Effect of high-order dispersion on three-soliton interactions for the variable-coefficients Hirota equation

Wenjun Liu,[*] Chunyu Yang, Mengli Liu, Weitian Yu, Yujia Zhang, and Ming Lei
State Key Laboratory of Information Photonics and Optical Communications, School of Science, and P. O. Box 91, Beijing University of Posts and Telecommunications, Beijing 100876, China

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The interactions of multiple solitons show different properties with two-soliton interactions. For the difficulty of deriving multiple soliton solutions, it is rare to study multiple soliton interactions analytically. In this paper, three-soliton interactions in inhomogeneous optical fibers, which are described by the variable coefficient Hirota equation, are investigated. Via the Hirota bilinear method and symbolic computation, analytic three-soliton solutions are obtained. According to the obtained solutions, properties and features of three-soliton interactions are discussed by changing the third-order dispersion (TOD) and other relevant coefficients, and some plentiful structure of three-soliton interactions are presented for the first time. The influences of TOD on the intensity and propagation distance of solitons are described, which can be used to realize the soliton control. Besides, the method that can achieve the phase reverse of solitons is suggested, and bound states of three solitons are observed, which have potential applications in the mode-locked fiber lasers. Furthermore, comparing to two-soliton interactions, a novel phenomenon of three-soliton interactions with a strong phase shift at $x = 0$ is revealed, which is potentially useful for optical logic switches.

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I. INTRODUCTION

Optical solitons, which can maintain their shape and velocity during the long-distance propagation due to the balance between the group velocity dispersion (GVD) and self-phase modulation (SPM) effects [1], have become an attractive research field on account of their potential applications in the optical communication systems and all-optical ultrafast switching devices [2–8]. It is well known that the soliton interaction is a unique phenomenon in the nonlinear fiber transmission. Solitons attract each other in the transmission process because of the existence of soliton interactions, which result in a serious reduction in the transmission rate [9]. Therefore, it is particularly important and urgent to study soliton interactions and find an effective method to control them.

Actually, the research of soliton interactions mostly focus on two solitons [10–13]. Mutual interactions of solitons can be reduced by the inclusion of the third-order dispersion (TOD) of optical fibers [14]. The bound two solitons can split into individual solitons traveling with different speeds by reason of higher-order effects [15]. Two solitons of a dark-soliton pair can also form a bound state, which can be applied to fabricate the optical coupler [16]. Besides, non-locality provides an attractive force between other repelling solitons, and can form their bound state [17]. More recently, interactions between bright and dark solitons can generate the dispersive radiation [18].

However, we notice that interactions among multiple solitons have not been discussed much. In some practical problems and applications, multiple solitons often transmit in a medium at the same time, whereas there are not only the interactions between two solitons, but also three and four soliton interactions. When the multiple solitons co-propagate and interact, there may be different interaction characteristics with two-soliton interactions. The propagation properties of multiple solitons could not be drawn simply from the results of two-soliton interactions. Besides, interactions of three solitons are inherently complex, plusing with the influence of the high-order dispersion effects on it, three soliton interactions will produce some novel phenomena. So it is inevitable to consider multiple soliton interactions. In addition, discrete solitons and soliton arrays that have attracted wide attention in recent years, which involve the co-propagation and interaction of a large number of solitons [19–24]. Furthermore, using multiple soliton interactions can give further impetus in constructing multistate logic, multi-input logic gates, memory storage devices, and so on [25]. Therefore, it is of great theoretical and practical significance to study the interaction rules of multiple solitons.

The nonlinear Schrödinger (NLS) equation can be used to study the properties and features of soliton interactions in nonlinear optics. In the mathematical point of view, the Hirota equation represents the integrable version of the NLS equation. The variable-coefficients Hirota equation can be used to describe soliton interactions in inhomogeneous optical fibers as follows [26]:

$$iq_x - \beta_2 q_{tt} + \gamma |q|^2 q + i\beta_3(x) q_{ttt} + i\tau(x)|q|^2 q_t + \mu q = 0. \tag{1}$$

Here, q is a complex function with x and t denoting the normalized propagation distance along the optical fiber and retarded time. The coefficients β_2, γ, $\beta_3(x)$, $\tau(x)$, and μ, respectively, represent the GVD, Kerr nonlinearity, TOD, time-delay related to the cubic term, and external potential [27,28].

However, three-soliton interactions in Eq. (1) have not been studied. In this paper, the plentiful structure of three-soliton interactions will be presented. We will directly analyze the

[*]Also at: Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China; jungliu@bupt.edu.cn

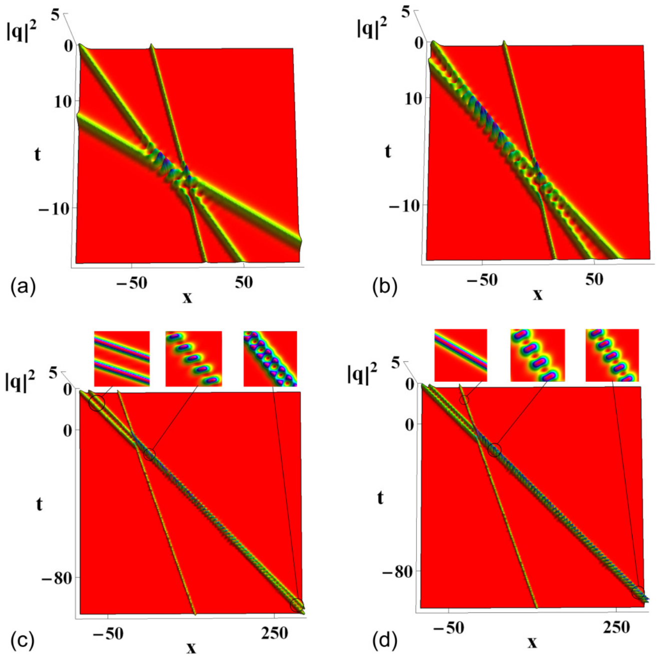

FIG. 1. The intensity and distance of three-soliton interactions affected by the change of $\beta_3(x)$. Parameters chosen as: $\xi_{11} = 8.4$, $\xi_{21} = 6.1$, $\xi_{31} = 9.4$, $\xi_{12} = 1$, $\xi_{22} = 3$, $\xi_{32} = 5$, $w_{11} = 1$, $w_{21} = 1$, $w_{31} = 1$, $w_{12} = -1.4$, $w_{22} = 1.9$, $w_{32} = -0.77$, $\beta_2 = -0.15$, $\mu = 0.13$, $\gamma = 0.3$ with (a) $\beta_3(x) = 0.07$; (b) $\beta_3(x) = 0.08$; (c) $\beta_3(x) = 0.09$; (d) $\beta_3(x) = 0.0885$.

effect of $\beta_3(x)$ on the intensity and distance of soliton interactions. Besides, the phase of solitons can be reversed by choosing suitable parameters. By setting appropriate values of TOD and corresponding parameters, we will find the bound state of three solitons, which have potential applications in the formation of mode-locked fiber lasers, and increase the transmission line bandwidth of optical communication systems. In addition, we will find a novel type of soliton interaction in which a strong phase shift occurs on both sides of $x = 0$, which is different from two-soliton interactions [29].

This paper is organized as follow. Section II is devoted to deriving an analytic three-soliton solution of Eq. (1) by the Hirota method. Section III is allotted to analyze the properties and features of three-soliton interactions, and investigate the effect on soliton interactions using different choices of TOD. Finally, Sec. IV is reserved for drawing a conclusion.

II. BILINEAR FORMS AND THREE-SOLITON SOLUTIONS

In order to derive the bilinear forms for Eq. (1), we introduce the dependent variable transformation $q(x,t) = g(x,t)/f(x,t)$ [30]. Under the constrains $\tau(x) = -3\gamma\beta_3(x)/\beta_2$, the bilinear forms for Eq. (1) can be derived as follows:

$$\begin{aligned} D_t^2 f\cdot f + \frac{\gamma}{\beta_2} g g^* &= 0,\\ \left[iD_x + \mu - \beta_2 D_t^2 + i\beta_3(x) D_t^3\right] g\cdot f &= 0, \end{aligned} \tag{2}$$

where $g(x,t)$ is assumed as a complex differentiable function while $f(x,t)$ is a real one. D_x and D_t are the bilinear derivative operators defined as [31]

$$\begin{aligned} &D_x^m D_t^n G(x,t)\cdot F(x,t)\\ &= \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^n G(x,t)F(x',t')|_{x'=x,t'=t} \end{aligned} \tag{3}$$

with $G(x,t)$ and $F(x',t')$ are the differentiable functions, while m and n are both non-negative integers.

To obtain soliton solutions of Eq. (1), we expand $g(x,t)$ and $f(x,t)$ with a formal expansion parameter ε as

$$\begin{aligned} g(x,t) &= \varepsilon g_1(x,t) + \varepsilon^3 g_3(x,t) + \varepsilon^5 g_5(x,t),\\ f(x,t) &= 1 + \varepsilon^2 f_2(x,t) + \varepsilon^4 f_4(x,t) + \varepsilon^6 f_6(x,t), \end{aligned}$$

where $g_i(x,t)(i = 1,3,5\ldots)$ are the complex deferential functions while $f_j(x,t)(j = 0,2,4,\ldots)$ are real ones. Without losing generality, setting $\varepsilon = 1$ and substituting them into bilinear form (2), we can construct three-soliton solutions for Eq. (1) as follows:

$$q(x,t) = \frac{g(x,t)}{f(x,t)} = \frac{g_1(x,t) + g_3(x,t) + g_5(x,t)}{1 + f_2(x,t) + f_4(x,t) + f_6(x,t)}, \tag{4}$$

where

$$\begin{aligned} g_1(x,t) &= e^{\theta_1} + e^{\theta_2} + e^{\theta_3},\\ g_3(x,t) &= \kappa_1(x)e^{\theta_1+\theta_2+\theta_1^*} + \kappa_2(x)e^{\theta_1+\theta_2+\theta_2^*} + \kappa_3(x)e^{\theta_1+\theta_2+\theta_3^*} + \kappa_4(x)e^{\theta_1+\theta_3+\theta_1^*} + \kappa_5(x)e^{\theta_1+\theta_3+\theta_2^*} + \kappa_6(x)e^{\theta_1+\theta_3+\theta_3^*}\\ &\quad + \kappa_7(x)e^{\theta_2+\theta_3+\theta_1^*} + \kappa_8(x)e^{\theta_2+\theta_3+\theta_2^*} + \kappa_9(x)e^{\theta_2+\theta_3+\theta_3^*},\\ g_5(x,t) &= \iota_1(x)e^{\theta_1+\theta_2+\theta_3+\theta_1^*+\theta_2^*} + \iota_2(x)e^{\theta_1+\theta_2+\theta_3+\theta_1^*+\theta_3^*}\iota_3(x)e^{\theta_1+\theta_2+\theta_3+\theta_2^*+\theta_3^*},\\ f_2(x,t) &= \phi_1(x)e^{\theta_1+\theta_1^*} + \phi_2(x)e^{\theta_1+\theta_2^*} + \phi_3(x)e^{\theta_1+\theta_3^*} + \phi_4(x)e^{\theta_2+\theta_1^*} + \phi_5(x)e^{\theta_2+\theta_2^*} + \phi_6(x)e^{\theta_2+\theta_3^*}\\ &\quad + \phi_7(x)e^{\theta_3+\theta_1^*} + \phi_8(x)e^{\theta_3+\theta_2^*} + \phi_9(x)e^{\theta_3+\theta_3^*},\\ f_4(x,t) &= \varphi_1(x)e^{\theta_1+\theta_2+\theta_1^*+\theta_2^*} + \varphi_2(x)e^{\theta_1+\theta_2+\theta_1^*+\theta_3^*} + \varphi_3(x)e^{\theta_1+\theta_2+\theta_2^*+\theta_3^*} + \varphi_4(x)e^{\theta_1+\theta_3+\theta_1^*+\theta_2^*}\\ &\quad + \varphi_5(x)e^{\theta_1+\theta_3+\theta_1^*+\theta_3^*} + \varphi_6(x)e^{\theta_1+\theta_3+\theta_2^*+\theta_3^*} + \varphi_7(x)e^{\theta_2+\theta_3+\theta_1^*+\theta_2^*} + \varphi_8(x)e^{\theta_2+\theta_3+\theta_1^*+\theta_3^*} + \varphi_9(x)e^{\theta_2+\theta_3+\theta_2^*+\theta_3^*},\\ f_6(x,t) &= \psi_1(x)e^{\theta_1+\theta_2+\theta_3+\theta_1^*+\theta_2^*+\theta_3^*} \end{aligned} \tag{5}$$

with

$$\theta_1 = [k_{11}(x) + ik_{12}(x)]x + (w_{11} + iw_{12})t + \xi_{11} + i\xi_{12}, \theta_2 = [k_{21}(x) + ik_{22}(x)]x + (w_{21} + iw_{22})t + \xi_{21} + i\xi_{22},$$
$$\theta_3 = [k_{31}(x) + ik_{32}(x)]x + (w_{31} + iw_{32})t + \xi_{31} + i\xi_{32}.$$
$$k_{11}(x) = \frac{\int [2w_{11}w_{12}\beta_2 - w_{11}^3\beta_3(x) + 3w_{11}w_{12}^2\beta_3(x)]dx}{x},\ k_{12}(x) = \frac{\int [\mu - w_{11}^2\beta_2 + w_{12}^2\beta_2 - 3w_{11}^2w_{12}\beta_3(x) + w_{12}^3\beta_3(x)]dx}{x},$$
$$k_{21}(x) = \frac{\int [2w_{21}w_{22}\beta_2 - w_{21}^3\beta_3(x) + 3w_{21}w_{22}^2\beta_3(x)]dx}{x},\ k_{22}(x) = \frac{\int [\mu - w_{21}^2\beta_2 + w_{22}^2\beta_2 - 3w_{21}^2w_{22}\beta_3(x) + w_{22}^3\beta_3(x)]dx}{x},$$
$$k_{31}(x) = \frac{\int [2w_{31}w_{32}\beta_2 - w_{31}^3\beta_3(x) + 3w_{31}w_{32}^2\beta_3(x)]dx}{x},\ k_{32}(x) = \frac{\int [\mu - w_{31}^2\beta_2 + w_{32}^2\beta_2 - 3w_{31}^2w_{32}\beta_3(x) + w_{32}^3\beta_3(x)]dx}{x},$$
$$\kappa_1(x) = -\frac{\gamma A_{11}^2}{8w_{11}^2\beta_2A_{21}^2},\quad \kappa_2(x) = -\frac{\gamma A_{11}^2}{8w_{21}^2\beta_2A_{31}^2},\ \kappa_3(x) = -\frac{\gamma A_{11}^2}{2\beta_2A_{32}^2A_{33}^2},\quad \kappa_4(x) = -\frac{\gamma A_{12}^2}{8w_{11}^2\beta_2A_{22}^2},$$
$$\kappa_5(x) = -\frac{\gamma A_{12}^2}{2\beta_2A_{23}^2A_{31}^2},\quad \kappa_6(x) = -\frac{\gamma A_{12}^2}{8w_{31}^2\beta_2A_{32}^2},\ \kappa_7(x) = -\frac{\gamma A_{13}^2}{2\beta_2A_{21}^2A_{22}^2},\quad \kappa_8(x) = -\frac{\gamma A_{13}^2}{8\beta_2A_{23}^2},$$
$$\kappa_9(x) = -\frac{\gamma A_{13}^2}{8\beta_2A_{33}^2},\quad \varphi_1(x) = \frac{\gamma^2B_{11}^2}{64w_{11}^2w_{21}^2\beta_2^2B_{12}^2},\ \varphi_2(x) = \frac{\gamma^2A_{11}^2A_{42}^2}{16w_{11}^2\beta_2^2A_{32}^2A_{33}^2A_{21}^2},\ \varphi_3(x) = \frac{\gamma^2A_{11}^2A_{43}^2}{16w_{21}^2\beta_2^2A_{32}^2A_{33}^2A_{31}^2},$$
$$\varphi_4(x) = \frac{\gamma^2A_{12}^2A_{41}^2}{16w_{11}^2\beta_2^2A_{23}^2A_{31}^2A_{22}^2},\varphi_5(x) = \frac{\gamma^2B_{21}^2}{64w_{11}^2w_{31}^2\beta_2^2B_{22}^2},\ \varphi_6(x) = \frac{\gamma^2A_{12}^2A_{43}^2}{16w_{31}^2\beta_2^2A_{23}^2A_{31}^2A_{32}^2},\ \varphi_7(x) = \frac{\gamma^2A_{13}^2A_{41}^2}{16w_{21}^2\beta_2^2A_{21}^2A_{22}^2A_{23}^2},$$
$$\varphi_8(x) = \frac{\gamma^2A_{13}^2A_{42}^2}{16w_{31}^2\beta_2^2A_{21}^2A_{22}^2A_{33}^2},\varphi_9(x) = \frac{\gamma^2B_{31}^2}{64w_{21}^2w_{31}^2\beta_2^2B_{32}^2},\ \iota_1(x) = \frac{\gamma^2A_{12}^2A_{13}^2B_{11}^2}{64w_{11}^2w_{21}^2\beta_2^2A_{21}^2A_{22}^2A_{23}^2A_{31}^2}, \tag{6}$$
$$\iota_2(x) = \frac{\gamma^2A_{11}^2A_{13}^2B_{21}^2}{64w_{11}^2w_{31}^2\beta_2^2A_{21}^2A_{22}^2A_{32}^2A_{33}^2},\ \iota_3(x) = \frac{\gamma^2A_{11}^2A_{12}^2B_{31}^2}{64w_{21}^2w_{31}^2\beta_2^2A_{23}^2A_{31}^2A_{32}^2A_{33}^2},$$
$$\psi_1(x) = -\frac{\gamma^3B_{11}^2B_{21}^2B_{31}^2}{512\mu^3w_{11}^2w_{21}^2w_{31}^2A_{21}^2A_{22}^2A_{23}^2A_{31}^2A_{32}^2A_{33}^2},\ A_{11} = w_{11} + iw_{12} - w_{21} - iw_{22},\ A_{12} = w_{11} + iw_{12} - w_{31} - iw_{32},$$
$$A_{13} = w_{21} + iw_{22} - w_{31} - iw_{32},\ A_{21} = w_{11} - iw_{12} + w_{21} + iw_{22},\ A_{22} = w_{11} - iw_{12} + w_{31} + iw_{32},$$
$$A_{23} = w_{21} - iw_{22} + w_{31} + iw_{32},\ A_{31} = w_{11} + iw_{12} + w_{21} - iw_{22},\ A_{32} = w_{11} + iw_{12} + w_{31} - iw_{32},$$
$$A_{33} = w_{21} + iw_{22} + w_{31} - iw_{32},\ A_{41} = w_{11} - iw_{12} - w_{21} + iw_{22},\ A_{42} = w_{11} - iw_{12} - w_{31} + iw_{32},$$
$$A_{43} = w_{21} - iw_{22} - w_{31} + iw_{32},\ B_{11} = (w_{11} - w_{21})^2 + (w_{12} - w_{22})^2,\ B_{12} = (w_{11} + w_{21})^2 + (w_{12} - w_{22})^2,$$
$$B_{21} = (w_{11} - w_{31})^2 + (w_{12} - w_{32})^2,\ B_{22} = (w_{11} + w_{31})^2 + (w_{12} - w_{32})^2,\ B_{31} = (w_{21} - w_{31})^2 + (w_{22} - w_{32})^2,$$
$$B_{32} = (w_{21} + w_{31})^2 + (w_{22} - w_{32})^2. \tag{7}$$

III. DISCUSSION

In order to better study the effect of high-order dispersion on three-soliton interactions, we take the following examples of relevant coefficients, especially the TOD parameter $\beta_3(x)$. Consequently, we analyze the dispersion management or various types of soliton control by choosing different parameters.

As shown in Figs. 1 and 2, interactions among solitons are the typical soliton elastic interactions when we choose the constant as the parameter of TOD. We can observe in Fig. 1 that the intensity and distance of soliton interaction can be controlled by changing the value of $\beta_3(x)$. In Fig. 1(a), when we set $\beta_3(x) = 0.07$, interactions among solitons are weakened after interacting a limited distance, and they separate in accordance with the original velocities and phases. With the decreasing of the value of $\beta_3(x)$, the interaction distance between solitons become longer in Figs. 1(b) and 1(c). As shown in Fig. 1(c), when $\beta_3(x) = 0.09$, before three-soliton interactions, each of them is transmitted in the original direction without affecting each other. After three-soliton interactions, two solitons attract each other and transmit for a long distance with a strong interaction intensity, but they will separate after interacting a certain distance. Especially, when choosing $\beta_3(x) = 0.0885$, two solitons merge together and transmit a longer distance after three-soliton interactions. This means that the transmitted distance of solitons in the fused state can be controlled by changing the value of TOD. Thus, we can control the distance and intensity of soliton interactions in a certain range by choosing the suitable value of $\beta_3(x)$. As expressions (6) show, $\beta_3(x)$ mainly affects the coefficient of the propagation distance x. So the reason the $\beta_3(x)$ effect the distance and intensity of soliton interactions can be explained by the $k_{ij}(x),(i = 1,2,3; j = 1,2)$ in expressions (6).

On the other hand, as we can see in the expressions (5)–(7), the relevant coefficients of the three-soliton solution

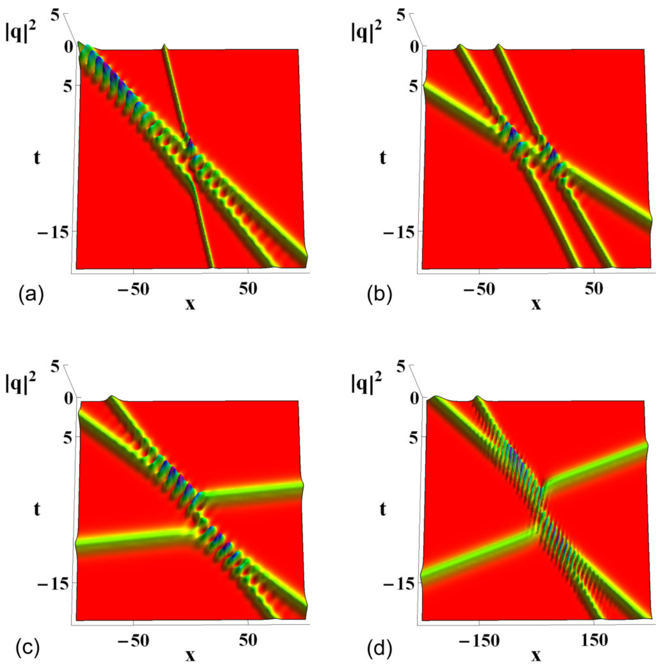

FIG. 2. Phase shift and propagation direction change of three solitons affected by the arbitrary parameters w_{12}, w_{22}, w_{32}. Parameters are the same as Fig. 1 but $\beta_3(x) = 0.085$ with (a) $w_{12} = -1.2$, $w_{22} = 1.7$, $w_{32} = -0.61$; (b) $w_{12} = 2$, $w_{22} = -0.44$, $w_{32} = 1.9$; (c) $w_{12} = -0.47$, $w_{22} = 1.8$, $w_{32} = -0.2$; (d) $w_{12} = -0.34$, $w_{22} = 1.6$, $w_{32} = -0.19$.

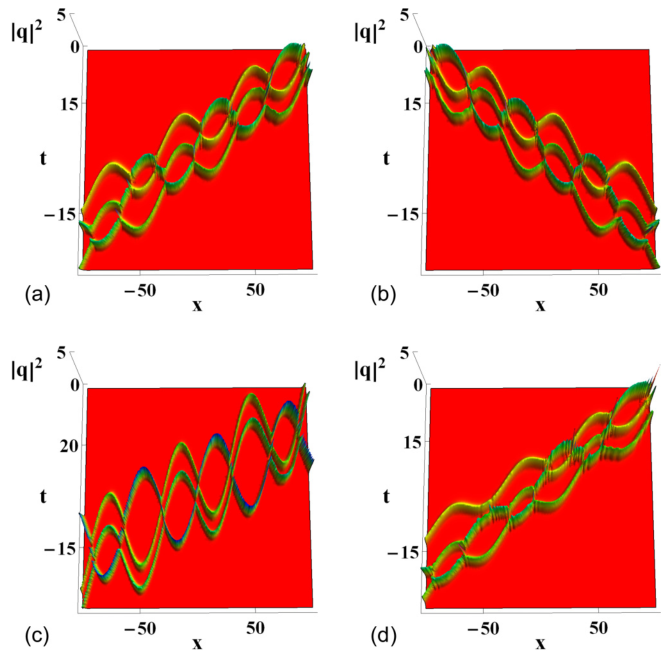

FIG. 3. Three-soliton interactions affected by the change of $\beta_3(x)$. Parameters are the same as Fig. 1 but $w_{11} = -1.5$, $w_{21} = -1.2$, $w_{31} = -1$ with (a) $w_{12} = 0.78$, $w_{22} = 0.8$, $w_{32} = 0.7$, $\beta_3(x) = \cos(0.1x)$; (b) $w_{12} = -0.78$, $w_{22} = -0.8$, $w_{32} = -0.7$, $\beta_3(x) = -\cos(0.1x)$; (c) $w_{12} = 0.58$, $w_{22} = 0.95$, $w_{32} = 0.9$, $\beta_3(x) = \cos(0.1x)$; (d) $w_{12} = 0.82$, $w_{22} = 0.76$, $w_{32} = 0.65$, $\beta_3(x) = \cos(0.1x)$.

are affected by the parameters $w_{ij}(i,j = 1,2,3)$. Therefore, we fix the value of $\beta_3(x) = 0.085$, and change the values of w_{12}, w_{22}, and w_{32} to observe their influence on three-soliton interactions in Fig. 2. We can observe clearly that the propagation direction of three solitons are different with each other through choosing different values of w_{12}, w_{22}, and w_{32}. Besides, it exhibits the phase shift after three-soliton interactions in Figs. 2(c) and 2(d). Particularly, the periodic oscillation has occurred between two relatively close solitons under appropriate parameter conditions in Fig. 2(d).

If we choose the TOD coefficient as periodic functions, such as $\beta_3(x) = \cos(0.1x)$, three solitons will interact periodically as shown in Fig. 3. Comparing Fig. 3(a) to Fig. 3(b), we can find that the phase of solitons is completely opposite if we replace the values of w_{12}, w_{22}, w_{32}, and $\beta_3(x)$ with the values that are opposite to their sign, which also can be seen in Figs. 4 and 5. In addition, their periods are almost the same. The amplitude of solitons and soliton separations in bound states can be adjusted by changing the values of w_{12}, w_{22}, and w_{32}, as shown in Figs. 3(c) and 3(d).

When $\beta_3(x)$ is a Gauss function, such as $\beta_3(x) = 0.07e^{-0.005x^2}$, three solitons attract and repel in a short distance, and a trigonometric structure appears in Fig. 4. In the process of transmission, the pulse trajectory of one soliton is no longer a straight line, but the appearance of an S-type. The phase of solitons in Figs. 4(a) and 4(b) are exactly opposite. Besides, we consider the TOD parameter as $\beta_3(x) = e^{-2.7x^2}$. As Fig. 5 shows, three solitons occur a strong phase shift on both sides of $x = 0$ and form a lattice structure. Through adjusting the values of w_{12}, w_{22}, and w_{32}, the phase shift intensity can be controlled as shown in Figs. 5(a) and 5(c). Similarly, the phase of three solitons reverses in Fig. 5 by changing the sign of w_{12}, w_{22}, w_{32}, and $\beta_3(x)$. Therefore, the parameters play an important role in determining the properties of solitons. By analyzing the influences of $\beta_3(x)$ and relevant parameters, not only can we avoid the disordered propagation of solitons, but also utilize the soliton interaction properties to realize mode-locked fiber lasers, optical logic switches, and path control.

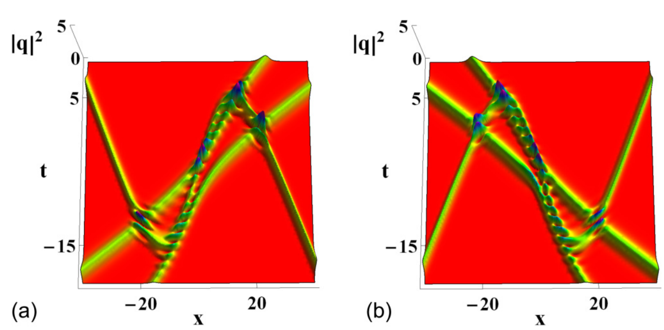

FIG. 4. Three-soliton interactions affected by the change of $\beta_3(x)$. Parameters are the same as Fig. 1 but (a) $w_{12} = 0.98$, $w_{22} = -3.3$, $w_{32} = 1.5$, $\beta_3(x) = -0.07e^{-0.005x^2}$; (b) $w_{12} = -0.98$, $w_{22} = 3.3$, $w_{32} = -1.5$, $\beta_3(x) = 0.07e^{-0.005x^2}$.

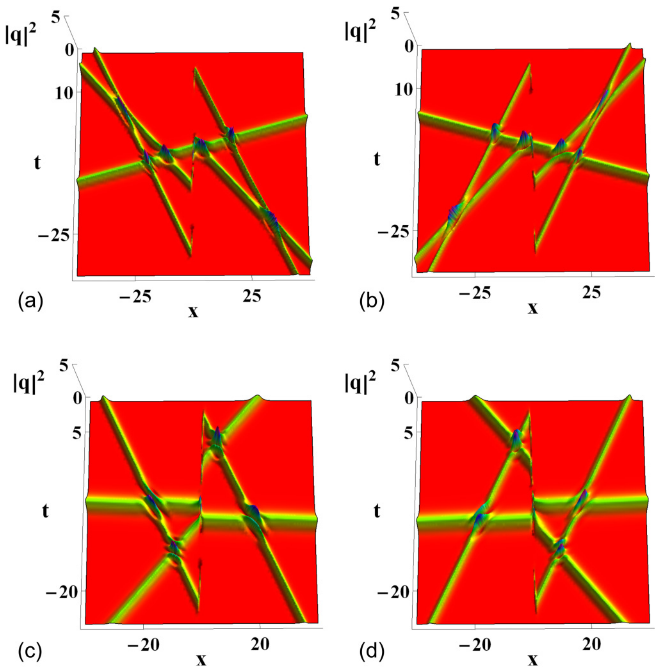

FIG. 5. Three-soliton interactions affected by the change of $\beta_3(x)$. Parameters are the same as Fig. 1 but (a) $w_{12} = -3.8$, $w_{22} = 0.5$, $w_{32} = -2$, $\beta_3(x) = -e^{-2.7x^2}$; (b) $w_{12} = 3.8$, $w_{22} = -0.5$, $w_{32} = 2$, $\beta_3(x) = e^{-2.7x^2}$; (c) $w_{12} = -3.1$, $w_{22} = 1.7$, $w_{32} = -0.063$, $\beta_3(x) = -e^{-2.7x^2}$; (d) $w_{12} = 3.1$, $w_{22} = -1.7$, $w_{32} = 0.063$, $\beta_3(x) = e^{-2.7x^2}$.

IV. CONCLUSIONS

Analytic three soliton solution (4) for Eq. (1) has been obtained with the bilinear method in this paper. The influences of different types of TOD and other corresponding parameters on the interactions of three solitons have been analyzed. By choosing different parameters, various three-soliton interaction properties and features have been presented. Through selecting the suitable value of TOD, we have found that the TOD can effect the soliton interaction intensity and distance. Furthermore, the phase of solitons can be reversed if the values of w_{12}, w_{22}, w_{32}, and $\beta_3(x)$ are replaced with the values that are opposite to their sign. Besides, the bound states of three solitons have been observed, and the amplitude of solitons has been adjusted by changing the values of w_{12}, w_{22}, and w_{32}. In addition, a new soliton interaction phenomenon, that the three-soliton interaction generates a strong phase shift at the both side of $x = 0$ by setting appropriate parameters, has been demonstrated. We hope that these results are helpful for soliton applications in such fields as mode-locked fiber lasers and optical logic switches.

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