Orthogonality catastrophe and fractional exclusion statistics

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We show that the N-particle Sutherland model with inverse-square and harmonic interactions exhibits orthogonality catastrophe. For a fixed value of the harmonic coupling, the overlap of the N-body ground state wave functions with two different values of the inverse-square interaction term goes to zero in the thermodynamic limit. When the two values of the inverse-square coupling differ by an infinitesimal amount, the wave function overlap shows an exponential suppression. This is qualitatively different from the usual power law suppression observed in the Anderson's orthogonality catastrophe. We also obtain an analytic expression for the wave function overlaps for an arbitrary set of couplings, whose properties are analyzed numerically. The quasiparticles constituting the ground state wave functions of the Sutherland model are known to obey fractional exclusion statistics. Our analysis indicates that the orthogonality catastrophe may be valid in systems with more general kinds of statistics than just the fermionic type.

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I. INTRODUCTION

The phenomenon of orthogonality catastrophe (OC) as discussed by Anderson [1] demonstrates that in the thermodynamic limit, the perturbed ground state of certain fermionic quantum systems is orthogonal to the ground state in the absence of the perturbation. The overlap between the two fermionic ground states is usually suppressed by a power law, which goes to zero in the thermodynamic limit. OC has been observed in Kondo systems [2,3] semiconductor quantum dots [4–6], graphene [7], Luttinger liquids [8–12], and various other physical systems. Recently a study of statistical OC [13] has led to the possibility of an exponential decay of the wave function overlap [14], in contrast to the usual power law suppression [1]. A particular way to introduce perturbations in a system is provided by a quench, in which the perturbation could be turned on suddenly or over a small period of time. For a sudden quench, the old ground state is no longer the ground state of the perturbed system, but can be expanded in terms of the complete set of eigenstates of the quenched Hamiltonian. The overlap of the ground states before and after the quench can be used to study the OC.

The OC has been studied primarily in fermionic systems which obey the Fermi-Dirac statistics. It is therefore interesting to ask if nonfermionic systems, such as those exhibiting fractional exclusion statistics [15–17] can also demonstrate OC. In certain fractional quantum Hall systems, where the Laughlin quasiparticles satisfy more general statistics, indirect effects of the OC have been observed mainly through the suppression of the conductance peaks in the thermodynamic limit [18,19]. However, to our knowledge there has been no direct demonstration of OC in terms of the suppression of the wave function overlap for systems with fractional exclusion statistics.

In this paper we want to investigate the existence of the OC in one-dimensional quantum systems with fractional exclusion statistics [15–17], which is considered as a generalization of the fermionic case. The N-body Calogero type systems [20–22] with inverse-square and harmonic interactions exhibit fractional exclusion statistics [23–26]. The inverse-square interaction is not merely a mathematical curiosity but actually appears in a wide variety of physical situations, including conformal quantum mechanics [27–29], polar molecules [30,31], quantum Hall effect [32], Tomonaga-Luttinger liquid [33], and black holes [34–37] as well as in graphene with a Coulomb charge [38–45]. Following the solutions originally obtained by Calogero [20–22], systems with inverse-square interactions have been analyzed with a variety of different techniques [46–51], and the study of OC with such an interaction is of potential interest for a wide class of physical systems.

Soon after the appearance of the Calogero model, Sutherland [52,53] proposed a variation of that which also exhibits fractional exclusion statistics [25,26] and is more convenient for our purpose. In this paper, we shall use the Sutherland model (SM) [52] as a prototype of a quantum system with the inverse-square interaction and the harmonic term. The parameters defining the SM include the inverse-square interaction strength μ, the harmonic confining strength ω, and the number N of particles that are interacting with each other. We start our analysis with a fixed value of N and quench the system parameters from (μ,ω) to (μ',ω'). The thermodynamic limit will be taken at the end of the calculation. We shall show that the overlap of the ground state of the SM before and after the quench decays exponentially in the thermodynamic limit. The wave functions of the SM exhibit fractional exclusion statistics

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[25,26]. Therefore the results obtained in this paper suggest that the phenomenon of OC might extend to systems with statistics more general than just the fermionic type.

Recent advances in ultra cold atoms and optical lattices have made it possible to experimentally realize physical models in lower dimensions. There has been a proposal to experimentally realize the SM with the help of Bose-Einstein condensates in the cold alkali atoms [54]. It is therefore plausible that the effects discussed in this paper could be observed in the laboratory in the future.

II. THE SUTHERLAND MODEL

The Hamiltonian of the N-particle SM [52] is given by

$$H_N = \frac{1}{2}\sum_{j=1}^{N}\left(-\partial_{x_j}^2 + \omega^2 x_j^2\right) + \sum_{j=2}^{N}\sum_{k=1}^{j-1}\left[\frac{\mu}{(x_j - x_k)^2}\right], \tag{1}$$

where ω is a natural frequency common to all the N particles and $\mu \geqslant 3/4$ is the coupling constant for the inverse-square interaction. The ground state wave function for this system has the form

$$\Psi_{\lambda,\omega}(\{x\}_N) = \mathcal{N}_{(\lambda,\omega)} z^{\lambda} e^{-\frac{\omega}{4}\sum_{j=1}^{N} x_j^2}, \tag{2}$$

where $\lambda \equiv (\sqrt{\mu+1}+1)/2$ is associated with fractional statistics [24] and $z \equiv \prod_{j<k}^{N}(x_j - x_k)$. The normalization constant $\mathcal{N}_{(\lambda,\omega)}$ can be obtained by using the Selberg's integral formula [55]

$$\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} z^{2\gamma}\prod_{j=1}^{N} e^{-a x_j^2}\, dx_j = (2\pi)^{N/2}(2a)^{-\gamma N(N-1)/2 - N/2}\prod_{j=1}^{N}\frac{\Gamma(1+j\gamma)}{\Gamma(1+\gamma)}. \tag{3}$$

Applying (3) to the ground state (2), we obtain

$$\mathcal{N}_{(\lambda,\omega)} = \left(\frac{\omega}{2\pi}\right)^{N/4}\omega^{\lambda\frac{N(N-1)}{4}}\prod_{j=1}^{N}\sqrt{\frac{\Gamma(1+\lambda)}{\Gamma(1+j\lambda)}}. \tag{4}$$

The dispersion relation of the Sutherland Hamiltonian (1) is that of N interacting Harmonic oscillators

$$E_N = \omega\sum_{j=1}^{N} n_i + \omega\lambda\frac{N(N-1)}{2} + \omega\frac{N}{2}, \tag{5}$$

where $n_j = 0,1,2,\ldots$ represents the energy level of the j-particle and satisfying the bosonic occupation rule $n_1 \leqslant n_2 \leqslant \cdots \leqslant n_N$. Observe that due to the introduction of the inverse-square interaction, the total energy E_N becomes superextensive, i.e., it is quadratic in the number of particles N.

If we define $\tilde{n}_j = n_j + \lambda(j-1)$, the energy (5) can be rewritten as

$$E_N = \omega\sum_{i=1}^{N}\tilde{n}_j + \omega\frac{N}{2}, \tag{6}$$

which can be interpreted as the spectrum of N free quasiparticles obeying the occupation rules:

$$\tilde{n}_j \leqslant \tilde{n}_{j+1} - \lambda. \tag{7}$$

They are a generalization of the Pauli exclusion principle, which corresponds to the particular case $\lambda = 1$. This is consistent with the fact that the Sutherland model exhibits fractional exclusion statistics [24,26]. Note that the superextensive term of E_N in (5) determines the form of the generalized exclusion rules (7).

III. SCALING OF THE OVERLAP BETWEEN GROUND STATES

For a fixed value of the particle number N, the ground state wave function (2) of the SM is characterized by the parameters ω and λ. We would like to quench the parameters of this system from (λ,ω) to (λ',ω'). Our primary focus is on the strength of the inverse-square interaction term, but we also consider the quench in the harmonic interaction as well [56]. To this end, we consider the overlap of the ground states of the SM with different (λ,ω) and (λ',ω'), which is given by

$$\mathcal{A}_{(\lambda,\omega),(\lambda',\omega')} \equiv [\Psi_{(\lambda,\omega)}(\{x\}_N),\Psi_{(\lambda',\omega')}(\{x\}_N)] = \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\overline{\Psi}_{(\lambda,\omega)}(\{x\}_N) \times \Psi_{(\lambda',\omega')}(\{x\}_N)\prod_{j=1}^{N} dx_j. \tag{8}$$

Using Eq. (2) in the above formula, we obtain

$$\mathcal{A}_{(\lambda,\omega),(\lambda',\omega')} = \mathcal{N}_{(\lambda,\omega)}\mathcal{N}_{(\lambda',\omega')} I(\lambda,\omega;\lambda',\omega'), \tag{9}$$

with

$$I(\lambda,\omega;\lambda',\omega') = \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} z^{\lambda+\lambda'}\prod_{j=1}^{N} e^{-\frac{\omega+\omega'}{4}x_j^2}\, dx_j = (2\pi)^{N/2}\left(\frac{\omega+\omega'}{2}\right)^{-(\lambda+\lambda')N(N-1)/4 - N/2} \times \prod_{j=1}^{N}\frac{\Gamma\left(1 + j\frac{\lambda+\lambda'}{2}\right)}{\Gamma\left(1+\frac{\lambda+\lambda'}{2}\right)}. \tag{10}$$

For this evaluation we have used the Selberg's integral formula (3) to get

$$\mathcal{A}_{(\lambda,\omega),(\lambda',\omega')} = \left[\frac{4\omega\omega'}{(\omega+\omega')^2}\right]^{N/4} \times 2^{(\lambda+\lambda')N(N-1)/4}\left[\frac{\omega^{\lambda}\omega'^{\lambda'}}{(\omega+\omega')^{\lambda+\lambda'}}\right]^{N(N-1)/4} \times \left\{\frac{\Gamma(1+\lambda)\Gamma(1+\lambda')}{\Gamma[1+(\lambda+\lambda')/2]^2}\right\}^{\frac{N}{2}} \times \prod_{j=1}^{N}\sqrt{\frac{\Gamma[1+j(\lambda+\lambda')/2]^2}{\Gamma(1+j\lambda)\Gamma(1+j\lambda')}}. \tag{11}$$

Notice that the first line gives the overlap due to the harmonic term (see Appendix A), the second line mixes both parameters, the natural frequency ω and the coupling λ, and the third line refers only to the overlap due to the inverse-square interaction strength λ. Also note that it is not allowed to take the limit $\lambda \to 1$ or equivalently $\mu \to 0$ as we have restricted the inverse-square coupling strength to $\mu \geqslant 3/4$. Below this value the analysis requires modified boundary conditions related to either self-adjoint extensions [49–51] or renormalization of the inverse-square interaction strength [57]. Such modified boundary conditions are not central to the purpose of the present work, and that is why we have restricted the analysis to $\mu \geqslant 3/4$. Moreover, the case $\omega = 0$ does not apply in this analysis, where normally the states would be non-normalizable.

For $\lambda = \lambda'$ the harmonic piece remains, while the terms under the product become unity. The mixture piece becomes

$$\left[\frac{4\omega\omega'}{(\omega+\omega')^2}\right]^{\lambda N(N-1)/4}, \tag{12}$$

which can be then combined with the harmonic piece to give

$$\mathcal{A}_{(\lambda,\omega),(\lambda,\omega')} = \left[\frac{4\omega\omega'}{(\omega+\omega')^2}\right]^{N/4+N(N-1)\lambda/4}. \tag{13}$$

In this last case, it is easy to see that the base is positive and smaller than one for all ω and ω', and the exponent is always positive. Therefore, when $N \to \infty$ the whole expression goes to zero in agreement with the OC. The dominant term decays exponentially as $\exp(-N^2)$ due to the Calogero coupling. This should be compared with the case of the pure harmonic oscillator (see Appendix A), which decays as $\exp(-N)$.

We now come to the main result of this analysis, for which we consider the case $\omega = \omega'$. The harmonic piece and the mixed term in (11) become unity. We are now essentially quenching the inverse-square interaction strength from λ to λ'. Following Anderson [1], let us first consider the case where $\lambda' = \lambda + \delta\lambda$, where $\delta\lambda \to 0$ is a small or even infinitesimal perturbation of the inverse-square interaction strength. In this case, the overlap between the ground states of the initial and the perturbed systems is given by (see Appendix B for details)

$$\mathcal{A}_{\lambda,\lambda+\delta\lambda} \sim e^{-\frac{\delta\lambda^2}{\lambda}\frac{N(N+1)}{16}}. \tag{14}$$

Therefore, we find that the overlap exponentially decays to zero with N^2, in contrast to the power law suppression as in the Anderson's original OC [1]. It may be noted that similar exponential suppression has also been recently obtained in a different context [14]. In Fig. 1 we compare the perturbative approximation (14) with the exact expression (11).

We now consider two arbitrarily different values of λ and λ', within the allowed parameter region $\mu \geqslant 3/4$, and analyze how the overlap between the ground states scales with the number of particles N in this case. For that, it is useful to apply the Stirling formula for the Gamma function, $\Gamma(z) \sim \sqrt{\frac{2\pi}{z}}z^z e^{-z}$, when $N \to \infty$ with λ, λ' keep fixed. We get

$$\prod_{j=1}^{N}\frac{\Gamma\left(1+j\frac{\lambda+\lambda'}{2}\right)}{\sqrt{\Gamma(1+j\lambda)\Gamma(1+j\lambda')}} \sim \left[\frac{(\lambda/2+\lambda'/2)^{\lambda+\lambda'}}{\lambda^\lambda\lambda'^{\lambda'}}\right]^{\frac{N(N+1)}{4}} \times\left(\frac{\lambda+\lambda'}{2\sqrt{\lambda\lambda'}}\right)^{N/2}. \tag{15}$$

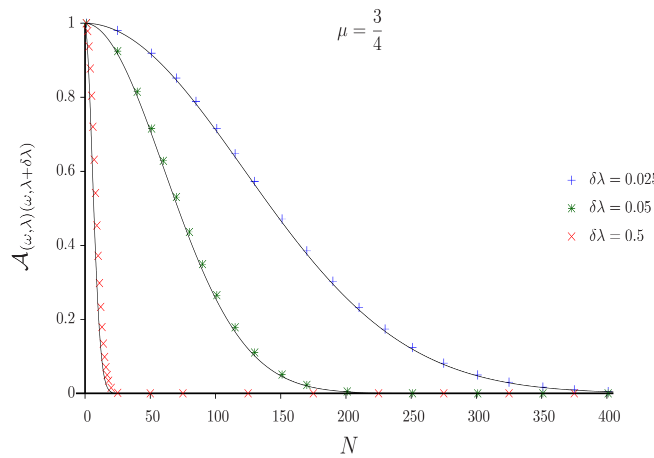

FIG. 1. Overlap between the ground state of a SM (1) with arbitrary ω and $\mu = 3/4$ and the ground state of another model (1) obtained by a perturbation of the inverse-square coupling from λ [recall $\lambda = (\sqrt{\mu+1}+1)/2)$] to $\lambda + \delta\lambda$ for different $\delta\lambda$. The harmonic term does not change in any case. The points were obtained with the exact expression (11), and the solid lines correspond to the perturbative approximation (14).

The final form of the overlap is obtained as

$$\mathcal{A}_{(\lambda,\omega),(\lambda',\omega)} \sim \left[\frac{\Gamma(1+\lambda)\Gamma(1+\lambda')}{\Gamma[1+(\lambda+\lambda')/2]^2}\right]^{N/2} \times\left\{\frac{[(\lambda+\lambda')/2]^{\lambda+\lambda'}}{\lambda^\lambda\lambda'^{\lambda'}}\right\}^{N(N+1)/4}\left(\frac{\lambda+\lambda'}{2\sqrt{\lambda\lambda'}}\right)^{N/2}. \tag{16}$$

In Fig. 2 we check the validity of this expansion comparing it with the exact result (11) for several quenches from a given λ to different λ'.

Observe that, as happens with the energy of the Hamiltonian (5), the exponent of the previous overlaps is also superextensive, being quadratic in the number of particles N. The inverse-square interaction seems to be the responsible of this behavior since the extensivity is recovered when we turn this interaction off as it is shown in Appendix A.

The SM can be used to describe a one-dimensional Bose gas in an harmonic potential where the particles interact with each other through an inverse-square potential. Recent advances in the field of ultracold atoms and optical lattices have opened up the possibility of simulating such a system in the laboratory [54,58–61]. In particular, it has been argued in Ref. [54] that the dipole-dipole interactions between certain Bose-Einstein condensates (BEC) in an optical lattice generates an inverse-square potential whose strength is proportional to the number of atoms within the BEC. Thus the coefficient of the inverse-square interaction in the SM model can be changed by tuning the number of atoms within the BEC. On the other hand, a quench in the harmonic trap of the SM model also leads to interesting effects [62], and it can be easily implemented in the laboratory.

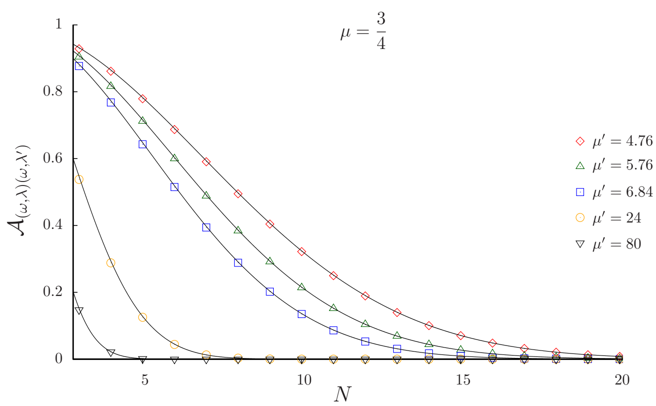

FIG. 2. Overlap between the ground state of a SM (1) with arbitrary ω and $\mu = 3/4$ and the ground state of other SMs (1) with the same harmonic coupling ω and different inverse-square strength μ'. Observe that the results do not depend on the value of ω. The points were computed employing the exact expression (11), and the solid lines correspond to the asymptotic expansion (16) obtained by applying the Stirling formula for the Gamma function when $N \to \infty$.

IV. SUMMARY AND OUTLOOK

In our analysis we have considered the N-body Sutherland model with the harmonic and inverse-square interactions as a prototype for systems with fractional exclusion statistics. We have obtained a general analytical expression for the overlap of the ground state wave functions when the system parameters are quenched from (λ,ω) to (λ',ω'). This overlap tends to zero in the thermodynamic limit as the number of particles $N \to \infty$. As a special case, we have considered the quench with the harmonic interaction fixed while the inverse-square interaction strength changes infinitesimally. Here we have shown that the ground state overlap goes to zero in an exponential fashion which is different from the usual power law overlap obtained in the usual Anderson's OC. In addition, the exponent is quadratic in the number of particles. We think that this is a sign of the superextensivity of the SM, whose dispersion relation is actually quadratic in the number of particles. This superextensivity is also related to the fractional exclusion statistics that this model exhibits. The leading term of the overlap or fidelity of two ground states of a many-body system can be employed to detect and study quantum phase transitions [63,64]. However, we are not aware of any quantum phase transition in this model. It may be noted that the Calogero-Sutherland system is gapped and there is no thermal phase transition [53].

Another interesting quantity in this context is the time dependence of the overlap of the wave functions before and after the quench, which yields rich information about the equilibrium properties of the quantum system. The Loschmidt echo [65–67]

$$L(t) = |\langle\phi_g|e^{iH_it}e^{-iH_ft}|\phi_g\rangle|^2, \tag{17}$$

where ϕ_g is the initial state before the quench and H_i, H_f denote the Hamiltonians before and after the quench, provides a characterization of such a time dependence and its behavior in the context of Anderson OC has attracted recent attention in the literature [68–72]. There have also been various proposals to empirically study the Loschmidt echo in setups with ultracold atoms [14,71–75], and a Ramsey interferometric-type experiment with dilute fermionic impurities has been performed recently [76]. It is known that the OC is related to the power law decay of the Loschmidt echo [72,77]. The calculation of the Loschmidt echo in the SM and its analysis in the time and frequency domains is presently under investigation.

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APPENDIX A: HARMONIC OSCILLATORS

In this Appendix, we recollect a simple example of orthogonality catastrophe. In one dimension, consider N noninteracting harmonic oscillators

$$H(\omega) = \frac{1}{2}\sum_{j=1}^{N}\left(p_j^2 + \omega^2 x_j^2\right), \tag{A1}$$

with $p_j = -i\partial_{x_j}$. The ground state of $H\Psi = E\Psi$ is

$$\Psi_\omega(\{x\}_N) = \left(\frac{\omega}{\pi}\right)^{N/4} e^{-\frac{\omega}{4}\sum_j^N x_j^2}. \tag{A2}$$

Let us consider the overlap of two system with different frequencies, ω and ϖ,

$$\begin{aligned}\mathcal{A}_{\omega\varpi,N} &\equiv (\Psi_\omega(\{x\}_N),\Psi_\varpi(\{x\}_N))\\ &= \int dx_1\cdots dx_N\overline{\Psi}_\omega(\{x\}_N)\Psi_\varpi(\{x\}_N).\end{aligned} \tag{A3}$$

Replacing (A2) in the above overlap and integrating the Gaussians,

$$\mathcal{A}_{\omega\varpi,N} = \left[\frac{4\omega\varpi}{(\omega+\varpi)^2}\right]^{N/4} = \left[\frac{4\eta}{(1+\eta)^2}\right]^{N/4}, \quad \eta \equiv \frac{\omega}{\varpi}. \tag{A4}$$

Now, notice that except when $\eta = 1$ (or $\omega = \varpi$), the ratio $4\eta/(1+\eta)^2$ is less than one. Therefore, in the limit $N \to \infty$,

$$\lim_{N\to\infty}\mathcal{A}_{\omega\varpi,N} = 0, \qquad \omega \neq \varpi. \tag{A5}$$

This is the orthogonality catastrophe. It is a remarkable fact that the study of the scaling of this exponential suppression is currently amenable to experiments in cold atoms.

APPENDIX B: PERTURBATIVE ANALYSIS FOR THE OC OF THE SUTHERLAND MODEL

For the case $\omega' = \omega$, the overlap between two ground states (2) with λ and λ' as a function of the number of particles N writes from (11) as

$$\mathcal{A}_{\lambda,\lambda'}(N) = R_1(\lambda,\lambda')^{\frac{N}{2}} \sqrt{\prod_{j=1}^{N} \frac{1}{R_j(\lambda,\lambda')}} = \exp\left[\frac{N}{2}\log R_1(\lambda,\lambda') - \frac{1}{2}\sum_{j=1}^{N}\log R_j(\lambda,\lambda')\right], \tag{B1}$$

where

$$R_j(x,y) \equiv \frac{\Gamma(1+jx)\Gamma(1+jy)}{\Gamma[1+j(x+y)/2]^2}. \tag{B2}$$

We are interested in the asymptotics of the overlap function $\mathcal{A}_{\lambda,\lambda'}(N)$ as $N \to \infty$. We will also consider a small perturbation around λ, that is, $\lambda' = \lambda + \delta\lambda$. The result below will be valid for the scaling limit:

$$N \to \infty,\ \delta\lambda \to 0, \quad N\delta\lambda^2 < 1. \tag{B3}$$

The series expansion around x of $R_j(x,x+\delta x)$ to smallest order is

$$R_j(x,x+\delta x) = 1 + j^2\psi'(1+jx)\frac{\delta x^2}{4} + O(\delta x^3), \tag{B4}$$

where

$$\psi'(z) = \sum_{k=0}^{\infty}\frac{1}{(k+z)^2}, \quad z \neq 0,-1,-2,\dots \tag{B5}$$

is the derivative of the Polygamma function

$$\psi(z) = \frac{\Gamma'(z)}{\Gamma(z)}. \tag{B6}$$

Notice that the argument in ψ' for our case is always positive. The asymptotics of ψ' as $z \to \infty$ is of the form

$$\psi'(z) \sim \frac{1}{z} + \frac{1}{2z^2} + \sum_{k=1}^{\infty}\frac{B_{2k}}{z^{2k}}, \tag{B7}$$

where B_n are the Bernoulli numbers.

Now we consider the case $j \to \infty$, and since x is held fixed, then we can take

$$\psi'(1+jx) \sim \frac{1}{j\left(x+\frac{1}{j}\right)} \sim \frac{1}{jx}. \tag{B8}$$

Replacing this result in (B4), we obtain

$$R_j(x,x+\delta x) \sim 1 + j\frac{\delta x^2}{4x}. \tag{B9}$$

Using the scaling limit $N\delta x^2 < 1$, for large N and small δx, we then Taylor expand the previous formula to first order as

$$\log R_j(x,x+\delta x) \sim \log\left(1 + j\frac{\delta x^2}{4x}\right) = j\frac{\delta x^2}{4x}. \tag{B10}$$

We can also consider that formulas (B9) and thus (B10) are valid for $j = 1$.

We now replace this formula in the sum over j in (B1). Although the above formula is valid for large j, we can start the sum with $j = 1$, which brings nothing more than a small error. Indeed, the smaller j terms are quite irrelevant w.r.t. the large ones (in a linear approximation). Thus,

$$\sum_{j=1}^{N}\log R_j(x,x+\delta x) \sim \frac{\delta x^2}{4x}\sum_{j=1}^{N} j = \frac{\delta x^2}{4x}\frac{N(N+1)}{2}. \tag{B11}$$

Summing up all the above in the overlap expression (B1), we obtain

$$\mathcal{A}_{\lambda,\lambda+\delta\lambda}(N) \sim e^{-\frac{\delta\lambda^2}{\lambda}\frac{N(N+1)}{16}}. \tag{B12}$$

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