Sinc noise for the Kardar-Parisi-Zhang equation

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In this paper we study the one-dimensional Kardar-Parisi-Zhang (KPZ) equation with correlated noise by field-theoretic dynamic renormalization-group techniques. We focus on spatially correlated noise where the correlations are characterized by a sinc profile in Fourier space with a certain correlation length ξ. The influence of this correlation length on the dynamics of the KPZ equation is analyzed. It is found that its large-scale behavior is controlled by the standard KPZ fixed point, i.e., in this limit the KPZ system forced by sinc noise with arbitrarily large but finite correlation length ξ behaves as if it were excited by pure white noise. A similar result has been found by Mathey *et al.* [S. Mathey *et al.*, Phys. Rev. E **95**, 032117 (2017)] for a spatial noise correlation of Gaussian type ($\sim e^{-x^2/2\xi^2}$), using a different method. These two findings together suggest that the KPZ dynamics is universal with respect to the exact noise structure, provided the noise correlation length ξ is finite.

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I. INTRODUCTION

The standard form of the Kardar-Parisi-Zhang (KPZ) equation introduced for modeling nonlinear growth processes reads [1]

$$\frac{\partial h(x,t)}{\partial t} = \nu\nabla^2 h(x,t) + \frac{\lambda}{2}[\nabla h(x,t)]^2 + \eta(x,t), \quad (1)$$

where $h(x,t)$ is a scalar height field (with x and t as space and time coordinates, respectively), ν is a surface tension parameter, and λ is a nonlinear coupling constant. Here $\eta(x,t)$ denotes an uncorrelated Gaussian noise with zero mean (white noise in space and time), meaning that the first and second moments of the noise are given by

$$\begin{aligned} \langle \eta(x,t)\rangle &= 0, \\ \langle \eta(x,t)\eta(x',t')\rangle &= 2D\delta^d(x-x')\delta(t-t'), \end{aligned} \quad (2)$$

where D is a constant amplitude and d is the spatial dimension. Besides white noise, various types of spatially and temporally correlated driving forces have been studied over the years [2,3].

With respect to spatial correlations, a widely studied type of driving forces is power-law correlated noise with Fourier-space correlations $\sim q^{-2\rho}$ and $\rho > 0$ as a free parameter [2,4,5]. The intriguing observation here is the emergence of a different noise fixed point for $\rho > 1/4$, in addition to the standard Gaussian and KPZ fixed points.

Recently the KPZ equation with spatially colored and temporally white noise decaying as $\sim e^{-x^2/2\xi^2}$ was studied by a nonperturbative dynamic renormalization-group (DRG) analysis in [6]. It was found that for small values of ξ the KPZ equation behaves in the large-scale limit as if it were driven by white noise, i.e., by a driving force with vanishing correlation length. Since the nonperturbative renormalization-group (RG) equations are difficult to solve analytically, the authors of [6] relied on numerical techniques.

In the present paper we study the case of spatially correlated noise, where the correlations are characterized by a sinclike profile in Fourier space. In contrast to [6], we solve the problem analytically using field-theoretic renormalization-group techniques. Our aim is to provide a complementary study of universality in the KPZ dynamics for finite correlation lengths ξ.

As already pointed out in [6], the assumption of ξ being finite is reasonable in many experimental settings and, on the other hand, we would expect that universal properties do not change if the system is scale invariant on large scales. In fact, Refs. [7–9] found evidence of universal behavior in the case of (1+1)-dimensional directed polymers based on scaling arguments and numerical techniques.

There are mainly two reasons why we focus on a driving noise with a sinc profile. First, in one spatial dimension this type of noise has a very intuitive real-space equivalent, namely, the rectangle function. Second, in Fourier space the noise correlations are mathematically well behaving and thus we expect to be able to solve the perturbation integrals analytically.

Of course, a sinc profile is not the only possible choice. In order to analytically solve the integrals, other types of driving forces may be possible as well; for instance, noise correlations following a Lorentzian profile also work.

The paper is organized as follows. For treating sinc-type noise, we first generalize the field-theoretic DRG formalism for the KPZ equation in such a way that we can handle homogeneous and isotropic noise distributions, whose correlations in momentum space are given by

$$\langle \eta(q,\omega)\eta(q',\omega')\rangle = 2D(|q|^2)\delta^d(q+q')\delta(\omega+\omega'). \quad (3)$$

Note that D does not depend on the frequency, i.e., the noise is spatially colored but temporally white.

For this class of noise correlations, which includes the power law ($\sim q^{-2\rho}$) and Gaussian ($\sim e^{-\xi^2 q^2/2}$) and sinc-type correlations, we set up the field-theoretic DRG formalism in the next section. With the theoretical framework laid out in Sec. II, the explicit sinc-noise excitation will be analyzed in Sec. III. In Sec. IV the results obtained in Sec. III will be discussed. Technical details are given in the Appendix.

II. GENERALIZED FIELD-THEORETIC RENORMALIZATION-GROUP PROCEDURE

A useful tool for building a field theory for stochastic differential equations of type (1) is the effective action $\mathcal{A}[\tilde{h},h]$, known as the Janssen–De Dominicis response functional [10,11]. Here the action depends on the original height field $h(x,t)$ and the Martin-Siggia-Rose response field $\tilde{h}(x,t)$.

To derive the effective action, it is useful to transform Eq. (3) into real space

$$\langle\eta(x,t)\eta(x',t')\rangle = 2D(x-x')\delta(t-t'), \tag{4}$$

where $D(x) = \mathcal{F}T^{-1}\{D(|q|^2)\}$ [4,5]. Using the abbreviations $\underline{y} = (x,t)$ and $\int_{\underline{y}} = \int d^dx \int dt$, the corresponding Gaussian noise probability distribution can be written as [12]

$$\mathcal{W}[\eta] \propto \exp\left[-\frac{1}{2}\int_{\underline{y}}\int_{\underline{y}'} \eta(\underline{y})\mathcal{M}(\underline{y};\underline{y}')\eta(\underline{y}')\right], \tag{5}$$

where $\mathcal{M}(x,t;x',t')$ is the inverse of the covariance operator

$$\mathcal{M}^{-1}(x,t;x',t') = \langle\eta(x,t)\eta(x',t')\rangle$$

given in (4), i.e.,

$$\int d^dx' \int dt' \mathcal{M}(x,t;x',t')\mathcal{M}^{-1}(y,\tau;x',t') = \delta^d(x-y)\delta(t-\tau). \tag{6}$$

Using (5) and following Refs. [13–15], the expectation value of any observable $\mathcal{O}[h]$ can be written as

$$\langle\mathcal{O}[h]\rangle = \int \mathcal{D}[h]\int \mathcal{D}[i\tilde{h}]\mathcal{O}[h]\exp\left[\int d^dx \int dt\, \tilde{h}(x,t)\Big(\partial_t h(x,t) - \nu\nabla^2 h(x,t) - \frac{\lambda}{2}[\nabla h(x,t)]^2\Big)\right]$$
$$\times \int \mathcal{D}[\eta]\exp\left[-\frac{1}{2}\int d^dx\int dt\Big(\int d^dx'\int dt' \eta(x,t)\mathcal{M}(x,t;x',t')\eta(x',t') - \tilde{h}(x,t)\eta(x,t)\Big)\right]. \tag{7}$$

Integrating out the noise, Eq. (7) can be rewritten in the form [4,5,10,11,16]

$$\langle\mathcal{O}[h]\rangle \propto \int \mathcal{D}[h]\mathcal{O}[h]\mathcal{P}[h] = \int \mathcal{D}[h]\mathcal{O}[h]\int \mathcal{D}[i\tilde{h}]e^{-\mathcal{A}[\tilde{h},h]}, \tag{8}$$

with the Janssen–De Dominicis functional [12,15,17,18]

$$\mathcal{A}[\tilde{h}(x,t),h(x,t)] = \int_{\underline{y}}\bigg\{\tilde{h}(\underline{y})\Big(\frac{\partial h(\underline{y})}{\partial t} - \nu\nabla^2 h(\underline{y}) - \frac{\lambda}{2}[\nabla h(\underline{y})]^2\Big) - \int d^dx'\tilde{h}(x,t)D(x-x')\tilde{h}(x',t)\bigg\}. \tag{9}$$

With this functional, one can carry out the usual field-theoretic perturbation expansion in λ (see, e.g., [17,19–21]).

Field-theoretic calculations can be simplified by exploiting the symmetries of the problem. In the present case the KPZ equation is known to be invariant under tilts (Galilei transformation) of the form [4,19]

$$\begin{aligned} h(x,t) &\to h'(x,t) = h(x+\alpha\lambda t,t) + \alpha\cdot x,\\ \tilde{h}(x,t) &\to \tilde{h}'(x,t) = \tilde{h}(x+\alpha\lambda t,t), \end{aligned} \tag{10}$$

where α is the tilting angle. This symmetry gives rise to two Ward-Takahashi identities. For this reason the KPZ equation has only two independent RG entities, namely, the amplitude of the noise correlation $D(x-x')$ and the surface tension ν [4,19]. These are renormalized by

$$D_R = Z_D D, \quad \nu_R = Z_\nu \nu, \tag{11}$$

where the multiplicative RG factors Z_ν and Z_D compensate for logarithmic UV divergences occurring in the perturbation integrals. The RG parameters are related to the vertex functions $\Gamma_{\tilde{h}h}$ and $\Gamma_{\tilde{h}\tilde{h}}$, which are given by functional derivatives

$$\Gamma_{\tilde{h}h} = \frac{\delta}{\delta\tilde{h}}\frac{\delta}{\delta h}\Gamma[\tilde{h},h,\tilde{j},j]\Big|_{\tilde{j}=0=j}, \tag{12}$$

$$\Gamma_{\tilde{h}\tilde{h}} = \frac{\delta^2}{\delta\tilde{h}\delta\tilde{h}}\Gamma[\tilde{h},h,\tilde{j},j]\Big|_{\tilde{j}=0=j} \tag{13}$$

of the generating functional $\Gamma[\tilde{h},h,\tilde{j},j]$, which is the Legendre transformation of the free energy $-\ln\mathcal{F}$ (see, e.g., [19,21–24]),

$$\Gamma[\tilde{h},h,\tilde{j},j] = -\ln\mathcal{F}[\tilde{j},j] + \int d^dx\int dt(\tilde{h}\tilde{j} + hj). \tag{14}$$

Here $\tilde{h} = \delta\ln\mathcal{F}[\tilde{j},j]/\delta\tilde{j}$ and $h = \delta\ln\mathcal{F}[\tilde{j},j]/\delta j$ are the respective new variables after the Legendre transformation and $\tilde{j}$ and j are artificial source fields (see [17,19,21]). In what follows the vertex functions $\Gamma_{\tilde{h}h}$ and $\Gamma_{\tilde{h}\tilde{h}}$ will be calculated to one-loop order in Fourier space for an arbitrary noise with correlations of the form (3).

Denoting the free propagator by $G_0(\underline{k})$ [$\underline{k} = (q,\omega)$] and the self-energy by $\Sigma(\underline{k})$, the analytic expressions for the diagrams shown in Fig. 1 are given by the Dyson equation $\Gamma_{\tilde{h}h}(\underline{k}) = G_0(-\underline{k})^{-1} - \Sigma(\underline{k})$ [21–23] and the expansion of the noise vertex $\Gamma_{\tilde{h}\tilde{h}}(\underline{k}) = -2D(q) + \cdots$, where the ellipsis denotes higher-order terms. Using the usual Feynman rules (see, e.g., [17]) and integrating out the inner frequencies, one obtains

$$\Gamma_{\tilde{h}h}(\underline{k}) = i\omega + \nu q^2 + \frac{\lambda^2}{2\nu^2}\int_p \frac{D(|p-q/2|^2)(q^2/2 - q\cdot p)}{(q/2-p)^2} \times \frac{q^2/4 - p^2}{\frac{i\omega}{2\nu} + q^2/4 + p^2} + O(\lambda^3), \tag{15}$$

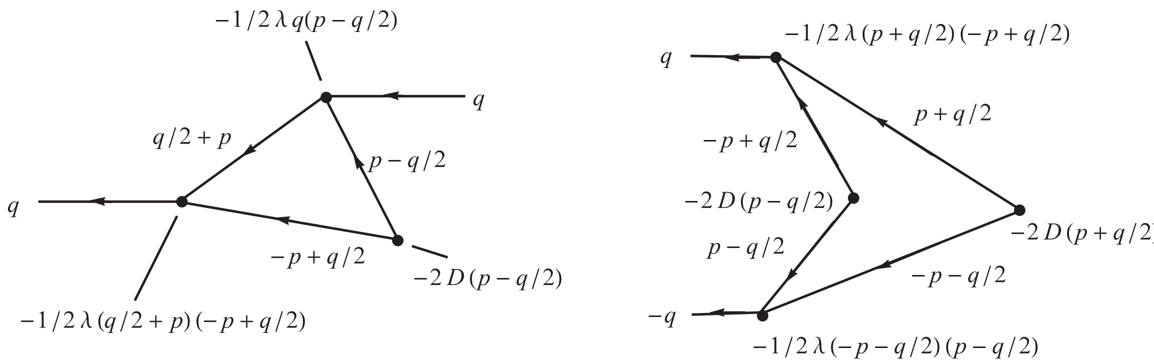

FIG. 1. Feynman diagrams for the KPZ equation in a style following [17]. The left-hand diagram shows the one-loop order expansion of the propagator vertex function $\Gamma_{\tilde{h}h}$, whereas the right-hand side depicts the one-loop order expansion of the noise vertex function $\Gamma_{\tilde{h}\tilde{h}}$. Here q denotes the outer momentum and p stands for the inner momentum. Note that a symmetrization has already been done, which leads to the noise amplitudes depending on $p \pm q/2$. For clarity the frequency components carried by each line corresponding to the different momenta are omitted.

$$\Gamma_{\tilde{h}\tilde{h}}(\underline{k}) = -2D(|q|^2) - \frac{\lambda^2}{2\nu^3}\int_p D(|p+q/2|^2)D(|p-q/2|^2) \times \Re\left[\frac{1}{\frac{i\omega}{2\nu}+q^2/4+p^2}\right] + O(\lambda^3), \tag{16}$$

where we used the abbreviation $\int_p = \frac{1}{(2\pi)^d}\int d^d p$. These integrals generalize those obtained, e.g., in [4,19] to arbitrary noise correlations of the form (3).

Evaluating (15) and (16), it is essential to avoid mixing ultraviolet and infrared divergences of the integrands. One way to keep those divergences separated is to introduce a so-called normalization point (NP). An indiscriminate yet very useful choice is given by [19]

$$\frac{\omega}{2\nu} = \mu^2, \quad q = 0, \tag{17}$$

where μ is an arbitrary momentum scale. One advantage of the choice in (17) is that the integrals (15) and (16) can be evaluated at $q = 0$ by expanding the general noise amplitude $D(|p \pm q/2|^2)$ about p for $|q| \ll 1$. Hence, to $O(|q|^2)$ the momentum-dependent noise amplitude reads

$$D\left(\left|p \pm \frac{q}{2}\right|^2\right) = D(|p|^2) \pm (p\cdot q)D'(|p|^2) + O(|q|^2). \tag{18}$$

Using the identities (d is the spatial dimension) [17,19]

$$\int_p (p\cdot q)^2 h(|p|,|q|) = \frac{q^2}{d}\int_p p^2 h(|p|,|q|),$$

$$\int_p p^2(p\cdot q)^2 h(|p|,|q|) = \frac{q^2}{d}\int_p p^4 h(|p|,|q|)$$

and inserting (18) into (15) implies, at the NP,

$$\left.\frac{\partial\Gamma_{\tilde{h}h}}{\partial q^2}\right|_{q=0} = \nu - \frac{\lambda^2}{4\nu^2}\frac{d-2}{d}\int_p \frac{D(|p|^2)}{i\mu^2+p^2} - \frac{\lambda^2}{2\nu^2}\frac{1}{d}\int_p \frac{p^2 D'(|p|^2)}{i\mu^2+p^2}. \tag{19}$$

The evaluation of (16) at the NP (17) leads, with (18), to

$$\Gamma_{\tilde{h}\tilde{h}} = -2D(|q|^2) - \frac{\lambda^2}{2\nu^3}\int_p [D(|p|^2)]^2\frac{p^2}{\mu^4+p^4}. \tag{20}$$

From (19) and (20) we obtain the renormalization factors

$$Z_\nu = 1 - \frac{\lambda^2}{4\nu^3}\frac{d-2}{d}\int_p \frac{D(|p|^2)}{i\mu^2+p^2} - \frac{\lambda^2}{2\nu^3}\frac{1}{d}\int_p \frac{p^2 D'(|p|^2)}{i\mu^2+p^2}, \tag{21}$$

$$Z_D = 1 + \frac{1}{D(|q|^2)|_{q=0}}\frac{\lambda^2}{4\nu^3}\int_p [D(|p|^2)]^2\frac{p^2}{\mu^4+p^4}. \tag{22}$$

These results allow us to compute the Wilson flow functions [4,17,19]

$$\gamma_D = \mu\frac{\partial}{\partial\mu}\ln Z_D, \tag{23}$$

$$\gamma_\nu = \mu\frac{\partial}{\partial\mu}\ln Z_\nu, \tag{24}$$

where the derivative is taken while keeping D and ν fixed. Likewise, the β function is given by

$$\beta_g = \mu\frac{\partial}{\partial\mu}g_R, \tag{25}$$

where

$$g_R = gZ_g\mu^{d-2} = gZ_D Z_\nu^{-3}\mu^{d-2} \sim \frac{\lambda^2 D}{4\nu^3}\mu^{d-2} \tag{26}$$

is a dimensionless effective coupling constant and $D = D(|q|^2 = 0)$ is given in Eq. (3).

The dimension of an effective coupling constant in the above form is (see, e.g., [19])

$$[g] = \left[\frac{\lambda^2 D}{4\nu^3}\right] = \mu^{2-d}.$$

This explains why g has to be multiplied by μ^{d-2} to render g_R dimensionless.

With the flow functions (23)–(25) a partial differential renormalization-group equation can be formulated. This RG equation may be solved by using the method of characteristics, where a flow parameter l and an l-dependent continuous momentum scale $\tilde{\mu}(l) = \mu l$ are introduced. Those solutions are then used to formulate a KPZ-specific scaling relation for, say, the two-point correlation function $C(q,\omega)$. This relation reads [19]

$$C(\mu,D_R,\nu_R,g_R,q,\omega) = q^{-4-2\gamma_\nu^*+\gamma_D^*}\hat{C}\left(\frac{\omega}{q^{2+\gamma_\nu^*}}\right), \tag{27}$$

where the asterisk superscript indicates that the Wilson flow functions are evaluated at the stable IR fixed point. A detailed explanation of how the scaling form in (27) is obtained can be found, e.g., in [17,19]. A comparison of (27) with the general scaling form for the KPZ two-point correlation function in Fourier space (see, e.g., [1,19,25,26]), i.e.,

$$C(q,\omega) = q^{-d-2\chi-z}\hat{C}\left(\frac{\omega}{q^z}\right), \tag{28}$$

leads to the following expressions for the dynamical exponent z and the roughness exponent χ [17,19]:

$$z = 2 + \gamma_\nu^*, \tag{29}$$

$$\chi = 1 - \frac{d}{2} + \frac{\gamma_\nu^* - \gamma_D^*}{2}. \tag{30}$$

These general considerations will be used in the next part to obtain the critical exponents z and χ for KPZ driven by sinc-type noise.

III. THE KPZ EQUATION WITH SINC-NOISE CORRELATION

We now apply these results to the case of the sinc-type noise with the correlations of the form (3) with

$$D(|q|^2) = D\frac{\sin(\xi|q|)}{\xi|q|}, \tag{31}$$

where D is a constant noise amplitude, $q \in \mathbb{R}^d$, and ξ defines the scale of the sinc profile. For simplicity, let us consider the case $d = 1$. Here the noise distribution transformed back to real space is a rectangle with size $2\xi \times D/\xi$ centered at $x = 0$, which tends to $\delta(x)$ (white noise) in the limit $\xi \to 0$ [1].

The first step now is to calculate explicit expressions for the renormalization factors from (21) and (22). Inserting Eq. (31), the renormalization factors evaluated in $d = 1$ to one-loop order read

$$Z_\nu = 1 + \frac{D\lambda^2}{4\nu^3}\frac{1}{\pi}\left[2\int_0^\infty dp\frac{\sin(\xi p)}{\xi p(i\mu^2 + p^2)} - \int_0^\infty dp\frac{\cos(\xi p)}{i\mu^2 + p^2}\right], \tag{32}$$

$$Z_D = 1 + \frac{D\lambda^2}{4\nu^3}\frac{1}{\pi}\int_0^\infty dp\frac{\sin^2(\xi p)}{\xi^2(\mu^4 + p^4)}. \tag{33}$$

The integrals occurring in (32) and (33) can be computed by means of the residue theorem, which leads to

$$Z_\nu = 1 + \frac{D\lambda^2}{4\nu^3}\frac{e^{-(1/\sqrt{2})\xi\mu}}{\xi\mu^2}\left[\sin\left(\frac{1}{\sqrt{2}}\xi\mu\right)\left(1 + \frac{\xi\mu}{2\sqrt{2}}\right) - \frac{\xi\mu}{2\sqrt{2}}\cos\left(\frac{1}{\sqrt{2}}\xi\mu\right)\right], \tag{34}$$

$$Z_D = 1 + \frac{D\lambda^2}{4\nu^3}\frac{e^{-\sqrt{2}\xi\mu}}{4\sqrt{2}\xi^2\mu^3} \times \{e^{\sqrt{2}\xi\mu} - [\sin(\sqrt{2}\xi\mu) + \cos(\sqrt{2}\xi\mu)]\}. \tag{35}$$

In the Appendix the derivation of these formulas is explained in more detail.

A. Small-correlation-length expansion

Let us now focus on small correlation lengths $\xi \ll 1$ and expand (34) and (35) in ξ up to $O(\xi^2)$. Introducing the effective coupling constants [2]

$$g = \frac{D\lambda^2}{4\nu^3}, \quad g_R = \frac{gZ_g}{2\sqrt{2}\mu}, \tag{36}$$

$$g_\xi = \frac{D\xi^2\lambda^2}{4\nu^3}, \quad g_{\xi,R} = \frac{g_\xi Z_g\mu}{2\sqrt{2}}, \tag{37}$$

with $Z_g = Z_D Z_\nu^{-3}$ the one-loop integrals simplify,

$$Z_\nu = 1 + \frac{g}{2\sqrt{2}\mu} - \frac{g_\xi\mu}{12\sqrt{2}}, \tag{38}$$

$$Z_D = 1 + \frac{g}{2\sqrt{2}\mu} - \frac{D\lambda^2}{12\nu^3}\xi + \frac{g_\xi\mu}{6\sqrt{2}}. \tag{39}$$

With the renormalized dimensionless effective coupling constants from (36) and (37) and using (25), one obtains the flow equations

$$\beta_g = g_R\left[2g_R + \tfrac{5}{6}g_{\xi,R} - 1\right], \tag{40}$$

$$\beta_{g_\xi} = g_{\xi,R}\left[2g_R + \tfrac{5}{6}g_{\xi,R} + 1\right]. \tag{41}$$

Solving (40) and (41) for their fixed points (g^*,g_ξ^*) yields three different possible solutions, namely,

$$(g_R^*,g_{\xi,R}^*) = \begin{cases} (0,0) & \text{(Gaussian)}, \\ \left(0,-\frac{6}{5}\right) & \\ \left(\frac{1}{2},0\right) & \text{(KPZ)}. \end{cases} \tag{42}$$

The second one is nonphysical, since $g_{\xi,R}^* < 0$, and thus there are two valid fixed points for the KPZ equation driven by sinc-type noise with $\xi \ll 1$.

To determine the stability of the two fixed points we carry out a linear stability analysis via the Jacobian of the two flow functions (40) and (41), i.e.,

$$\mathcal{J} = \begin{pmatrix} \partial_{g_R}\beta_g & \partial_{g_{\xi,R}}\beta_g \\ \partial_{g_R}\beta_{g_\xi} & \partial_{g_{\xi,R}}\beta_{g_\xi} \end{pmatrix} = \begin{pmatrix} 4g_R + 5/6g_{\xi,R} - 1 & 5/6g_R \\ 2g_{\xi,R} & 2g_R + 5/3g_{\xi,R} + 1 \end{pmatrix}. \tag{43}$$

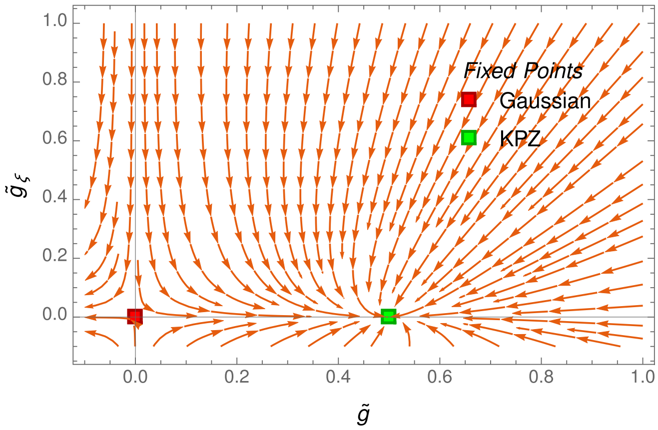

FIG. 2. Renormalization-group flow of the two effective coupling constants (36) and (37) for small values of the noise correlation length ξ in $d = 1$ spatial dimension. As can be seen, the only stable infrared fixed point is the KPZ fixed point at $(\tilde{g},\tilde{g}_\xi) = (1/2,0)$.

By evaluating (43) at the respective fixed points it turns out that for the Gaussian fixed point $\mathcal{J}$ is indefinite and for the KPZ fixed point $\mathcal{J}$ is positive definite. Since the condition for asymptotic stability in this framework is positive definiteness of (43), only the KPZ fixed point is stable in the infrared limit and the Gaussian fixed point is unstable.

To provide a simple graphical representation, we analyzed the RG flow in Wilson's picture [27]. Transforming the running parameter l to Wilson's representation by (see, e.g., [19])

$$l_W = -\ln l, \tag{44}$$

one obtains the flow equations

$$\frac{d\tilde{g}(l_W)}{dl_W} = -\beta_g \overset{(40)}{=} -\tilde{g}(l_W)\left[2\tilde{g}(l_W) + \tfrac{5}{6}\tilde{g}_\xi(l_W) - 1\right], \tag{45}$$

$$\frac{d\tilde{g}_\xi(l_W)}{dl_W} = -\beta_\xi \overset{(41)}{=} -\tilde{g}_\xi(l_W)[2\tilde{g}(l_W) + \tfrac{5}{6}\tilde{g}_\xi(l_W) + 1]. \tag{46}$$

The corresponding RG flow is shown in Fig. 2.

The critical exponents z and χ are obtained via (29) and (30). Here the fixed-point values of the Wilson flow functions

$$\gamma_\nu = -g_R - \frac{g_{\xi,R}}{6} + O\left(g_R^2, g_{\xi,R}^2, g_R g_{\xi,R}\right), \tag{47}$$

$$\gamma_D = -g_R + \frac{g_{\xi,R}}{3} + O\left(g_R^2, g_{\xi,R}^2, g_R g_{\xi,R}\right) \tag{48}$$

are given by

$$\gamma_\nu^* = \gamma_\nu\left(g_R = \tfrac{1}{2}, g_{\xi,R} = 0\right) = -\tfrac{1}{2}, \tag{49}$$

$$\gamma_D^* = \gamma_D\left(g_R = \tfrac{1}{2}, g_{\xi,R} = 0\right) = -\tfrac{1}{2}. \tag{50}$$

Hence the dynamical exponent z and the roughness exponent χ read

$$z = \tfrac{3}{2}, \quad \chi = \tfrac{1}{2}. \tag{51}$$

They coincide with those in the white-noise case and confirm the KPZ exponent identity $z + \chi = 2$ (see, e.g., [1,2,4,19,28]).

B. Arbitrary-correlation-length calculation

So far, we have assumed the correlation length ξ to be small. In the following we show that the same result can be derived for arbitrary correlation lengths ξ in $d = 1$ dimensions, although the calculations are technically more involved.

Inserting (34) and (35) into (24) and (23) and expanding to lowest order in the effective coupling constant $g = D\lambda^2/4\nu^3$, the Wilson flow functions γ_i can be written as

$$\begin{aligned}\gamma_\nu &= \mu\frac{\partial \ln Z_\nu}{\partial\mu} = -\frac{g_R}{Z_\nu}\frac{e^{-(1/\sqrt{2})\xi\mu}}{\xi\mu}\left[(3\xi\mu + 4\sqrt{2})\sin\frac{\xi\mu}{\sqrt{2}} - \xi\mu(\sqrt{2}\xi\mu + 3)\cos\frac{\xi\mu}{\sqrt{2}}\right]\\ &= -g_R\frac{e^{-(1/\sqrt{2})\xi\mu}}{\xi\mu}\left[(3\xi\mu + 4\sqrt{2})\sin\frac{\xi\mu}{\sqrt{2}} - \xi\mu(\sqrt{2}\xi\mu + 3)\cos\frac{\xi\mu}{\sqrt{2}}\right] + O\left(g_R^2\right),\end{aligned} \tag{52}$$

$$\begin{aligned}\gamma_D &= \mu\frac{\partial \ln Z_D}{\partial\mu} = \frac{g_R}{Z_D}\frac{e^{-\sqrt{2}\xi\mu}}{2\xi^2\mu^2}[(2\sqrt{2}\xi\mu + 3)\sin\sqrt{2}\xi\mu - 3(e^{\sqrt{2}\xi\mu} - \cos\sqrt{2}\xi\mu)]\\ &= g_R\frac{e^{-\sqrt{2}\xi\mu}}{2\xi^2\mu^2}[(2\sqrt{2}\xi\mu + 3)\sin\sqrt{2}\xi\mu - 3(e^{\sqrt{2}\xi\mu} - \cos\sqrt{2}\xi\mu)] + O\left(g_R^2\right),\end{aligned} \tag{53}$$

where we introduced the dimensionless form of the renormalized couplings

$$g_R = \frac{gZ_g}{2\sqrt{2}\mu}, \quad Z_g = Z_D Z_\nu^{-3}. \tag{54}$$

The corresponding β function (25) reads

$$\beta_g = g_R[g_R(\tilde{\gamma}_D - 3\tilde{\gamma}_\nu) - 1], \tag{55}$$

where $\tilde{\gamma}_i = \gamma_i/g_R$ and γ_ν and γ_D are taken from (52) and (53), respectively.

Again the flow of the effective coupling constant is modeled via the flow parameter l used for the solution of the RG equations by the method of characteristics. This leads to a continuous momentum scale $\tilde{\mu}(l)$, effective coupling constant $\tilde{g}(l)$, and thus to an l-dependent flow equation (see, e.g., [4,19])

$$\beta_g(l) = l\frac{d\tilde{g}(l)}{dl}. \tag{56}$$

Hence a fixed point is characterized by $\beta_g(l) = 0$. Applying this fixed-point condition to (55) and solving for g_R leads to two separate infrared fixed-point solutions $g^*_{R,i}$:

$$g^*_{R,1} = 0, \tag{57}$$

$$g^*_{R,2} = \lim_{l\to 0} \frac{1}{\tilde{\gamma}_D(l) - 3\tilde{\gamma}_\nu(l)}. \tag{58}$$

Here (57) represents the trivial Gaussian fixed point, while the second solution in the limit $l \to 0$ [17] yields the nontrivial KPZ fixed point

$$g^*_{R,2} = \lim_{l\to 0} \frac{1}{\tilde{\gamma}_D(l) - 3\tilde{\gamma}_\nu(l)} = \frac{1}{2}. \tag{59}$$

Again the fixed points are stable, if $d\beta_g/dg_R > 0$. Since (52), (53), and (55) imply that

$$\beta'_g = \frac{d\beta_g(l)}{d\tilde{g}(l)} = 2\tilde{g}(l)[\tilde{\gamma}_D(l) - 3\tilde{\gamma}_\nu(l)] - 1 \overset{l\to 0}{=} 4g_R - 1, \tag{60}$$

we find that

$$\beta'_g = \begin{cases} -1 < 0 & \text{for } g^*_R = 0 \quad \text{(unstable)} \\ 1 > 0 & \text{for } g^*_R = \frac{1}{2} \quad \text{(stable).} \end{cases}$$

Hence there is one stable infrared fixed point $g^*_R = \frac{1}{2}$ at which the critical exponents of the KPZ universality class can be calculated. We obtain the critical exponents in $d = 1$ dimensions again as

$$z = 2 + \gamma^*_\nu = 2 - \tfrac{1}{2} = \tfrac{3}{2}, \tag{61}$$

$$\chi = \tfrac{1}{2} + \frac{-\frac{1}{2}+\frac{1}{2}}{2} = \tfrac{1}{2}. \tag{62}$$

C. Perturbation series

An expectation value of the form in (8), in general, cannot be evaluated in closed form. Thus one usually has to rely on perturbative computations. To this end the Janssen–De Dominicis functional (9) will be split into its Gaussian ($\mathcal{A}_0$) and nonlinear ($\mathcal{A}_{\text{int}}$) parts $\mathcal{A} = \mathcal{A}_0 + \mathcal{A}_{\text{int}}$. This provides the possibility to express (8) in terms of purely Gaussian expectation values of $\mathcal{O}e^{-\mathcal{A}_{\text{int}}}$. Performing an expansion with respect to the interaction functional $\mathcal{A}_{\text{int}}$ then leads to Gaussian expectation values of terms like $\mathcal{O}\mathcal{A}^l_{\text{int}}, l \in \mathbb{N}_0$. This resembles an expansion in powers of the nonlinear coupling constant λ. It turns out, however, that an expansion in powers of the effective dimensionless coupling constant g_R from (26) is the natural choice for the problem at hand (see, e.g., [4,17]). It is also well known that the standard technique of an ϵ expansion, being viable if the fixed-point value of g_R is proportional to $\epsilon = d_c - d$, fails for the KPZ equation in $d < d_c = 2$ dimensions [4,17,19,29].

In this paper we have focused on the KPZ equation in $d = 1$ dimensions due to the physical significance of this case (e.g., its equivalence to the one-dimensional Burgers equation), but we are indeed able to derive (not shown here in detail) a more general d-dependent result for the nontrivial KPZ fixed point in, e.g., (42), which reads

$$(g^*_R, g^*_{\xi,R}) = \left(\frac{d\cos\left(\frac{d\pi}{4}\right)}{\sqrt{2}(3-2d)}, 0\right), \quad 0 < d < \frac{3}{2}. \tag{63}$$

Setting $d = 1$ in (63) yields the results obtained in the two previous sections. Moreover, letting $d \to 0$ implies $g^*_R \to 0$. Following the argument made in [30], we arrive at a controlled perturbation expansion with respect to the small parameter $g_R \sim d$. Hence, our expansion in the effective coupling constant g_R is in fact an expansion in the spatial dimension d about $d = 0$, which leads to the following behavior of the solutions: The Gaussian fixed point (0,0) is always repellent and for sufficiently small $d > 0$ a branch of nontrivial stable fixed-point solutions $(g^*_R(d), 0)$ [see (63)] bifurcates, which lead to nontrivial scaling behavior. This branch was analytically continued up to $d = 1$. As can be inferred from (63), its domain is bounded from above by $d = \frac{3}{2}$. Via a two-loop calculation this upper bound is however believed to be extendable to $d = 2$, which is the physical critical dimension of the system (see, e.g., [19,30] for the white-noise case).

At this point we would like to include a general remark concerning the applicability of perturbative DRG schemes to the KPZ problem. While it is generally accepted that the usual ϵ expansion fails for the KPZ problem, as mentioned above, there are also claims that any perturbative DRG technique is invalid for treating the KPZ problem, regardless of the choice of the expansion parameter. It is certainly true that there is no mathematically rigorous proof justifying the application of perturbative DRG method to the KPZ problem. However, in numerous works the perturbative DRG method applied to the Burgers-KPZ problem has led to a wealth of interesting and sometimes exact results; see [1–4,19], to name just a few. Thus we believe that once a small parameter has been identified for organizing a controlled perturbation expansion, the perturbative DRG method is indeed a viable approach. This is exactly what we did in this paper and our result is in full agreement with a similar one in [6], which was found via a nonperturbative RG scheme according to, e.g., [18] (see also Sec. IV).

D. Comparison between sinc noise and white noise

Let us now compare our fixed-point result for arbitrary d ($0 < d < \frac{3}{2}$) and sinc noise from (63) with the results obtained by Frey and Täuber in [19] for uncorrelated noise. To this end, we briefly summarize the relevant equations needed in the white-noise case. A good starting point is the renormalization factors in the following form [19] [see also (21) and (22) with $D(|p|^2) \equiv D = \text{const}$]:

$$Z_\nu = 1 - \frac{D\lambda^2}{4\nu^3}\frac{d-2}{d}\int_p \frac{1}{i\mu^2 + p^2}, \tag{64}$$

$$Z_D = 1 + \frac{D\lambda^2}{4\nu^3}\Re\left[\int_p \frac{1}{i\mu^2 + p^2}\right]. \tag{65}$$

These expressions can be evaluated by means of dimensional regularization (for details see, e.g., [4,19]) and the introduction of an effective coupling constant $g = D\lambda^2/4\nu^3$,

$$Z_\nu = 1 + gK_d\frac{d-2}{d}\frac{\kappa_d}{\epsilon}\mu^\epsilon, \tag{66}$$

$$Z_D = 1 - gK_d\frac{\kappa_d}{\epsilon}\mu^\epsilon, \tag{67}$$

where we used the abbreviations

$$K_d = \frac{S_d}{(2\pi)^d} = \frac{\frac{2\pi^{d/2}}{\Gamma(d/2)}}{(2\pi)^d} = \frac{1}{\Gamma(d/2)2^{d-1}\pi^{d/2}},$$

$$\kappa_d = \Gamma\left(2-\frac{d}{2}\right)\Gamma\left(\frac{d}{2}\right)\sin\left(\frac{d\pi}{4}\right),$$

$$\epsilon = d-2.$$

Using (66) and (67), the Wilson flow functions expanded to linear order in the effective coupling constant g read

$$\gamma_\nu = \mu\partial_\mu \ln Z_\nu = gK_d\frac{d-2}{d}\kappa_d\mu^\epsilon + O(g^2), \quad (68)$$

$$\gamma_D = \mu\partial_\mu \ln Z_D = -gK_d\kappa_d\mu^\epsilon + O(g^2). \quad (69)$$

Choosing the renormalized dimensionless effective coupling constant

$$g_R = gZ_gK_d\mu^\epsilon, \quad (70)$$

the Wilson flow functions simplify to $\gamma_\nu = g_R(d-2)\kappa_d/d$ and $\gamma_D = -g_R\kappa_d$, leading to the β-flow function

$$\beta_g = \mu\partial_\mu g_R = g_R[\gamma_D - 3\gamma_\nu + d - 2]. \quad (71)$$

Inserting (68) and (69) in (71) and solving for the fixed points g_R^* yields the trivial Gaussian fixed point $g_R^* = 0$ as well as the nontrivial KPZ fixed point

$$g_{R,FT}^* = \frac{d(d-2)}{2\kappa_d(2d-3)}. \quad (72)$$

Using the identities $\Gamma(x)\Gamma(1-x) = \pi/\sin(\pi x)$ and $\sin(d\pi/2)/\sin(d\pi/4) = 2\cos(d\pi/4)$, we arrive at

$$g_{R,FT}^* = -2\frac{d}{2d-3}\cos\left(\frac{d\pi}{4}\right) \overset{(63)}{=} 2\sqrt{2}g_R^*.$$

The difference in the prefactor is easily explained by the slightly different definitions of the dimensionless effective coupling constants in the sinc-noise and white-noise cases, respectively. For the sinc noise we chose to include a numerical prefactor of $1/(2\sqrt{2})$ for the sake of convenience. We thus arrive at the conclusion that the KPZ equation driven by spatially correlated noise with a finite correlation length is described by the standard KPZ fixed point not only in $d = 1$ but for any dimension in the range $0 < d < \frac{3}{2}$.

IV. DISCUSSION

In the present work we have studied the field-theoretic DRG formalism of the KPZ equation for correlated noise of sinc type, which is characterized by a finite correlation length ξ. The fixed points of the KPZ DRG flow have been calculated in two different manners, namely, first for small correlation lengths ξ and via two effective coupling constants g and g_ξ [see Sec. III A and (36) and (37)] and then using only one effective coupling constant g (see Sec. III B) for arbitrary values of the correlation length ξ. Both methods yield the same results, i.e., we obtain an unstable Gaussian fixed point and the stable KPZ fixed point [see (42) and (57)–(59)].

It might be argued that the second method is somewhat redundant since the small-ξ expansion can also be interpreted to be valid for arbitrary values of ξ in the infrared limit as in this regime $\mu \to 0$ and hence $\xi\mu =: \delta \ll 1$. The expansion would then be done for the parameter δ. Nevertheless, the method used in Sec. III B is a reassuring confirmation of the results obtained in Sec. III A.

Using these fixed points, we computed the critical exponents characterizing the KPZ universality class, i.e., the dynamical exponent $z = 3/2$ and the roughness exponent $\chi = 1/2$ [see (51), (61), and (62)]. These values coincide with the standard KPZ exponents in one spatial dimension, where the system is driven by white noise (see, e.g., [1,19]). Hence, for every finite noise correlation length ξ the system behaves to one-loop order as if it was driven by standard uncorrelated Gaussian noise of the form (2).

To determine the behavior to two-loop order, a very involved calculation needs to be done. However, using the findings of [19] and modifying these results with the momentum-dependent noise amplitude from (3) and (31), respectively, a first cursory but by no means complete analysis suggests that the correlated noise does not produce new UV singular contributions in the two-loop-order approximation. This would imply, if true for all perturbation integrals, that the one-loop-order approximation covers all the interesting physics.

This result corresponds nicely with [6], where a different spatial noise correlation was analyzed. The authors there found that for small values of the noise correlation length the KPZ equation behaves as if it was driven by uncorrelated white noise. Combining the findings of [6] with the present ones, based on different noise functions and different methods, we arrive at the intuitively expected conjecture that the large-scale KPZ dynamics is independent of the details of the noise structure, provided that the correlation length ξ is finite.

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APPENDIX: EXPLICIT EVALUATION OF THE RENORMALIZATION FACTORS

To obtain (34) and (35) from the expressions in (32) and (33), respectively, we use the residue theorem. To this end, the integrals are first rewritten in a more easily accessible form.

1. Evaluation of Eq. (32)

The first integral needed for the calculation of Z_ν reads

$$\int_0^\infty dp\frac{\sin(\xi p)}{\xi p(i\mu^2 + p^2)}. \quad \text{(A1)}$$

This may be rewritten as

$$\int_0^\infty dp\frac{\sin(\xi p)}{\xi p(i\mu^2 + p^2)} = -\frac{i}{2}\int_{-\infty}^\infty dp\frac{e^{i\xi p}}{\xi p(i\mu^2 + p^2)}. \quad \text{(A2)}$$

The integrand on the right-hand side in (A2) has three simple poles which are given by $z_{1/2} = \pm\mu e^{i3\pi/4}$ and $z_3 = 0$. Those and the chosen integration contour are shown in Fig. 3. Hence

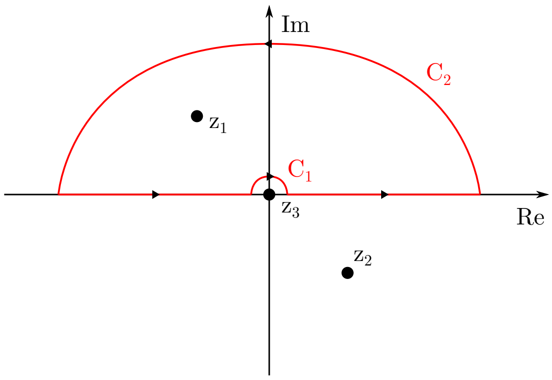

FIG. 3. Integration contour C for the evaluation of (A1). Here C_1 and C_2 are circles about $z = 0$ with radii ϵ and R, respectively.

the residue theorem yields

$$\begin{aligned}\int_C dz \frac{e^{i\xi z}}{\xi z(i\mu^2+z^2)} &= \int_{-R}^{-\epsilon} dz \frac{e^{i\xi z}}{\xi z(i\mu^2+z^2)} + \int_{C_1} dz \frac{e^{i\xi z}}{\xi z(i\mu^2+z^2)} \\ &\quad + \int_{\epsilon}^{R} dz \frac{e^{i\xi z}}{\xi z(i\mu^2+z^2)} + \int_{C_2} dz \frac{e^{i\xi z}}{\xi z(i\mu^2+z^2)} \\ &= 2\pi i \lim_{z\to\mu e^{i3\pi/4}} \frac{(z-\mu e^{i3\pi/4})e^{i\xi z}}{\xi z(z-\mu e^{i3\pi/4})(z+\mu e^{i3\pi/4})} \\ &= -\frac{\pi e^{-(1/\sqrt{2})\xi\mu}}{\xi\mu^2}\left[\cos\left(\frac{1}{\sqrt{2}}\xi\mu\right) - i\sin\left(\frac{1}{\sqrt{2}}\xi\mu\right)\right].\end{aligned}$$

To obtain the integral on the real axis from minus to plus infinity, the contributions of the integrals over the two circular paths have to be computed. Therefore, the parametrization

$$z = \epsilon e^{i\varphi} \Leftrightarrow dz = i\epsilon e^{i\varphi} d\varphi$$

is used, which yields, for the integral over C_1 with $\epsilon \to 0$,

$$\begin{aligned}\lim_{\epsilon\to 0}\int_{C_1} dz \frac{e^{i\xi z}}{\xi z(i\mu^2+z^2)} &= -\lim_{\epsilon\to 0}\int_0^{\pi} d\varphi \frac{i e^{i\xi\epsilon e^{i\varphi}}}{\xi(i\mu^2+\epsilon^2 e^{2i\varphi})} \\ &= -\int_0^{\pi} d\varphi \lim_{\epsilon\to 0} \frac{i e^{i\xi\epsilon e^{i\varphi}}}{\xi(i\mu^2+\epsilon^2 e^{2i\varphi})} = -\frac{\pi}{\xi\mu^2}.\end{aligned}$$

For the integration over the contour C_2 a similar parametrization is used

$$z = Re^{i\varphi} \Leftrightarrow dz = iRe^{i\varphi} d\varphi. \tag{A3}$$

The contribution from this integral vanishes for $R \to \infty$,

$$\begin{aligned}\lim_{R\to\infty}\left|\int_{C_2} dz \frac{e^{i\xi z}}{\xi z(i\mu^2+z^2)}\right| &= \lim_{R\to\infty}\left|\int_0^{\pi} d\varphi \frac{iRe^{i\xi Re^{i\varphi}} e^{i\varphi}}{\xi Re^{i\varphi}(i\mu^2+R^2e^{2i\varphi})}\right| \\ &\leqslant \lim_{R\to\infty}\int_0^{\pi} d\varphi \frac{|e^{i\xi Re^{i\varphi}}|}{|\xi(i\mu^2+R^2e^{2i\varphi})|} \\ &= \lim_{R\to\infty}\int_0^{\pi} d\varphi \frac{e^{-\xi R\sin\varphi}}{\xi R^2\left|i\frac{\mu^2}{R^2}+e^{2i\varphi}\right|} = 0,\end{aligned}$$

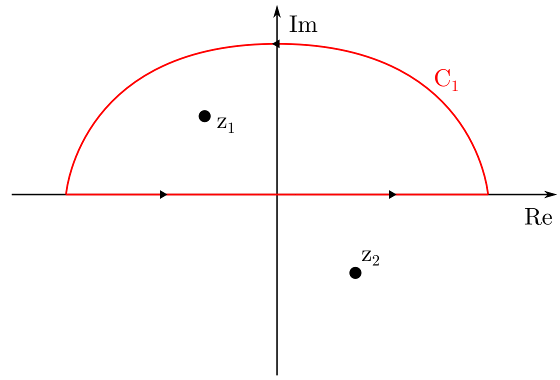

FIG. 4. Contour C of integration for (A6). Here C_1 denotes a circle about $z = 0$ with radius R.

since $\sin\varphi > 0$ for $0 < \varphi < \pi$. Thus, in the limits $R \to \infty$ and $\epsilon \to 0$ the residue theorem results in

$$\begin{aligned}\int_{-\infty}^{\infty} dz \frac{e^{i\xi z}}{\xi z(i\mu^2+z^2)} &= \frac{\pi e^{-(1/\sqrt{2})\xi\mu}}{\xi\mu^2}\left\{e^{(1/\sqrt{2})\xi\mu}\right. \\ &\quad \left. -\left[\cos\left(\frac{1}{\sqrt{2}}\xi\mu\right) - i\sin\left(\frac{1}{\sqrt{2}}\xi\mu\right)\right]\right\}.\end{aligned}$$

The integral from (A1) is therefore given by

$$\begin{aligned}\int_0^{\infty} dp \frac{\sin(\xi p)}{\xi p(i\mu^2+p^2)} &= \pi\frac{e^{-(1/\sqrt{2})\xi\mu}}{2\xi\mu^2}\left\{\sin\left(\frac{1}{\sqrt{2}}\xi\mu\right)\right. \\ &\quad \left. + i\left[\cos\left(\frac{1}{\sqrt{2}}\xi\mu\right) - e^{(1/\sqrt{2})\xi\mu}\right]\right\}.\end{aligned} \tag{A4}$$

The second integral needed for the evaluation of (32) is given by

$$\int_0^{\infty} dp \frac{\cos(\xi p)}{i\mu^2+p^2}. \tag{A5}$$

As for the calculation of (A1), the integral will be rewritten according to

$$\int_0^{\infty} dp \frac{\cos(\xi p)}{i\mu^2+p^2} = \frac{1}{2}\int_{-\infty}^{\infty} dp \frac{e^{i\xi p}}{i\mu^2+p^2}. \tag{A6}$$

The integrand on the right-hand side of (A6) has two simple poles at $z_{1/2} = \pm\mu e^{i3\pi/4}$. For the integration contour shown in Fig. 4, the residue theorem leads to

$$\begin{aligned}\int_C dz \frac{e^{i\xi z}}{i\mu^2+z^2} &= \int_{-R}^{R} dz \frac{e^{i\xi z}}{i\mu^2+z^2} + \int_{C_1} dz \frac{e^{i\xi z}}{i\mu^2+z^2} \\ &= 2\pi i \lim_{z\to\mu e^{i3\pi/4}} \frac{(z-\mu e^{i3\pi/4})e^{i\xi z}}{(z-\mu e^{i3\pi/4})(z+\mu e^{i3\pi/4})}\end{aligned}$$

$$= \frac{\pi e^{-(1/\sqrt{2})\xi\mu}}{\sqrt{2}\mu}\left\{\cos\left(\frac{1}{\sqrt{2}}\xi\mu\right) - \sin\left(\frac{1}{\sqrt{2}}\xi\mu\right) - i\left[\cos\left(\frac{1}{\sqrt{2}}\xi\mu\right) + \sin\left(\frac{1}{\sqrt{2}}\xi\mu\right)\right]\right\}.$$

Choosing again the parametrization (A3), it is readily shown that its contribution vanishes for $R \to \infty$:

$$\lim_{R\to\infty}\left|\int_{C_1} dz \frac{e^{i\xi z}}{i\mu^2 + z^2}\right| = \lim_{R\to\infty}\left|\int_0^\pi d\varphi \frac{iRe^{i\xi Re^{i\varphi}}e^{i\varphi}}{i\mu^2 + R^2e^{2i\varphi}}\right| \leqslant \lim_{R\to\infty}\int_0^\pi d\varphi \frac{|iRe^{i\xi Re^{i\varphi}}e^{i\varphi}|}{|i\mu^2 + R^2e^{2i\varphi}|} = \lim_{R\to\infty}\int_0^\pi d\varphi \frac{e^{-\xi R\sin\varphi}}{R|\frac{i\mu^2}{R^2} + e^{2i\varphi}|} = 0.$$

Hence the sought integral reads

$$\int_0^\infty dp \frac{\cos(\xi p)}{i\mu^2 + p^2} = \pi \frac{e^{-(1/\sqrt{2})\xi\mu}}{2\sqrt{2}\mu}\left\{\cos\left(\frac{1}{\sqrt{2}}\xi\mu\right) - \sin\left(\frac{1}{\sqrt{2}}\xi\mu\right) - i\left[\cos\left(\frac{1}{\sqrt{2}}\xi\mu\right) + \sin\left(\frac{1}{\sqrt{2}}\xi\mu\right)\right]\right\}. \tag{A7}$$

Taking the real parts [4] of (A4) and (A7) and inserting the results into (32) leads to the expression in (34).

2. Evaluation of Eq. (33)

The integral (33) reads

$$\frac{1}{\pi}\int_0^\infty dp \frac{\sin^2(\xi p)}{\mu^4 + p^4} = \frac{1}{2\pi}\left[\int_0^\infty dz \frac{1}{\mu^4 + z^4} - \int_0^\infty dz \frac{\cos(2\xi z)}{\mu^4 + z^4}\right]. \tag{A8}$$

The integrands of both integrals in (A8) have simple poles at $z_k = \mu e^{i(\pi/4 + \pi k/2)}$, with $k = 0,1,2,3$, and the contour of integration is shown in Fig. 5. The first of the two integrals on the right-hand side of (A8) is readily solved with the aid of the residue theorem [again it can be shown that $\int_{C_1} dz/(\mu^4 + z^4) = 0$ for $R \to \infty$]:

$$\int_0^\infty dz \frac{1}{\mu^4 + z^4} = \frac{1}{2}\int_{-\infty}^\infty dz \frac{1}{\mu^4 + p^4} = \frac{1}{2}2\pi i\left[\lim_{z\to z_0}\frac{z - z_0}{(z - z_0)(z - z_1)(z - z_2)(z - z_3)} + \lim_{z\to z_1}\frac{z - z_1}{(z - z_0)(z - z_1)(z - z_2)(z - z_3)}\right] = \frac{\pi}{2\sqrt{2}\mu^3}. \tag{A9}$$

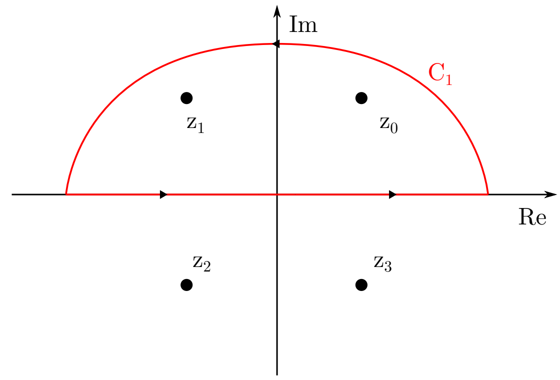

FIG. 5. Representation of the zeros and the contour C of integration for the two integrals on the right-hand side of (A8). Again, C_1 is a circle with radius R about $z = 0$.

For the second integral it is again used that

$$2\int_0^\infty dz \frac{\cos(2\xi z)}{\mu^4 + z^4} = \int_{-\infty}^\infty \frac{e^{2i\xi z}}{\mu^4 + z^4} \quad (z \in \mathbb{R}).$$

With the integration contour shown in Fig. 5, we arrive at

$$\int_{-R}^R dz \frac{e^{2i\xi z}}{\mu^4 + z^4} + \int_{C_1} dz \frac{e^{2i\xi z}}{\mu^4 + z^4} = 2\pi i\left[\lim_{z\to z_0}\frac{(z - z_0)e^{2i\xi z}}{(z - z_0)(z - z_1)(z - z_2)(z - z_3)} + \lim_{z\to z_1}\frac{(z - z_1)e^{2i\xi z}}{(z - z_0)(z - z_1)(z - z_2)(z - z_3)}\right] = \frac{\pi e^{-\sqrt{2}\xi\mu}}{\sqrt{2}\mu^3}[\cos(\sqrt{2}\xi\mu) + \sin(\sqrt{2}\xi\mu)].$$

As $\int_{C_1} dz e^{2i\xi z}/(\mu^4 + z^4)$ tends to zero for $R \to \infty$,

$$\lim_{R\to\infty}\left|\int_0^\pi d\varphi \frac{iRe^{i\varphi}e^{2i\xi Re^{i\varphi}}}{\mu^4 + R^4e^{4i\varphi}}\right| \leqslant \lim_{R\to\infty}\int_0^\pi d\varphi \frac{|iRe^{i\varphi}e^{2i\xi Re^{i\varphi}}|}{|\mu^4 + R^4e^{4i\varphi}|} = \lim_{R\to\infty}\int_0^\pi d\varphi \frac{e^{-2\xi R\sin\varphi}}{R^3\left|\frac{\mu^4}{R^4} + e^{4i\varphi}\right|} = 0,$$

it is found that

$$\int_0^\infty dz \frac{\cos(2\xi z)}{\mu^4 + z^4} = \frac{\pi e^{-\sqrt{2}\xi\mu}}{2\sqrt{2}\mu^3}[\cos(\sqrt{2}\xi\mu) + \sin(\sqrt{2}\xi\mu)]. \tag{A10}$$

The results from (A9) and (A10), inserted into (33), yield the expression (35).

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