Collective spin correlations and entangled state dynamics in coupled quantum dots

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Here we demonstrate that the dynamics of few-electron states in a correlated quantum-dot system coupled to an electronic reservoir is governed by the symmetry properties of the total system leading to the collective behavior of all the electrons. Time evolution of two-electron states in a correlated double quantum dot after coupling to the reservoir has been analyzed by means of kinetic equations for pseudoparticle occupation numbers with constraint on possible physical states. It was revealed that the absolute value of the spin correlation function and the degree of entanglement for two-electron states could considerably increase after coupling to the reservoir. The obtained results demonstrate the possibility of a controllable tuning of both the spin correlation function and the concurrence value in a coupled quantum-dot system by changing of the gate voltage applied to the barrier separating the dots.

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I. INTRODUCTION

Entangled spin correlated states physics is a rapidly growing area of research in semiconductor nanostructure science because it is of great importance for the development of quantum information technologies [1]. Creation, manipulation, and detection of entangled electrons is the central problem which opens the possibility of integrating solid-state two-level (qubit) systems in quantum circuits applicable for quantum information and quantum computing processing [2,3]. Few-electron quantum-dot (QD) systems are currently being actively investigated both theoretically and experimentally since their single- and two-electron states can be well initialized, investigated, and processed [4–12].

Entangled state properties are mostly analyzed in the stationary case. But time evolution of spin and charge configurations in correlated QDs after coupling to the reservoir should be also investigated, as nonstationary characteristics provide even more complete information about the properties of ultra-small-size systems [13–20]. Preparation of different initial states with several electrons in correlated systems [21–24] from simple product states to complex entanglements is one of the most interesting and vital problems in solid-state physics. It was demonstrated that spatially separated electrons can form entangled states with a particular spin configuration in QDs [25,26]. For proper treatment of such states, nonstationary currents flowing through QD systems have to be analyzed. In double-correlated QDs, an entangled state can appear as an eigenstate with a particular number of electrons [7,27,28]. Moreover, entangled states in correlated quantum dots can be controlled by changing the applied bias and gate voltage [29,30] or by external laser pulses [31,32]. It has been demonstrated that the preparation of the initial state in QDs strongly affects the entanglement dynamics, which can be further enhanced during the time evolution of a two-qubit system [33].

In the present paper we analyze the time evolution of the spin correlation function and the degree of entanglement for two-electron states initially prepared in a correlated double QD after it has been coupled to the electronic reservoir. We consider the kinetic equations for pseudoparticle occupation numbers with a constraint on possible physical states. Typically it is assumed that coupling to the reservoir destroys the correlations in the subsystem with localized interacting electrons. However, we demonstrate that the absolute value of the spin correlation function and the degree of entanglement for two-electron states can considerably increase after coupling to the reservoir due to collective dynamics of localized and reservoir electrons governed by the symmetry properties of the whole system. The possibility of control by changing the gate voltage for relative occupation between the singlet and triplet states has been revealed. Our results demonstrate that spin correlations and the degree of entanglement can be controllably tuned by changing the gate voltage applied to the barrier separating the quantum dots as well as by tuning the bias voltage applied to the reservoir.

II. THEORETICAL MODEL

We consider a system of two coupled single-level quantum dots with energy levels ε_1 and ε_2 connected to an electronic reservoir. The Hamiltonian of the system,

$$\hat{H} = \hat{H}_{\rm dot} + \hat{H}_{\rm res} + \hat{H}_{\rm tun}, \tag{1}$$

is written as the sum of the Hamiltonian describing the quantum dots

$$\hat{H}_{\rm dot} = \sum_{l\sigma} \varepsilon_l \hat{c}^+_{l\sigma}\hat{c}_{l\sigma} + U_1 \hat{n}^{\sigma}_{11}\hat{n}^{-\sigma}_{11} + U_2 \hat{n}^{\sigma}_{22}\hat{n}^{-\sigma}_{22} + \sum_{\sigma} T(\hat{c}^+_{1\sigma}\hat{c}_{2\sigma} + \hat{c}^+_{2\sigma}\hat{c}_{1\sigma}), \tag{2}$$

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the reservoir part

$$\hat{H}_{\rm res} = \sum_{k\sigma} (\varepsilon_k - eV)\, \hat{c}^+_{k\sigma} \hat{c}_{k\sigma}, \tag{3}$$

and the tunneling Hamiltonian

$$\hat{H}_{\rm tun} = \sum_{lk\sigma} t(\hat{c}^+_{k\sigma} \hat{c}_{l\sigma} + \hat{c}^+_{l\sigma} \hat{c}_{k\sigma}), \tag{4}$$

where ε_l ($l = 1,2$) are the spin-degenerate single-electron levels and U_l is the on-site Coulomb repulsion for double occupation of quantum dots. The creation or annihilation of an electron with spin $\sigma = \pm 1$ within a dot is denoted by operators $\hat{c}^+_{l\sigma}$ and $\hat{c}_{l\sigma}$, and $\hat{n}^\sigma_{ll}$ is the corresponding occupation number operator. Operator $\hat{c}^+_{k\sigma}$ ($\hat{c}_{k\sigma}$) creates (annihilates) an electron with spin σ and momentum k in the reservoir. The coupling between dots T and the tunneling transfer amplitude to the reservoir t are considered to be independent of the momentum and spin. eV is the value of the external bias voltage applied to the reservoir. For simplicity in what follows we consider nearly identical QDs ($\varepsilon_1 = \varepsilon_2 = \varepsilon$ and $U_1 = U_2 = U$) and assume the symmetric coupling to the reservoir: electrons can transfer from both quantum dots to the reservoir and back with the same tunneling amplitude t. The degree of entanglement of a two-qubit system in the transport regime strongly depends on the way of coupling to the reservoir [34].

When the interaction between QDs exceeds the coupling strength to the reservoir, it is reasonable to use the basis of exact eigenfunctions and eigenvalues of coupled QDs without interaction with the reservoir. In this case all energies of single- and multielectron states are well known.

In case of one electron in the system, there exist two single-electron states with energies $\varepsilon_i = \varepsilon \pm T$ ($i = a,s$) and the wave function

$$\Psi^\sigma_i = \mu_i |0\uparrow\rangle|00\rangle + \nu_i |00\rangle|0\uparrow\rangle, \tag{5}$$

where $|0\uparrow\rangle|00\rangle$ and $|00\rangle|0\uparrow\rangle$ are the basis functions corresponding to the presence of a single electron in each quantum dot. Six two-electron states are present in the system: two states with the same spin direction $T^+ = |\uparrow 0\rangle|\uparrow 0\rangle$, $T^- = |\downarrow 0\rangle|\downarrow 0\rangle$, which correspond to the existence of two electrons localized in two different quantum dots and four states with the opposite spins and the corresponding wave function

$$\begin{aligned}\Psi^{\sigma-\sigma}_j = {} & \alpha_j |\uparrow\downarrow\rangle|00\rangle + \beta_j |\downarrow 0\rangle|0\uparrow\rangle \\ & + \gamma_j |0\uparrow\rangle|\downarrow 0\rangle + \delta_j |00\rangle|\uparrow\downarrow\rangle,\end{aligned} \tag{6}$$

where functions $|\uparrow\downarrow\rangle|00\rangle$, $|00\rangle|\uparrow\downarrow\rangle$ describe two electrons localized in the same dot with the opposite directions of the spin, and functions $|\downarrow 0\rangle|0\uparrow\rangle$, $|0\uparrow\rangle|\downarrow 0\rangle$ correspond to electrons localized in different dots.

For identical quantum-dot states $T^\pm$ with energies 2ε, there also exist four two-electron states with opposite spins and energies $E^{\sigma\sigma'}_j$: 2ε, $2\varepsilon + U$, and $2\varepsilon + \frac{U}{2} \pm \sqrt{\frac{U^2}{4} + 4T^2}$. These states are low-energy singlet S^0 and triplet T^0 states and excited singlet and triplet states S^{0*} and T^{0*}.

There also exist two three-electron states with the wave function

$$\begin{aligned}\Psi^{\sigma\sigma-\sigma}_m = {} & p_m |\uparrow\downarrow\rangle|\uparrow 0\rangle + q_m |\uparrow 0\rangle|\uparrow\downarrow\rangle, \\ & m = \pm 1.\end{aligned} \tag{7}$$

Index m determines the relative sign of the coefficients p_m and q_m. The basis functions $|\uparrow\downarrow\rangle|\uparrow 0\rangle$ and $|\uparrow 0\rangle|\uparrow\downarrow\rangle$ represent one dot occupied by two opposite spin electrons and another quantum dot with the single electron correspondingly. Finally, a single four-electron state is present in the system, and its wave function

$$\Psi_n = |\uparrow\downarrow\rangle|\uparrow\downarrow\rangle \tag{8}$$

describes the situation when both quantum dots are fully occupied (two opposite spin electrons are present in each dot).

Correlated QD kinetics can be analyzed by means of the pseudoparticle formalism [35,36]. In this theoretical approach, pseudoparticles are introduced for each eigenstate of the system in question. Consequently, the electron operator $\hat{c}^+_{l\sigma}$ ($l = 1,2$) should be written as a combination of pseudoparticle operators:

$$\begin{aligned}\hat{c}^+_{l\sigma} = {} & \sum_i \Theta^{\sigma l}_i \hat{f}^+_{i\sigma} \hat{b} + \sum_{ji\sigma} \Phi^{\sigma-\sigma l}_{ji} \hat{d}^{+\sigma-\sigma}_j \hat{f}_{i-\sigma} \\ & + \sum_{i\sigma} \Phi^{\sigma\sigma l}_i \hat{d}^{+\sigma\sigma} \hat{f}_{i\sigma} + \sum_{mj\sigma} \Omega^{\sigma\sigma-\sigma l}_{mj} \hat{\psi}^+_{m-\sigma} \hat{d}^{\sigma-\sigma}_j \\ & + \sum_{m\sigma} \Omega^{\sigma-\sigma-\sigma l}_m \hat{\psi}^+_{m\sigma} \hat{d}^{-\sigma-\sigma} + \sum_{m\sigma} \Pi^{\sigma-\sigma-\sigma l}_m \hat{\varphi}^+ \hat{\psi}_{m\sigma},\end{aligned} \tag{9}$$

with constraint on possible physical states

$$\hat{N}_b + \sum_{i\sigma} \hat{N}_{i\sigma} + \sum_{j\sigma\sigma'} \hat{N}^{\sigma\sigma'}_j + \sum_{m\sigma} \hat{N}_{\psi m\sigma} + \hat{N}_\varphi = 1, \tag{10}$$

where $\hat{f}^+_\sigma$ ($\hat{f}_\sigma$) and $\hat{\psi}^+_\sigma$ ($\hat{\psi}_\sigma$) are pseudofermion creation (annihilation) operators for electronic states with one and three electrons correspondingly. $\hat{b}^+(\hat{b})$, $\hat{d}^{+\sigma}(\hat{d}^\sigma)$, and $\hat{\varphi}^+(\hat{\varphi})$ are slave boson operators, corresponding to states without electrons, with two electrons or four electrons. Operators $\hat{\psi}^+_{m-\sigma}$ describe a system configuration with three electrons: two of them are spin up electrons σ and one is a spin down electron $-\sigma$ in spatially symmetric and asymmetric states. $\Theta^{\sigma l}_i$, $\Phi^{\sigma-\sigma l}_{ji}$, $\Phi^{\sigma\sigma l}_i$, $\Omega^{\sigma\sigma-\sigma l}_{mj}$, $\Omega^{\sigma-\sigma-\sigma l}_m$, and $\Pi^{\sigma-\sigma-\sigma l}_m$ are matrix elements of the creation operators $\hat{c}^+_{l\sigma}$ between the states with n and $n+1$ electrons [37]. They can be simply expressed through eigenvectors defined by expressions (5)–(8).

In the presence of Coulomb interactions, the excited double-occupied electron states as well as three- and four-particle states are separated by a Coulomb gap from single- and low-energy two-electron states. Consequently, all terms containing operators $\hat{\varphi}^+$ and $\hat{\psi}^+_{m-\sigma}$ in expressions (7)–(10) can be omitted.

The total electron occupation in coupled QDs can be expressed in terms of the pseudoparticle occupation numbers:

$$\sum_{l,\sigma} \hat{n}^\sigma_{ll} = \sum_{i=a,s,\sigma} \hat{N}^\sigma_i + 2 \sum_j \hat{N}^{\sigma-\sigma}_j + 2 \sum_{j\sigma} \hat{N}^{\sigma\sigma}_j. \tag{11}$$

One can derive equations for the pseudoparticle occupation numbers N^σ_i, $N^{\sigma-\sigma}_j$, $N^{\sigma\sigma}_j$, and N_b by averaging equations of motion for the operators and by decoupling the electron occupation numbers in quantum dots from the reservoir occupation numbers. Such a decoupling procedure is reasonable provided that Kondo correlations can be neglected [38,39]. So, after taking into account the constraint on possible physical states, the following nonstationary system of equations can be

obtained for the pseudoparticle occupation numbers:

$$\frac{\partial N_j^{\sigma-\sigma}}{\partial t} = -\sum_{i\sigma}\left[\lambda_{ji}^{\sigma-\sigma}\left(1-N_{k-\sigma}^{ji}\right)N_j^{\sigma-\sigma} - \lambda_{ji}^{\sigma-\sigma}N_{k-\sigma}^{ji}N_i^{\sigma}\right],$$

$$\frac{\partial N_i^{\sigma}}{\partial t} = \sum_{j}\left[\lambda_{ji}^{\sigma-\sigma}\left(1-N_{k-\sigma}^{ji}\right)N_j^{\sigma-\sigma} + \lambda_{ji}^{\sigma\sigma}\left(1-N_{k\sigma}^{ji}\right)N_j^{\sigma\sigma}\right] - \sum_{j}\left[\lambda_{ji}^{\sigma-\sigma}N_{k-\sigma}^{ji}N_i^{\sigma} - \lambda_i\left(1-N_{k\sigma}^{i}\right)N_i^{\sigma} + \lambda_i N_{k\sigma}^{i}N_b - \lambda_{ji}^{\sigma\sigma}N_{k\sigma}^{ji}N_i^{\sigma}\right],$$

$$\frac{\partial N_b}{\partial t} = \sum_{i\sigma}\lambda_i\left[N_i^{\sigma}\left(1-N_{k\sigma}^{i}\right) - N_{k\sigma}^{i}N_b\right],$$

$$\frac{\partial N_j^{\sigma\sigma}}{\partial t} = -\sum_{i}\left[\lambda_{ji}^{\sigma\sigma}\left(1-N_{k\sigma}^{ji}\right)N_j^{\sigma\sigma} - \lambda_{ji}^{\sigma\sigma}N_{k\sigma}^{ji}N_i^{\sigma}\right], \quad (12)$$

where kinetic coefficients are

$$\lambda_i = \lambda_{ji}^{\sigma\sigma} = 2\gamma|\mu_i+\nu_i|^2,$$
$$\lambda_{ji}^{\sigma-\sigma} = 2\gamma|\alpha_j\mu_i+\beta_j\nu_i+\delta_j\nu_i+\gamma_j\mu_i|^2. \quad (13)$$

Index $i=a,s$ and the relaxation rate $\gamma=\pi\nu_0 t^2$ (ν_0 is the electronic density of states in the reservoir). Functions $N_{k-\sigma}^{ji}$ and $N_{k\sigma}^{i}$ depend on reservoir properties and have the form

$$N_{k-\sigma}^{ji} = \frac{1}{2\pi}i\int d\varepsilon_k f_k^{\sigma}(\varepsilon_k) \times\left[\frac{1}{E_j^{\sigma\sigma'}-\varepsilon_i+i\gamma_{ji}-\varepsilon_k} - \frac{1}{E_j^{\sigma\sigma'}-\varepsilon_i-i\gamma_{ji}-\varepsilon_k}\right],$$
$$N_{k\sigma}^{i} = \frac{1}{2\pi}i\int d\varepsilon_k f_k^{\sigma}(\varepsilon_k) \times\left[\frac{1}{\varepsilon_i+i\gamma_i-\varepsilon_k} - \frac{1}{\varepsilon_i-i\gamma_i-\varepsilon_k}\right], \quad (14)$$

where

$$\gamma_i = \lambda_i/2,$$
$$\gamma_{ji} = \lambda_{ji}^{\sigma-\sigma}/2, \quad (15)$$

and $f_k^{\sigma}(\varepsilon_k)$ is the Fermi distribution function of the electrons in the reservoir. Depending on the tunneling barrier width and height, the typical tunneling coupling strength γ can vary from 10 μeV [40] to 1–5 meV [41].

Kinetic equations (12) conserve the total number of quasiparticles; consequently, the constraint

$$N_b + \sum_{\sigma,i=a,s} N_i^{\sigma} + \sum_{j,\sigma,\sigma'} N_j^{\sigma,\sigma'} = 1 \quad (16)$$

is satisfied at any time moment provided that it is satisfied at $t=0$. The system of kinetic equations has to be solved with the initial conditions for each pseudoparticle occupation number. In case of symmetric coupling to the reservoir, the system of equations (12) can be solved as two independent systems of equations. One of them contains only the equations for the occupation numbers N_{T^0} and $N_{a^\pm}$ and the other one describes the dynamics of the occupation numbers $N_{T^\pm}$, $N_{S^0}(0)$, $N_{S^\pm}$, and N_b. So it is also reasonable to group the initial conditions for each system and determine them as $N_{\rm I}(0)$ and $N_{\rm II}(0)$. Due to the constraint on possible physical states one has $N_{\rm I}(0)+N_{\rm II}(0)=1$. Consequently, the corresponding initial conditions are

$$N_{\rm I}(0) = N_{T^0}(0) + 2N_{a^\pm}(0),$$
$$N_{\rm II}(0) = 2N_{T^\pm}(0) + N_{S^0}(0) + 2N_{S^\pm}(0) + N_b(0). \quad (17)$$

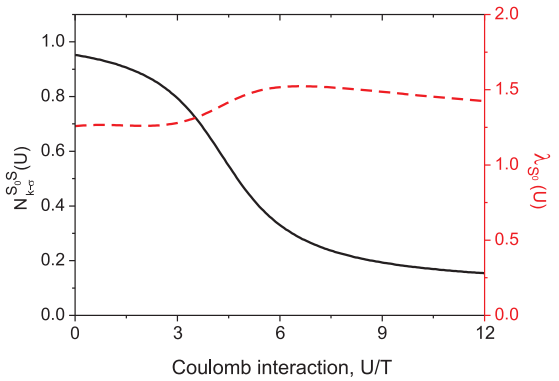

FIG. 1. $N_{k-\sigma}^{S^0S}(U)$ (black solid line) and $\lambda_{S^0}(U)$ (dashed red line) vs the ratio U/T between the Coulomb interaction strength and that of coupling between quantum dots. $\varepsilon_1/\gamma=\varepsilon_2/\gamma=-5$, $T/\gamma=15$, and $\gamma=1$.

III. MAIN RESULTS

The reservoir occupation numbers depend on the Fermi level position and thus they can be tuned by changing the applied bias voltage. We consider the situation when the Fermi level is localized between the single-electron states with energies ε_S and ε_a ($\varepsilon_S<\varepsilon_F$ and $\varepsilon_a>\varepsilon_F$). Moreover, we assume the regime of weak coupling to the reservoir ($\frac{|\varepsilon_{S(a)}-\varepsilon_F|}{\gamma}\gg 1$). It means that the functions $N_{k\sigma}^{S}$ and $N_{k-\sigma}^{T^0a}$ have their values very close to unity, while the values of the functions $N_{k-\sigma}^{T^\pm S}$ and $N_{k\sigma}^{a}$ asymptotically approach zero [see Eq. (14)]. In this case, the stationary occupation numbers of the states $T^\pm$ and b become zero ($N_{T^\pm}^{\rm st}\to 0$, $N_b^{\rm st}\to 0$), while the stationary occupation of the states S^0, T^0, $a^\pm$, and $S^\pm$ are determined as $N_{S^0}^{\rm st}=\frac{N_{\rm II}(0)N_{k-\sigma}^{S^0S}}{2-N_{k-\sigma}^{S^0S}}$, $N_{T^0}^{\rm st}=\frac{N_{\rm I}(0)N_{k-\sigma}^{T^0a}}{2-N_{k-\sigma}^{T^0a}}$, $N_{a^\pm}^{\rm st}=\frac{N_{\rm I}(0)(1-N_{k-\sigma}^{T^0a})}{2-N_{k-\sigma}^{T^0a}}$, and $N_{S^\pm}^{\rm st}=\frac{N_{\rm II}(0)(1-N_{k-\sigma}^{S^0S})}{2-N_{k-\sigma}^{S^0S}}$ (see the Appendix for pseudoparticle occupation numbers and their stationary values). So, the pseudoparticle occupation numbers in the stationary state are determined by the reservoir occupation functions $N_{k-\sigma}^{S^0S}$ and $N_k^{T^0a}$, which depend on the energies $E_{S^0}-\varepsilon_S$ and $E_{T^0}-\varepsilon_a$ correspondingly. Moreover, the triplet state T^0 does not decay for any value of the Coulomb interaction in the QD system, because the decay process is determined by the time evolution of the function $N_{k-\sigma}^{T^0a}$, which becomes equal to unity in the considered situation. The reservoir occupation function $N_{k-\sigma}^{S^0S}$ depends on the ratio between the Coulomb interaction strength U and the strength of QD coupling T. The value of T and thus the ratio U/T can be easily tuned by changing the gate voltage applied to the barrier separating the quantum dots. Figure 1 shows the occupation function $N_{k-\sigma}^{S^0S}$ plotted as a function of

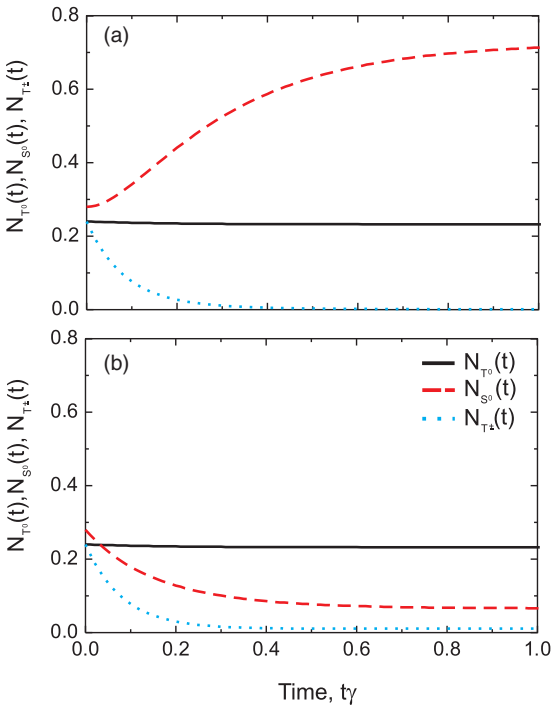

FIG. 2. Time evolution of partial pseudoparticle occupation numbers. Black curve - $N_{T^0}(t)$; dashed red curve - $N_{S^0}(t)$; and dotted blue curve - $N_{T^\pm}(t)$. (a) $U/T = 3.5$; (b) $U/T = 6.5$. Parameters $\varepsilon_1/\gamma = \varepsilon_2/\gamma = -5.0, T/\gamma = 15.0$, and $\gamma = 1$ are the same for all the figures. Initial conditions are $N_{S^0}(0) = 0.28$; $N_{T^0}(0) = N_{T^\pm}(0) = 0.24$.

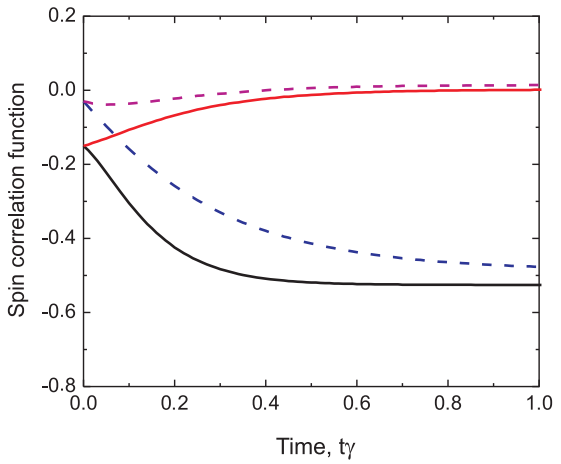

FIG. 3. Time evolution of the spin correlation function. Black and blue curves correspond to the ratio $U/T = 3$, while red and magenta curves to the ratio $U/T = 6.5$. The parameters $\varepsilon_1/\gamma = \varepsilon_2/\gamma = -5.0$, $T/\gamma = 15$, and $\gamma = 1$ are the same for all figures. The initial conditions for the solid curves are $N_{S^0}(0) = 0.4$; $N_{T^0}(0) = N_{T^\pm}(0) = 0.2$. The initial conditions for the dashed curves are $N_{S^0}(0) = 0.28$; $N_{T^0}(0) = N_{T^\pm}(0) = 0.24$.

the ratio U/T (see the black curve). Increasing this ratio leads to decrease of the stationary occupation of the singlet state S^0.

The time evolution of pseudoparticle occupation numbers for different values of Coulomb interaction (U/T) is demonstrated in Fig. 2. In the case of identical QDs with the Fermi level of the reservoir localized between single-electron states ($\varepsilon_S < \varepsilon_F$ and $\varepsilon_F < \varepsilon_a$), the triplet state T^0 does not decay at all (see the black curve in Fig. 2). The occupation of the singlet two-electron state S^0 depends on both the applied bias voltage and the ratio U/T (see the dashed red curve in Fig. 2). A particular regime could be realized in our system, when the occupation of the singlet state S^0 increases during the relaxation [see the dashed red curve in Fig. 2(a)]. For identical quantum dots with $\lambda_a = 0$, one can distinguish different inverse time scales, which determine the time evolution of the initial two-electron states:

$$\lambda_{T^0} = \lambda_S\left(1 - \frac{N^{T^0 a}_{k-\sigma}}{2}\right) \tag{18}$$

determines the time evolution of occupation numbers in the T^0 and a states. The kinetics of pseudoparticle occupation numbers N_{S^0}, $N_{T^\pm}$, and N_S is governed by the time scales λ_S, $2\lambda_S$ [see expression (13)], and

$$\lambda_{S^0} = 2\lambda_S|\alpha + \beta|^2\left(1 - \frac{N^{S^0 S}_{k-\sigma}}{2}\right). \tag{19}$$

The time evolution of the initial state depends on both the value of the applied bias and the ratio between Coulomb interaction and QD coupling strengths. The dependence of λ_{S^0} on the ratio U/T is shown in Fig. 1 as the dashed red curve.

We would like to mention that real systems are not perfect, so they are not ideally symmetrical. An asymmetry of the system gives rise to the appearance of one more typical relaxation time scale, which can be several orders larger than time scales of relaxation effects in the symmetric system. For example, the presence of a detuning between the energy levels in the quantum dots (detuning means that the quantum dots are not identical) results in the appearance of a nonzero relaxation rate $\lambda_a = \frac{\Delta\varepsilon^2}{T^2}\lambda_S$, which leads to a very slow decay of the triplet two-electron T_0 state [42]. But for $t < \lambda_a^{-1}$ the presence of an asymmetry does not influence the system behavior. So, the growth of the entanglement can be observed in this time interval.

For the two-electron mixed state one can determine the spin autocorrelation function

$$F = \langle S_1(t)S_2(t)\rangle = -\tfrac{3}{4}N_{S^0} + \tfrac{1}{4}(N_{T^0} + N_{T^+} + N_{T^-}). \tag{20}$$

The spin autocorrelation function time evolution is shown in Fig. 3. The absolute value of the spin correlation function could considerably increase after switching on of the interaction with the reservoir. Such behavior is not evident, because typically one expects that the interaction with the reservoir destroys correlations between localized spins. In our case the dynamics of the initial state is determined by the collective effects governed by the symmetry of the whole system. The spin symmetry properties of two-electron states are closely connected with the spatial symmetry of the QD-reservoir system. Consequently, more correlated spin state can appear after switching on of the "symmetric" coupling with the reservoir. One more interesting regime can be obtained when the initially single-electron state with energy ε_a is excited $\sum_\sigma N^\sigma_a(0) = 1$. Experimentally such a situation can be achieved by excitation of quantum dots via

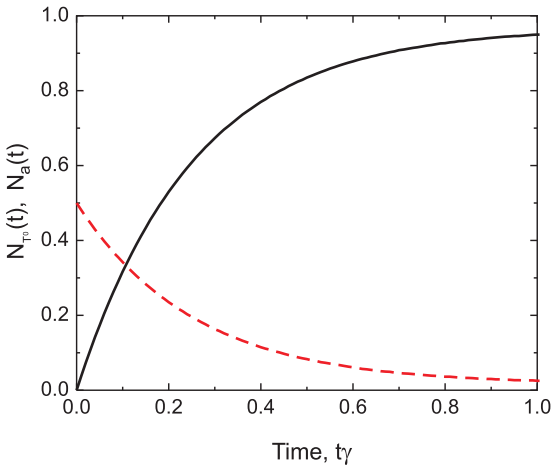

FIG. 4. Time evolution of partial pseudoparticle occupation numbers. The black curve - $N_{T^0}(t)$; the dashed red curve - $N_a(t)$. The parameters $U/T = 6.5, \varepsilon_1/\gamma = \varepsilon_2/\gamma = -5.0, T/\gamma = 15$, and $\gamma = 1$ are the same for all figures. The initial conditions are $N_a = 0.5$, $N_{S^0}(0) = N_{T^0}(0) = N_{T^\pm}(0) = 0.0$.

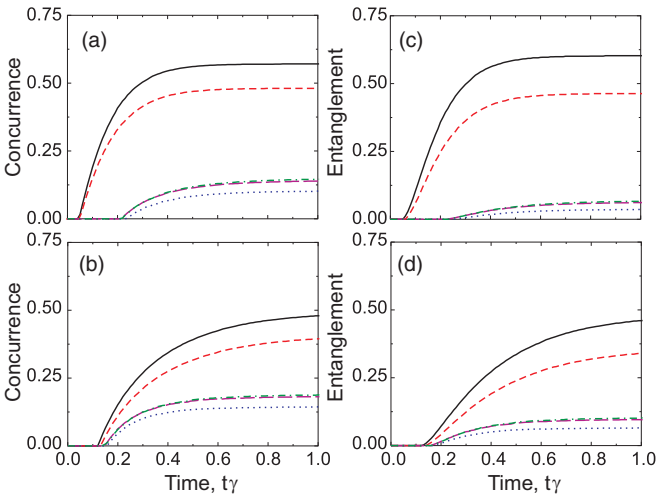

FIG. 5. Time evolution of concurrence and entanglement for different values of the ratio between the Coulomb interaction and quantum-dot coupling. Black curve - $U/T = 3.5$; dashed red curve - $U/T = 3$; dotted blue curve - $U/T = 6.5$; long dashed maroon curve $U/T = 10$ and dash-dotted green curve - $U/T = 13$. The parameters $\varepsilon_1/\gamma = \varepsilon_2/\gamma = -5.0$, $T/\gamma = 15.0$, and $\gamma = 1$ are the same for all figures. The initial conditions are (a) $N_{S^0}(0) = 0.4$; $N_{T^0}(0) = N_{T^\pm}(0) = 0.2$; (b) $N_{S^0}(0) = 0.28$; $N_{T^0}(0) = N_{T^\pm}(0) = 0.24$.

a resonant external field. After switching on of the symmetric coupling to the reservoir, the system evolves to the correlated two-electron triplet state T^0 (see Fig. 4). In this case, the stationary state is an almost pure triplet state T^0.

One can follow the changes of concurrence (the degree of entanglement) during the time evolution of the quantum-dot system. A standard measure of entanglement is the concurrence [33,34,43–47]. For each pure state of two qubits $|\psi\rangle$, the entanglement can be written as

$$E(\psi) = F(C(\psi)) = -\eta \log_2 \eta - (1-\eta)\log_2(1-\eta), \tag{21}$$

where $\eta = \frac{1}{2}[1 + \sqrt{1 - C^2(\psi)}]$ and $C(\psi) = |\langle\psi|\tilde{\psi}\rangle|$ ($|\tilde{\psi}\rangle$ is the "spin-flipped" pure state). In this case the entanglement is equal to zero for $C(\psi) = 0$. For the mixed state, the concurrence is denoted as $C(\rho) = \max\{0, \lambda_1 - \sum_i \lambda_i\}$, where $\{\lambda_i\}$ are the square roots of the matrix $\widetilde{\rho}\rho$ ($\widetilde{\rho}$ are the eigenvalues of the spin-flipped matrix ρ) arranged in decreasing order [33,34,43,47]. For the mixed state ρ of two qubits, the entanglement is determined as

$$E(\rho) = F(C(\rho)) = -\eta \log_2 \eta - (1-\eta)\log_2(1-\eta), \tag{22}$$

where $\eta = \frac{1}{2}[1 + \sqrt{1 - C^2(\rho)}]$. In both cases $F(C)$ is a monotonically increasing function as C goes from zero to unity [$F(0) = 0$], so one can take concurrence as a measure of entanglement in its own right. In the frame of the pseudoparticle approach, the two-electron density matrix λ_i can be simply expressed through the pseudoparticle occupation numbers corresponding to two-electron states. The two-electron mixed state concurrence can be also determined through the spin autocorrelation function F. When only the two-electron states are available, one has $C = \max\{0, -2F - \frac{1}{2}\}$. The time evolution of concurrence for the initially mixed state demonstrates the threshold behavior (see Fig. 5). The time at which a nonzero concurrence first appears in the system can be tuned by changing the gate voltage (the system parameter U/T). Moreover, the concurrence value increases during time evolution and reaches its maximum value in the steady state. The stationary value of the concurrence is determined as $C_{\rm st} = \max\{0, -2F_{\rm st} - 1/2\}$. The steady-state value $F_{\rm st}$ can be obtained from Eq. (20), where the stationary pseudoparticle occupation numbers are involved (see the Appendix for expressions determining the stationary pseudoparticle occupation numbers). Figures 5(c) and 5(d) show time evolution of entanglement in the system initially prepared in a mixed state obtained from Eq. (22). Figures 5(a)–5(d) reveal a very similar behavior for two different definitions of entanglement and, consequently, prove that our criteria are correct.

We would like to mention that in bipartite composite systems of distinguishable particles there exist several equivalent criteria for revealing the entanglement, and there is no consensus about how to quantify the entanglement of identical particle systems. In the case of identical fermion particles, the true entanglement should be carefully evaluated by eliminating the natural nonseparability imposed by the Pauli principle. In such systems, unavoidable correlations arising from the truly indistinguishable nature of particles are sometimes confused with the genuine entanglement due to the correlations. A very important question is whether the fermionic system is initially entangled due to the fermionic nature of electrons. In our theoretical approach we applied the second quantization formalism, so that the Pauli principle is explicitly included in commutation relations for fermionic creation and annihilation operators. Moreover, our results imply that only by choosing the proper basis of eigenstates does one distinguish truly entangled states from the false ones [33,34]. The wave function for two-electron states with opposite spins is described by Eq. (7). Without Coulomb correlations, basic two-electron states can be

represented as a simple product of single-electron states. This means that this state is not entangled. The presence of Coulomb correlations allows the possibility of a real entanglement in the system. In this case two-electron states cannot be represented as a simple product of single-electron states. By using the second quantization approach and choosing the correct basic functions we exclude the possibility of a false entanglement of a two-qubit fermionic system.

IV. CONCLUSIONS

We demonstrated that both spin correlations and the degree of entanglement in the system of two correlated QDs increase after it has been coupled to the reservoir. This effect appears because the dynamics of several-electron states in the system of correlated quantum dots is governed by the symmetry properties of the whole system, giving rise to a collective behavior of all electrons. The typical relaxation time scales and the stationary electron distribution was calculated by means of kinetic equations for the pseudoparticle occupation numbers considering constraint on the possible physical states. It was revealed that depending on the ratio between the strength of Coulomb interactions and that of coupling between quantum dots, not only the initial triplet state T^0 but also the singlet state S^0 can be approximately stable. Thus one can control the relative occupation of singlet and triplet states by changing the gate voltage. Our results demonstrate the possibility of controllable concurrence tuning by changing the gate voltage applied to the barrier separating the quantum dots and the bias voltage applied to the reservoir.

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APPENDIX

In the case of identical QDs, the pseudoparticle occupation numbers obtained from Eqs. (12) have the form

$$\begin{aligned}
N_{T^0}(t) &= N_{T^0}^{\mathrm{st}} + \left[N_{T^0}(0) - N_{T^0}^{\mathrm{st}}\right] e^{-\lambda_{T^0} t},\\
N_{S^0}(t) &= N_{S^0}^{\mathrm{st}} + \left[N_{S^0}(0) - N_{S^0}^{\mathrm{st}}\right] e^{-\lambda_{S^0} t},\\
N_{a^\pm}(t) &= N_{a^\pm}^{\mathrm{st}} + \left[N_{a^\pm}(0) - N_{a^\pm}^{\mathrm{st}}\right] e^{-\lambda_{T^0} t},\\
N_{S^\pm}(t) &= N_{S^\pm}^{\mathrm{st}} + \left[N_{S^\pm}(0) - N_{S^\pm}^{\mathrm{st}}\right] e^{-\lambda_{S^0} t},\\
N_{T^\pm}(t) &= N_{T^\pm}^{\mathrm{st}} + \left[N_{T^\pm}(0) - N_{T^\pm}^{\mathrm{st}}\right] e^{-\lambda_{S^0} t},\\
N_b(t) &= N_b^{\mathrm{st}} + \left[N_b(0) - N_b^{\mathrm{st}}\right] e^{-\lambda_{S^0} t}.
\end{aligned} \tag{A1}$$

The stationary values of partial pseudoparticles occupation numbers are

$$\begin{aligned}
N_{T^\pm}^{\mathrm{st}} &= \frac{N_{\mathrm{II}}(0) N_{k\sigma}^{S} N_{k-\sigma}^{T^\pm S}\left(1 - N_{k-\sigma}^{S^0 S}\right)}{Z},\\
N_{S^0}^{\mathrm{st}} &= \frac{N_{\mathrm{II}}(0) N_{k\sigma}^{S} N_{k-\sigma}^{S^0 S}\left(1 - N_{k-\sigma}^{T^\pm S}\right)}{Z},\\
N_{S^\pm}^{\mathrm{st}} &= \frac{N_{\mathrm{II}}(0) N_{k\sigma}^{S}\left(1 - N_{k-\sigma}^{S^0 S}\right)\left(1 - N_{k-\sigma}^{T^\pm S}\right)}{Z},\\
N_{b}^{\mathrm{st}} &= \frac{N_{\mathrm{II}}(0)\left(1 - N_{k\sigma}^{S}\right)\left(1 - N_{k-\sigma}^{S^0 S}\right)\left(1 - N_{k-\sigma}^{T^\pm S}\right)}{Z},\\
N_{T^0}^{\mathrm{st}} &= \frac{N_{\mathrm{I}}(0) N_{k-\sigma}^{T^0 a}}{2 - N_{k-\sigma}^{T^0 a}},\\
N_{a^\pm}^{\mathrm{st}} &= \frac{N_{\mathrm{I}}(0)\left(1 - N_{k-\sigma}^{T^0 a}\right)}{2 - N_{k-\sigma}^{T^0 a}},
\end{aligned} \tag{A2}$$

and

$$\begin{aligned}
Z = {} & 2\left(1 - N_{k-\sigma}^{S^0 S}\right)\left(1 - N_{k-\sigma}^{T^\pm S}\right) N_{k\sigma}^{S} + N_{k-\sigma}^{S^0 S} N_{k\sigma}^{S}\left(1 - N_{k-\sigma}^{T^\pm S}\right)\\
& + 2 N_{k-\sigma}^{T^\pm S} N_{k\sigma}^{S}\left(1 - N_{k\sigma}^{S^0 S}\right)\\
& + \left(1 - N_{k\sigma}^{S}\right)\left(1 - N_{k-\sigma}^{T^\pm S}\right)\left(1 - N_{k-\sigma}^{S^0 S}\right).
\end{aligned} \tag{A3}$$

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