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Liquidlike sloshing dynamics of monodisperse granulate

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Analogies between fluid flows and granular flows are useful because they pave the way for continuum treatments of granular media. However, in practice it is impossible to predict under what experimental conditions the dynamics of fluids and granulates are qualitatively similar. In the case of unsteadily driven systems no such analogy is known. For example, in a partially filled container subject to horizontal oscillations liquids slosh, whereas granular media of complex particles exhibit large-scale convection rolls. We here show that smooth monodisperse steel spheres exhibit liquidlike sloshing dynamics. Our findings highlight the role of particle material and geometry for the dynamics and phase transitions of the system.

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Flowing fluid and granular media are ubiquitous in nature and set the efficiency and limitations of many engineering applications. For fluid flows the governing equations are well established for over a century, allowing for very accurate predictions of their dynamics. In contrast, granular flows are notoriously difficult to describe with continuum approaches, which hinders a fundamental understanding of their physics and makes even qualitative predictions of their dynamics challenging. A central problem here is to determine under what conditions granular media can be treated like liquids or rather like solids [1]. Despite recent progress in understanding similarities in the global dynamics of liquids and granulates in some configurations [2–5], for systems subjected to unsteady driving there seems to be no analogy. An important example here is the sloshing of a liquid in an accelerated open container, which is a daily nuisance for people walking with a full cup of coffee [6] and challenges space engineers attempting to minimize undesired motions of cryogenic fuel in the tank of rockets [7,8].

In fundamental studies the sloshing of a liquid is often driven by a harmonic horizontal oscillation and occurs typically in the form of standing or traveling waves of the liquid surface (or their nonlinear interaction). However, if granulate is exposed to such an agitation the dominating patterns are convection rolls in the bulk of the system, provided that the acceleration is large enough to overcome a solidlike behavior [9–15]. The profile and mean flow of the surface layer are dominated by the underlying convection rolls, while a periodic displacement of grains with the driving frequency was also observed [14]. The granulate in these experiments consists typically of polydisperse complex shaped grains like poppy seeds [11,13] or sand [10,14,15]. Few experiments use monodisperse particles with a well-defined shape. Ristow *et al.* [16] investigated, for example, the transition thresholds between solid and fluidlike behavior of three to five layers of glass ballotonies in a quasi-two-dimensional system, but the dynamics of the spheres was not elucidated. In the special case of a monolayer of monodisperse spheres the granulate transitions for sufficiently strong driving from a solidlike directly to a gaslike state [17].

In this Rapid Communication we show that the dynamics of monodisperse steel spheres is qualitatively different from that of complex-shaped granulate and that sloshing phenomena play a key role here. We uncover a remarkable resemblance of the dynamics of monodisperse spheres to the dynamics of liquids and present a granular state, which is analogous to liquid sloshing. We describe the state of the system (solidlike and liquidlike sloshing dynamics and gas-like state) as a function of the layer thickness and the acceleration of the external driving. The boundaries delimiting regions of qualitatively different dynamics show surprisingly simple scaling and call for a detailed theoretical study of the physics governing the transitions.

Our experimental setup consists of a rectangular box which is horizontally oscillated and partially filled with spheres (see Fig. 1). The horizontal driving is harmonic $x(t) = A \sin(\omega t)$, where A is the amplitude and ω the angular frequency, and is accomplished by a step motor. The control parameter is the nondimensional acceleration $\Gamma = A\omega^2/g$, where g is the gravitational acceleration. During measurements the angular velocity is held constant at $\omega = 2\pi \times 3$ Hz, while A is varied between 8 and 27 mm, resulting in $\Gamma \in [0.29\, 1]$. The box is made of polycarbonate to allow for visualization and has an inside length of 97.8 mm parallel to the direction of oscillation, a width of 51.4 mm, and a height of 52.2 mm. The top plate can be opened between measurements to change the granulate [17,18], which consisted of monodisperse steel spheres with diameter $d = (4 \pm 0.003)$ mm of grade 10 from Spherotech. A monolayer of granulate consists of 337 spheres. In one measurement spheres with a diameter of $d = (2 \pm 0.005)$ mm (grade 20) were used.

In our system the dynamics of the bulk can be inferred from the dynamics of the top two visible layers, which we use to characterize the states. With visible layers we here refer to the number of spheres on top of each other which are visible from the side of the box and not to the number of spheres in the system, which is counted as multiples of the amount of spheres in a monolayer. A camera with a sampling rate of 240 Hz and a resolution of 640×480 pixels recorded the dynamics allowing us to identify the states unambiguously. Snapshots of the different dynamical states were also taken with a digital camera (see Figs. 2 and 3).

The occurrence of the different granular states as a function of acceleration Γ and number of spheres is summarized in the phase diagram of Fig. 4. The diagram was obtained by changing the amplitude of the oscillation in steps of

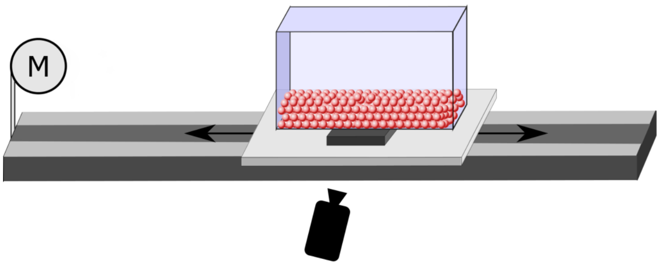

FIG. 1. Sketch of the experimental setup. A rectangular container is partly filled with monodisperse steel spheres and mounted on a sledge. The sledge is harmonically driven by a step motor with a frequency of 3 Hz, while the amplitude is varied to change the acceleration. The dynamics of the spheres is monitored by a camera from the front. In most of the measurements the diameter of the spheres is 4 mm.

1 mm, whereas for each measurement the flow dynamics was monitored for 30 s from the front of the experiment. At low acceleration, most of the spheres move like a solid body with the oscillation of the box and only a few spheres at the top layer may have a weak relative motion (with a displacement that is smaller than three sphere diameters d) with a frequency corresponding to the driving. Starting from this solidlike state we observe for increasing driving a swelling of the sloshing motion of a few spheres of the top layer. At $\Gamma \approx 0.5$ the granular sloshing state shown in the consecutive series of snapshots in the left panel of Fig. 2 is reached. Here the spheres of the top layer slosh substantially (relative displacement of the spheres $\gtrsim 3d$ within an oscillation of the box) and may pile up at the walls of the box perpendicular to the oscillation direction. The spheres of the second visible layer from the top may have a weak harmonic motion. An important feature of this state is that the ensembling surface of the top layer (which can be imagined as, e.g., a flexible plastic layer resting on top of the spheres) is rather flat except for the rising at the box wall. We refer to this dynamics as granular sloshing state because it is well known from previous granular investigations [14].

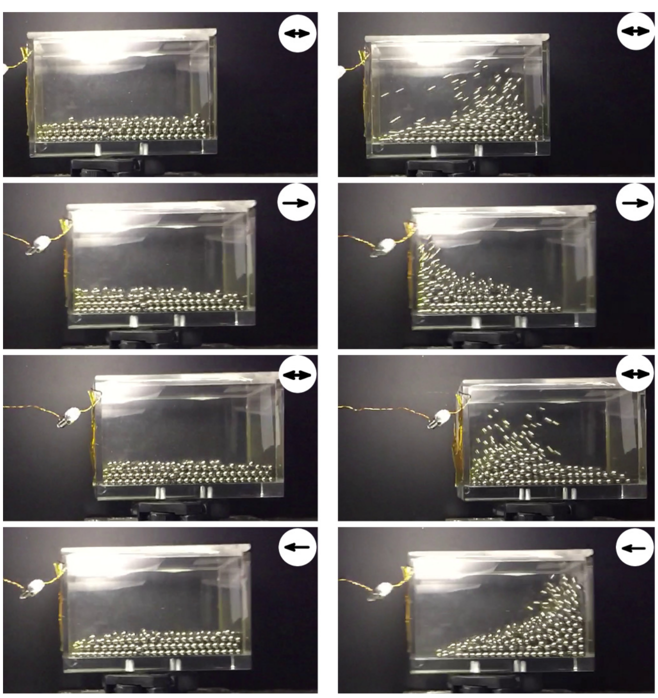

FIG. 2. Sequences of snapshots at about $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ of the driving period for three layers of monodisperse spheres with $d = 4$ mm. The phase of 0 corresponds to the maximum leftward displacement of the box, where the direction reversal takes place (indicated by a double-headed arrow). Single-headed arrows illustrate the instantaneous direction of the movement of the box. The granular sloshing (left panel) occurs at $\Gamma \approx 0.62$ and a dynamics of projectilelike motion similar to the gaslike state (right panel) of a monolayer [17] dominates at $\Gamma \approx 1.00$.

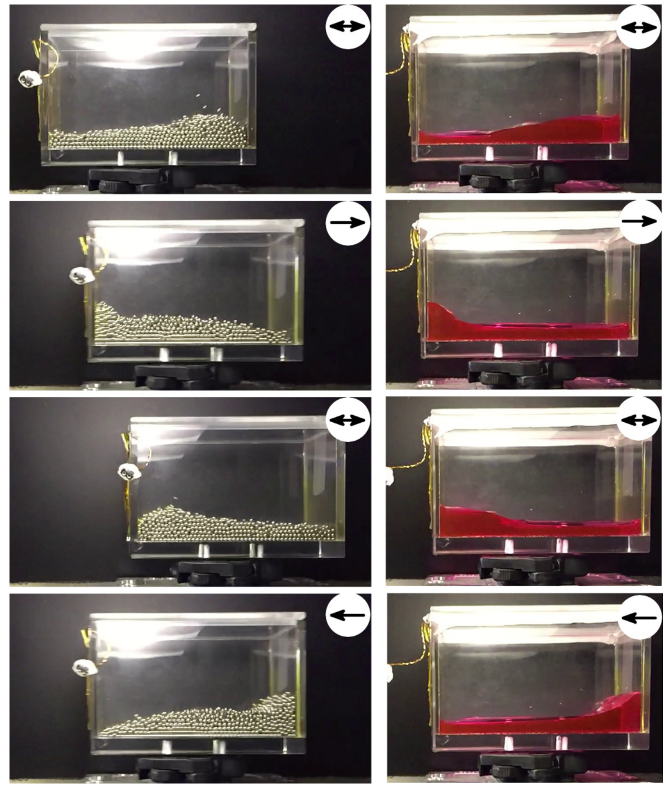

FIG. 3. Direct comparison of the liquidlike sloshing state (five layers of spheres with $d = 2$ mm at $\Gamma \approx 0.80$) with the sloshing dynamics of a liquid ($A = 7$ mm, $f = 1.7$ Hz). As in Fig. 2, snapshots are taken at about $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ of the driving period, where the phase 0 is defined as the maximum leftward displacement.

As the acceleration is further increased, a transition to a liquidlike sloshing state occurs (see the left panel of Fig. 3). Here the spheres of at least the top two visible layers pile up at the box wall and slosh in the form of a wave with the next oscillation, while most of the spheres stay in sliding contact to their neighbors. Interestingly, the wave is often triggered by the fluidization of one of the lower layers and sets in at $\Gamma \approx 0.63$ independently of the number of layers. The enveloping surface of the spheres resembles the surface of a sloshing liquid as is displayed in the right panel of Fig. 3 for comparison. This similarity is striking, since the liquid used (water colored with ink) and the parameters of the driving ($\Gamma = 0.08$ with $A = 7$ mm and $f = 1.7$ Hz) were rather randomly chosen. We therefore believe that the similarity exists in a large parameter space.

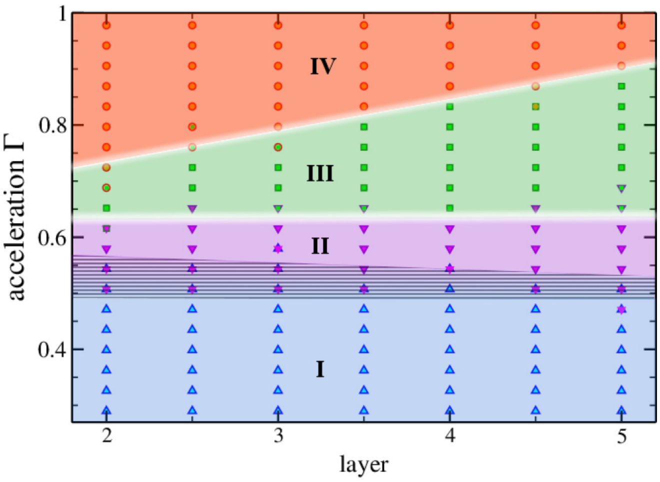

FIG. 4. Phase diagram of monodisperse spheres of diameter 4 mm summarizing the occurrence of the (I) solidlike state (△), the (II) granular (▽) and (III) liquidlike sloshing (□), and the (IV) gaslike state (○). The diagram was obtained for increasing and decreasing Γ, indicated by the full and empty symbols of slightly different color that are on top of each other. A hysteresis arises between the solidlike state and the granular sloshing, which is indicated by the horizontal stripe pattern for better recognition. One layer consists of 337 particles, so that, e.g., 3.5 layers correspond to $3.5 \times 337 \approx 1180$ spheres.

At large accelerations a wave of at least two visible layers of spheres is driven by the oscillation and splashes against the box wall, so that several spheres launch into projectile motion over at least half the box length. We refer to this dynamics as a gaslike state in accordance to [17]. It is shown in the snapshots in the right panel of Fig. 2. For the two-layer system the liquidlike sloshing is rather fragile and the gaslike state with flying spheres sets in already at $\Gamma = 0.72$. As the number of layers increases we observe a linear widening of the liquidlike sloshing regime up to $\Gamma = 0.91$ for five layers. It is interesting to note that this tendency is observed despite our characterization, which is based on the most energetic spheres of only the top two visible layers and therefore not directly dependent on the dynamics of the deeper layers.

Starting from the gaslike state, we observe for decreasing acceleration a qualitatively similar transition scenario in the reverse order. However, a small but systematic hysteresis is found between the solidlike state and the granular sloshing (indicated by the horizontal stripe pattern in Fig. 4 between states I and II), which seems to be caused by different packings of the spheres. At the start of each experiment the spheres are randomly packed after they have been poured in the (stationary) container. Therefore the packing is often quite loose giving plenty of space to the spheres to move and leading to a transition from the solidlike state to the granular sloshing at rather low accelerations. The backward transition for decreasing acceleration occurs instead at a larger Γ. An analysis of the dynamics reveals that the fluidlike sloshing of the top layers causes a crystal-like packing of the lower layers. This self-organizing process penetrates from the bulk to the surface for decreasing driving until the solidlike state is reached. The final packing in this state is then substantially denser than at the start of the measurement. This suggests an interesting connection to the experiments of Pouliquen *et al.*, who achieved the so far densest random packing of spheres by pouring them slowly in an oscillating container [19]. The phase diagram in Fig. 4 was obtained with spheres with a diameter of 4 mm (with a reproducibility in driving amplitude of ± 1 mm). We also performed measurements with five layers of spheres of 2 mm diameter and observed not only qualitatively but also quantitatively the same scenario.

The measurements presented so far were done for the "idealized" case of monodisperse polished steel spheres. However, we also performed a few "dirty" experiments with imperfect rough steel spheres. These imperfect spheres were generated by stirring the previously used 4 mm spheres together with stones to roughen their surface. After this procedure the stones and dust were removed and measurements with the resulting imperfect spheres performed. Only few experiments were possible before the polycarbonate of the box became nontransparent due to scratches, but in these the global dynamics remained qualitatively unchanged. No convection rolls were obtained. This raises the question of the key mechanism driving the liquidlike sloshing dynamics instead of the convection rolls occurring for, e.g., sand or poppy seeds. A main difference is the inertia of the individual particles, which is substantially larger for the steel spheres due to their higher density and larger size. Another aspect is the rather spherical shape of our particles compared to, e.g., sand or poppy seeds used in previous studies [10,11,13–15]. This strongly enhances rolling of the particles and the loose packing gives in addition more space for relative motion. Both aspects decrease the friction and damping of the system, which might be important ingredients for the sloshing to occur.

In summary, we have shown that multiple layers of monodisperse spheres in a horizontally oscillated container do not exhibit the well-known convection roll patterns described in many experimental and numerical studies. For intermediate accelerations, the dynamics resembles instead the sloshing motion of a liquid. This liquidlike sloshing state is distinct from the granular sloshing appearing at lower accelerations and a gaslike state at larger accelerations. In the parameter space spanned by acceleration and layer thickness, the boundaries between regions of qualitatively different dynamics are straight lines. One open point is to explain why the transition from the liquidlike sloshing state to the gaslike state is delayed as the number of layers increases. The simplicity of our setup combined with the precisely defined boundaries between regimes make our results ideally suited to improve and benchmark models describing the dynamics of granular media as used in particle simulations [9,12] or continuum models [20,21] based on liquid dynamics [22]. A remaining outstanding challenge for future modeling approaches is to relate particle inertia, geometry, material, and heterogeneity of the granular medium to its global dynamics. This can be investigated in our setup by examining under what conditions convection rolls or liquid sloshing is observed.

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