

Heavy inertial particles in turbulent flows gain energy slowly but lose it rapidlyAkshay Bhatnagar,^{1,2,*} Anupam Gupta,^{3,†} Dhruvadya Mitra,^{2,‡} and Rahul Pandit^{1,§}¹Centre for Condensed Matter Theory, Department of Physics, Indian Institute of Science, Bangalore 560012, India²Nordita, KTH Royal Institute of Technology and Stockholm University, Roslagstullsbacken 23, 10691 Stockholm, Sweden³Mechanical Science and Engineering, University of Illinois, 1206 West Green Street, Urbana, Illinois 61801, USA

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We present an extensive numerical study of the time irreversibility of the dynamics of heavy inertial particles in three-dimensional, statistically homogeneous, and isotropic turbulent flows. We show that the probability density function (PDF) of the increment, $W(\tau)$, of a particle's energy over a time scale τ is non-Gaussian, and skewed toward negative values. This implies that, on average, particles gain energy over a period of time that is longer than the duration over which they lose energy. We call this *slow gain* and *fast loss*. We find that the third moment of $W(\tau)$ scales as τ^3 for small values of τ . We show that the PDF of power-input p is negatively skewed too; we use this skewness Ir as a measure of the time irreversibility and we demonstrate that it increases sharply with the Stokes number St for small St ; this increase slows down at $St \simeq 1$. Furthermore, we obtain the PDFs of t^+ and t^- , the times over which p has, respectively, positive or negative signs, i.e., the particle gains or loses energy. We obtain from these PDFs a direct and natural quantification of the slow gain and fast loss of the energy of the particles, because these PDFs possess exponential tails from which we infer the characteristic loss and gain times t_{loss} and t_{gain} , respectively, and we obtain $t_{\text{loss}} < t_{\text{gain}}$ for all the cases we have considered. Finally, we show that the fast loss of energy occurs with greater probability in the strain-dominated region than in the vortical one; in contrast, the slow gain in the energy of the particles is equally likely in vortical or strain-dominated regions of the flow.

DOI: [10.1103/PhysRevE.97.033102](https://doi.org/10.1103/PhysRevE.97.033102)**I. INTRODUCTION**

Heavy inertial particles (or heavy particles) advected by turbulent flows are found in many natural phenomena and industrial processes; examples include dust particles in a storm [1], water droplets in a turbulent cloud [2], pollutant dispersions, the formation of planetesimals [3], and turbulent mixing in chemical reactions [4–8]. These heavy particles cannot be modeled as tracers because of their finite size and inertia. Many experimental, numerical, and theoretical studies have been carried out to understand the statistics of these particles in turbulent flows (see, e.g., Refs. [9–11] for reviews). Such a system of heavy particles also displays many intriguing features that are of interest in nonequilibrium statistical mechanics.

Some recent studies have investigated the time irreversibility of fluid turbulence by using the statistics of Lagrangian-tracer particles [12–15]. Fully developed Navier-Stokes turbulence occurs in the limit of infinite Reynolds number or zero viscosity. The rate of energy dissipation ε does not go to zero, but it remains constant even at the highest values of the Reynolds numbers (Re) that have been obtained in experiments and numerical simulations. The hypothesis $\varepsilon > 0$ as $Re \rightarrow \infty$, which lies at the core of the Kolmogorov theory

(K41) of turbulence, is known as the zeroth law of turbulence [16]. Fully developed forced turbulence is a nonequilibrium, statistically stationary state, which displays a constant average flux of energy from large to small length scales, where it is dissipated by viscosity. Hence, obviously, such turbulence is irreversible in time. However, this is not immediately obvious to our eyes if we look at movies of the advection of Lagrangian tracers. By following the evolution of the kinetic energy of a single tracer particle, Ref. [12] shows that, on average, these tracers decelerate faster than they accelerate. This phenomenon of *slow gain* and *fast loss* of energy has been suggested to be the signature of irreversible, turbulent dynamics in the trajectory of a single Lagrangian tracer, and it has been quantified, indirectly, in Ref. [12] by the negative third moment of the probability density function (PDF) of the particle's energy increments and the negative skewness of the PDF of the power input p to the particles by the flow. This observation suggests a violation of the principle of detailed balance in turbulent flows.

It is straightforward to understand this slow-gain and fast-loss phenomenon qualitatively via the K41 phenomenology of turbulence: The turbulent cascade in the inertial range conserves energy. The energy is injected into the fluid at the large, *integral length scale* and dissipated significantly at the small length scales that lie below the Kolmogorov dissipation scale. The eddies at the largest length scales evolve most slowly, and those at the smallest length scales are the fastest, hence the dynamics of a single tracer particle shows the *slow-gain* and *fast-loss* features described above; the resulting irreversibility is, therefore, related to the aforementioned separation of time scales in turbulent flows.

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We extend these ideas to heavy particles in turbulent flows by carrying out an extensive numerical study of the time irreversibility of the dynamics of heavy inertial particles in three-dimensional (3D) statistically homogeneous and isotropic turbulent flows. In addition to being advected by the time-irreversible turbulent flow, heavy particles experience a drag force that introduces an additional source of dissipation. Nevertheless, it is still impossible to distinguish visually between forward-in-time and backward-in-time trajectories of individual particles. We illustrate this in videos V1 and V2 [17] for representative heavy-particle trajectories in statistically stationary turbulent flows that are homogeneous and isotropic; video V1 runs forward in time and V2 runs backward. However, merely by looking at the two videos, it is not possible to tell which one is which. Following Ref. [12], which considers Lagrangian tracers, we first characterize the irreversibility of the trajectories of heavy particles by the following two quantities: (a) The energy difference of a particle across a time scale τ ,

$$W(\tau) = E(t + \tau) - E(t), \quad (1)$$

where $E(t)$ is the energy per unit mass of the particle at time t ; and (b) the skewness of the PDF of the power input p to the particle by the flow. We show that the PDF of the increment, $W(\tau)$, of a particle's energy over a time scale τ is non-Gaussian and skewed toward negative values. This implies that, on average, particles gain energy over a period of time that is longer than the duration over which they lose energy. We call this *slow gain* and *fast loss*. We find that the third moment of the PDF of $W(\tau)$ is negative and scales as τ^3 for small values of τ . Next, we calculate the PDFs of times over which the power p retains the same sign. In particular, we show that the PDF of p is negatively skewed; we use this skewness Ir as a measure of the time-irreversibility, and we demonstrate that it increases sharply with the particle Stokes number St (see below) for small St ; this increase slows down at $\text{St} \simeq 1$. Furthermore, we obtain the PDFs of t^+ and t^- , the times over which p has, respectively, positive or negative signs, i.e., the particle gains or loses energy. From these PDFs we obtain a direct and natural quantification of the slow-gain and fast-loss feature, because these PDFs possess exponential tails from which we infer the characteristic loss and gain times t_{loss} and t_{gain} , respectively. We obtain $t_{\text{loss}} < t_{\text{gain}}$ for all the cases we have considered. It is well known that, in 3D turbulent flows, every point in the flow can be classified into two topological classes [18,19]: vortical regions or saddles, which are strain-dominated, depending on whether the discriminant of the velocity-gradient matrix is positive or negative. By using this discriminant, we show that the slow gain in energy of the particles is equally likely in vortical or

strain-dominated regions of the flow; in contrast, the fast loss of energy occurs with greater probability in the latter than in the former.

The remainder of this paper is organized as follows. In Sec. II, we introduce the models we use and the numerical methods we employ to study them. Section III is devoted to a presentation of our results. We discuss our results in Sec. IV.

II. MODEL AND NUMERICAL METHODS

If the flow velocity at the position of the particle is \mathbf{u} , then the motion of a heavy particle is governed by the following equations:

$$\dot{\mathbf{X}} = \mathbf{v}, \quad (2a)$$

$$\dot{\mathbf{v}} = \frac{1}{\tau_p} [\mathbf{u}(\mathbf{X}, t) - \mathbf{v}]. \quad (2b)$$

Here $\mathbf{v}(t)$ and $\mathbf{X}(t)$ denote, respectively, the velocity and position of the particle at time t , and $\tau_p = (2a^2\rho_p)/(9\nu\rho_f)$ is the Stokes or response time of the particle, with a and ρ_p the radius and material density of the particle, respectively. Equation (2) is valid if (a) the radius of the particle $a \ll \eta$, with η the Kolmogorov dissipation scale of the advecting fluid (or the particle-scale Reynolds number is very small), (b) interactions between particles are negligible (e.g., at low number densities of particles), (c) the particle density $\rho_p \gg \rho_f$ (the fluid density), (d) typical particle accelerations are much larger than the acceleration because of gravity, and (e) the fluid velocity is not affected by the particles.

A. Three-dimensional Navier-Stokes turbulence

We consider the motion of the particles described by Eqs. (2) in 3D, homogeneous, and isotropic turbulent flows. The velocity field $\mathbf{u}(\mathbf{x}, t)$ is obtained by solving the 3D, incompressible, Navier-Stokes equation, i.e.,

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = \nu \nabla^2 \mathbf{u} - \nabla p + \mathbf{f}, \quad (3a)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (3b)$$

where p , \mathbf{f} , and ν are the pressure, the external force, and the kinematic viscosity, respectively. To solve Eq. (3) numerically, we use a pseudospectral method [20] with a cubical simulation domain with periodic boundary conditions and the 2/3 dealiasing rule. Table I gives the parameters for our DNSs of the 3D Navier-Stokes equation [21]. The Stokes number that we use is $\text{St} = \tau_p/t_\eta$.

TABLE I. Parameters for our 3D runs **R1** and **R2** with N^3 collection points, ν the coefficient of kinematic viscosity, δt the time step, N_p the number of particles, k_{max} the largest wave number in the simulation, ε the mean rate of energy dissipation, η and τ_η the dissipation length and time scales, respectively, λ the Taylor microscale, Re_λ the Taylor-microscale Reynolds number, I_l the integral length scale, and T_{eddy} the large-eddy turnover time.

Run	N	ν	δt	N_p	Re_λ	$k_{\text{max}}\eta$	ε	η	λ	I_l	τ_η	T_{eddy}
R1	256	3.8×10^{-3}	5×10^{-4}	40 000	43	1.56	0.49	1.82×10^{-2}	0.16	0.51	8.76×10^{-2}	0.49
R2	512	1.2×10^{-3}	2×10^{-4}	100 000	79	1.21	0.69	7.1×10^{-3}	0.08	0.47	4.18×10^{-2}	0.41

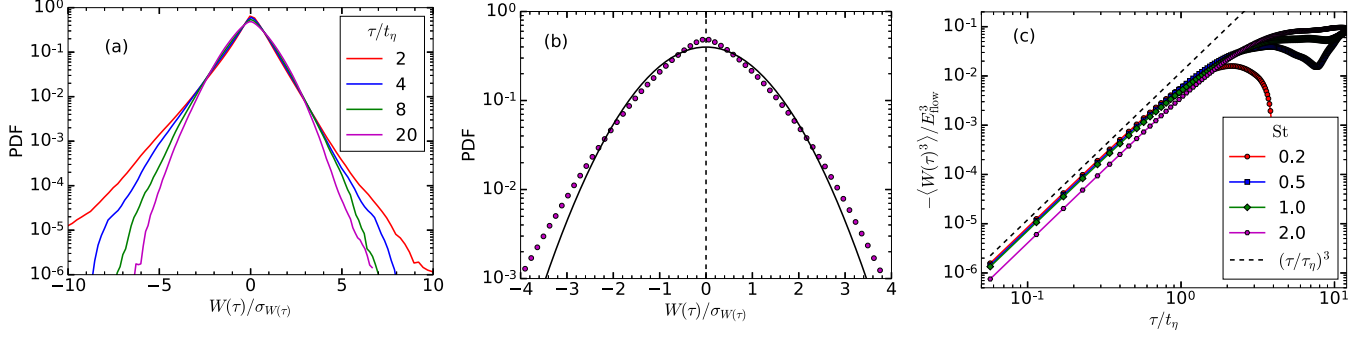


FIG. 1. (a) Probability density functions of the energy increments, $W(\tau)$, for the different values of the time lags, τ , for $St = 1$. (b) Probability density function of $W(\tau)$ for $\tau = 20t_\eta$ and $St = 1$ (magenta circles), compared with a normal distribution with zero mean and unit variance (solid black line). (c) The third moment of the PDF of $W(\tau)$ as a function of τ for different values of St .

III. RESULTS

We first allow the flow to develop until it reaches a statistically stationary turbulent state, and then we introduce the particles. We also ignore the transients until the heavy particles reach a nonequilibrium statistically stationary state, which we monitor via the temporal evolution of the total energy of the particles. In this nonequilibrium state, the PDF of any component v_k of the velocity of a heavy particle is a Gaussian with zero mean and a variance

$$\langle v^2 \rangle \approx \frac{u_{\text{rms}}^2}{1 + St_T}, \quad (4)$$

where St_T is the Stokes number defined with respect to the large-eddy-turnover time. The autocorrelation function $C(t) \equiv \langle v_k(0)v_k(t) \rangle / \langle v_k^2 \rangle$, at large t , decays with a time scale that is shorter than the large-eddy-turnover time of the flow (see Appendix A 1 for details). Equation (4) is derived for a random velocity field \mathbf{u} that has a Gaussian distribution and finite-time correlation [22].

As we have mentioned above, we follow the Lagrangian-tracer studies of Ref. [12], and we characterize the irreversibility of the dynamical system formed by the particles by calculating the statistics of the energy increments W and the power p :

$$W(\tau) \equiv E(t + \tau) - E(t), \quad (5a)$$

$$p \equiv \mathbf{v} \cdot \frac{d\mathbf{v}}{dt}, \quad (5b)$$

where $E \equiv (1/2) |\mathbf{v}|^2$ is the energy per unit mass.

A. Statistics of energy increments

In Fig. 1(a), we plot the PDF of the energy increment, $W(\tau)$, across a time scale τ (normalized by the dissipation time t_η), for several different values of τ and $St = 1$. A careful look at this figure shows that this PDF is asymmetric about zero, with an asymmetry that is most pronounced for small τ . Even for large τ , these PDFs do not approach a Gaussian distribution, as we demonstrate in Fig. 1(b). In Fig. 1(c), we plot the simplest characterization of the asymmetry of the PDF of $W(\tau)$, namely, its third moment $\langle W^3(\tau) / E_{\text{flow}}^3 \rangle$, as a function of τ for different values of St , where the characteristic energy of the flow, $E_{\text{flow}} \equiv (1/2) \langle \mathbf{u}^2 \rangle$, is used to nondimensionalize W .

As we expect [12], at small τ the third moment scales as τ^3 because $W(\tau)$ is smooth, so it can be Taylor-expanded at small τ .

B. Statistics of the power input

We plot in Fig. 2(a) the PDF of the power input p to the particle per unit mass; p is normalized by ε , the rate-of-energy dissipation of the flow. A careful look at the figure shows that the tails of the PDF are negatively skewed; they fall off more slowly on the negative side than on the positive side. This can be quantified by plotting the skewness of these PDFs, which,

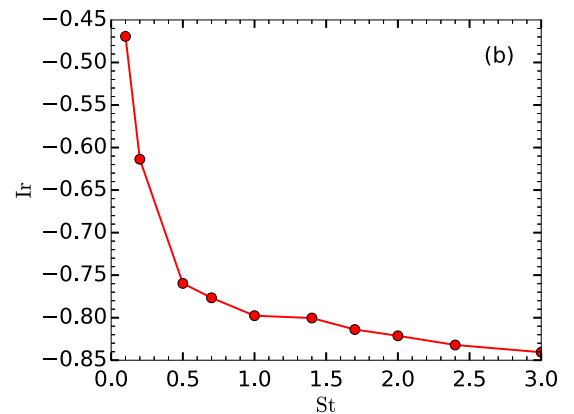
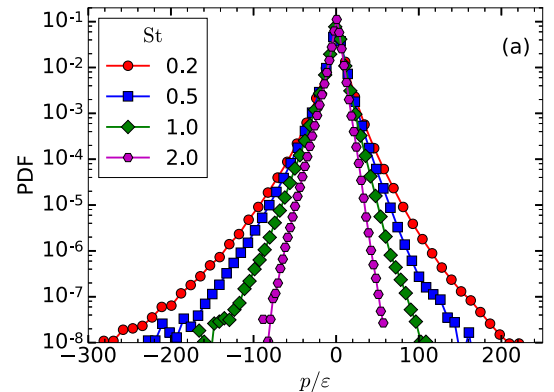


FIG. 2. (a) The PDF of the nondimensionalized power input p/ε to the particle by flow. (b) The measure of time irreversibility Ir , defined in Eq. (6), as a function of St .

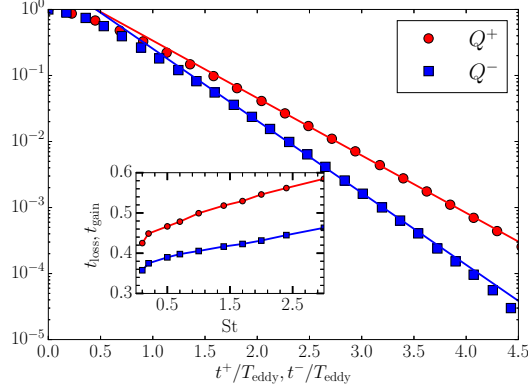


FIG. 3. The two cumulative PDFs, Q^+ and Q^- , of the times for which the p remains, respectively, positive (red) and negative (blue), for $St = 1$. The two straight lines are linear fits to the tail of the data. The slopes of these straight lines are t_{gain} and t_{loss} . These are plotted as a function of St in the inset, t_{gain} (red circles) and t_{loss} (blue squares). These time scales are scaled by a large eddy turnover time of the flow T_{eddy} .

following Ref. [12], we define as the irreversibility parameter:

$$\text{Ir} = \frac{\langle p^3 \rangle}{\langle p^2 \rangle^{3/2}}. \quad (6)$$

In Fig. 2(b), we plot Ir as a function of St . As $St \rightarrow 0$, we expect that Ir should approach its value for Lagrangian tracers. We find that Ir remains negative for all St ; in particular, its magnitude increases sharply at small St , but this increase slows down at about $St \simeq 1$.

C. Time scales of the gain and loss of energy

We now provide a direct and natural quantification of the slow-gain and fast-loss phenomenon by analyzing the time series of p as follows: Let t^+ (t^-) be the time over which p has a positive (negative) sign, i.e., the particle gains (loses) energy. These times are the first-passage times, from positive to negative values or vice versa, of the random variable p . The PDFs of such first-passage times are called persistence PDFs; if a persistence PDF has a power-law tail, the exponent of the power law is called the persistence exponent (see, e.g., Refs. [23,24] for the use of persistence in various problems of nonequilibrium statistical mechanics). The same idea has been used to calculate the persistence PDFs of residence times of tracers [25] and heavy inertial particles in topological structures in 2D [26] and 3D [27] turbulent flows.

From the time series of p we calculate the cumulative probability distribution (CDF) of both t^+ and t^- , which we denote by Q^+ and Q^- , respectively [28]. These two CDFs, for $St = 1$, are plotted in the main panel of Fig. 3 on log-linear scales. Clearly both Q^+ and Q^- have exponential tails, with characteristic time scales t_{gain} and t_{loss} , respectively. This implies that the corresponding PDFs also possess exponential tails, with the same characteristic time scales. These two time scales are plotted, as functions of St , in the inset of Fig. 3, from which we infer that, for all St , $t_{\text{gain}} < t_{\text{loss}}$, which is a natural quantification of the slow-gain and fast-loss feature.

D. Irreversibility and the topology of the flow

The topology of a 3D vector field can be characterized by its gradient matrix. A 3×3 matrix \mathcal{B} has three invariants, namely its trace $\text{Tr } \mathcal{B} = \lambda_1 + \lambda_2 + \lambda_3$, $Q \equiv \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1$, and its determinant $\det \mathcal{B} = \lambda_1\lambda_2\lambda_3$, where λ_1 , λ_2 , and λ_3 are the three eigenvalues of \mathcal{B} . If the vector field is incompressible, like our flow velocity field, there are only two invariants because the trace of the velocity-gradient matrix is zero everywhere. We consider incompressible turbulent flows, so the velocity-gradient matrix is a random matrix with zero trace; it is conventional [18,19] to denote its two invariants by the symbols Q and $R \equiv -\det \mathcal{B}$. Depending on the values of Q and R , four different types of flow topologies are possible: two are elliptic (or vortical) points, with a third stable or unstable direction, and two are saddles, with axial or biaxial strain. Whether the flow at a point is a topological vortex or a saddle depends on the sign of the discriminant, $\Delta \equiv (27/4)R^2 + Q^3$, of the characteristic equation of the velocity-gradient matrix; it is positive in vortical regions and negative in strain-dominated saddles. We have argued above that the particles lose energy to fast, small-length-scale eddies and gain energy from large-length-scale eddies. The topological structures are small-length-scale properties; hence, by the usual assumption of length-scale-separation in turbulence, we expect that the gain in energy, which occurs in large-scale eddies, does not depend

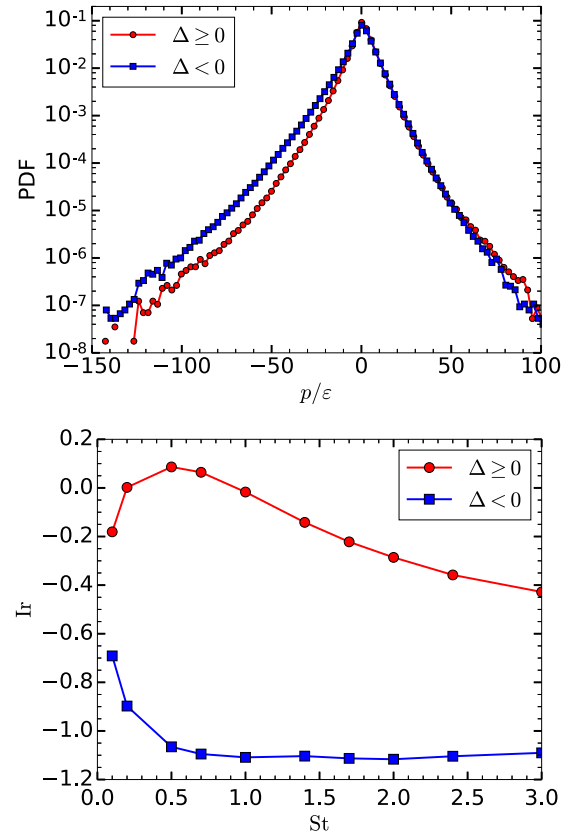


FIG. 4. Top panel: The PDFs of the nondimensionalized power input p/ϵ obtained separately from vortical (red) and strain-dominated (blue) regions of the flow, for $St = 1$. Bottom panel: Contributions to the irreversibility parameter Ir from vortical (red) and strain-dominated (blue) regions of the flow.

on the topology of the flow. By contrast, the loss in energy occurs in small-length-scale eddies, which are intimately connected with the topological structures we have described above. It has been established recently that heavy particles, in 3D turbulent flows, spend more time in strain-dominated regions than in vortical regions [27]; consequently, we expect that the loss of energy occurs more in strain-dominated regions than in vortical regions in the flow. To check the validity of this expectation, we plot, in the top panel of Fig. 4, the PDFs of p , obtained separately from regions with saddles and vortices. There is no distinction between these two PDFs for positive p , i.e., when the particles gain energy. By contrast, when p is negative, i.e., when the particles lose energy, this loss is more likely to occur in strain-dominated flow regions than in vortical ones. This is also confirmed in the bottom panel of Fig. 4, where we plot the contribution to the irreversibility parameter Ir , obtained separately from vortical and strain-dominated regions, for several different values of St ; in particular, the contribution from the vortices is significantly smaller in magnitude than that from the saddles, which shows that the dominant contribution to the skewness of the PDF of the power comes from the saddles.

IV. CONCLUSIONS

We have carried out a detailed numerical study of the time irreversibility of the dynamics of heavy particles in 3D, statistically homogeneous, and isotropic turbulent flows. We have shown that these particles, which follow Eq. (2), reach nonequilibrium statistically stationary states. We have characterized these states by calculating a variety of PDFs and autocorrelation functions. The simplest of these are PDFs and autocorrelation functions of the velocity components; we have shown that these PDFs are close to Gaussian. We have also computed the PDFs of the increments of the particle's energy $W(\tau)$ for different values of τ , and we have shown that these PDFs are non-Gaussian and skewed toward negative values. This implies that, on average, particles gain energy over a period of time that is longer than the duration over which they lose energy. For passive Lagrangian tracers, this phenomenon has been called a flight-crash effect in Ref. [12]; we simply refer to it as slow gain and fast loss. We have also found that the third moment of $W(\tau)$ scales as $\sim \tau^3$ at small values of τ .

We have computed the PDFs of the scaled power input p for different values of St , and we showed that it is negatively skewed. This negative skewness provides us with a measure of the time irreversibility Ir . We have demonstrated that the magnitude of Ir increases with St , sharply for small St but more slowly thereafter (at about $St \approx 1$). These qualitative features can also be captured by models in which the flow velocity is obtained from stochastic models with nonzero correlation time [29].

Our study has led to a direct and natural measure of the slow gain and fast loss of energy. Specifically, we have calculated the PDFs of t^+ and t^- , the times over which p has, respectively, positive or negative signs. These PDFs have exponential tails from which we have inferred the characteristic loss and gain times t_{loss} and t_{gain} , respectively. We have shown $t_{\text{loss}} < t_{\text{gain}}$ for all the values of St that we have considered. Furthermore,

we have shown that the slow gain in energy of the particles is equally likely in vortical or strain-dominated regions of the flow; in contrast, the fast loss of energy occurs with greater probability in the latter than in the former.

Time irreversibility for Lagrangian tracers, advected by turbulent flows, arises solely because of the time-irreversible nature of such flows. In contrast, for the case of heavy particles, time irreversibility arises because of two reasons: (a) turbulent flows, which advect such particles, are irreversible; and (b) the Stokes drag, exerted by the flow on the particle, is dissipative. The separation of the effects of particle inertia and turbulence on time irreversibility is nontrivial. Our study has shown how the effect of inertia can be captured clearly by the dependence of Ir on St , which we have shown in Fig. 2.

The time irreversibility for Lagrangian tracers, advected by turbulent flows, has been studied theoretically, numerically, and experimentally (see, e.g., Ref. [12]). Our study has carried out analogous theoretical and numerical studies for heavy particles advected by turbulent flows, and we have obtained clear signatures for such irreversibility, which can be measured in heavy-particle-laden flows. We hope, therefore, that our studies will stimulate experimental investigations of time irreversibility in such heavy-particle-laden flows.

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APPENDIX

1. Characterization of the statistically stationary turbulent state

In Fig. 5 we show plots of PDFs of x component v_x of the velocity of the particle (left panel). These PDFs are close to a Gaussian distribution. The middle panel shows the mean of v^2 plotted as a function of the Stokes number defined, by T_{eddy} , as $St_T = \tau_p/T_{\text{eddy}}$; the black solid line shows the plot of $\langle u^2 \rangle / (1 + St_T)$ as a function of St_T . The right panel shows the autocorrelation function

$$C(t) \equiv \frac{\langle v_x(t)v_x(0) \rangle}{\langle v_x^2 \rangle} \quad (\text{A1})$$

of the x component of \mathbf{v} . The autocorrelation functions decay at large times. The characteristic decay time decreases with St .

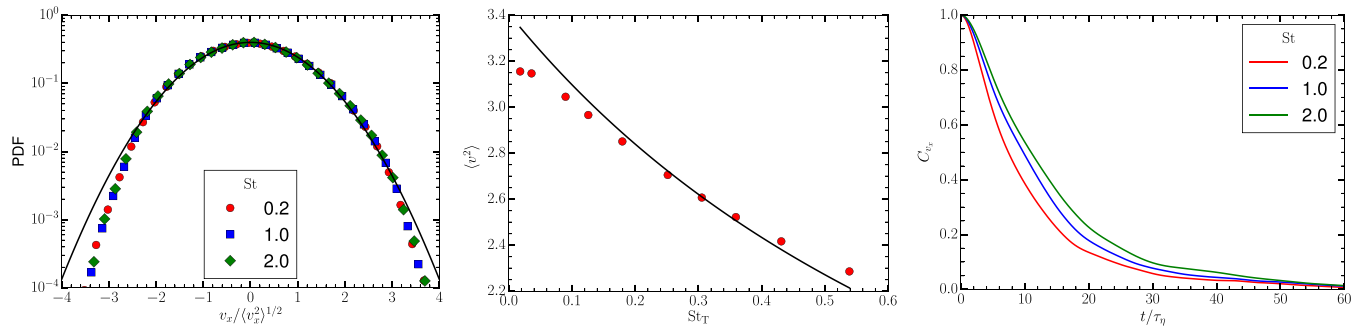


FIG. 5. Left panel: The PDFs of the x component of the velocity of the particle. The black solid line shows a normal distribution with mean zero and standard deviation unity. Middle panel: The mean of v^2 plotted as a function of the Stokes number defined, by T_{eddy} , as $St_T = \tau_p / T_{\text{eddy}}$; the black solid line shows the plot of $\langle v^2 \rangle / (1 + St_T)$ as a function of St_T . Right panel: The autocorrelation function C_{v_x} of the x component of the velocity of the particle.

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