Dark gap solitons in exciton-polariton condensates in a periodic potential

Szu-Cheng Cheng[1] and Ting-Wei Chen[2,*]
[1]Department of Optoelectric Physics, Chinese Culture University, Taipei 11114, Taiwan, Republic of China
[2]Department of Electrophysics, National Chiayi University, Chiayi city 60004, Taiwan, Republic of China

(Received 10 November 2017; published 22 March 2018)

We show that dark spatial gap solitons can occur inside the band gap of an exciton-polariton condensate (EPC) in a one-dimensional periodic potential. The energy dispersions of an EPC loaded into a periodic potential show a band-gap structure. Using the effective-mass model of the complex Gross-Pitaevskii equation with pump and dissipation in an EPC in a periodic potential, dark gap solitons are demonstrated near the minimum energy points of the band center and band edge of the first and second bands, respectively. The excitation energies of dark gap solitons are below these minimum points and fall into the band gap. The spatial width of a dark gap soliton becomes smaller as the pump power is increased.

DOI: 10.1103/PhysRevE.97.032212

I. INTRODUCTION

Solitons are solitary waves [1,2] that preserve their shape structures during propagating if the nonlinearity compensates the dispersion-induced broadening. In the past decades, the experimental and theoretical achievements in Bose-Einstein condensates (BECs) of atoms [3–6] have led to an attention on nonlinear matter waves. By tuning the chemical potential of a BEC in an optical lattice, the bright or dark soliton (DS) occurring inside the band gap has been predicted and observed in atomic BECs [7–9]. While interactions between atoms are repulsive, the matter wave is favorable to form a bright soliton and DS if the effective mass of a BEC in an optical lattice is negative and positive, respectively.

Recently, exciton-polariton condensates (EPCs) occurring in semiconductor microcavities [10] showed some interesting phenomena not observed in equilibrium BECs. Exciton polaritons are bosonic particles arising from the strong coupling between photons and excitons in semiconductor microcavities. Owing to its intrinsically out-of-equilibrium nature determined by the balance between nonlinear interaction, pumping, and decay [11], the EPC is a nonresonantly pumped and nonequilibrium system. It is a good system to study the physical properties of a nonequilibrium quantum fluid. Due to the nonequilibrium character of an EPC, a DS in a uniform EPC is unstable and displays an abrupt decay [12–14]. Moreover, the unstable and decaying DS in an EPC can be stabilized or pinned by the presence of a defect potential [15].

To create bright (dark) solitons, the mass of an EPC has to be negative (positive) to balance the nonlinearity from repulsive interactions between exciton polaritons. The negative (positive) effective mass is found near the maximum (minimum) point of the first Brillouin-zone edge (center) of an EPC in a periodic potential. This exciting soliton research area is a solitonlike state, called gap soliton, which exists in periodic media [16]. The EPCs in a static [17,18] or tunable [19,20] periodic potential have been realized in experiments on studying the s- and p-type wave functions which could have different energies, symmetry, and spatial coherence. Bright gap solitons have been discovered inside the band gap of an EPC [21]. Meanwhile, the resonantly excited bright gap solitons in two-dimensional lattices have been reported [22,23]. However, no research is focused on dark gap solitons (DGSs) occurring inside the band gap of an EPC. A recent article reported the existence of stable DSs in a polariton condensate with an incoherent periodic pump [24]. The formation of a stable and regular dark-soliton train due to a local abrupt change of self-interaction strength of the condensate was theoretically proposed within experimentally accessible schemes [25]. It is worth studying the effects of periodicity, pump, and loss on the formation of a DS in an EPC. In this paper, the effects of spatial localization of EPCs in a one-dimensional periodic potential are investigated by illuminating the system in a homogeneous fashion. Due to the effective mass of polaritons being positive near the minimum energy points of the band center and band edge of the first and second bands, respectively, we demonstrate the existence of spatially localized DSs if the excitation energies of DSs are below these minimum points and fall into the band gap. This kind of a dark soliton is named as the dark gap soliton. There are two kinds of DGSs. One is a DGS which is occurring below the minimum energy of the first band center, whose density shows a dip in a periodic density background. Another is a DGS which is occurring below the minimum energy of the second band edge, whose density has a gap in a periodic density distribution.

The present paper is organized as follows. In Sec. II, we introduce the model to describe the dynamics of the polaritons in a period potential. Bloch's theorem is applied to deduce the linear band structure without pump and loss. In Sec. III, the effective-mass theory is then used as an intermediate step to obtain the analytic solutions of DGSs. In Secs. IV and V, we solve the complex Gross-Pitaevskii equation in a periodic potential numerically to find the DGSs below the minimum energy points of the band center and band edge of the first and second bands, respectively. Finally, conclusions are given in Sec. VI.

*twchen@mail.ncyu.edu.tw

II. MEAN-FIELD MODEL AND NONLINEAR BLOCH WAVES

In the theoretical modeling of the polariton matter waves, we rely on the mean-field complex Gross-Pitaveskii equation (cGPE) incorporating the trapping potential, interparticle interactions, pumping, and decay [26]:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi + U|\Psi|^2\Psi + i(\gamma_{\mathrm{eff}} - \Gamma|\Psi|^2)\Psi, \tag{1}$$

where Ψ is the wave function and $\hbar$ and m are Planck's constant and polariton mass, respectively. The trapping potential, $V(x)$, here is a one-dimensional periodic potential given by $V(x) = V_0\sin^2(\pi x/a)$, with potential depth V_0 and lattice constant a. The third term on the right-hand side of the equation represents the normalized two-body interaction with U being the strength of the two-body interaction potential. γ_{eff} represents the linear net gain describing the balance between the stimulated scattering of polaritons into the condensate and the linear loss of polaritons out of the cavity. Γ is the coefficient of gain saturation. By choosing the length, time, and energy scales in units of $2/a$, ω, and $\hbar\omega = 4\hbar^2/ma^2$, respectively, and further rescaling the wave function $\Psi \rightarrow \sqrt{\hbar\omega/U}\Psi$ with respect to U, we can obtain

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{1}{2}\frac{\partial^2\Psi}{\partial x^2} + V_0\sin^2(\pi x/2)\Psi + |\Psi|^2\Psi + i(\alpha - \sigma|\Psi|^2)\Psi, \tag{2}$$

where α is the homogeneous pumping rate in units of $\hbar\omega$; $\sigma = \Gamma/U$ is a factor of the nonlinear effective loss. The sigma term, i.e., $-\sigma|\Psi|^2\Psi$, is a nonlinear loss due to the gain saturation. Its physical origin can be from the depletion of the population of reservoir polaritons, and may also include an effect due to an increase of condensate chemical potential reducing scattering from the reservoir. If no such nonlinear process exists in the equation, the system would be unstable, which means any pumping rate α would bring the population to go to infinity. Therefore, there should be some mechanism whereby the effective pump strength would decrease as the density $|\Psi|^2$ increases. When the pump is higher, more population is generated, but the loss is also increased; eventually a steady-state solution can be reached when the pumping is equal to the dissipation. Interested readers can refer to the article by Szymańska, Keeling, and Littlewood [27] for how Eq. (1) is obtained. The mean-field theory of the nonequilibrium system describes a self-consistent steady state which can be written in terms of the nonlinear complex susceptibility. Making a gradient and Taylor expansion of the nonlinear complex susceptibility, a complex Gross-Pitaevskii equation with an imaginary term like Eq. (1) is then available. Here $\sigma = 0.52$ is chosen in this article so that the energy difference between the lowest and highest energy of the first band is 1 meV, which is consistent with Ref. [17]. The steady state solution is shown by the wave function $\Psi(x,t) = \psi(x)e^{-iEt}$, where E is the chemical potential or polariton energy of the system. Then Eq. (2) becomes

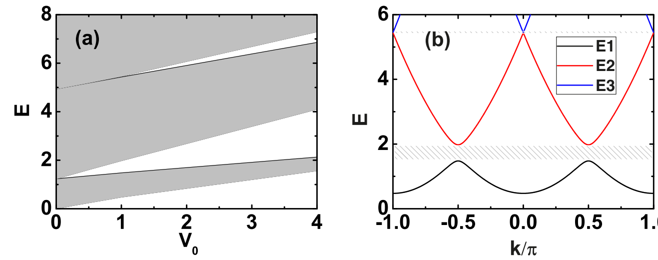

FIG. 1. (a) Energy dispersion versus V_0 at $k = \pi/2$ and (b) band diagram at $V_0 = 1$.

$$E\psi = -\frac{1}{2}\frac{\partial^2\psi}{\partial x^2} + V_0\sin^2(\pi x/2)\psi + |\psi|^2\psi + i(\alpha - \sigma|\psi|^2)\psi. \tag{3}$$

The stationary solutions are found by applying Bloch's theorem, which states that the wave functions have the form $\psi_{n,k}(x) = e^{ikx}u_{n,k}(x)$, where k is the quasimomentum and n indicates the band index. The functions $u_{n,k}(x)$ are periodic with period a, i.e., $u_{n,k}(x+a) = u_{n,k}(x)$. In the case of an EPC without interactions, pump, and loss, the condensate wave function can be presented as a superposition of Bloch waves, $\psi_{n,k}(x) = e^{ikx}\sum_m C_m^n e^{imGx}$, where C_m^n are the expansion coefficients of the Bloch wave and $G = 2\pi/a$ is the reciprocal-lattice vector. Therefore, the energy spectrum of an EPC consists of several bands of eigenvalues $E_n(k)$ in which k is a real wave number of Bloch waves. The bands are separated by gaps in which $\mathrm{Im}(k) \neq 0$.

Figure 1(a) shows the energy band diagram for different potential depths under the conditions of neglecting the nonlinear, pump, and loss effects. If we select $V_0 = 1$, the energy band diagram is shown as Fig. 1(b). Without the periodic potential, $V_0 = 0$, there is no band gap. The existence of the periodic potential splits the energy dispersion into multiple bands with the zone folding happening at $k = \pi/2$. Gaps between the bands are getting wider for a deeper potential depth (larger V_0). The lowest and intermediate energy bands are the first and second bands, respectively. There are two wide open gaps below the first band center at $k = 0$ and the second band edge at $k = \pi/2$.

III. NONEQUILIBRIUM AND NONLINEAR EFFECTIVE MASS FORMALISM

We assume a wave packet of condensate $\Psi(x,t)$ with a small momentum distribution centered around k_0 in one specific band n is described by a slowly varying envelope function $F(x,t)$ (on the scale of several lattice constants) multiplied by the linear Bloch wave function $\psi_{n,k_0}^L(x)$ as

$$\Psi(x,t) = F(x,t)\psi_{n,k_0}^L(x)e^{iE_n(k_0)t}. \tag{4}$$

In the case of weakly interacting polaritons and negligible band mixing (interband transition), a nonlinear Schrödinger equation for $F(x,t)$ can be derived employing the effective mass approximation [28,29]. The reduced differential equation has the same form as the Gross-Pitaevskii equation but with modified linear dispersion and nonlinear terms:

$$i\left[\frac{\partial F(x,t)}{\partial t}+v_g\frac{\partial F(x,t)}{\partial x}\right] = \frac{-1}{2m^*}\frac{\partial^2 F(x,t)}{\partial x^2}+E_n(k_0)F(x,t) + \lambda|F(x,t)|^2F(x,t)+i(\alpha-\sigma\lambda|F(x,t)|^2)F(x,t), \quad (5)$$

where $m^*(k_0)=[\frac{\partial^2 E_n(k)}{\partial k^2}|_{k=k_0}]^{-1}$ is the effective mass of the polariton and $\lambda=\int_{-1}^{1}|\psi^L_{n,k_0}(x)|^4dx/\int_{-1}^{1}|\psi^L_{n,k_0}(x)|^2dx$ is a renormalization factor to renormalize the nonlinear interaction strength of an EPC in a periodic potential. The periodic potential also leads to a group velocity of the envelope $F(x,t)$ determined by the energy band via $v_g(k_0)=\frac{\partial E_n(k)}{\partial k}|_{k=k_0}$. The effect of the periodicity is manifested by parameters m^*, λ, and v_g at $k=k_0$. Note that the nonlinear effects (nonlinear interaction and saturated loss) are considered as a small perturbation here. Inserting $F(x,t)=F_0(x)e^{iEt}$ with energy eigenvalue E into Eq. (5), we obtain a time-independent differential equation of F_0:

$$EF_0(x)=-iv_g\frac{dF_0(x)}{dx}-\frac{1}{2m^*}\frac{d^2F_0(x)}{dx^2}+E_n(k_0)F_0(x) + \lambda|F_0(x)|^2F_0(x)+i(\alpha-\sigma\lambda|F_0(x)|^2)F_0(x). \quad (6)$$

To find the positive effective mass near the band center at $k=0$ or band edge at $k=\pi/2$, we know $v_g=0$ and Eq. (6) becomes

$$\frac{1}{2m^*}\frac{d^2F_0}{dx^2}+\delta F_0-\lambda|F_0|^2F_0-i(\alpha-\sigma\lambda|F_0|^2)F_0=0, \quad (7)$$

where $\delta=E-E_n(k_0)$ is the energy detuning parameter at the energy extreme points of the nth band. In order to address the existence of dark polariton solitons, we first substitute $|F_0|^2$ in Eq. (6) by $\tilde{F}_0^{\,2}$. Then the intermediate solution $\tilde{F}_0$ of a toy model of nonequilibrium EPCs can be found through the following equation:

$$\frac{d^2\tilde{F}_0}{dx^2}+2m^*(\delta-i\alpha)\tilde{F}_0-2m^*\lambda(1-i\sigma)\tilde{F}_0^{\,3}=0. \quad (8)$$

A hyperbolic tangent solution $\tilde{F}_0=C\tanh(Bx)$ with $B=\sqrt{m^*(\delta-i\alpha)}$ and $C=\sqrt{(\delta-i\alpha)/[\lambda(1-i\sigma)]}$ can be found to satisfy Eq. (8) with a positive effective mass. Using this analytical $\tilde{F}_0$ as the initial trial solution of Eq. (7), we then apply the Newton-Raphson method to find the numerical solution of F_0 in Eq. (7). Finally, the numerical solution of Eq. (3) can be obtained by guessing $\psi(x)=F_0\psi^L_{n,k_0}(x)$ as the initial trial solution of Eq. (3).

IV. DARK GAP SOLITONS UNDER THE FIRST BAND

The effective mass near the first-band center at $k=0$ is positive. There is no abnormal dispersion to balance the repulsive interaction between exciton polaritons. A dark soliton can form spontaneously due to the defocusing nonlinearity. There is no stable dark soliton in a pure EPC [12]. However, the dark soliton pinned by a defect is stable [15]. For an EPC loaded into a periodic potential, the external periodic potential provides periodic defects to pin a dark soliton. We believe that a dark soliton inside the band gap can exist in an EPC in a periodic potential.

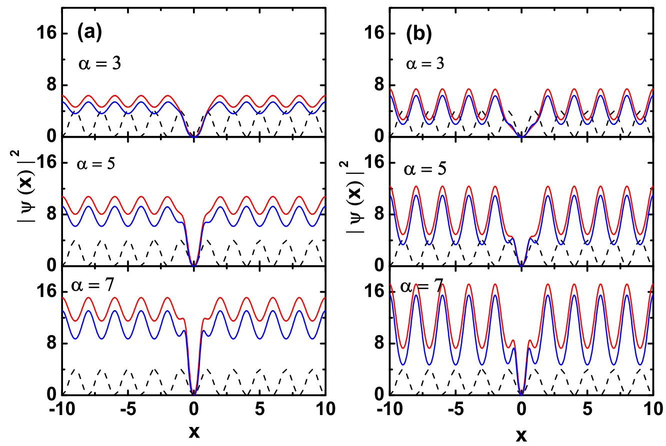

FIG. 2. (a) Numerical (red solid lines) and analytic (blue solid lines) density distributions of dark gap solitons for pump power $\alpha=3,5,7$ under $V_0=1$; (b) numerical (red solid lines) and analytic (blue solid lines) density distributions of dark gap solitons for pump power $\alpha=3,5,7$ under $V_0=3$. The periodic potential is shown by black dashed line. The energy detuning is $\delta=-0.25$, which is located below the first band.

To find the steady state of a DGS, we solve Eq. (3) numerically using the Newton-Raphson method. Figure 2 shows the density distributions of DGSs whose energy is below the first-band center at $k=0$. Only a very low pump power, $\alpha=0.4$ for $\delta=-0.25$, can start generating a density dip on a periodically modulated density background. The threshold pump power of generating a DGS is decreasing as the excitation energy is getting closer from below to the minimum energy point of the first band and the value of the detuning parameter $|\delta|$ becomes smaller. Nevertheless, changing the value of $|\delta|$ has a minor effect on the density-dip structure of the DGS in this regime. Therefore, we only show density plots of DGSs for only one detuning value, $\delta=-0.25$, in Fig. 2. With increasing the pump power, the nonzero density background is increased and the width of the density dip of the DGS becomes smaller and deeper. The periodically modulated strength of the background keeps approximately 30% of the background density. The structure of the density dip reveals two humps if the pump power is strong enough (see density plots for $\alpha=7$, Fig. 2). Moreover, the periodically modulated strength of the background is bigger as the strength of the external potential goes stronger. Then we can see that the contrast of the dip becomes effectively smaller even though the interior structure is not significantly affected [compare density distributions shown in Figs. 2(a) and 2(b)].

V. DARK GAP SOLITONS INSIDE THE ENERGY GAP BETWEEN THE FIRST AND SECOND BANDS

After showing the first kind of DGSs under the first band, there is another energy gap between the first and second bands. The effective mass near the second-band edge at $k=\pi/2$ is positive. The nonlinearity of an EPC is defocusing. Then we intend to show the existence of the second kind of DGSs occurring inside the energy gap between the first and second band edges at $k=\pi/2$. The density distributions of these

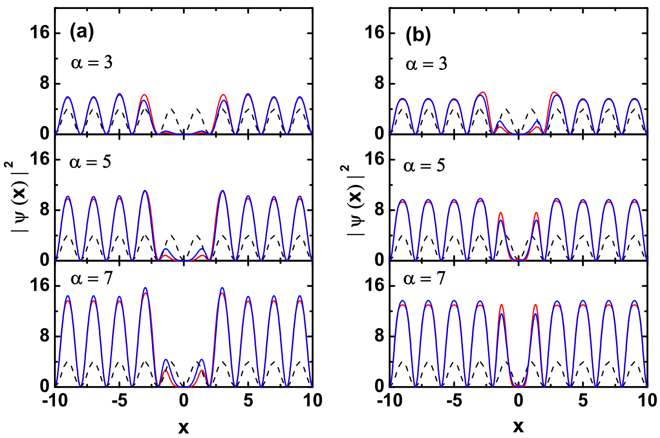

FIG. 3. (a) Numerical (red solid lines) and analytic (blue solid lines) density distributions of dark gap solitons for pump power $\alpha = 3,5,7$ under $V_0 = 1$; (b) numerical (red solid lines) and analytic (blue solid lines) density distributions of dark gap solitons for pump power $\alpha = 3,5,7$ under $V_0 = 3$. The periodic potential is shown by black dashed line. The energy detuning is $\delta = -0.25$ inside the band gap of first and second band.

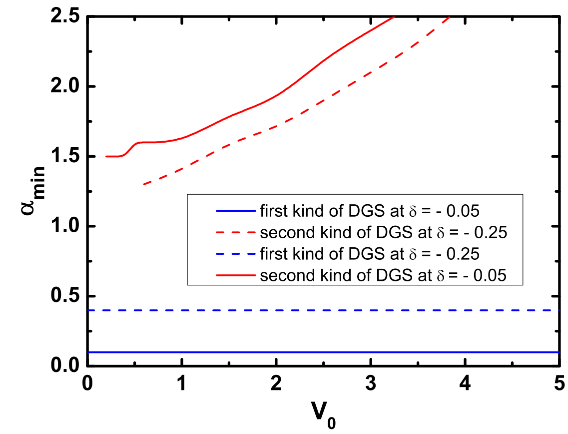

FIG. 4. Minimum required pump power ($\alpha_{\min}$) as a function of the potential depth at $\delta = -0.05$ and $\delta = -0.25$.

DGSs are shown in Fig. 3. There is a density gap around the center of the periodic density distribution without a nonzero density background. A minimum pump strength is required to obtain the DGS, and the threshold pump power increases with the potential depth. On increasing the potential depth, the width of the DGS becomes narrower. However, the periodic density distribution is not significantly affected. We find that the density structure of the DGS is strongly affected by the pump strength. As the pump power is increasing, the width of the DGS also becomes narrower. By either increasing the potential depth or pump power, two humps inside the density gap grow up gradually and the inner density structure of the DGS becomes narrower.

In contrast to the nonzero density background for the first kind of DGSs, the background density is changing between zero and nonzero densities periodically. The second kind of a DGS here is generated on the top of a p-type (antisymmetric) Bloch wave function coming from the minimum of the second band at $k = \pi/2$ and this wave function has nodes on the valleys of the potential. Every π-phase shift of the wave function near a space point causes a density notch near that point. In the center of the DGS, the density notch together with an area of a π-phase change can be observed. A higher pump power shrinks the area, and thus a narrower DGS width has happened. Although no typical density and phase structure of a DS in a uniform system is shown in the system, the localized nature of a DGS with the distinct phase gradient across its notch occurs here. The defocusing nonlinearity and phase gradient tend to reduce and enlarge the width of the notch of a DGS, respectively. A DGS is formed due to the balance between the defocusing nonlinearity and phase gradient. Note that, as we reduce the energy detuning parameter $|\delta|$ to the minimum energy point at the second band edge, the threshold pump power to find a DGS would be increasing. On the other hand, a changed $|\delta|$ has a minor effect on changing the interior structure of DGSs.

The threshold pump power ($\alpha_{\min}$) for the existence of DGSs as a function of the potential depth V_0 is depicted in Fig. 4. The DGSs below the first band start forming as the pump power is larger than $\gtrsim 0.4$ for $\delta = -0.25$. For a smaller detuning of $\delta = -0.05$, $\alpha_{\min}$ decreases to $\gtrsim 0.1$. However, increasing the potential depth doesn't affect the threshold power. On the other hand, the threshold power of the second kind of DGSs increases with the potential depth. $\alpha_{\min}$ decreases as the excitation energy of a DGS lies deep inside the band gap.

VI. CONCLUSIONS

The exciton-polariton condensate in a one-dimensional periodic potential has been studied using the effective-mass approach. We predicted the existence of dark gap solitons of exciton-polariton condensates in semiconductor microcavities. Two kinds of dark gap solitons exist in the system with lower potential depths. One is the dark gap soliton with a density dip in a nonzero density background. Another is the dark gap soliton with a density gap in an oscillating density background. Both dark gap solitons can exist for various potential depths and pump powers. Our studies reveal methods for creating and controlling dark solitons in a nonequilibrium and nonlinear system.

We mentioned two kinds of dark gap solitons; the third kind of dark gap soliton can also be found inside the gap between the second and third bands at the zone center. It can be excited from the relaxation of the high-energy exciton polaritons to the low-energy band. However, the dark gap soliton of the higher band can be found only at a deeper lattice potential and higher pump powers. The band gap of the higher band opens only at a deeper lattice potential as shown in Fig. 1(a). On increasing the lattice potential depth, the interior structure of a dark gap soliton is not affected. Furthermore, the width of a dark gap soliton becomes narrower as the pump power becomes stronger.

ACKNOWLEDGMENT

We acknowledge the financial support from the Ministry of Science and Technology of the Republic of China under Contracts No. MOST105-2112-M-034-001-MY3 and No. MOST105-2112-M-415-009-MY3.

[1] G. I. Stegeman and M. Segev, Science **286**, 1518 (1999).
[2] J. S. Russell, Report of the 14th Meeting of the British Association for the Advancement of Science XLVII–LVII, 1845 (unpublished), p. 311.
[3] F. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. **71**, 463 (1999).
[4] A. J. Leggett, Rev. Mod. Phys. **73**, 307 (2001).
[5] E. A. Cornell and C. E. Wieman, Rev. Mod. Phys. **74**, 875 (2002).
[6] W. Ketterle, Rev. Mod. Phys. **74**, 1131 (2002).
[7] B. Eiermann, T. Anker, M. Albiez, M. Taglieber, P. Treutlein, K. P. Marzlin, and M. K. Oberthaler, Phys. Rev. Lett. **92**, 230401 (2004).
[8] B. Eiermann, P. Treutlein, T. Anker, M. Albiez, M. Taglieber, K. P. Marzlin, and M. K. Oberthaler, Phys. Rev. Lett. **91**, 060402 (2003).
[9] P. Meystre, *Atom Optics* (Springer-Verlag, New York, 2001).
[10] J. Kasprzak *et al.*, Nature (London) **443**, 409 (2006).
[11] H. Deng, H. Haug, and Y. Yamamoto, Rev. Mod. Phys. **82**, 1489 (2010).
[12] Y. Xue and M. Matuszewski, Phys. Rev. Lett. **112**, 216401 (2014).
[13] A. Amo *et al.*, Science **332**, 1167 (2011).
[14] M. Sich *et al.*, Nat. Photon. **6**, 50 (2012).
[15] T. W. Chen, W. F. Hsieh, and S. C. Cheng, Opt. Express **23**, 24974 (2015).
[16] D. Mandelik, R. Morandotti, J. S. Aitchison, and Y. Silberberg, Phys. Rev. Lett. **92**, 093904 (2004).
[17] C. W. Lai *et al.*, Nature (London) **450**, 529 (2007).
[18] N. Y. Kim *et al.*, Nat. Phys. **7**, 681 (2011).
[19] E. A. Cerda-Méndez, D. N. Krizhanovskii, M. Wouters, R. Bradley, K. Biermann, K. Guda, R. Hey, P. V. Santos, D. Sarkar, and M. S. Skolnick, Phys. Rev. Lett. **105**, 116402 (2010).
[20] D. N. Krizhanovskii *et al.*, Phys. Rev. B **87**, 155423 (2013).
[21] D. Tanese, H. Flayac, D. Solnyshkov, A. Amo, A. Lematre, E. Galopin, R. Braive, P. Senellart, I. Sagnes, G. Malpuech *et al.*, Nat. Commun. **4**, 1749 (2013).
[22] E. A. Cerda-Méndez, D. Sarkar, D. N. Krizhanovskii, S. S. Gavrilov, K. Biermann, M. S. Skolnick, and P. V. Santos, Phys. Rev. Lett. **111**, 146401 (2013).
[23] J. V. T. Buller, R. E. Balderas-Navarro, K. Biermann, E. A. Cerda-Méndez, and P. V. Santos, Phys. Rev. B **94**, 125432 (2016).
[24] X. Ma, O. A. Egorov, and S. Schumacher, Phys. Rev. Lett. **118**, 157401 (2017).
[25] F. Pinsker and H. Flayac, Phys. Rev. Lett. **112**, 140405 (2014).
[26] J. Keeling and N. G. Berloff, Phys. Rev. Lett. **100**, 250401 (2008).
[27] M. H. Szymańska, J. Keeling, and P. B. Littlewood, in *Quantum Gases: Finite Temperature and Non-equilibrium Dynamics*, edited by N. P. Proukakis, S. Gardiner, M. J. Davis, and M. H. Szymańska (Imperial College Press, London, 2013), Part III, p. 451.
[28] V. A. Brazhnyi, V. V. Konotop, and V. M. Pérez-García, Phys. Rev. Lett. **96**, 060403 (2006).
[29] C. M. de Sterke, D. G. Salinas, and J. E. Sipe, Phys. Rev. E **54**, 1969 (1996).