Dissipative solitons in the discrete Ginzburg-Landau equation with saturable nonlinearity

Fatkhulla Kh. Abdullaev[1] and Mario Salerno[2]

[1]*Physical-Technical Institute, Uzbek Academy of Sciences, 100084 Tashkent, Uzbekistan*

[2]*Dipartimento di Fisica E.R. Caianiello and INFN, Gruppo Collegato di Salerno, Universita di Salerno, Via Giovanni Paolo II, 84084 Fisciano, Salerno, Italy*

(Received 7 January 2018; published 15 May 2018)

The modulational instability of nonlinear plane waves and the existence of periodic and localized dissipative solitons and waves of the discrete Ginzburg-Landau equation with saturable nonlinearity are investigated. Explicit analytic expressions for periodic solutions with a zero and a finite background are derived and their stability properties investigated by means of direct numerical simulations. We find that while discrete periodic waves and solitons on a zero background are stable under time evolution, they may become modulationally unstable on finite backgrounds. The effects of a linear ramp potential on stable localized dissipative solitons are also briefly discussed.

DOI: 10.1103/PhysRevE.97.052208

I. INTRODUCTION

A great deal of attention is presently devoted to the investigation of localized states in the discrete Ginzburg-Landau equation (DGLE) [1–7]. This equation is similar to the well known discrete nonlinear Schrödinger equation (DNLSE) but includes also nonconservative terms which allow one to model dissipative and amplification effects. It appears in connection with several physical systems, including arrays of waveguides with amplification and damping, arrays of semiconductor lasers [8], arrays of exciton-polariton condensates [9], etc. While solitons solutions of the DNLSE emerge from the balance between the discrete dispersion and nonlinearity, dissipative discrete solitons of the DGLE require also additional balance between dissipation and amplification. Discrete dissipative solitons have been investigated for power-law (nonsaturable) nonlinearities and in particular for complex extensions of the Ablowitz-Ladik equation [3,10] and DNLS-type equations with cubic-quintic nonlinearities [11–14]. In the continuous case, dissipative solitons and breathers were investigated for the nonlinear Schrödinger equation with different types of complex periodic potentials [15].

In the case of saturable nonlinearities, the study of solitons has been restricted mainly to the conservative case. In particular, discrete solitons for the DNLSE with saturable nonlinearity were investigated in [16,17] and discrete breathers for the same type of equation in [18]. In spite of the relevance of this type of nonlinearity for optics, the existence and stability of dissipative solitons in the presence of a saturable nonlinearity have not been thoroughly investigated. In the continuum case saturable nonlinearities have been recently considered in the one-dimensional complex Ginzburg-Landau equation (GLE) for both scalar and vectorial cases [19]. Modulational instability and stopping of Kerr self-focusing induced by nonconservative effects have also been investigated in the multidimensional continuous complex Ginzburg-Landau-type equation with nonlinear saturation [20,21].

It is then interesting to consider possible extensions of the above conservative results by including complex terms in the DNLSE that correspond to amplification and damping. This leads to the consideration of the DGLE with saturable nonlinearity.

The aim of the present paper is to investigate both analytically and numerically modulational instability of nonlinear plane waves, as well as the existence and stability of dissipative solitons of the DGLE with saturable nonlinearity. We start from the consideration of the dispersion relation and properties of the nonlinear plane-wave solution. Analysis of the instability of these waves under weak modulations (the so-called modulational instability problem) allows us to define the region of parameters where solitons and a train of solitons can be formed. This will allow us to construct soliton solutions and nonlinear periodic waves of the DGLE with saturable nonlinearity. In particular, we provide explicit analytic expressions for periodic dissipative solitons solutions in the form of elliptic functions on both a zero and a finite background. The stability properties of these solutions are investigated by means of direct numerical simulations of their time evolution under the DGLE. As a result, we show that while discrete periodic waves and solitons on a zero background are stable under time evolution, they become modulationally unstable on a finite background. The effects of a linear ramp potential on stable localized dissipative solitons will be also briefly considered.

II. MODEL

Let us consider the discrete complex Ginzburg-Landau equation with saturable nonlinearity

$$i A_{n,t} + (1 - i\alpha)(A_{n+1} + A_{n-1}) + (\nu - i\gamma)\frac{|A_n|^2}{1 + \mu|A_n|^2}A_n - i\delta A_n = 0, \qquad (1)$$

where parameters μ and ν control the saturation and the strength of the nonlinearity, respectively, α is the discrete filter parameter, and γ, δ denote the amplification and

 ©2018 American Physical Society

the linear damping parameters, respectively. This model arises in connection with several interesting physical phenomena such as the dynamics of nonlinear excitations in dissipative photorefractive crystals, pulse propagation in optical fibers with dopants, and arrays of cavities with exciton-polariton condensates [22,23]. Notice that in Eq. (1) a missing term $-2A_n$ in the discrete Laplacian can always be introduced by means of the transformation $A_n \to A_n e^{i2t}$ and $\delta \to \delta - 2\alpha$.

It is easy to check that Eq. (1) supports nonlinear plane-wave solutions of the form $A_n = A\exp[i(kn-\omega t)]$, with the amplitude A, wave numbers k, and frequency ω satisfying the nonlinear dispersion relations

$$A^2 = -\frac{\delta + 2\alpha\cos(k)}{\delta\mu + \gamma + 2\mu\alpha\cos(k)},$$
$$k \neq \pm\arccos\left(\frac{\delta\mu+\gamma}{2\mu\alpha}\right) \pm \pi, \tag{2}$$
$$\omega = 2\cos(k)\left(\frac{\nu\alpha}{\gamma} - 1\right) + \frac{\nu\delta}{\gamma}.$$

For $\omega > 0$ there are two possibilities for plane-wave existence; e.g., (i) for $\nu/\mu > 0$, the frequency must vary in the interval $2 - \nu/\mu < \omega < 2$, and (ii) for $\nu/\mu < 0$, the frequency must vary in the interval $2 < \omega < 2 + |\nu/\mu|$. In the case of $\omega < 0$ we find that the frequency must be varied in the interval $2 - \nu/\mu < \omega < 0$, with $\nu/\mu > 2$. Taking into account that ω is defined by Eq. (2), one can easily derive a restriction on parameters for the existence of plane waves at special points of k space: at $k = 0$ (unstaggered solution) and at $k = \pi$ (staggered solution). For the staggered solution we find the restriction $\delta \leqslant 2\alpha$, while for the unstaggered $k = 0$ solution we find that $\nu(2\alpha+\delta)/\gamma \leqslant 4$ must be satisfied.

III. MODULATIONAL INSTABILITY ANALYSIS

As is well known, the modulational instability is a fundamental dynamical phenomenon responsible for soliton and pattern generation in nonlinear systems [24]. In comparison to the continuous (nonperiodic) case, the modulational instability (MI) in the discrete case displays different properties since the discrete diffraction makes it possible to have MI also for defocusing (e.g., repulsive) nonlinearity. Since the parameter region where plane waves become modulationally unstable coincides with the existence region of solitons, one has that nonlinear lattices can support solitons also for defocusing interactions. This fact is true also in the continuous case, if a periodic potential is present [25]. It is worth mentioning here that different discretizations of the same continuous nonlinearity can have different effects on the MI and correspondingly can lead to different conditions for the existence of soliton solutions [26–28]. We also remark that in the DNLSE for small wave numbers of the nonlinear plane wave, all modulations become unstable if the power exceeds a threshold value [18]. For the DNLSE with a saturable nonlinearity the gain and critical frequency are decreased in comparison with the Kerr nonlinearity model [12].

Experimentally discrete MI has been observed in the array of nonlinear optical waveguides [29] and in photovoltaic crystals [30]. To analyze MI in the model (1), we look for solutions of the form

$$A_n = [A + \psi_n(t)]\exp[i(kn - \omega t)], \quad \psi \ll A. \tag{3}$$

By substituting into Eq. (1) and using the dispersion relation (2), we get the equation for ψ_n, to the linear order, as

$$i\psi_{n,t} + (1 - i\alpha)[\psi_{n+1}e^{ik} + \psi_{n-1}e^{-ik} - 2\cos(k)\psi_n]$$
$$+ (\nu - i\gamma)\frac{A^2}{(1+\mu A^2)^2}(\psi_n + \psi_n^*) = 0. \tag{4}$$

By looking for solutions of Eq. (4) of the form

$$\psi_n = Be^{i(Qn - \Omega t)} + C^* e^{-i(Qn - \Omega^* t)}, \tag{5}$$

with B, C, and Ω complex numbers, one can readily check that the dispersion relation

$$\Omega^2 - \Lambda_1\Omega - \Lambda_2 = 0 \tag{6}$$

is obtained where

$$\Lambda_1 = 2[2S + i(2\alpha\Delta + \gamma D)],$$
$$\Lambda_2 = 4(1+\alpha^2)(\Delta^2 - S^2) + 4D\Delta(\nu + \alpha\gamma)$$
$$+ 4i(\alpha\nu - \gamma)DS, \tag{7}$$

with

$$\Delta = \cos(k)[\cos(Q) - 1],$$
$$D = \frac{A^2}{(1+\mu A^2)^2},$$
$$S = \sin(k)\sin(Q).$$

From these equations the MI gain $g(Q,k) = |\mathrm{Im}[\Omega(Q,k)|]$ is found as

$$g(Q,k) = \left|(2\alpha\Delta + \gamma D) + \frac{1}{\sqrt{2}}\sqrt{-F + \sqrt{G^2 + F^2}}\right|, \tag{8}$$

with the functions F and G given by

$$F = [4S^2 - (2\alpha\Delta + \gamma D)^2] + 4(1+\alpha^2)(\Delta^2 - S^2)$$
$$+ 4D\Delta(\nu + \alpha\gamma),$$
$$G = 4S\alpha[(2\Delta + \nu D)].$$

It is interesting to link the above MI results with the ones of the continuous GLE with saturable nonlinearity

$$iA_t + (1 - i\alpha)(A_{xx} + 2A) + (\nu - i\gamma)\frac{|A|^2A}{1 + \mu|A|^2} - i\delta A = 0. \tag{9}$$

Continuous MI results can be obtained from our expressions in the long-wave limit $Q \ll 1$ and $k \ll 1$. In this case, as it follows from Eq. (8), the expression for the gain has the form

$$g = (\gamma D - \alpha Q^2) + \sqrt{2Dk^2\nu Q^2 - Q^4 + \gamma^2 D^2}. \tag{10}$$

From this it follows that the continuous plane-wave solutions

$$A = A_0 e^{-i\omega t}, \quad \omega = -2 - \frac{\nu A_0^2}{1 + \mu A_0^2},$$
$$A_0^2 = -\frac{2\alpha + \delta}{\delta\mu + \gamma + 2\alpha\mu} \tag{11}$$

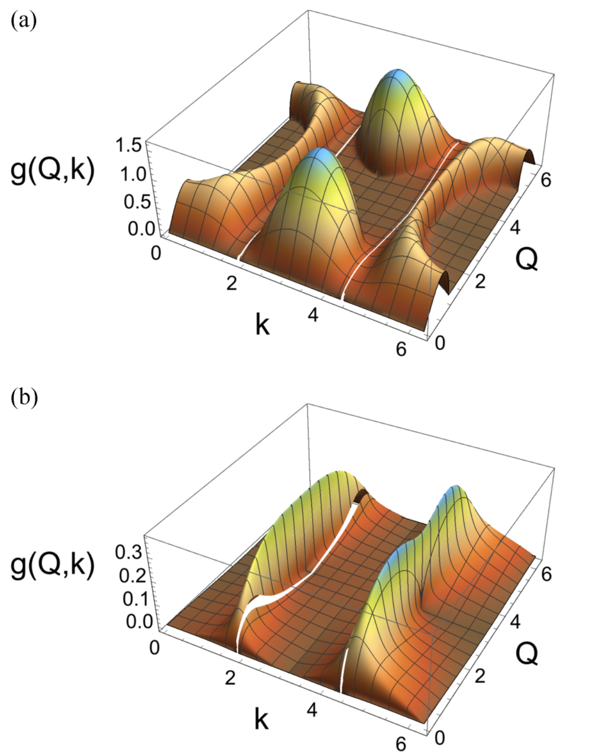

FIG. 1. The MI gain $g(Q,k)$ in Eq. (8) versus wave numbers Q and k for focusing and defocusing cases with (a) $\nu=3$ and (b) $\nu=-3$, respectively. Other parameters are fixed as $\mu=1$, $\alpha=0.01/3$, $\delta=-0.01$, and $\gamma=0.012$.

are unstable with respect to modulations with wave numbers

$$Q < \sqrt{D(\nu+\sqrt{\nu^2+\gamma^2})}, \quad Q < \sqrt{\gamma D/\alpha}. \tag{12}$$

Note that very recently the MI in the continuous GLE for the scalar and vector form has been studied in [19] and our analysis is consistent with the derivations therein.

The expressions of the MI gain for some specific value of the wave number k can be also easily derived.

(i) Case $k=0$ (unstaggered wave). The expression of the MI gain in this case is

$$g(Q) = \left| -4\alpha\sin^2\left(\frac{Q}{2}\right) + \gamma D + \frac{1}{\sqrt{2}}\sqrt{-F_0 \pm \sqrt{F_0^2+G_0^2}}\right|, \tag{13}$$

where F_0 and G_0 are values at $k=0$. The wave is unstable when the wave numbers satisfy the condition

$$\sin\left(\frac{Q}{2}\right) < \frac{1}{2}\sqrt{D(\nu+\sqrt{\nu^2+\gamma^2})}. \tag{14}$$

(ii) Case $k=\pi/2$. In this case the MI gain is given by Eq. (8) with

$$F = 4\alpha^2\sin^2 Q - \gamma^2 D, \quad G = 4\alpha\nu D. \tag{15}$$

The region of instability is given by

$$Q < \sin^{-1}\left(\frac{\gamma D}{2\alpha}\right). \tag{16}$$

(iii) Staggered wave $k=\pi$. The expression of the MI gain is the same as in the case of $k=0$ but with $\Delta = 2\sin^2(Q/2)$. The MI imposes the condition that $F<0$. Then we obtain that for the staggered wave the modulations with the wave numbers

$$Q < \sin^{-1}[D(-\nu+\sqrt{\nu^2+\gamma^2})] \tag{17}$$

are unstable.

In Fig. 1 we show the typical dependence of the MI gain on wave vectors Q and k for the focusing [Fig. 1(a)] and defocusing nonlinearities [Fig. 1(b)]. Notice that the white open regions visible in the figures correspond to the lines $k = \pm\cos^{-1}(\frac{\gamma+\delta\mu}{2\alpha\mu})+\pi$ on which the wave vector k is not defined [see Eq. (2)]. From this analysis we expect that nonlinear localized and extended solutions can exist in the DGLE with saturable nonlinearity. In the next two sections we will confirm the existence of dissipative solitons and cnoidal wave solutions by providing a few exact solutions and by investigating their stability by direct numerical integrations of the DGLE.

IV. EXACT DISSIPATIVE SOLITON SOLUTIONS

In the following we report different types of exact soliton solutions of the DGLE with saturable nonlinearity.

(i) The single dissipative soliton solution is sought in the form

$$A_n = \frac{\sinh(\beta)}{\cosh(\beta n)}e^{-i\omega t}. \tag{18}$$

Using the relation

$$\operatorname{sech}(z+\beta)+\operatorname{sech}(z-\beta) = 2\frac{\cosh(z)\cosh(\beta)}{\cosh^2(z)+\sinh^2(\beta)}, \tag{19}$$

we obtain that it is the exact solution of Eq. (1) if

$$\beta = \cosh^{-1}\left(\frac{\gamma}{2\alpha}\right), \quad \omega = -\frac{\gamma}{\alpha}, \quad \mu = 1, \quad \omega = -\nu, \quad \gamma = -\delta. \tag{20}$$

(ii) The nonlinear periodic solution is sought in the form

$$A_n = \frac{\mathrm{sn}(\beta,m)}{\mathrm{cn}(\beta,m)}e^{i\omega t}\mathrm{dn}(\beta n,m). \tag{21}$$

Taking into account the relation for the cnoidal functions

$$\mathrm{dn}(z+\beta)+\mathrm{dn}(z-\beta) = 2\frac{\mathrm{dn}(z)\mathrm{dn}(\beta)}{1-m^2\mathrm{sn}^2(z)\mathrm{sn}^2(\beta)}, \tag{22}$$

we find that the solution parameters should be taken as

$$\omega = \frac{\delta}{\alpha} = -\nu, \quad \frac{\mathrm{dn}\beta}{\mathrm{cn}^2(\beta)} = \frac{\gamma}{2\alpha}, \quad \gamma = -\delta. \tag{23}$$

(iii) The second type of nonlinear periodic solution is

$$A_n = \sqrt{m}\frac{\mathrm{sn}(\beta,m)}{\mathrm{dn}(\beta,m)}\mathrm{cn}(\beta n)e^{i\omega t}. \tag{24}$$

Taking into account the relation

$$\mathrm{cn}(z+\beta)+\mathrm{cn}(z-\beta) = 2\frac{\mathrm{cn}(z)\mathrm{cn}(\beta)}{m\,\mathrm{cn}^2(z)\mathrm{sn}^2(\beta)+\mathrm{dn}^2(\beta)}, \tag{25}$$

we find that in this case the parameters must satisfy

$$\omega = \frac{\delta}{\alpha} = -\nu, \quad \frac{\mathrm{cn}\beta}{\mathrm{dn}^2(\beta)} = \frac{\gamma}{2\alpha}, \quad \gamma = -\delta. \tag{26}$$

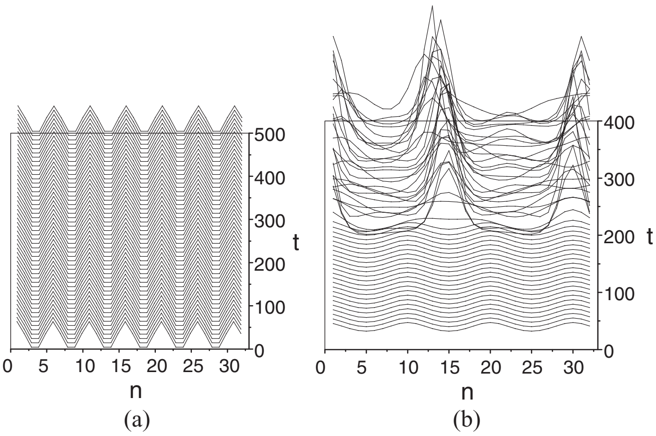

FIG. 2. Time evolution of the modulo square of the periodic dissipative soliton trains in (a) Eq. (24) and (b) Eq. (21) as obtained from direct numerical integration of Eq. (1). Parameter values are fixed as in (a) Eq. (26) with $\alpha = 0.001$, $m = 0.5$, and $\beta = 2K(m)/N_p$ and (b) Eq. (23) with $\alpha = 0.01$, $m = 0.32$, and $\beta = 4K(m)/N_p$. In both cases the number of lattice points per period is $N_p = 10$ and the total number of points along the line is 30. The cnoidal solution remains stable and the dnoidal solution displays modulational instability.

We remark that for the periodic solutions the above parameter relations must be complemented with the periodicity condition $\beta N_p = X_p$, where N_p is the number of points per spatial period $X_p = 2K(m)$ for (22). Also $X_p = 4K(m)$ for (24), with $K(m)$ the complete elliptic integral of first kind.

V. NUMERICAL RESULTS

To check the stability of the dissipative solitons derived in the preceding section we have performed direct numerical integrations of Eq. (1), taking as initial conditions the exact solutions with a small noise component added in order to accelerate the emergence of eventual instabilities. In Fig. 2 we show the time evolution of the periodic dissipative soliton trains in Eqs. (22) and (24). We see that while the cnoidal solution remains stable over a long time, the dnoidal solution display modulational instability at time $t \approx 200$ out of which two single-hump dissipative solitons are created.

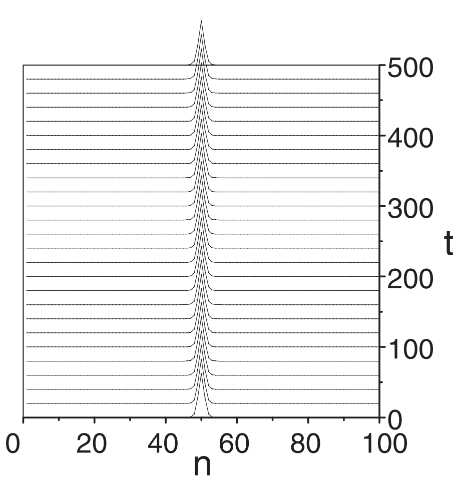

FIG. 3. Time evolution of the modulo square of the dissipative soliton in Eq. (18) as obtained from direct numerical integration of Eq. (1), for parameter values $\gamma = 0.01$, $\nu = 3$, and $\delta = -0.01$. Other parameters are derived from Eq. (20) as $\beta = 0.962\,424$ and $\alpha = 0.01/3$.

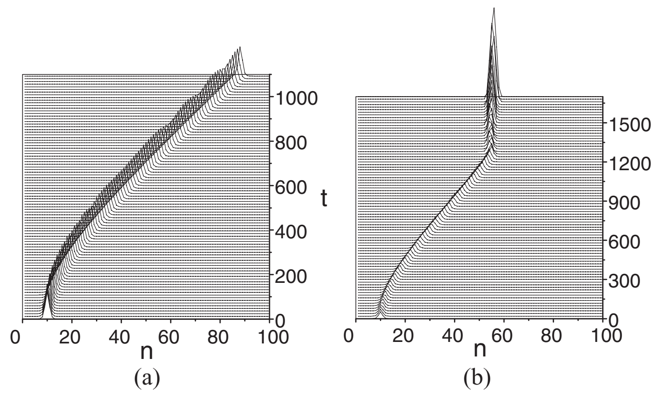

FIG. 4. Time evolution of a dissipative soliton of Eq. (1) in the presence of a linear force $\epsilon n A_n$ of strength (a) $\varepsilon = 0.0002$ and (b) $\varepsilon = 0.0001$. Other parameters are fixed as in Fig. 3.

Notice from Fig. 2(b) that the dnoidal solution can be seen as a uniform $k = 0$ background with a superimposed plane wave of wave number $Q = 0.628$, in correspondence to which the analysis of the preceding section predicts a MI gain of approximately 0.561. This also correlates with the fact that out of the instability emerge three bright solitons, as expected for an attractive (focusing) interaction and from the fact that the wave number of the modulation satisfies the relation $QL/2\pi = 3$, where L is the length of the chain (in our case $L = 30$).

The single-hump dissipative soliton centered on a site in Eq. (18), which is the limit of an infinite period ($m \to 1$) of the soliton trains in Eqs. (24) and (22), is found to be also stable over a long time, as one can see from Fig. 3. We remark that this soliton can exist only due to the perfect balance between the linear damping ($\delta < 0$) and the nonlinear amplification, a condition which can be realized only in the stationary case. As soon as one deviates from stationarity, as is the case, for example, when external forces or potentials try to put the soliton in motion, the soliton may become dynamically unstable under time evolution. To investigate this dynamical instability we add a linear potential of the type $\epsilon n A_n$ on the right-hand side of Eq. (1) which can be implemented in an optical context by a curved optical fiber. The resulting dynamics of the dissipative soliton is depicted in Fig 4. We see that, apart from small oscillations, the soliton can survive the acceleration process for a long time without significative changes in its shape. By reducing the strength of the linear potential, pinning phenomena become possible [see Fig. 4(b)]. In this case an on-site symmetric soliton becomes pinned to a lattice site in a state for which the perfect balance between damping and amplification is not realized, this leading to the instability of the state.

VI. CONCLUSION

In this paper we have investigated the modulational instability of nonlinear plane waves and the existence of dissipative solitons and cnoidal waves of the complex discrete Ginzburg-Landau equation with saturable nonlinearity. We showed that in the region of the parameter space where the MI gain is positive, generation of solitons and nonlinear periodic wave structures is possible. By taking specific parameters in these regions, we derived exact soliton periodic-wave analytical solutions of the GLE with saturable nonlinearity. We found that while discrete periodic waves and solitons of cnoidal type are stable, solutions of dnoidal type with a finite background are modulationally unstable. We also considered the effect of a linear ramp on a stable localized dissipative soliton and showed that the soliton could survive such a disturbance for a relatively long time.

ACKNOWLEDGMENTS

The authors are grateful to R. Conte, R. Galimzyanov, N. N. Rozanov, E. N. Tsoy, and B. A. Umarov for useful discussions. M.S. acknowledges partial support from the Ministero dell'Istruzione, dell'Universitá e della Ricerca through the Programmi di Ricerca Scientifica di Rilevante Interesse Nazionale Grant No. 2015K7KK8L-007.

[1] F. Kh. Abdullaev, A. A. Abdumalikov, and B. A. Umarov, Phys. Lett. A **305**, 371 (2002).
[2] N. K. Efremidis and D. N. Christodoulides, Phys. Rev. E **67**, 026606 (2003).
[3] K. Maruno, A. Ankiewicz, and N. Akhmediev, Opt. Commun. **221**, 199 (2003); Phys. Lett. A **347**, 231 (2005).
[4] U. Peschel, O. Egorov, and F. Lederer, Opt. Lett. **29**, 1909 (2004).
[5] O. A. Egorov, F. Lederer, and Yuri S. Kivshar, Opt. Express **15**, 4149 (2007).
[6] J. Garnier, F. Kh. Abdullaev, and M. Salerno, Phys. Rev. E **75**, 016615 (2007).
[7] M. Johansson, J. E. Prilepsky, and S. A. Derevyanko, Phys. Rev. E **89**, 042912 (2014).
[8] S. S. Wang and H. G. Winful, Appl. Phys. Lett. **52**, 1774 (1988).
[9] A. A. Tikhomirov, O. I. Kanakov, B. L. Altshuler, and M. V. Ivanchenko, Eur. Phys. J. B **88**, 7 (2015).
[10] F. Kh. Abdullaev, in *Dissipative Solitons*, edited by N. Akhmediev and A. Ankiewicz (Springer, Berlin, 2005), pp. 50–60.
[11] J. Cuevas, J. C. Eilbeck, and N. I. Karachalios, Discret. Cont. Dyn. Syst. **21**, 445 (2008).
[12] M. Stepic, C. E. Ruter, D. Kip, A. Maluckov, and L. Hadzievski, Opt. Commun. **267**, 229 (2006).
[13] F. Kh. Abdullaev, A. Bouketir, A. Messikh, and B. A. Umarov, Physica D **232**, 54 (2007).
[14] B. B. Baizakov, A. Bouketir, A. Messikh, and B. A. Umarov, Phys. Rev. E **79**, 046605 (2009).
[15] F. Kh. Abdullaev, V. V. Konotop, M. Salerno, and A. V. Yulin, Phys. Rev. E **82**, 056606 (2010).
[16] A. Khare, K. Ø. Rasmussen, M. Salerno, M. R. Samuelsen, and A. Saxena, Phys. Rev. E **74**, 16607 (2006).
[17] R. Conte and K. W. Chow, J. Nonlinear Math. Phys. **15**, 398 (2008).
[18] L. Hadžievski, A. Maluckov, M. Stepić, and D. Kip, Phys. Rev. Lett. **93**, 033901 (2004); M. Stepić, D. Kip, L. Hadžievski, and A. Maluckov, Phys. Rev. E **69**, 066618 (2004).
[19] R. A. Van Gorder, A. L. Krause, F. Brosa Planella, and A. M. Burton, arXiv:1710.07968.
[20] S. Champeaux and L. Bergé, Phys. Rev. E **71**, 046604 (2005).
[21] S. Champeaux, L. Bergé, D. Gordon, A. Ting, J. Peñano, and P. Sprangle, Phys. Rev. E **77**, 036406 (2008).
[22] C. W. Lai *et al.*, Nature (London) **450**, 529 (2007).
[23] K. G. Lagoudakis, B. Pietka, M. Wouters, R. André, and B. Deveaud-Plédran, Phys. Rev. Lett. **105**, 120403 (2010).
[24] F. Kh. Abdullaev, S. A. Darmanyan, and J. Garnier, Prog. Opt. **44**, 303 (2002).
[25] V. V. Konotop and M. Salerno, Phys. Rev. A **65**, 021602(R) (2002).
[26] Yuri S. Kivshar and M. Salerno, Phys. Rev. E **49**, 3543 (1994).
[27] J. F. Ravoux, S. L. Dizès, and P. L. Gal, Phys. Rev. E **61**, 390 (2000).
[28] A. Mohamadou and T. C. Kofané, Phys. Rev. E **73**, 046607 (2006).
[29] J. Meier, G. I. Stegeman, D. N. Christodoulides, Y. Silberberg, R. Morandotti, H. Yang, G. Salamo, M. Sorel, and J. S. Aitchison, Phys. Rev. Lett. **92**, 163902 (2004).
[30] M. Stepić, C. Wirth, C. E. Rüter, and D. Kip, Opt. Lett. **31**, 247 (2006).