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Inverse Bremsstrahlung current drive

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The generation of the plasma current resulting from Bremsstrahlung absorption is considered. It is shown that the electric current is higher than the naive estimates assuming that electrons absorb only the photon momentum and using the Spitzer conductivity would suggest. The current enhancement is in part because electrons get the recoil momentum from the Coulomb field of ions during the absorption and in part because the electromagnetic power is absorbed asymmetrically within the electron velocity distribution space.

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I. INTRODUCTION

In the presence of external electromagnetic field, colliding electrons and ions absorb the incoming radiation through the process known as inverse Bremsstrahlung. In Bremsstrahlung absorption, the electron receives additional recoil momentum from the ion besides the momentum of the photon. Therefore, plasma electrons absorb more than just the photon momentum from the incoming radiation. The generated current is then larger than one would get by assuming that electrons absorb just the photon momentum. It was shown in [1] that this increase in current is equal to $8/5$.

However, the recoil is not the only mechanism that will increase the current. Plasma electrons absorb the radiation asymmetrically in velocity space; specifically, electrons co-moving with the incoming photons will absorb slightly more power than electrons going in the opposite direction. Even in the absence of net momentum absorption, this asymmetric absorption in power can lead to current drive. This is because the collision frequency in plasma is speed dependent. Thus, upon absorbing energy, electrons going in the direction of the incoming radiation will experience less resistance from the plasma than electrons going in the opposite direction, resulting in current. This is called the asymmetric resistivity current drive effect and is mostly known with respect to cyclotron absorption used to drive toroidal current in tokamaks [2,3]. Moreover, even without the asymmetric resistivity effect the fluid approximation is less precise in considering current generation as opposed to momentum input, because it assumes that all electrons get equal push in the same direction, which is not the case for Bremsstrahlung absorption. In fact, the ability of electrons to retain current is sensitive to both its location in velocity space and the direction in which it is being pushed.

In this paper we rederive the result for the momentum absorption rate and calculate the additional increase in current due to the current drive effect. To derive the current drive effect, it will be necessary to consider in detail how exactly the momentum is absorbed within the electron velocity space. To do this, we use the formalism developed by Tsytovich *et al.* [4–6].

II. PROBABILITY OF BREMSSTRAHLUNG

Consider Bremsstrahlung absorption for particles α (electrons) due to the Coulomb collisions with much heavier particles β (ions). To satisfy the conservation laws of momentum and energy, in each act of the Bremsstrahlung absorption some recoil momentum must be transferred from the electron to the ions. We can write the momentum balance during inverse Bremsstrahlung as

$$\mathbf{p}'_\alpha = \mathbf{p}_\alpha + \hbar\mathbf{k} - \hbar\mathbf{q}, \tag{1}$$

$$\mathbf{p}'_\beta = \mathbf{p}_\beta + \hbar\mathbf{q}, \tag{2}$$

where the primed values correspond to the quantities after the absorption, $\mathbf{k}$ is the wave vector of the photon, and $\mathbf{q}$ is the recoil wave vector transferred from the electron to the ion. The conservation of energy is

$$\varepsilon^\alpha_{\mathbf{p}_\alpha} + \varepsilon^\beta_{\mathbf{p}_\beta} + \hbar\omega_{\mathbf{k}} = \varepsilon^\alpha_{\mathbf{p}_\alpha+\hbar\mathbf{k}-\hbar\mathbf{q}} + \varepsilon^\beta_{\mathbf{p}_\beta+\hbar\mathbf{q}}. \tag{3}$$

Here, we will use the diffusion approximation, when $\hbar\mathbf{k}$, $\hbar\mathbf{q}$ are small in comparison with the particle momentum ($\hbar\mathbf{k}$, $\hbar\mathbf{q} \ll \mathbf{p}_\alpha$). In this approximation, the energy conservation is simplified to

$$\omega_{\mathbf{k}} = (\mathbf{k} - \mathbf{q})\mathbf{v}_\alpha + \mathbf{q}\mathbf{v}_\beta. \tag{4}$$

Now consider the direct process of spontaneous Bremsstrahlung emission. The momentum balance can be written as

$$\mathbf{p}'_\alpha = \mathbf{p}_\alpha - \hbar\mathbf{k} + \hbar\mathbf{q}, \tag{5}$$

$$\mathbf{p}'_\beta = \mathbf{p}_\beta - \hbar\mathbf{q}. \tag{6}$$

With such a definition of the recoil momentum $\mathbf{q}$ (notice different signs in the definition of $\mathbf{q}$ for emission and absorption), the energy conservation yields the same relationship between velocities of the particles and parameters of the photon as for the inverse process [Eq. (4)].

A schematic diagram of the two processes is shown in Fig. 1. Essentially, inverse Bremsstrahlung can be considered as Compton scattering, by the incoming electron, of the incoming photon $\mathbf{k}$ into the virtual photon of the Coulomb field $\mathbf{q}$ [see Fig. 1(a)], while the Bremsstrahlung emission can be considered as Compton scattering of the virtual photons of the Coulomb field on the incoming electron [see Fig. 1(b)].

It is clear, that due to time-reversal symmetry, the transition probability of the inverse and direct processes are related to each other:

$$w^{I\mathrm{Br}}_{\mathbf{p}_\alpha,\mathbf{p}_\beta}(\mathbf{k},\mathbf{q}) = w^{\mathrm{Br}}_{\mathbf{p}_\alpha+\hbar\mathbf{k}-\hbar\mathbf{q},\mathbf{p}_\beta+\hbar\mathbf{q}}(\mathbf{k},\mathbf{q}). \tag{7}$$

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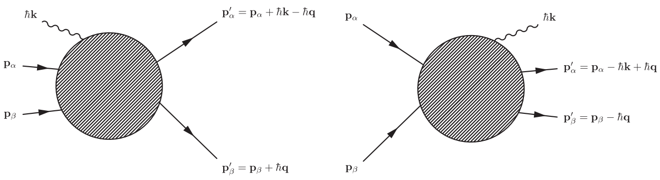

(a) (b)

FIG. 1. Schematic diagram of Bremsstrahlung absorption (a) and emission (b).

Here $w^{\mathrm{Br}}_{\mathbf{p}_\alpha,\mathbf{p}_\beta}(\mathbf{k},\mathbf{q})$ and $w^{I\mathrm{Br}}_{\mathbf{p}_\alpha,\mathbf{p}_\beta}(\mathbf{k},\mathbf{q})$ are the probabilities of spontaneous Bremsstrahlung emission and inverse Bremsstrahlung per unit time within $d\mathbf{k}d\mathbf{q}$. Note that these probabilities must contain condition (4) as the argument of the δ function.

One must remember that, in the presence of external radiation, the true absorption due to inverse Bremsstrahlung is always accompanied by the process of stimulated emission. For example, for electromagnetic waves ($\omega = kc$) and infinitely massive ions ($\mathbf{v}_\beta = 0$), condition (4) implies that for inverse Bremsstrahlung the change in the parallel momentum of the electron is approximately $\hbar\omega/v$, while for stimulated Bremsstrahlung emission this change is approximately $-\hbar\omega/v$. However, these two processes do not completely compensate each other because their probabilities are slightly different.

More generally, the evolution of the distribution function $f^\alpha_{\mathbf{p}_\alpha}$ due to the processes of inverse Bremsstrahlung and stimulated Bremsstrahlung emission is described by [4]

$$\begin{aligned}\frac{\partial f^\alpha_{\mathbf{p}_\alpha}}{\partial t} = &-\int w^{I\mathrm{Br}}_{\mathbf{p}_\alpha,\mathbf{p}_\beta}(\mathbf{k},\mathbf{q}) f^\alpha_{\mathbf{p}_\alpha} f^\beta_{\mathbf{p}_\beta} n_{\mathbf{k}} d\mathbf{k}d\mathbf{q}d\mathbf{p}_\beta \\ &+\int w^{I\mathrm{Br}}_{\mathbf{p}_\alpha-\hbar\mathbf{k}+\hbar\mathbf{q},\mathbf{p}_\beta-\hbar\mathbf{q}}(\mathbf{k},\mathbf{q}) f^\alpha_{\mathbf{p}_\alpha-\hbar\mathbf{k}+\hbar\mathbf{q}} f^\beta_{\mathbf{p}_\beta-\hbar\mathbf{q}} n_{\mathbf{k}} d\mathbf{k}d\mathbf{q}d\mathbf{p}_\beta \\ &-\int w^{\mathrm{Br}}_{\mathbf{p}_\alpha,\mathbf{p}_\beta}(\mathbf{k},\mathbf{q}) f^\alpha_{\mathbf{p}_\alpha} f^\beta_{\mathbf{p}_\beta} n_{\mathbf{k}} d\mathbf{k}d\mathbf{q}d\mathbf{p}_\beta \\ &+\int w^{\mathrm{Br}}_{\mathbf{p}_\alpha+\hbar\mathbf{k}-\hbar\mathbf{q},\mathbf{p}_\beta+\hbar\mathbf{q}}(\mathbf{k},\mathbf{q}) f^\alpha_{\mathbf{p}_\alpha+\hbar\mathbf{k}-\hbar\mathbf{q}} f^\beta_{\mathbf{p}_\beta+\hbar\mathbf{q}} n_{\mathbf{k}} d\mathbf{k}d\mathbf{q}d\mathbf{p}_\beta.\end{aligned} \tag{8}$$

Following Tsytovich [4–6], after Taylor expansion for $\hbar\mathbf{k}, \hbar\mathbf{q} \ll \mathbf{p}_\alpha$ we get the Fokker-Planck equation for the evolution of $f^\alpha_{\mathbf{p}_\alpha}$,

$$\frac{\partial f^\alpha_{\mathbf{p}_\alpha}}{\partial t} = \frac{\partial}{\partial \mathbf{p}_\alpha}\cdot \mathbf{S}_{\mathbf{p}_\alpha} = \frac{\partial}{\partial \mathbf{p}_\alpha}\cdot\left(\widehat{D}_\alpha \frac{\partial f^\alpha_{\mathbf{p}_\alpha}}{\partial \mathbf{p}_\alpha} + \mathbf{F}_\alpha f^\alpha_{\mathbf{p}_\alpha}\right), \tag{9}$$

where

$$\widehat{D}_\alpha = \int \hbar^2(\mathbf{k}-\mathbf{q})(\mathbf{k}-\mathbf{q}) w^{\mathrm{Br}}_{\mathbf{p}_\alpha,\mathbf{p}_\beta} n_{\mathbf{k}} f^\beta_{\mathbf{p}_\beta} d\mathbf{k}d\mathbf{q}d\mathbf{p}_\beta, \tag{10}$$

$$\mathbf{F}_\alpha = \int \hbar^2(\mathbf{k}-\mathbf{q})\left(\mathbf{q}\cdot\frac{\partial f^\beta_{\mathbf{p}_\beta}}{\partial \mathbf{p}_\beta}\right) w^{\mathrm{Br}}_{\mathbf{p}_\alpha,\mathbf{p}_\beta} n_{\mathbf{k}} d\mathbf{k}d\mathbf{q}d\mathbf{p}_\beta. \tag{11}$$

The normalization is such that the density of particles is $n_\alpha = \int f^\alpha_{\mathbf{p}_\alpha} d\mathbf{p}_\alpha = \int f^\alpha_{\mathbf{v}_\alpha} d\mathbf{v}_\alpha$, the total number of photons per volume is $N_{\mathrm{ph}} = \int n_{\mathbf{k}} d\mathbf{k}$, and $n_{\mathbf{k}}$ is the number of photons within $d\mathbf{k}$.

The probability of spontaneous Bremsstrahlung emission for electromagnetic waves ($\omega = kc$) keeping terms of the order of $\mathbf{k}\mathbf{v}/\omega \sim v/c$ is given by [6]

$$\begin{aligned}w^{\mathrm{Br}}_{\mathbf{p}_\alpha,\mathbf{p}_\beta}(\mathbf{k},\mathbf{q}) = &\frac{2e_\alpha^4 e_\beta^2 \delta[\omega_{\mathbf{k}} - (\mathbf{k}-\mathbf{q})\mathbf{v}_\alpha - \mathbf{q}\mathbf{v}_\beta]}{\hbar\pi^2 m_\alpha^2 q^4 (\omega_{\mathbf{k}} - \mathbf{k}\mathbf{v}_\alpha)^2 \frac{\partial(\varepsilon\omega^2)}{\partial\omega}\big|_{\omega=\omega_{\mathbf{k}}} \varepsilon^2_{\mathbf{q},\mathbf{q}\mathbf{v}_\beta}} \\ &\times \left|[\mathbf{e}_k \times \mathbf{q}] + \frac{\mathbf{k}\mathbf{q}}{\omega_{\mathbf{k}} - \mathbf{k}\mathbf{v}_\alpha}[\mathbf{e}_k \times \mathbf{v}]\right|^2.\end{aligned} \tag{12}$$

This expression is only correct for Bremsstrahlung ignoring the polarization effects. By polarization effects we mean that the plasma environment in which the electron finds itself is influenced by the presence of the electron. This approximation is good for dilute plasma. In general, the probability of Bremsstrahlung is proportional to $|[\mathbf{e}_k \times (\mathbf{M}^\alpha + \mathbf{M}^\beta + \mathbf{M}^{\alpha\beta})]|^2$, where $\mathbf{M}^\alpha$ is the emission due to oscillation of α particles in the screened field of β charges, $\mathbf{M}^\beta$ is the emission due to oscillation of β particles in the screened field of α charges, and $\mathbf{M}^{\alpha\beta}$ is the emission due to oscillation of the polarization clouds around particles α and β. While $\mathbf{M}^\beta$ is small due to the high ion mass, the term $\mathbf{M}^{\alpha\beta}$ can be comparable with $\mathbf{M}^\alpha$. Moreover, polarization effects may make electron-electron and ion-ion collisions important as well. The polarization effects are especially important for longitudinal waves and must be almost always taken into account for them (we consider only transverse electromagnetic waves here) [4–6]. In Eq. (12) the polarization effects are ignored, and only the $\mathbf{M}^\alpha$ term is retained; this requires the plasma to be tenuous enough. Another approximation used in Eq. (12) is nonrelativistic velocities. In all subsequent calculations, we also take a unity dielectric function ($\varepsilon \approx 1$), which is a good approximation for tenuous plasma. We will also ignore plasma dispersive effects, take $\omega_{\mathbf{k}} = \omega = kc$, and assume an infinite ion mass and set $\mathbf{v}_\beta = 0$, $\mathbf{v}_\alpha = \mathbf{v}$.

III. MOMENTUM CHANGE

In this section let us calculate the rate of momentum change for electrons during Bremsstrahlung absorption.

From Eq. (9) we can calculate the rate of momentum absorption due to Bremsstrahlung as

$$\frac{d\mathbf{p}^\alpha_V}{dt} = -\int \mathbf{S}_{\mathbf{p}_\alpha} d\mathbf{p}_\alpha, \tag{13}$$

so $-\mathbf{S}_{\mathbf{p}_\alpha}$ has the meaning of the rate of momentum absorption per $d\mathbf{p}_\alpha$ by electrons with momentum between $\mathbf{p}_\alpha$ and $\mathbf{p}_\alpha + d\mathbf{p}_\alpha$.

For plasma with a spherically symmetric distribution function and infinitely massive ions ($\mathbf{v}_\beta = 0$) we can take advantage of condition (4) and write

$$\frac{d\mathbf{p}^\alpha_V}{dt} = \int \hbar(\mathbf{k}-\mathbf{q})\frac{\hbar\omega_{\mathbf{k}}}{v_\alpha}\frac{\partial f^\alpha_{\mathbf{p}_\alpha}}{\partial p_\alpha} w^{\mathrm{Br}}_{\mathbf{p}_\alpha,\mathbf{p}_\beta} f^\beta_{\mathbf{p}_\beta} n_{\mathbf{k}} d\mathbf{k}d\mathbf{q}d\mathbf{p}_\beta d\mathbf{p}_\alpha. \tag{14}$$

This suggests that the probability of the total absorption (inverse Bremsstrahlung plus stimulated Bremsstrahlung emission) in plasma with a spherically symmetric distribution function is proportional to the probability of spontaneous Bremsstrahlung emission and is $(\hbar\omega_{\mathbf{k}}/v_\alpha)(\partial \ln f^\alpha_{\mathbf{p}_\alpha}/\partial p_\alpha) w^{\rm Br}_{\mathbf{p}_\alpha,\mathbf{p}_\beta}(\mathbf{k},\mathbf{q})$. For plasma near equilibrium with Maxwell distribution function, which for convenience we will consider, this probability becomes $(\hbar\omega_{\mathbf{k}}/T) w^{\rm Br}_{\mathbf{p}_\alpha,\mathbf{p}_\beta}(\mathbf{k},\mathbf{q})$ and is actually correct even for the finite ion mass.

Consider the incoming electromagnetic radiation that consists of photons with $\mathbf{k} = k\mathbf{e}_z$ and of the total intensity $I = c\int \hbar\omega n_{\mathbf{k}} d\mathbf{k}$. Because of the condition (4) the recoil momentum can be divided into the parts parallel and perpendicular to the velocity component:

$$\mathbf{q} = -\frac{\omega - \mathbf{k}\mathbf{v}}{v^2}\mathbf{v} + \mathbf{q}_\perp. \quad (15)$$

Then the rate of momentum absorption directed along the z axis can be written as

$$\frac{d\mathbf{p}^\alpha_{V,z}}{dt} = \int \hbar\left(k + \frac{\omega}{v}\frac{v_z}{v} - \frac{\mathbf{k}\mathbf{v}}{v}\frac{v_z}{v} - q_{\perp_z}\right) \times \frac{\hbar\omega}{T} w^{\rm Br}_{\mathbf{p}_\alpha,\mathbf{p}_\beta} f^\alpha_{\mathbf{p}_\alpha} f^\beta_{\mathbf{p}_\beta} n_{\mathbf{k}} d\mathbf{k} d\mathbf{q} d\mathbf{p}_\beta d\mathbf{p}_\alpha. \quad (16)$$

To calculate the probability of Bremsstrahlung (12), we express

$$\left|[\mathbf{e}_k \times \mathbf{q}] + \frac{\mathbf{k}\mathbf{q}}{\omega - \mathbf{k}\mathbf{v}}[\mathbf{e}_k \times \mathbf{v}]\right|^2 = \left|[\mathbf{e}_z \times \mathbf{q}_\perp] + \left(-\frac{\omega}{v^2} + \frac{\mathbf{k}\mathbf{q}_\perp}{\omega - \mathbf{k}\mathbf{v}}\right)[\mathbf{e}_z \times \mathbf{v}]\right|^2 = q_\perp^2 - q_{\perp_z}^2 + \frac{\omega^2}{v^2}\frac{v_\perp^2}{v^2} + 2\frac{\omega}{v^2} q_{\perp_z}(v_z - v_\perp\beta_\perp) - 2q_{\perp_z}^2\beta_z, \quad (17)$$

where we introduced $\boldsymbol{\beta} = \mathbf{v}/c$, used the expression for the scalar quadruple product $[\mathbf{e}_z \times \mathbf{q}_\perp]\cdot[\mathbf{e}_z \times \mathbf{v}] = -q_{\perp_z} v_z$, and kept only the first-order terms.

We can write the z-axis projection of the perpendicular to the velocity component of the recoil momentum as $q_{\perp_z} = q_\perp \sin\theta \sin\varphi_{q_\perp}$, where θ is the angle between velocity and the z axis, i.e., $v_z = v\cos\theta$ and $v_\perp = v\sin\theta$, while $\varphi_{q_\perp}$ is the polar angle of $q_\perp$ in the plane perpendicular to $\mathbf{v}$. We then integrate over $\varphi_{q_\perp}$ from 0 to 2π and over $dq_\| q_\perp dq_\perp$. When we integrate over $dq_\perp$, it is necessary to introduce a cutoff to get rid of a logarithmic divergence. For definiteness, we will use the quantum mechanical cutoff ($q_{\rm max} = m_\alpha v/\hbar$), which is correct when the Born approximation can be applied ($v \gg e^2/\hbar$). In the opposite classical limit ($v \ll e^2/\hbar$) the proper cutoff is $q_{\rm max} = m_\alpha v^2/e_\alpha e_\beta$, and the conclusions of the paper should remain true, but all logarithmic factors should be replaced with $\ln(m_\alpha v^3/\omega e_\alpha e_\beta)$.

Keeping only the leading logarithmic terms, the probability of Bremsstrahlung integrated over $d\mathbf{q}$ is then

$$\int w^{\rm Br}_{\mathbf{p}_\alpha,\mathbf{p}_\beta}(\mathbf{k},\mathbf{q}) d\mathbf{q} \approx \frac{e_\alpha^4 e_\beta^2}{\pi\hbar m_\alpha^2 \omega^3 v}\left(1 + \frac{v_z^2}{v^2} + 4\beta_z\frac{v_z^2}{v^2}\right)\ln\left(\frac{m_\alpha v^2}{\hbar\omega}\right), \quad (18)$$

which determines the absorbed power, and

$$\int q_{\perp_z} w^{\rm Br}_{\mathbf{p}_\alpha,\mathbf{p}_\beta}(\mathbf{k},\mathbf{q}) d\mathbf{q} \approx \frac{\omega}{c}\frac{e_\alpha^4 e_\beta^2}{\pi\hbar m_\alpha^2\omega^3 v} 2\frac{v_\perp^2}{v^2}\left(\frac{c v_z}{v^2} + 2\frac{v_z^2}{v^2} - \frac{v_\perp^2}{v^2}\right)\ln\left(\frac{m_\alpha v^2}{\hbar\omega}\right), \quad (19)$$

which determines the amount of momentum change in the direction perpendicular to the velocity. This is needed to calculate the current. Note that while it is not necessary to retain the first-order terms in Eq. (18) to calculate the absorbed power, one needs to keep them while calculating current. Note also in Eq. (18) that electrons moving in the direction of the photon ($\beta_z > 0$) are more likely to absorb energy than electrons moving in the opposite direction ($\beta_z < 0$). This is consistent with the picture that an electron moving in the direction of the photon can absorb its energy through a smaller angle scatter than would an electron moving in the opposite direction.

From Eqs. (18) and (19) we can write the rate of momentum absorption as

$$\frac{d\mathbf{p}^\alpha_{V,z}}{dt} = \int \frac{\hbar\omega}{c}\left(1 + \frac{c v_z}{v^2} - \frac{v_z^2}{v^2}\right)\frac{\hbar\omega}{T} \times \frac{n_\beta e_\alpha^4 e_\beta^2}{\pi\hbar m_\alpha^2\omega^3 v}\left(1 + \frac{v_z^2}{v^2} + 4\beta_z\frac{v_z^2}{v^2}\right)\ln\left(\frac{m_\alpha v^2}{\hbar\omega}\right) f^\alpha_{\mathbf{p}} d\mathbf{p} n_{\mathbf{k}} d\mathbf{k} - \int \frac{\hbar\omega}{c}\frac{\hbar\omega}{T}\frac{n_\beta e_\alpha^4 e_\beta^2}{\pi\hbar m_\alpha^2\omega^3 v} \times 2\frac{v_\perp^2}{v^2}\left(\frac{c v_z}{v^2} + 2\frac{v_z^2}{v^2} - \frac{v_\perp^2}{v^2}\right)\ln\left(\frac{m_\alpha v^2}{\hbar\omega}\right) f^\alpha_{\mathbf{p}} d\mathbf{p} n_{\mathbf{k}} d\mathbf{k}. \quad (20)$$

Integrating over angle θ, we get

$$\frac{d\mathbf{p}^\alpha_{V,z}}{dt} = \frac{32}{15}\int \frac{\hbar\omega}{T}\frac{n_\beta e_\alpha^4 e_\beta^2}{\pi c m_\alpha^2 \omega^2 v}\ln\left(\frac{m_\alpha v^2}{\hbar\omega}\right) f^\alpha_v d\mathbf{v} n_{\mathbf{k}} d\mathbf{k}. \quad (21)$$

Therefore,

$$\frac{d\mathbf{p}^\alpha_{V,z}}{dt} = \frac{8}{5}\frac{\alpha I}{c}. \quad (22)$$

Here α is the effective absorption coefficient,

$$\alpha \approx \frac{4}{3}\sqrt{\frac{2}{\pi}}\frac{n_\alpha n_\beta e_\alpha^4 e_\beta^2}{\pi c m_\alpha^3 \omega^2 v_{\rm th}^3}\ln\left(\frac{2T}{\hbar\omega}\right), \quad (23)$$

where $v_{\rm th}^2 = T/m_\alpha$. This absorption coefficient determines the total absorbed power density: $P_V^{\rm abs} = \alpha I$.

If we ignored the recoil momentum and assumed that electrons absorb just the incoming photon momentum $\hbar\mathbf{k}$, then the rate of momentum change would be

$$\frac{d\mathbf{p}^{\mathbf{k}}_{V,z}}{dt} = \int \hbar k\frac{\hbar\omega}{T} w^{\alpha,\beta}_{\mathbf{p}_\alpha,\mathbf{p}_\beta} f^\alpha_{\mathbf{p}_\alpha} f^\beta_{\mathbf{p}_\beta} n_{\mathbf{k}} d\mathbf{k} d\mathbf{q} d\mathbf{p}_\beta d\mathbf{p}_\alpha = \frac{\alpha I}{c}. \quad (24)$$

Thus, due to the recoil, electrons get 8/5 times more momentum than they would have got absorbing only the photon momentum, which is consistent with the result obtained in [1]. This conclusion is true for any spherically symmetric distribution function, not just a Maxwellian. This additional momentum absorbed by electrons (as a whole) is in the direction of the incoming radiation. The ions (as a whole), on the other hand, absorb momentum in the direction opposite to the incoming radiation, such that the total rate of momentum absorption for plasma is equal to the rate of photon momentum absorption:

$$\frac{d\mathbf{p}^{\alpha}_{V,z}}{dt} + \frac{d\mathbf{p}^{\beta}_{V,z}}{dt} = \frac{8}{5}\frac{\alpha I}{c} - \frac{3}{5}\frac{\alpha I}{c} = \frac{d\mathbf{p}^{\mathbf{k}}_{V,z}}{dt} = \hbar k \frac{dN^{\rm abs}_{\rm ph}}{dt} = \frac{\alpha I}{c}. \quad (25)$$

It is curious that, after averaging for spherically symmetric distribution functions, the last two terms in Eq. (16) cancel each other, and the rate of momentum absorption becomes just

$$\frac{d\mathbf{p}^{\alpha}_{V,z}}{dt} = \int \hbar\left(k + \frac{\omega}{v}\frac{v_z}{v}\right) \times \frac{\hbar\omega}{T} w^{\rm Br}_{\mathbf{p}_\alpha,\mathbf{p}_\beta} f^{\alpha}_{\mathbf{p}_\alpha} f^{\beta}_{\mathbf{p}_\beta} n_{\mathbf{k}} d\mathbf{k} d\mathbf{q} d\mathbf{p}_\beta d\mathbf{p}_\alpha, \quad (26)$$

where integration of $w^{\rm Br}_{\mathbf{p}_\alpha,\mathbf{p}_\beta}(\mathbf{k},\mathbf{q})$ over $d\mathbf{q}$ can be done independently to get (18). $\int w^{\rm Br}_{\mathbf{p}_\alpha,\mathbf{p}_\beta}(\mathbf{k},\mathbf{q})d\mathbf{q}$ has a zero-order term, which is even in v_z, and a first-order term $O(\beta_z)$, which is odd in v_z. In Eq. (26) the first term $k = \omega/c$ is the momentum of the absorbed photon, and it is much smaller than the momentum coming from the recoil $(\omega/v)(v_z/v)$. However, the photon term $k = \omega/c$ is the same for all electrons and is multiplied by the zero-order term in $\int w^{\rm Br}_{\mathbf{p}_\alpha,\mathbf{p}_\beta}(\mathbf{k},\mathbf{q})d\mathbf{q}$, while the recoil term, which depends on the velocity projection v_z, has contribution only from the first-order term in $\int w^{\rm Br}_{\mathbf{p}_\alpha,\mathbf{p}_\beta}(\mathbf{k},\mathbf{q})d\mathbf{q}$, because the zero-order term is the same for oppositely going electrons and so gives zero contribution after averaging over the distribution function. Thus, after multiplication by the probability both terms give contributions of equal order. The coefficient next to the first-order term in $\int w^{\rm Br}_{\mathbf{p}_\alpha,\mathbf{p}_\beta}(\mathbf{k},\mathbf{q})d\mathbf{q}$ is positive, which comes from the fact that Bremsstrahlung emission is the most pronounced in the direction of the electron velocity [7]. Since also the recoil term is proportional to v_z, we can immediately conclude that the averaged momentum gained by electrons due to the recoil is in the positive z-axis direction.

IV. INVERSE BREMSSTRAHLUNG CURRENT

The time evolution of the current density can be put as

$$\frac{d\mathbf{j}}{dt} = -\frac{e}{m_e}\frac{d\mathbf{p}^{e}_{V}}{dt} - \nu_{\rm Sp}\mathbf{j}. \quad (27)$$

This is a fluid approach, since it takes into account only how much momentum is absorbed by electrons, not which electrons absorb the momentum.

The collision frequency $\nu_{\rm Sp}$ in Eq. (27) corresponds to the Spitzer conductivity and can be approximated by the following empirical formula [8],

$$\nu_{\rm Sp} = \frac{Z}{3}\sqrt{\frac{2}{\pi}}\left(0.295 + \frac{0.39}{0.85+Z}\right)\frac{\Gamma}{v^3_{\rm th}}, \quad (28)$$

where $\Gamma = \omega^4_p \ln\Lambda/4\pi n$ and Z is the ion charge. From Eq. (27) the stationary current density is

$$\mathbf{j}_{\rm fluid} = -\frac{e}{m_e}\nu^{-1}_{\rm Sp}\frac{d\mathbf{p}^{e}_{V}}{dt}. \quad (29)$$

Since the current density in the fluid approximation is proportional to the rate of momentum absorption, the current corrected for the recoil is 8/5 times higher than the simple fluid estimate ignoring the recoil and is equal to

$$j_{\rm fluid} = -\frac{8}{5}\frac{e}{m_e}\frac{\alpha I}{c}\nu^{-1}_{\rm Sp} = -\frac{20.4}{Z\left(1+\frac{1.32}{0.85+Z}\right)}\frac{e v^3_{\rm th}}{m_e\Gamma}\frac{\alpha I}{c}. \quad (30)$$

However, the Spitzer conductivity is strictly applicable only to the current produced by a dc electric field, when all electrons get equal acceleration in the same direction. The current generation due to inverse Bremsstrahlung is not equivalent to the action of a dc electric field because different electrons absorb different amounts of power and are pushed in different directions.

One example of the kinetic effects is the additional current due to asymmetric absorption of radiation. Figure 2 shows the integrated probability of absorption within $d\theta$ given by Eq. (18) for electrons lying on the circle with radius $\beta = 0.08$ in velocity space. We see that the electrons going in the direction of the incoming photons $(0 \leqslant \theta < \pi/2)$ absorb more radiation than electrons going in the opposite direction $(\pi/2 < \theta \leqslant \pi)$. This asymmetric absorption will create additional current because the collision frequency in plasma is speed dependent, and thus electrons going in the direction of the incoming radiation will experience less resistance from the plasma than electrons going in the opposite direction, resulting in more current.

Figure 3 shows, averaged over all possible recoils, the rate of momentum absorption along the z axis by an electron with

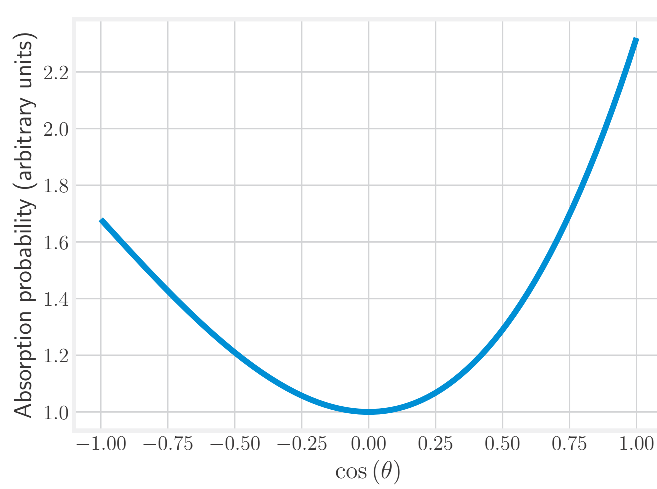

FIG. 2. The probability of Bremsstrahlung absorption in arbitrary units versus the angle between the electron velocity and the incoming photon direction $\cos\theta = v_z/v$ for $\beta = 0.08$.

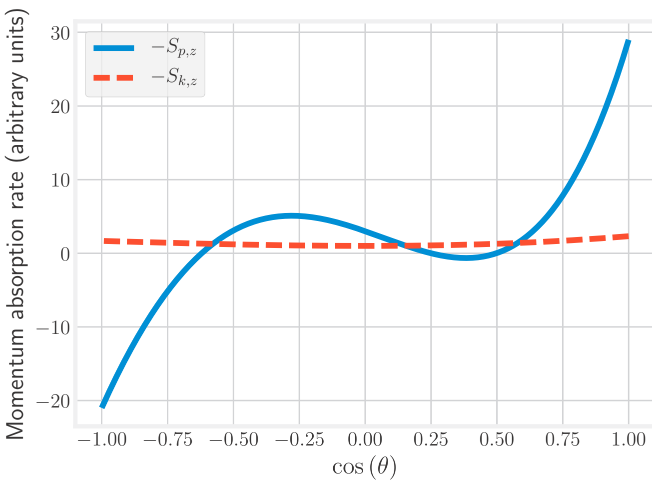

FIG. 3. The momentum absorption rate per electron as a function of $\cos\theta = v_z/v$ for $\beta = 0.08$: along the z axis taking into account the recoil (solid blue line), along the z-axis taking into account only the photon momentum (dashed red line).

$\beta = 0.08$ versus $\cos\theta = v_z/v$. $-S_{p,z}$ is defined by Eq. (14) and determines the rate of momentum absorption taking into account the recoil effect. $-S_{k,z}$ is defined by Eq. (24) and determines the rate of momentum absorption assuming that only the photon momentum is absorbed. We can see that the recoil effect not only changes the integrated (average) rate of momentum absorption but radically alters the distribution of the absorbed momentum in velocity space. For $-S_{k,z}$ the momentum absorption rate is always positive, i.e., along the z axis, and does not strongly depend on $\cos\theta$, while for $-S_{p,z}$ the momentum absorption rate varies greatly with $\cos\theta$ in both magnitude and sign. In considering Bremsstrahlung absorption by a particular electron, the natural directions are along the electron velocity and perpendicular to the electron velocity. When $|\cos(\theta)|$ is close to 1, the velocity of the electron is either parallel or antiparallel to the direction of the incoming photon, and so the change in momentum along the z axis is determined mostly by the recoil parallel to the velocity, which is about $(\hbar\omega/v)(v_z/v)$ in each act of the Bremsstrahlung, as was shown previously. For smaller values of $|\cos(\theta)|$ the change in momentum along the z axis is mostly determined by the recoil perpendicular to the electron velocity. This is why the absorption rate shown in Fig. 3 changes sign.

In general, the distribution function will evolve both under the influence of Bremsstrahlung absorption and under the influence of collisions,

$$\frac{\partial f^e_{\mathbf{p}}}{\partial t} = \left(\frac{\partial f^e_{\mathbf{p}}}{\partial t}\right)_{\mathrm{Br}} + \left(\frac{\partial f^e_{\mathbf{p}}}{\partial t}\right)_{\mathrm{coll}}, \tag{31}$$

and the time evolution of the current should be described more completely than Eq. (27) does by

$$\frac{d\mathbf{j}}{dt} = -e\int \mathbf{v}\frac{\partial f^e_{\mathbf{p}}}{\partial t}d\mathbf{p}. \tag{32}$$

Following [3] we can write the current density at time t as the rate of pushing electrons times the ensemble-averaged current difference:

$$\begin{aligned} j_{\mathrm{cd}}(t) &= \sum_{\mathbf{v},\triangle\mathbf{v}}\int_0^t d\tau \frac{P_V(\tau,\mathbf{v},\triangle\mathbf{v})}{\triangle\varepsilon} \\ &\quad\times \langle q_e v_z(t-\tau,\mathbf{v}+\triangle\mathbf{v}) - q_e v_z(t-\tau,\mathbf{v})\rangle \\ &\underset{\triangle\mathbf{v}\to 0}{=} \sum_{\mathbf{v},\triangle\mathbf{v}}\int_0^t d\tau \frac{P_V(\tau,\mathbf{v},\triangle\mathbf{v})}{\triangle\varepsilon}\triangle\mathbf{v}\cdot\frac{\partial\langle q v_z(t-\tau,\mathbf{v})\rangle}{\partial\mathbf{v}}. \end{aligned} \tag{33}$$

If the power is independent of time, we can put integration inside the ensemble-averaged current and write for a steady-state current

$$j_{\mathrm{cd}} = \int\left[-\frac{e}{m_e}\frac{\hbar(\mathbf{k}-\mathbf{q})\cdot\partial\chi/\partial\mathbf{v}}{\hbar\omega}\right]dP_V(\mathbf{v},\mathbf{k},\mathbf{q}), \tag{34}$$

where we expressed infinitesimal changes in energy and velocity through ω, $\mathbf{k}$, $\mathbf{q}$, changed from summation to integration, and introduced a Green's function: $\chi = \int_0^\infty \langle v_z(\tau,\mathbf{v})\rangle d\tau$. In most cases it is possible to express the Green's function as $\chi(\mathbf{v}) = v_z\nu^{-1}(v)$, where ν^{-1} can be thought of as an effective collision frequency [9].

The expression in square brackets in Eq. (34) can be understood as incremental current drive efficiency. Thus, to find the generated current, one needs to average the incremental current drive efficiency over the power density absorbed:

$$\begin{aligned} j_{\mathrm{cd}} &= \int\left(\frac{\delta j_z}{\delta P_V}\right)dP_V \\ &= \frac{e}{m_e}\int\frac{(\mathbf{k}-\mathbf{q})\cdot\partial\chi/\partial\mathbf{v}}{\omega}\frac{m_e v^2}{2} \\ &\quad\times\frac{\partial}{\partial\mathbf{v}}\cdot\hbar(\mathbf{k}-\mathbf{q})\frac{\hbar\omega}{m_e T}w^{\mathrm{Br}}_{\mathbf{p}_\alpha,\mathbf{p}_\beta}f^\alpha_{\mathbf{p}_\alpha}f^\beta_{\mathbf{p}_\beta}n_{\mathbf{k}}d\mathbf{k}d\mathbf{q}d\mathbf{p}_\beta d\mathbf{p}_\alpha \\ &= \frac{e}{m_e}\int\left[\frac{\nu^{-1}}{\omega}(k_z-q_z) + \frac{\partial\nu^{-1}}{\partial v}\frac{v_z}{v}\right]\frac{m_e v^2}{2} \\ &\quad\times\frac{\partial}{\partial\mathbf{v}}\cdot\hbar(\mathbf{k}-\mathbf{q})\frac{\hbar\omega}{m_e T}w^{\mathrm{Br}}_{\mathbf{p}_\alpha,\mathbf{p}_\beta}f^\alpha_{\mathbf{p}_\alpha}f^\beta_{\mathbf{p}_\beta}n_{\mathbf{k}}d\mathbf{k}d\mathbf{q}d\mathbf{p}_\beta d\mathbf{p}_\alpha. \end{aligned} \tag{35}$$

The first term in square brackets of Eq. (35), which is proportional to $k_z - q_z$, is the usual current due to momentum injection along the z axis, while the second term, which is proportional to $\partial\nu^{-1}/\partial v$, is the current due to asymmetric absorption.

One might want to calculate the generated current by summing the incremental currents instead,

$$\begin{aligned} j_{\mathrm{cd,res}} &= \int\delta j_z = -\frac{e}{m}\int\hbar(\mathbf{k}-\mathbf{q})\cdot\frac{\partial\chi}{\partial\mathbf{v}} \\ &\quad\times\frac{\hbar\omega}{T}w^{\mathrm{Br}}_{\mathbf{p}_\alpha,\mathbf{p}_\beta}f^\alpha_{\mathbf{p}_\alpha}f^\beta_{\mathbf{p}_\beta}n_{\mathbf{k}}d\mathbf{k}d\mathbf{q}d\mathbf{p}_\beta d\mathbf{p}_\alpha \\ &= e\int\mathbf{S}_{\mathbf{v}}\cdot\frac{\partial\chi}{\partial\mathbf{v}}d\mathbf{v}, \end{aligned} \tag{36}$$

where we used the wave induced flux in velocity space $\mathbf{S}_{\mathbf{v}} = m_e^2\mathbf{S}_{\mathbf{p}}$. Equation (36) follows from Eq. (33) if the power absorbed is localized around certain velocity. Therefore, Eqs. (35) and (36) are identical when the absorption is localized in the velocity space, but they produce different results otherwise. In the present problem all electrons are pushed by the incoming electromagnetic field, and Eq. (36) miscalculates the generated current density.

After integration by parts, Eq. (35) can be written as

$$j_{\rm cd} = -\frac{e}{2}\int \frac{\partial\left(v\frac{\partial \nu^{-1}}{\partial v}\right)}{\partial v}\frac{v_z}{v}\hbar\omega\frac{\hbar\omega}{m_eT}N_{\rm ph}n_\beta w^{\rm Br}_{\mathbf{p}_\alpha,\mathbf{p}_\beta}f^e_{\mathbf{v}}d\mathbf{q}d\mathbf{v} - e\int\frac{\partial(v\nu^{-1})}{\partial v}\hbar(k_z-q_z)\frac{\hbar\omega}{m_eT}N_{\rm ph}n_\beta w^{\rm Br}_{\mathbf{p}_\alpha,\mathbf{p}_\beta}f^e_{\mathbf{v}}d\mathbf{q}d\mathbf{v}. \tag{37}$$

The Green's function and the corresponding effective collision frequency ν, generally speaking, can be found only numerically. However, the high-velocity approximation exists [3,10]:

$$\nu^{-1} = \frac{v^3}{\Gamma(5+Z)} + \frac{9v_{\rm th}^2 v}{\Gamma(5+Z)(3+Z)}. \tag{38}$$

This expression has two shortcomings. First, it uses the high-velocity approximation for both electron-electron and electron-ion collisions. While for electron-ion collisions this approximation is always good, it is less so for electron-electron collisions. Since it is mostly thermal electrons that absorb through Bremsstrahlung, the high-velocity approximation will noticeably underestimate the current for low-Z plasma. Second, this expression violates the momentum conservation in electron-electron collisions. Thus, we expect that Eq. (38) is a good approximation for high-Z plasma, but for low-Z plasma the error in the current can be appreciable.

After straightforward calculations using ν defined by Eq. (38) we obtain from Eq. (35)

$$j_{\rm cd} = -\frac{34.2}{5+Z}\frac{ev_{\rm th}^3}{m_e\Gamma}\frac{\alpha I}{c} - \frac{39.5}{(5+Z)(3+Z)}\frac{ev_{\rm th}^3}{m_e\Gamma}\frac{\alpha I}{c}, \tag{39}$$

while Eq. (36) would only give factors 12.8 and 24.8, respectively, in the above formula.

For comparison, in the fluid approximation the current density corrected for the recoil, which is given by Eq. (30), can be represented as

$$j_{\rm fluid} = e\nu_{\rm Sp}^{-1}\int S_{v,z}d\mathbf{v}. \tag{40}$$

We can clearly see that Eq. (35) has an additional term that is responsible for the current due to asymmetric absorption.

Because of the use of the high-velocity and momentum conservation violating approximation for ν, Eq. (39) underestimates the current, especially for small Z. Reckoning that electron-electron collisions conserve current, to remedy this problem we propose an alternative hybrid expression, where the part of the current in Eq. (35) proportional to $k_z - q_z$ is substituted by the fluid expression Eq. (30), while the part proportional to $\partial\nu^{-1}/\partial v$ is left unchanged:

$$j_{\rm hybrid} = j_{\rm fluid} - \frac{e}{m_e}\int\frac{\partial\nu^{-1}}{\partial v}\frac{v_z}{v}dP_V(\mathbf{v},\mathbf{k},\mathbf{q}) = j_{\rm fluid} - \frac{19.2}{5+Z}\frac{ev_{\rm th}^3}{m_e\Gamma}\frac{\alpha I}{c} - \frac{12.4}{(5+Z)(3+Z)}\frac{ev_{\rm th}^3}{m_e\Gamma}\frac{\alpha I}{c}. \tag{41}$$

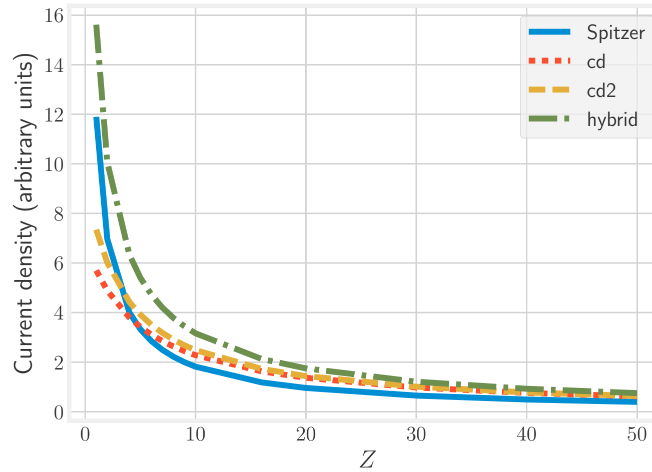

FIG. 4. The generated current density versus the ion charge Z: fluid approximation with the Spitzer conductivity given by Eq. (30) (solid blue line), current drive approximation keeping only the first term in Eq. (39) (dotted red line), current drive approximation keeping both terms in Eq. (39) (dashed orange line), hybrid current given by Eq. (41) (dash-dotted green line).

If all electrons were to absorb equal amounts of power, then the part of the current in Eq. (35) proportional to $k_z - q_z$ would be exactly given by the fluid expression Eq. (30). In case of Bremsstrahlung absorption it is mostly thermal electrons that absorb radiation and the fluid formula overestimates the corresponding part of the current. On the other hand, the second part of Eq. (41) underestimates the current because of the high-velocity limit for ν. So, all in all, Eq. (41) can be a decent approximation for the current for all values of Z.

Figure 4 shows the generated current given by the fluid formula (30), by the current drive formula (39) keeping one and two terms in Eq. (39), and by the hybrid expression (41) versus the ion charge Z. We see that for small Z the current drive formula substantially underestimates current, making it even lower than the fluid prediction. However, starting already with $Z = 4$ the current drive estimate (39) gives higher current. For higher Z, when electron-electron collisions become negligible, the ratio of the current drive prediction to the Spitzer becomes stable and for infinite Z is around 1.7, so that for high Z the generated current with the recoil and kinetic effects taken into account is at least 2.7 higher than the naive fluid estimate without recoil would suggest. The hybrid expression is 1.3 times larger than the fluid estimate even for $Z = 1$, and for Z going to infinity the increase is about 2. To get better and definite results for small Z plasma, it is necessary to use an estimate of the effective collision frequency ν more accurate than Eq. (38) or perform computer simulations.

V. SUMMARY

We analytically considered the generation of the plasma current resulting from electron-ion Bremsstrahlung absorption using the following approximations: the polarization effects in Bremsstrahlung are negligible; velocities are nonrelativistic; recoil and photon momenta are small in comparison with the electron momentum; ions have infinite mass; waves are electromagnetic with the dispersion relation $\omega = kc$; and the plasma dielectric function is close to one. The laser intensity is not too high, so the quiver velocity $eE/m\omega$ is much smaller than the thermal velocity. We also note that the logarithmic dependence on velocity has been ignored throughout the paper and $\ln(m_\alpha v^2/\hbar\omega)$ has been substituted with $\ln(2T/\hbar\omega)$ in all the equations.

We investigated how the momentum and energy are absorbed by electrons within the velocity space and confirmed the result obtained in [1], namely that the averaged momentum absorption by electrons with the recoil taken into account is $8/5$ times higher than the momentum absorption assuming that electrons absorb just the photon momentum. In addition, we demonstrated that for high-Z plasma the actual current with the kinetic effects taken into account is at least 2.7 times higher than the naive fluid estimates without recoil would suggest, both because electrons get the recoil momentum from the Coulomb field of ions during the absorption and because electrons absorb power asymmetrically. We also proposed a hybrid expression of fluid and kinetic descriptions for the current that can be a good approximation for all values of Z.

The calculation of the current generated from Bremsstrahlung absorption is a fundamental problem of the basic plasma physics. Thus, the results here ought to be of interest in the different areas where radiation-driven currents and the generated magnetic fields are important. Areas in which these effects might be important include the radiation-driven magnetic field in astrophysics [11–13] and laboratory experiments that use lasers to drive current [14], in particular for applications to inertial confinement fusion.

ACKNOWLEDGMENT

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