present different dynamical regimes with days of normal activity and days with large price variations, characterized by a different behavior of volatility. In order to consider these different dynamical regimes and feedback effects on the price fluctuations, a Langevin approach to the market dynamics was already proposed in Refs. [5,23,24,78,79], where a nonlinear stochastic dynamical equation with one or more metastable states was considered. The evolution inside the metastable state represents the normal market behavior, while the escape from the metastable state represents the beginning of large price variations, which is related to the "clustering" phenomenon of volatility [23,24].

Before introducing our nonlinear model in Sec. IV B, based on a nonlinear generalization of the Heston model, we briefly introduce the Heston model in Sec. IV A.

A. Heston model

One of the most widely used stochastic volatility models, being broadly accepted as a reasonable explanation for many empirical observations, the so-called *stylized facts*, is the Heston model [31,80–82], which is based on two-dimensional diffusion processes. The model, introduced by Heston in 1993 [80], is a closed-form solution for pricing options that seeks to overcome the shortcomings in the Black-Scholes option pricing model related to return skewness and strike-price bias. This model introduces a dynamics for the underlying asset which can take into account the asymmetry and excess kurtosis that are typically observed in financial assets returns.

The Heston model [31,39,80,83], which describes the dynamics of stock prices $p(t)$ as a geometric Brownian motion with the volatility given by a mean-reverting process, known as the Cox, Ingersoll, and Ross (CIR) process [30,39,84,85], is defined by the following Itô stochastic differential equations,

$$dp(t) = \mu p dt + \sigma(t) p dW_1(t), \tag{1}$$

$$dv(t) = a\,(b - v(t))dt + c\sqrt{v(t)}dW_2(t), \tag{2}$$

where $\sigma(t)$ is the time-dependent volatility, $v(t) = \sigma^2(t)$, and $W_i(t)$ are uncorrelated Wiener processes with the usual statistical properties,

$$\langle dW_i \rangle = 0, \quad \langle dW_i(t) dW_j(t') \rangle = dt\, \delta(t - t') \delta_{i,j}. \qquad (3)$$

The CIR process, known in mathematical statistics as the Feller process [37], and later introduced in mathematical finance [84], represents the term structure of interest rates and it successfully evaluates bond prices [84,85]. Moreover, the Feller process also appears to describe the default intensity rate [86], and the growth stock [87]. The process defined as its square root is connected to the square of a δ-dimensional Bessel process [88]. Stochastic volatility obeying the Feller model jointly with a log-Brownian stochastic dynamics for the asset price evolution gives rise to the two-dimensional diffusion market process of Eqs. (1) and (2) [80,83], which is a useful model for option pricing [39,80,85]. Recently, a study of the mean first passage time (or mean first hitting time) for two well-known mean-reverting processes, that is, the square root process of Feller and the GARCH diffusion process, was done in Refs. [38,39]. Specifically, in Ref. [38], the asymptotic expansions of the MFPT around the starting position and the boundary points of GARCH and Feller processes as well as the sensitivity analysis of MFPT to changes in the relevant parameters were investigated. In Ref. [39], the first passage and escape problems for the Feller process have been fully addressed.

In Eq. (1), μ represents a drift at macroeconomic scales. In Eq. (2), the volatility $\sigma(t) = \sqrt{v(t)}$ reverts towards a macroeconomic long time term given by the mean squared value b, with a relaxation time a^{-1}. Here, c is the amplitude of volatility fluctuations often called the *volatility of volatility*.

By introducing log returns $x(t) = \ln[p(t)/p(0)]$ in a time window $[0,t]$ and using Itô's formula [89], we obtain the stochastic differential equation (SDE) for $x(t)$,

$$dx(t) = [\mu - v(t)/2]dt + \sqrt{v(t)} dW_1(t). \qquad (4)$$

We note that the Heston model gives a good reproduction of the price returns probability density function (PDF), but does not reproduce a long-range volatility correlation [40]. On the contrary, the GARCH model provides a basic way to model the volatility correlation, by modeling the high memory of the volatility [90], but gives a rather poor fitting of the return PDF (see Refs. [40,41,90]). Moreover, the statistical properties of the returns and FPTs for models with stochastic volatility, such as the Heston and the discrete GARCH (1,1) model, have been investigated in Refs. [40,41], finding that the PDF of both stock price returns and FPTs obtained with the Heston model exhibit a better agreement with real market data than those calculated in the GARCH discrete model [91].

B. Nonlinear Heston model

Here, we employ a generalization of the Heston model [80] proposed by Ref. [30], where the geometric Brownian motion is replaced by a random walk in the presence of a cubic nonlinearity. This theoretical approach considers the financial market as an out-of-equilibrium system, whose dynamical evolution can be described by a nonlinear Heston model defined by the following Itô stochastic differential equations [89],

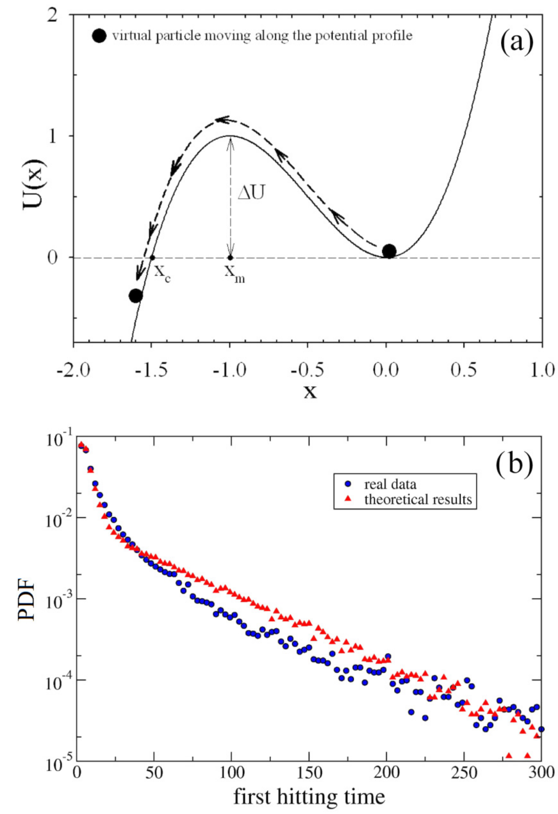

FIG. 4. (a) Cubic potential used in the dynamic equation for the variable $x(t)$. The black circle denotes the starting position ($x_0 = 0.0$) used to obtain the theoretical results. The potential parameters are $m = 2$ and $n = 3$. (b) PDF of the first hitting times of the returns for real data (blue circles) and the model (red triangles).

$$dx(t) = -\left(\frac{\partial U}{\partial x} + \frac{v(t)}{2}\right)dt + \sqrt{v(t)}\, dW_1(t), \qquad (5)$$

$$dv(t) = a[b - v(t)]\, dt + c\sqrt{v(t)}\, dW_2(t), \qquad (6)$$

with the volatility $v(t)$ given by the mean-reverting CIR process [37,83–85], and $U(x) = mx^3 + nx^2$ is the effective cubic potential with a metastable state [Fig. 4(a)] [79]. As explained in Ref. [79], the parameters m and n, identified in line with the consideration of liquid markets, are influenced by the degree of risk aversion, the market depth, and the "friction" of prices to changes in demand and supply in the market. In Eq. (5), $x(t) = \ln[p(t)/p(0)]$ is the return in the time window $[0,t]$, $p(t)$ is the price, and W_i are uncorrelated Wiener processes with the usual statistical properties $\langle dW_i(t) \rangle = 0$, $\langle dW_i(t)\, dW_j(t') \rangle = dt\, \delta_{i,j} \delta(t - t')$.

We note that the particle is governed by a nonstationary dynamics since it is subject to a noise source $dW_1(t)$, whose intensity is given by the volatility $v(t)$, which is itself a stochastic process [see Eq. (6)]. In addition, due to the presence of this noise source, the particle can leave the metastable well also for low volatility, crossing the potential barrier and moving to nonequilibrium positions along the potential profile.

For the daily returns we have

$$x_d(t) = \ln[p(t)/p(0)] - \ln[p(t-1)/p(0)]$$

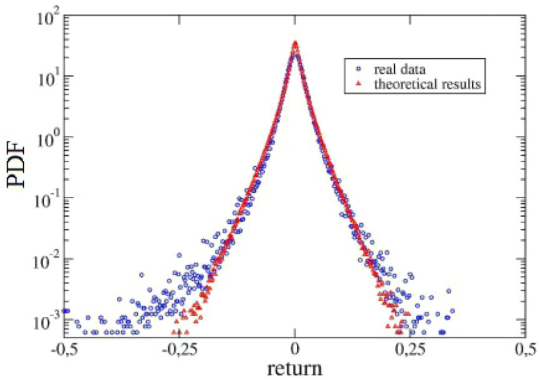

FIG. 5. PDF of the stock price returns for real data (blue circles) and the model (red triangles).

$$= \ln[p(t)] - \ln[p(t-1)]$$
$$\simeq [p(t) - p(t-1)]/[p(t-1)] = r(t), \qquad (7)$$

where we use $\ln x \simeq x - 1$.

We solve Eqs. (5) and (6) numerically, obtaining a number of time series of returns equal to 1071, with an initial position $x_0 = 0.0$ and CIR stochastic process $v(t)$ defined by $v_{\rm start} = 8.62 \times 10^{-5}$, $a = 2.00$, $b = 0.01$, and $c = 0.83$. These values, used to obtain the data shown in Figs. 2–6, were obtained by best fitting between theoretical and empirical results for all the statistical features investigated, by performing both χ^2 and Kolmogorov-Smirnov (KS) goodness-of-fit tests. Since we are focusing on the daily returns we have $x(t) \simeq r(t)$. We fix again the two thresholds, $\Theta_i = \mp 0.1\bar{\sigma}^r$ and $\Theta_f = \mp 1.5\bar{\sigma}^r$ ($-$ for microcrash, $+$ for rally), where $\bar{\sigma}^r = 0.023\,83$ is the average standard deviation calculated over the numerical time series. This yields, for the MFHT, the nonmonotonic behavior shown in Figs. 2(a) and 3(a) (red triangles), which exhibits a very close agreement with the real data (blue circles).

To quantitatively characterize the observed empirical results, we determine the probability distribution function (PDF) of the FHTs of the daily returns, calculated by setting $\Theta_i = -0.1\bar{\sigma}^r$ and $\Theta_f = -1.5\bar{\sigma}^r$, and compare it with the corresponding theoretical PDF, obtaining a good qualitative agreement [Fig. 4(b)]. Performing both χ^2 and KS goodness-of-fit tests, we get $\chi^2 = 0.01668$, $\tilde{\chi}^2 = 0.00018$ (reduced χ^2), and $D = 0.149$, $P = 0.198$. D and P are respectively the maximum difference between the cumulative distributions and the corresponding probability for the KS test. The results indicate that the two distributions are not significantly different [see Fig. 4(b)].

Finally, we note that in Ref. [30] the dynamical regimes corresponding to different parameter values of a, b, and c of the CIR process were studied and the parameter region was found to observe a nonmonotonic behavior of the MFHT.

V. STATISTICAL CHARACTERISTICS

Some of the well-established statistical properties of the financial time series are the probability distribution of stock price returns, the PDF of volatility, the return correlation, and the absolute return correlation. In Fig. 5 we show the PDF of the stock price returns for real data (blue circles) and the model (red triangles). The agreement between theoretical results and real data of the PDFs of returns is quite good, except at high values of the returns. This can be ascribed to the failure of the proposed nonlinear model for returns higher than or comparable to the height of the metastable state barrier. For these values of returns, other mechanisms, which we have not taken into account in the model, come into play [30,79].

To quantitatively characterize the PDF of returns (Fig. 5) with respect to their average, width, asymmetry, and fatness, we consider the whole set of N_T values of the daily returns and calculate, both for real data and theoretical results, the four moments of the PDF, that is, the mean value $\langle r \rangle$, the variance σ_r, the skewness κ_3, and the kurtosis κ_4, obtaining the following values: $\langle r \rangle^{\rm expt} = -1.91 \times 10^{-5}$, $\sigma_r^{\rm expt} = 0.025$, $\kappa_3^{\rm expt} = -4.30$, and $\kappa_4^{\rm expt} = 442$, and $\langle r \rangle^{\rm theor} = -4.76 \times 10^{-5}$, $\sigma_r^{\rm theor} = 0.024$, $\kappa_3^{\rm theor} = -1.96$, and $\kappa_4^{\rm theor} = 105$.

The quantitative statistical characterization of the shape of the PDF of returns (Fig. 5) shows that the model reproduces the asymmetry and leptokurtic distribution observed for the real market data [4,5]. In Fig. 6(a) we show the PDF of the volatility both for real market data and theoretical results, finding a log-normal behavior in both cases. The agreement is quite good, as confirmed by the Kolmorogov-Smirnov test: $D = 0.2178$ and $P = 0.014$. The values of the volatility shown in Fig. 6(a) are those corresponding to the hitting time events observed for $\Theta_i = -0.1\bar{\sigma}^r$ and $\Theta_f = -1.5\bar{\sigma}^r$ (see Fig. 2).

Another important characteristic, emerging from the statistical analysis of price variations in various types of financial markets [78], is the absence of return autocorrelation. This property also ensures a no-arbitrage condition. To verify that our model fulfills this requirement, we calculate the autocorrelations of the asset returns, and compare them with those obtained from the real data. The results are shown in Fig. 6(b). The autocorrelations from the model [red triangles in Fig. 6(b)] are insignificant except for very short times where microstructure effects possibly come into play. This result is again in close agreement with the autocorrelations calculated for the real data [blue circles in Fig. 6(b)]. A similar behavior, but with a slow decay to zero, is displayed by the correlation function of the absolute returns [see the inset of Fig. 6(b)].

Finally, we note that the "clustering" phenomenon of volatility is important for understanding the instabilities in price returns. We had clear evidence of this phenomenon looking at the time series of the returns used in our empirical analysis. Moreover, the contemporaneous presence, in the time series of returns, of *clustering* and *spikes* of volatility gives rise to the nonmonotonic behavior observed in Figs. 2 and 3. Specifically, the simultaneous presence of two neighboring spikes is correlated with the presence of low MFHTs at low volatility, while a spike close to a cluster is related to low MFHTs at high volatility values. Pairs of *clusters* and/or *spikes* spaced from a nearly laminar or "tranquil" regime give rise to an increase of the MFHT (intermediate region of volatility values) with the presence of a maximum. This gives rise to the observed nonmonotonic behavior of the MFHT vs volatility.

We note that this clustering phenomenon is also observed in the time series analysis of the seismic activity of earthquakes [92,93]. Indeed, the clustering in these time series

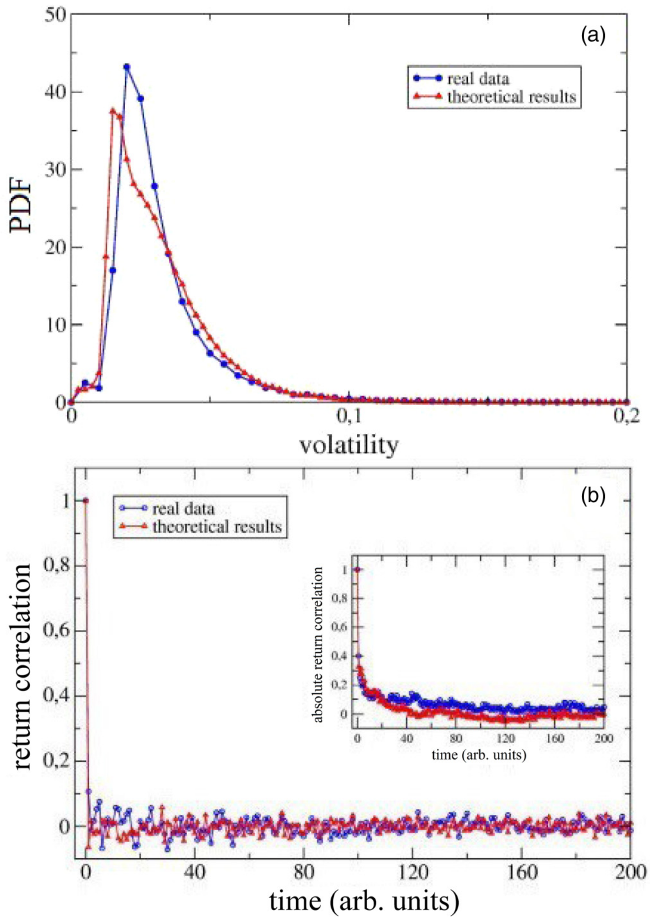

FIG. 6. (a) Probability distribution of the volatility for real data (blue circles) and the model (red triangles). We consider the whole set of price returns consisting of $N_T = 3030 \times 1071 = 3\,245\,130$ values and calculate, both for real data and theoretical results, the related PDFs. (b) Correlation function of the returns for real data and the model. Inset: Correlation function of the absolute returns for real data and the model. The values of the parameters are the same as in Fig. 3.

shows that earthquakes represent a dynamical process involving many spatiotemporal scales. However, the analysis of the similarity between the behavior of stock returns around microcrashes and rallies and the dynamics of earthquake sequences would clearly require a deeper investigation and could be a potentially interesting extension for future work.

VI. CONCLUSIONS

In summary, we have proposed using the MFHT as an indicator of price return stability and looking at its relationship with return volatility. In an empirical analysis carried out on stocks traded in the New York Stock Exchange, the time series of daily returns show limited fluctuations, that is, high stability, when volatility increases. In particular, there is an intermediate range of volatility values where price returns show higher stability according to the proposed indicator, with a maximum in the nonmonotonic behavior of the MFHT versus the volatility. In addition, the proposed measure of price return stability is applicable and observed not only for negative variations of price returns (crashes) but also for positive variations (rallies).

Moreover, our nonlinear Heston model appears to satisfy some of the well-established properties of financial markets and is able to reproduce the statistical properties of the hitting times of daily returns in real stocks. The model is also able to describe the dynamics of price returns by considering an analogy between the metastability in the market and that occurring in a variety of physical and complex systems [22,30], [47–75]. Our findings show that lower stability (smaller mean first hitting times) can be the result not only of large volatility, as it would be expected during periods of market "turbulence" [27], but also of small volatility, which is usually considered an indicator of "tranquil" periods. This result could bear important implications both for practitioners and policymakers responsible for market stability. Further, the proposed measure can be considered as an additional useful indicator to monitor market stability and to support *risk control* functions.

It is worth mentioning that the clustering phenomenon of volatility is important for understanding the instabilities in price returns and the nonmonotonic behavior of the MFHT versus volatility. In fact, the contemporaneous presence, in the time series of returns, of pairs of clustering and/or spikes spaced from a nearly laminar or tranquil regime gives rise to a nonmonotonic increase of the MFHT, with a presence of a maximum, in the intermediate region of volatility values.

While this study has considered a sufficiently long time span and number of stocks to make our results sufficiently general, for the benefit of practice and policy, future extensions could consider more recent data or even a comparative analysis across different markets.

Finally, we note that the applications of our definition of stability based on the concept of first hitting time can help to quantitatively characterize the resilience of different complex systems, both in physics and biology (such as in neuronal activity and population dynamics), to variations of a given feature.

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