Velocity correlations and spatial dependencies between neighbors in a unidirectional flow of pedestrians

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The aim of the paper is an analysis of self-organization patterns observed in the unidirectional flow of pedestrians. On the basis of experimental data from Zhang *et al.* [J. Zhang *et al.*, J. Stat. Mech. (2011) P06004], we analyze the mutual positions and velocity correlations between pedestrians when walking along a corridor. The angular and spatial dependencies of the mutual positions reveal a spatial structure that remains stable during the crowd motion. This structure differs depending on the value of n, for the consecutive nth-nearest-neighbor position set. The preferred position for the first-nearest neighbor is on the side of the pedestrian, while for further neighbors, this preference shifts to the axis of movement. The velocity correlations vary with the angle formed by the pair of neighboring pedestrians and the direction of motion and with the time delay between pedestrians' movements. The delay dependence of the correlations shows characteristic oscillations, produced by the velocity oscillations when striding; however, a filtering of the main frequency of individual striding out reduces the oscillations only partially. We conclude that pedestrians select their path directions so as to evade the necessity of continuously adjusting their speed to their neighbors'. They try to keep a given distance, but follow the person in front of them, as well as accepting and observing pedestrians on their sides. Additionally, we show an empirical example that illustrates the shape of a pedestrian's personal space during movement.

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I. INTRODUCTION

When considering unidirectional flow of pedestrians a basic question appears: Can we identify any specific behavioral rules between neighbors during such a flow? We believe that research on this topic is very important in the context of modeling of unidirectional flow.

The aim of this paper is to address this question with respect to self-organized flow of pedestrians. On the basis of calculated velocity correlations and identified spatial distributions, we propose a set of basic rules that drive individual behavior in a unidirectional flow.

In order to address the issue we have analyzed experimental data from Zhang *et al.* [1]. We take into consideration correlations of velocity and spatial dependencies between neighbors in a unidirectional flow. The results of the analysis reveal the existence of stable patterns, which can be explained by a set of universal rules that drive the pedestrians behavior in a crowd.

The article is organized as follows. Section II describes related works. Section III includes an analysis of the spatial distribution among pedestrians. Velocity correlations between close neighbors in a crowd are presented in Sec. IV. Section V contains a discussion of the obtained results. Section VI summarizes the work.

II. RELATED WORKS

The issue of flow patterns in crowd dynamics, as well as the relation between the velocity and density in different contexts, has been a subject of research by different authors. Spatial relations between people was discussed in [2], where the idea of social distances and the theory of proxemics were introduced.

Helbing *et al.* [3] presented basic ideas related to different patterns of pedestrians' motion such as shock waves in dense crowds, lanes of uniform walking directions during counterflows, circulating flows at intersections, and clogging effects or oscillatory flows at bottlenecks. Several experiments in normal and paniclike conditions of collective phenomena such as lane formation in corridors and oscillations at a bottleneck were presented in their work. The paper rightly emphasizes that such spatiotemporal patterns emerge due to the pedestrians' nonlinear interactions and the interactions are

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not based on strategic considerations, but on the reactions of particular units or groups.

Rio and Warren [4] carried out some experiments with real and virtual groups of people, namely, they tested the occurrence of local visual coupling between neighbors in a crowd. They concluded that pedestrians in a crowd are (unidirectionally) coupled to their neighbors and they confirmed that the influence of multiple neighbors is linearly combined. Zanlungo *et al.* [5] proposed a potential to estimate the dynamics of the relative motion of two pedestrians who socially interact in a group while walking. They confirmed that the two-dimensional probability distribution of the relative distances between pedestrians is determined by their proposed potential using a Boltzmann distribution.

Zanlungo *et al.* [6] analyzed the relation between the velocity and spatial relations in different sizes of pedestrian social groups, such as dyads and triads, as well as the formation of different patterns, for example, V shapes and lines. In their experimental research they found that an observed extension of the pedestrian group's size in the direction orthogonal to motion decreases linearly with the pedestrian density around them. They confirmed in their observations that an individual pedestrian usually walks faster than two-person groups, while two-person groups are faster than three-person groups. They concluded that the observed differences in velocities were weakly affected by density.

Vizzari *et al.* [7] analyzed mechanisms of group cohesion preservation. Their paper refers to popular group patterns, distances around members of groups, and correlations between occupied area and the size of groups. A simulation model of group behavior related to crowd density was proposed. Mechanisms of automatic tracking of small groups in a crowd were presented by Ge *et al.* [8]. They proposed an application of bottom-up hierarchical clustering using a generalized symmetric Hausdorff distance defined with respect to pairwise proximity and velocity.

Mussaïd *et al.* [9] analyzed walking speeds and patterns in pedestrian flow using laboratory experiments and numerical simulations. During the laboratory experiments the authors observed structural instabilities in the pedestrian flow with alternation of organized and disorganized states. It was highlighted that the lifetime of well-organized clusters of pedestrians follows a stretched exponential relaxation law. It was confirmed that a key variable at the origin of the observed traffic perturbations is the interpedestrian variability of comfortable walking speeds.

The concept of the nth-nearest neighbor was proposed by Ballerini *et al.* [10], where such an approach was applied to the analysis of a bird flock. The authors compared the behavior of nearby birds in observed and simulated flocks using topological and metric distances. They concluded that interactions in a flock are driven by topological distance. Preliminary results on the nth-nearest-neighbor analysis applied to crowd spatial structure were presented in our previous paper [11].

Among papers that analyze pedestrians' behavior according to their neighbors, it is worthwhile to note that of Seitz *et al.* [12]. The authors defined a set of simple rules (step or wait, tangential evasion, sideways evasion, follower) that determines the next possible steps. A similar approach was presented by Moussaïd *et al.* [13], who used two heuristics based on visual information that determines desired walking directions and walking speed.

III. SPATIAL RELATIONS BETWEEN NEIGHBORS

In the following sections we use experimental data for unidirectional flow from the Hermes project [1]. In this paper we focus on an experiment where 349 people walk through a 300-cm-wide corridor. This experiment uses the widest corridor among all 28 runs, thus it was chosen for further analysis in order to reduce the influence of boundaries. Nevertheless, our preliminary results [11] show that the observed phenomenon occurs also for other geometries.

The duration of this experiment is approximately 83 s and it corresponds to 1325 frames that have been processed. The retrieved trajectories of particular pedestrians allow one to check basic physical characteristics such as velocity, flow, density, and individual distances in any time and any place. A comprehensive description of this experiment is provided in [1]. The detailed methodology of collecting trajectories of moving pedestrians is presented in [14].

In this paper we focus on the analysis of unidirectional flow, usually observed in cases of evacuation or egress under normal conditions. Thus, the results and conclusions refer to this kind of crowd movement, where the density is about 3 people/m^2.

We consider the nth-nearest neighbor as a pedestrian in a crowd with the nth-shortest Euclidean distance to a given occupant. The closest person to a given pedestrian is the first-nearest neighbor, while a pedestrian with the nth-shortest distance is the nth-nearest neighbor. This concept is presented in Fig. 1. It is worth noting that being someone's nth-nearest neighbor is not a mutual relation (i.e., the fact that person A is the closest pedestrian in a crowd to person B does not imply that B is the closest person to A).

Additionally, we define the angle Θ between the pedestrian and the neighbor as an angle between the direction of motion and a line connecting the positions of two people (see angle Θ in Fig. 1). Thus, the neighbor in front of a pedestrian is at 0°, while the one exactly on the right is at 90°.

In the analyzed experiment, considering the sum over all frames, we detected approximately 79 800–79 600 relations of nth-nearest neighbors for each $n \leqslant 8$. A histogram of distances for the nth-nearest neighbor is shown in Fig. 2. The peak of the histogram for the first-nearest neighbor is at 54 cm, for the second-nearest neighbor it is at 66 cm, for the third-nearest

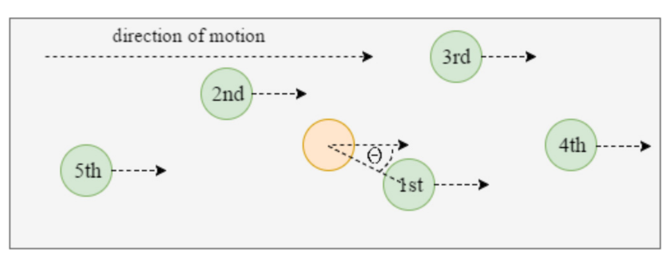

FIG. 1. Concept of the nth-nearest neighbor and angle between neighbors in a unidirectional flow. For the central pedestrian (orange), the pedestrian on the right is the first-nearest neighbor. Consecutive neighbors are marked on the figure. Here Θ denotes the angle between the orange pedestrian and the first-nearest neighbor.

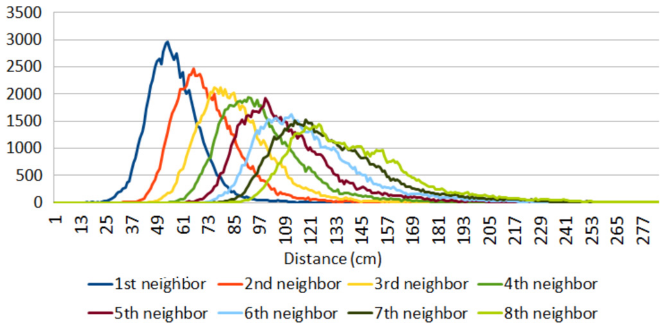

FIG. 2. Histogram of distances for the nth-nearest neighbor.

neighbor it is at 79 cm, for the fourth-nearest neighbor it is at 92 cm, for the fifth-nearest neighbor it is at 100 cm, for the sixth-nearest neighbor it is at 112 cm, for the seventh-nearest neighbor it is at 119 cm, and for the eighth-nearest neighbor it is at 125 cm. For the first-nearest neighbor, the histogram is similar to the normal distribution, while for the higher n the distribution becomes asymmetric (log-normal with positive skew). With increasing n, both the skewness of the distribution and the standard deviation are increasing.

A more detailed image of a pedestrian's behavior according to the neighbors is given in the form of a spatial distribution histogram of the nth-nearest neighbor in Fig. 3. For each n the shape of this distribution is circular and condensed. For higher n, nonzero probability covers a larger area and the condensation is lower (this corresponds to the heavy tailed distance distribution for such values of n in Fig. 2).

The spatial distribution for the first-nearest neighbor is visibly different from others. Pedestrians strongly prefer the closest neighbor to be on their side rather than on the axis of movement. For the second-nearest neighbor no significant preference is visible, but one can observe the area around the pedestrian (50–100 cm from the pedestrian's center) where neighbors are usually located. For nth-nearest neighbors, where $n \geqslant 3$ the most probable location of the neighbor is in front of or behind the pedestrian.

An aggregated spatial distribution histogram for $n \leqslant 4$ is presented in Fig. 4. In order to increase readability, we introduce the black ellipse (major axis, 46 cm; minor axis, 28 cm), which represents the typical size of a pedestrian [15]. The elliptical red area depicts the most probable locations of near neighbors. In contrast to the histograms for a given n, for the aggregate histogram there is no preference for specific angles.

The white area in the center depicts a pedestrian's personal zone, where no other pedestrians are allowed. A single point (pixel) is marked in the white area only if during the whole experiment there is no single moment (video frame) with one of the four closest neighbors in such a position (we consider

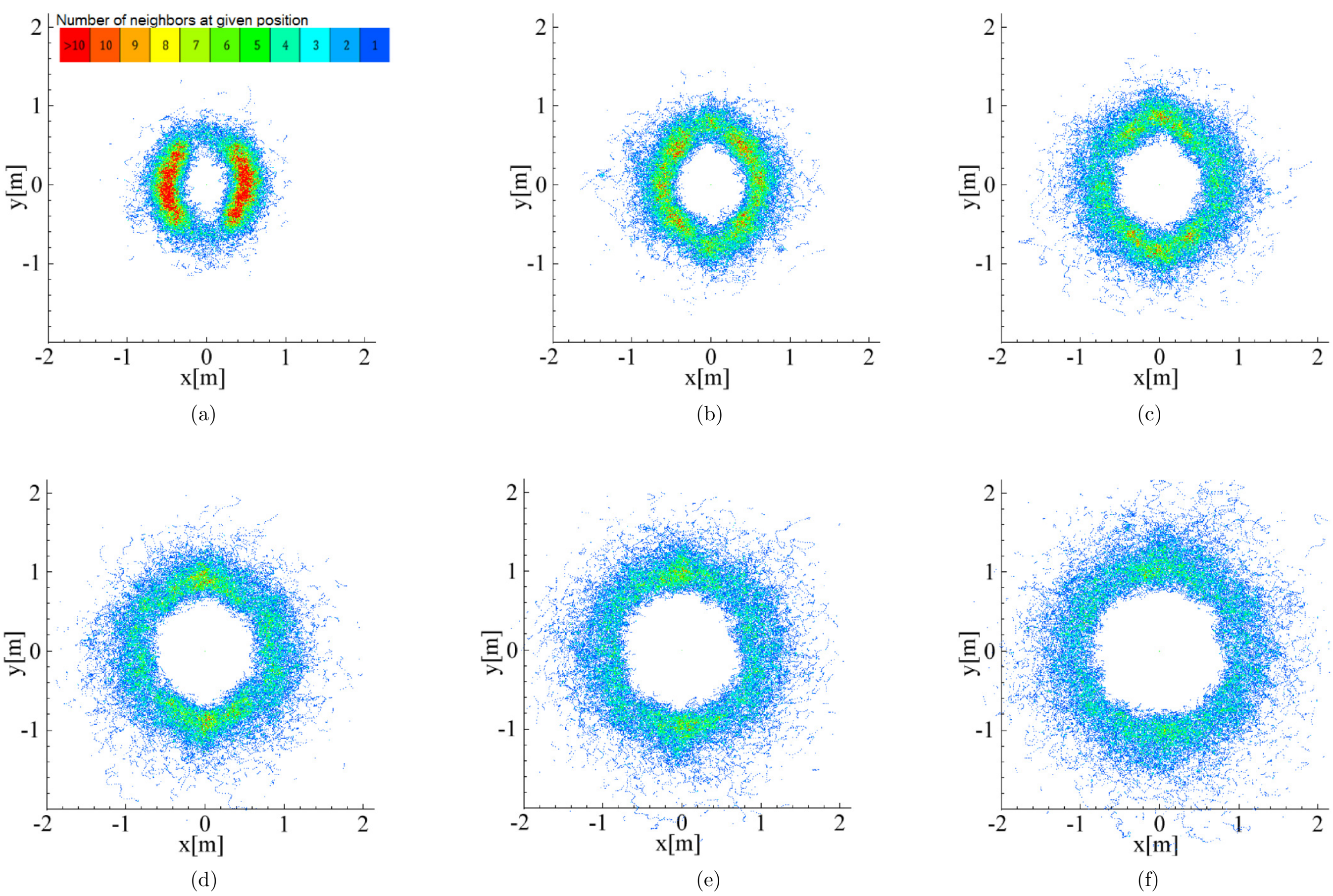

FIG. 3. Spatial distribution of the (a) first-, (b) second-, (c) third-, (d) fourth-, (e) fifth-, and (f) sixth-nearest neighbors around moving pedestrians in the analyzed experiment. The direction of motion corresponds to the top of the plot. Each point on the pixel corresponds to 1 cm^2. The redder the color is the more often the nth-nearest neighbor was detected at this particular position.

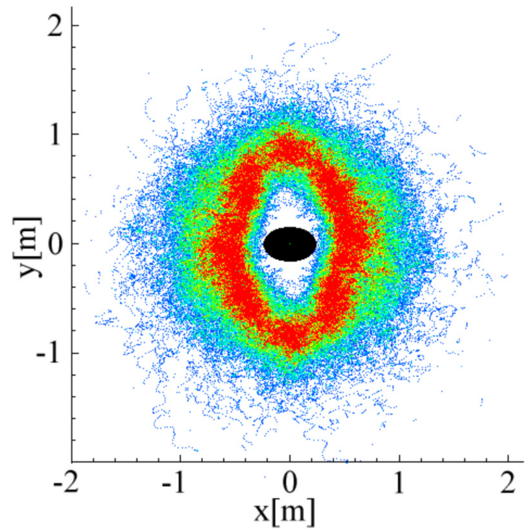

FIG. 4. Sum of the spatial distribution histogram for four nearest neighbors $n \in \langle 1,4 \rangle$. The black ellipse in the center represents the typical body size [15]. The central white area is forbidden for other pedestrians.

over 159 000 mutual positions of the four closest neighbors during the whole experiment). The observed asymmetry in the axis of motion can be explained by the influence of triads, the left-right asymmetry of which has been shown in other research [6].

Finally, the angular distribution presented in Fig. 5 provides the analysis. One can observe a sharp change in preference for the first-nearest neighbor between 35° and 40° from the direction of motion. For smaller angles [$\Theta \in (-35^\circ, 35^\circ)$] pedestrians tend to strongly avoid the first neighbor, while for larger angles [$\Theta \in (-140^\circ, -40^\circ) \cup (40^\circ, 140^\circ)$] a neighbor is accepted. This sharp transition in behavior occurs within just a few degrees.

On the other hand, in the case of $n \geqslant 3$ the preference for neighbors at Θ close to 0° and 180° is visible, however the transition is not as sharp as for $n = 1$. For $n = 2$ we observe an intermediate state, which is more or less isotropic. Clear stability of this angular structure has been observed: The nearest neighbor of a pedestrian, once determined for a given angle, maintains this angle with good accuracy. More generally, the stability of the nth neighborhoods of the pedestrians is an important feature of the experimental data.

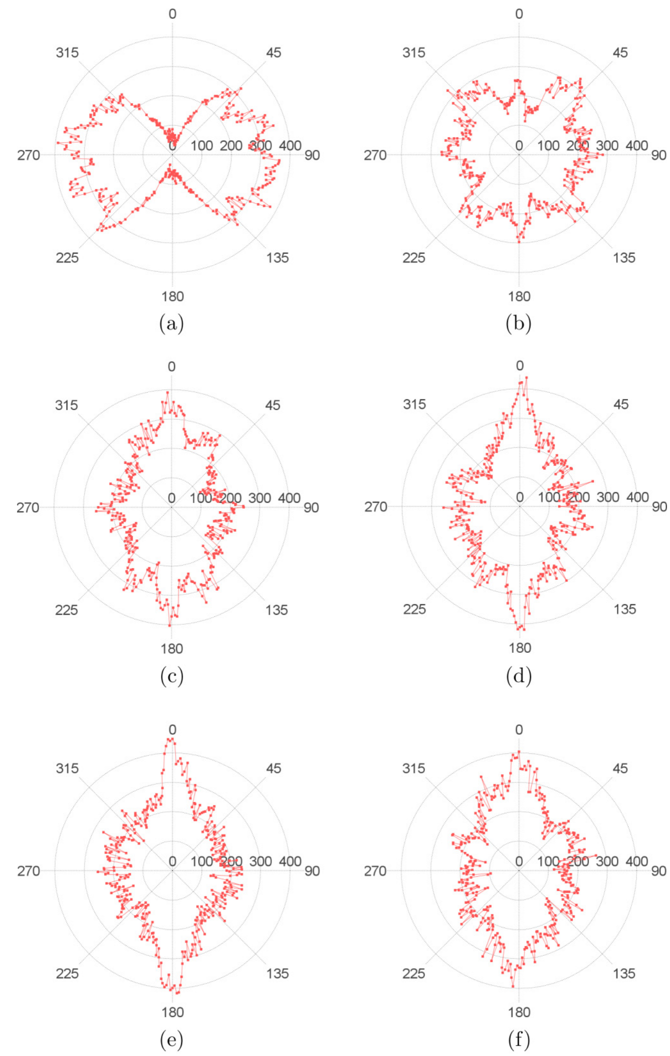

FIG. 5. Histogram of angular distribution of the (a) first-, (b) second-, (c) third-, (d) fourth-, (e) fifth-, and (f) sixth-nearest neighbors according to Θ for $n \leqslant 6$. A zero angle (top) corresponds to the direction of motion. Angular discretization is 1°. The radius of the plot corresponds to 400 detected positions of the nth neighbor at a given Θ. Assuming approximately 79 800 total nth neighbors detected for each n corresponds to a 0.5% chance of finding the nth neighbor at this Θ.

IV. VELOCITY CORRELATIONS

The positions of the pedestrians have been collected 16 times per second [1]. The changes of positions between successive time steps are considered as velocities; there are differences in positions divided by $\Delta t = \frac{1}{16}$ s.

A. Definitions

For each pedestrian i and for each time t he or she is present in the corridor, the component of his or her velocity $v_i(t)$ parallel to the corridor is found. In the following, we consider only these components. The velocity components perpendicular to the corridor axis come from the specific way humans walk, moving not only ahead, but also left and right, shifting the body center of gravity above the leg placed on the ground. Here we treat this kind of motion as unavoidable and therefore not dependent on any specific characteristics of the crowd motion.

The time dependence of the velocities v_i of two sample pedestrians is shown in Fig. 6. We note that two effects are visible there: the oscillations due to the rhythm of striding, of a frequency of about 2 Hz, and slower variations in the time scale of about 3–4 s. In Fig. 7 the Fourier spectrum is shown for a sample pedestrian. There, the strongest maximum appears near $f = 2$ Hz.

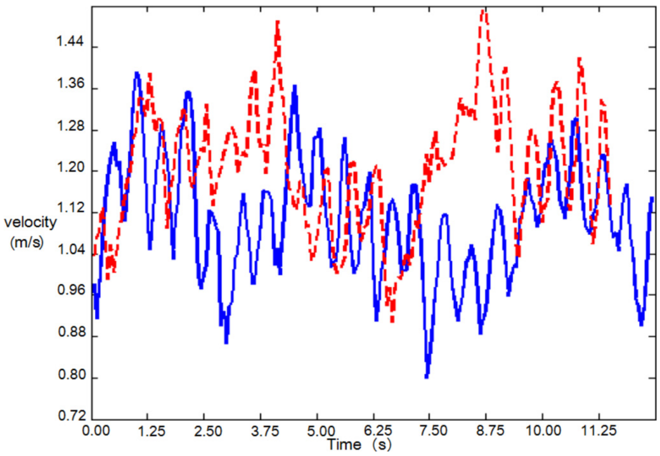

FIG. 6. Time dependence of the velocity component parallel to the corridor, for two neighboring pedestrians.

For each pedestrian the first and second moments of velocity distribution is found, as

$$\langle v_i \rangle = \frac{\sum_{t=1}^{T_i} v_i(t)}{T_i}, \tag{1}$$

$$\langle v_i^2 \rangle = \frac{\sum_{t=1}^{T_i} v_i^2(t)}{T_i}, \tag{2}$$

where T_i is the length of the time period when i is present in the corridor. Accordingly, the variance $\sigma_i^2 = \langle v_i^2 \rangle - \langle v_i \rangle^2$.

The correlation function is calculated as follows. For each pedestrian i we identify the nearest neighbor $j(i)$. As noted in the preceding section, the angle Θ formed by the pair with the corridor axis remains approximately constant during the time evolution, therefore we can assign an angle to each pair. The velocity correlation for the pair (i,j) is

$$\rho_{i,j(i),\tau} = \frac{\sum_{t=1}^{T_{ij}} [v_i(t+\tau) - \langle v_i \rangle][v_j(t) - \langle v_j \rangle]}{T_{ij}\sigma_i \sigma_j}, \tag{3}$$

where T_{ij} is the length of the time period when both i and j are present in the corridor and τ is the delay. Note that T_{ij} in the denominator plays the role of normalization of the time average. Finally, the angle- and delay-dependent correlation function $\rho(\Theta,\tau)$ is

$$\rho(\Theta,\tau) = \frac{\sum_i \rho_{i,j,\tau}}{N(\Theta)}, \tag{4}$$

where the sum in the numerator is taken over those pairs of nearest neighbors that form an angle Θ with the corridor axis and $N(\Theta)$ is the number of such pairs. The correlation functions for the second- and third-nearest neighbors are calculated in the same way.

B. Analysis of the results

In Figs. 8(a)–8(c) the obtained correlations are shown as dependent on the delay time τ for different angles Θ. It is evident that, as expected, the correlations with first-nearest neighbors are stronger than those with second- and third-nearest neighbors. This can be seen by a comparison of the results for zero delay, where the maximal values are 0.26 ($\Theta = 30^\circ$), 0.22 ($\Theta = 80^\circ$), and 0.17 ($\Theta = 50^\circ$) for the first-, second-, and third-nearest neighbors, respectively. The second result is that the correlations decrease with the delay; this was expected as well. A different result is that almost all correlations show a kind of oscillation with the delay τ, with the period 0.6–0.9 s. This effect can be explained as a kind of beat, when two oscillations appear with almost the same frequency. Actually, this effect can be observed in Fig. 6, where two plots of velocity vs time meet once with the same phase and once with the opposite phase. For a finite-time range, a shift in phase and a small change of frequency have similar consequences.

We have made an attempt to remove the beat effect at least partially, by filtering out the main frequency related to the striding. As is shown in Fig. 7, the Fourier spectrum of the velocity of a sample pedestrian shows a strong maximum near $f = 2$ Hz. Such a value of frequency correlates with the most common rhythm of walking. For each pedestrian, the characteristic frequency of striding has been filtered out and the correlation functions have been calculated again. The results are shown in Figs. 8(d)–8(f). A comparison of upper and lower plots shows that when the striding frequency is filtered out, the correlations are reconstructed only slightly. For the first neighbor [Fig. 8(d)], the curve for 0° is shifted upward by about 0.05 and this correlation becomes slightly larger than those for other angles. For the second neighbor, the curve for 80° is shifted in the same way, being even more outstanding [Fig. 8(e)]. Similarly, for the third neighbor [Fig. 8(f)], the curve for 90° is shifted upward by about 0.1.

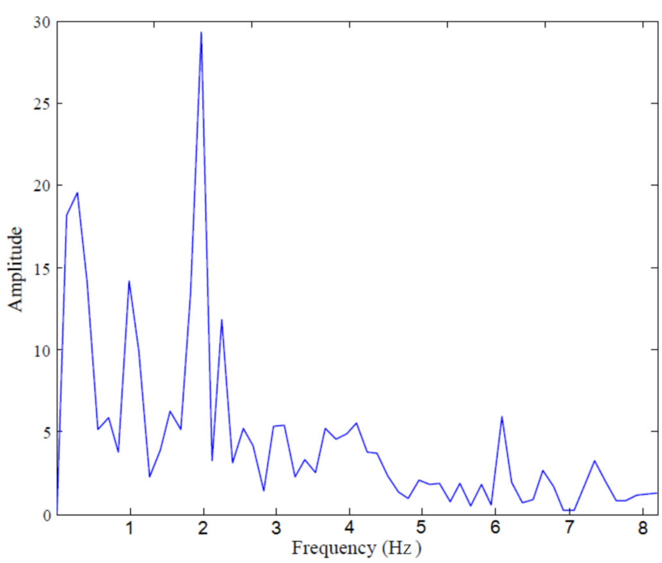

FIG. 7. Fourier spectrum of the velocity of an exemplary pedestrian.

V. DISCUSSION OF THE RESULTS

Taking into account the distribution of pedestrians during a unidirectional flow as shown in Fig. 3, some

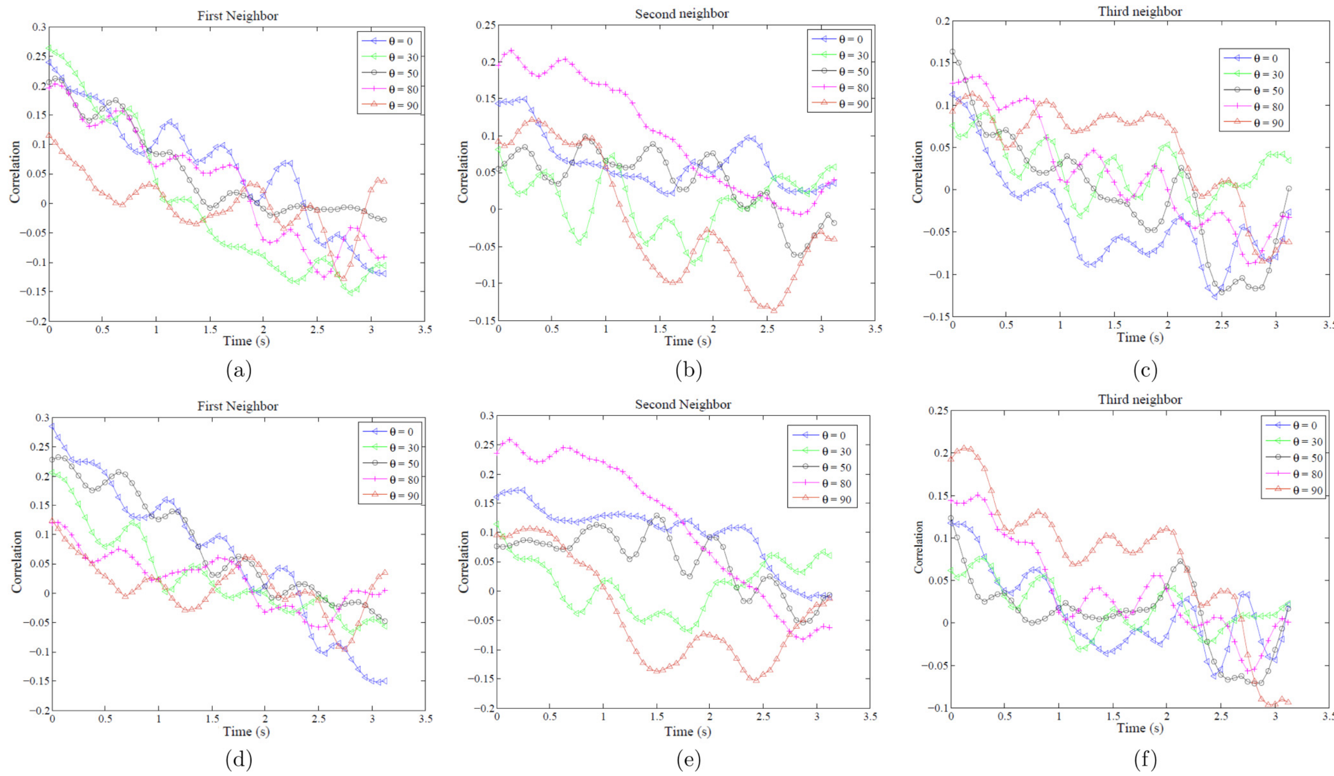

FIG. 8. Angle- and delay-dependent velocity correlations between (a) first-, (b) second-, and (c) third-nearest neighbors within a given Θ and (d) first-, (e) second-, and (f) third-nearest neighbors within a given Θ after the frequency of striding is filtered out.

characteristic patterns can be observed. Regarding the first and the second neighbor of a pedestrian, one can notice that they are located on both sides of the pedestrian. In general, people tend to keep a distance between themselves and their predecessors; next neighbors, namely, the third, fourth, fifth, etc., are usually located in front of or behind the pedestrian. Moreover, neighbors on the sides of pedestrian are allowed closer than those ahead of or behind them. One can formulate a general rule regarding formulating patterns in unidirectional movement: One should keep at least a given distance between oneself and the person ahead, follow closely the person directly in front, and pedestrians on the sides are acceptable.

The occurrence of the sharp transition in angular distribution for the first-nearest neighbor in Fig. 5(a) is worth noting. Between 35° and 40° from the axis of movement one can observe a transition from acceptance for other pedestrians to strong avoidance. Such a clear pattern suggests the existence of some internal factor that determines this. One explanation seems to be the horizontal angle of human binocular vision, which is necessary for depth perception. This angle typically equals 114° [16], i.e., up to 57° from the axis of movement. Thus, a possible explanation for the observed phenomenon is a compromise between the need for free space in front of a pedestrian (to have an area for the next step) and the ability to perceive at least one neighbor on the side with both eyes (in order to determine distance). However, this hypothesis requires further research.

Regarding the patterns, one can observe good compatibility with the theory of social distances by Hall [2], where different distances between people were taken into account. Figure 4 clearly illustrates the existence and the exact shape of the personal space around a pedestrian when moving in a unidirectional flow. Similar results were obtained in [4]; however, in this case the personal space around pedestrians was almost circular. This difference can be explained by the fact that in the latter experiment participants were instructed to move in the room in random directions.

There is also an analogy between the observed results and forming groups of people, due to the fact that people prefer to walk in small groups of two or three people rather than alone. Typical patterns formed by groups are abreast, diagonal, and riverlike [17,18]. A riverlike pattern is typical for a dense crowd in bidirectional movement, while the other two are explainable from a physical as well as a psychological point of view in unidirectional flow. These two patterns are in good agreement with the observed results, namely, the positions of the first- and second-nearest neighbors, which correspond to the movement in dyads and triads. The detailed analysis of mutual pedestrians position in dyads and triads [5,6] is in good agreement with the results presented in this paper. It is worth noting that [5] also proposed a crowd dynamics model that uses a discomfort potential field to model stable dyads; the minimum value of this field corresponds to the most probable positions of the first-nearest neighbor in the histograms of the spatial distribution presented in Fig. 3.

Our intention when investigating the delay was to detect the cause and effect relation between variations in the velocities of a pair of pedestrians: one in front (cause) and the other behind (effect). Here we see that this relation is hidden by two phenomena. The first phenomenon is the beat effect, which overshadows the correlations of other origins. The second is that pedestrians prefer to have open space before them, which is shown in Fig. 4. Then they try to move towards the free space between two people in front of them. As a consequence, the distance between pedestrians along the corridor axis is usually larger than the average and the correlation is weaker, just along the direction where the cause and effect relation should be the strongest. In other words, pedestrians are able to self-organize themselves so as to minimize the necessity to modify their velocity because of the velocity variations of other pedestrians. The same effect is common in car traffic, where drivers find it more convenient to occupy the left lane because it is supposed to be free.

The decrease of the correlations with increasing n is rather weak. This result disproves the expectation that it is only the nearest neighborhood that is relevant to the motion. However, as we see in Fig. 3, the first-nearest neighbors are usually situated on the side of a pedestrian and not in front and therefore they are supposed to be observed by the pedestrian less carefully. Then again, the weak decay of the correlations can be due to the self-organization of the crowd structure, where the motion of a pedestrian correlates more with more distant neighbors.

Another result is that the maximum of correlations expected at a delay of about 0.3 s is invisible. The expectation is based on the time of human reaction [19]. If a person in a queue takes a step, the subsequent person behind also takes a step to fill the gap and the delay of the second step with respect to the first one cannot be shorter than the reaction time of a human. However, having removed the striding frequency, we see only a weak maximum of the correlation with the first neighbor for a Θ of about 10°, at a delay τ of about 0.9 s. This maximum is rather a result of oscillations of the correlations due to the beating effect, one of the series at 0, 0.9, and 1.8 s. Apparently, the pedestrians adjust their mutual positions; the queue analogy is not appropriate in a sparse crowd.

VI. SUMMARY

While the existence of self-organization phenomena in a crowd are well known [3], the exact preferences or driving factors of individuals that are the origins of such phenomena are still a subject of research. The results described in this paper widen our knowledge of mutual spatial relations between individuals in a crowd. They can be used in modeling crowd dynamics, especially for microscopic models, where each individual is simulated independently. Increasing the efficiency of computers allows the development of models that use cognitive heuristics, like in [5,12,13], where knowledge of factors that drive pedestrians were used directly.

Correlations between the positions and velocities of pedestrians in the laminar motion of a crowd in a corridor were calculated from some experimental data. We have analyzed and discussed spatial dependencies and velocity correlations between pedestrians.

Analysis of spatial relations reveals a number of clear patterns: the preference for a position on the side for the first neighbor and for positions close to the axis of motion for the nth neighbor for $n \geqslant 3$. We have shown a sharp transition in the acceptable angle for the first neighbor and the shape of a pedestrian's personal space in unidirectional motion. The observed phenomenon can be explained by simple rules.

The dependence of the correlation on the delay shows oscillations, which can be ascribed to a superposition of striding frequencies of the pedestrians. However, if the individual frequencies of striding are identified by the Fourier transform and removed from the data, the oscillations are modified only slightly. The results are interpreted in terms of a self-organization of positions and velocities of pedestrians.

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