Endoreversible quantum heat engines in the linear response regime

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We analyze general models of quantum heat engines operating a cycle of two adiabatic and two isothermal processes. We use the quantum master equation for a system to describe heat transfer current during a thermodynamic process in contact with a heat reservoir, with no use of phenomenological thermal conduction. We apply the endoreversibility description to such engine models working in the linear response regime and derive expressions of the efficiency and the power. By analyzing the entropy production rate along a single cycle, we identify the thermodynamic flux and force that a linear relation connects. From maximizing the power output, we find that such heat engines satisfy the tight-coupling condition and the efficiency at maximum power agrees with the Curzon-Ahlborn efficiency known as the upper bound in the linear response regime.

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I. INTRODUCTION

The Carnot efficiency $\eta_C = 1 - T_c^r/T_h^r$, obtained from a Carnot heat engine infinitely slowly working in the quasistatic limit with vanishing irreversibility, is an upper limit of the efficiency of all the existing heat engines working between two heat reservoirs with constant temperatures T_h^r and T_c^r. The Carnot heat engine with maximum efficiency produces vanishing power, though some specific models outputting finite power at the Carnot efficiency have been found [1–9]. The issue of a heat engine working at maximum power, with a sacrifice of efficiency, was studied by Curzon and Ahlborn [10]. In their paper, adopting the endoreversible assumption and using Newton's heat transfer law, the efficiency at maximum power η^* for a finite-time Carnot cycle is given by the Curzon-Ahlborn (CA) efficiency $\eta_{\rm CA} = 1 - \sqrt{T_c^r/T_h^r}$. The finite-time performance has been subsequently studied in autonomous (steady-state) [11–22] or cyclic (periodically driven) heat engines [4,12,14,23–45] based on classical [4,11,13,18,24–28,33,34,41–43,45] and quantum [17,19–23,29,31–33,36,37] systems, with special emphasis on the issue of efficiency at maximum power and its bound(s). Linear response theory was applied to the description of heat engines [12,14,16,24,27,40], proving that the CA efficiency [12,14,24,27] is the upper bound of efficiency at maximum power η^*, i.e.,

$$\eta^* \leqslant \frac{\Delta T^r}{2T^r} = \eta_{\rm CA} + \mathcal{O}[\Delta(T^r)^2], \tag{1}$$

where $\Delta T^r \equiv T_h^r - T_c^r$ and $T \equiv (T_h + T_c)/2$.

The efficiency at maximum power η^* achieves its upper bound when the heat engine works under the tight-coupling (no-heat-leakage) condition. Under the endoreversible assumption, the thermodynamic quantities can be well defined and the fundamental thermodynamic relation holds well between the thermodynamic variables of the working substance even in a finite-time thermodynamic process. This endoreversible condition was used to describe the classical heat engines proceeding from specific heat transfer laws [25,27,28,34], which, however, are phenomenological. Linear irreversible thermodynamics was recently introduced by Izumida and Okuda [27] to study the classical endoreversible Carnot cycle consisting of two adiabatic and two isothermal processes, where the heat conduction is assumed to be the Fourier law. Nevertheless, the liner response description of endoreversible quantum heat engines, which are not restricted to the Carnot model and where the relaxation dynamics (instead of phenomenological heat transfer laws) are used, is still lacking. For this reason, we study the finite-power performance of (generalized) quantum engines under the endoreversible condition and analyze the efficiency at maximum power.

The present study follows a tradition of analyzing quantum cycle models of heat engines, applying the endoreversible condition under which the total entropy production is solely due to heat transfer between the system and the heat reservoirs. Without loss of generality, the working substance of the engine is composed of an ensemble of noninteracting harmonic oscillators or spin-$\frac{1}{2}$ subsystems, two types of systems in the universe: bosons and fermions. The dynamics and thermodynamics of these quantum systems are described in Sec. II. The quantum version of a cyclic heat engine is optimized in Sec. III, where the efficiency at maximum power is analyzed. Section IV develops the optimization within the framework of linear irreversible thermodynamics, confirming the result obtained in Sec. III. Section V summarizes the main conclusions.

II. QUANTUM DYNAMICS AND THERMODYNAMICS OF AN ISOTHERMAL PROCESS

For a quantum system with a magnetic moment **M** trapped in a magnetic field **B**, its direction can be assumed to be constant and along the positive-z axis, with the magnitude of the magnetic field changing over time and being not vanishing. The system Hamiltonian can be given by $H(t) = -\mathbf{M}\cdot\mathbf{B} = 2\mu_B\mathbf{S}\cdot\mathbf{B} = 2\mu_B B_z(t)S_z$, where μ_B is the Bohr magnetron and **S** is the spin angular momentum. The system Hamiltonian can be written as $H = \omega(t)S_z$ by setting $\omega(t) = 2\mu_B B_z(t)$. The average spin polarization S can be given by ($k_B \equiv 1$) $S = \langle S_z\rangle = -\frac{1}{2}\tanh(\frac{\beta\omega}{2})$, where $\beta = 1/T$ is the inverse temperature of the system. Throughout the paper, β rather than T refers to the temperature. The

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expectation of the system Hamiltonian can be written as a function of the temperature β and the external field ω ($\hbar \equiv 1$),

$$\langle H \rangle = \omega S = -\frac{1}{2}\omega \tanh\left(\frac{\beta\omega}{2}\right). \tag{2}$$

The Hamiltonian of a harmonic oscillator is determined by

$$H = \omega(t)\big(\hat{N} + \tfrac{1}{2}\big), \tag{3}$$

where the number operator is $\hat{N} = \hat{a}^\dagger\hat{a}$, with $\hat{a}^\dagger$ and $\hat{a}$ being the Bosonic creation and annihilation operators, respectively, and its expectation value

$$\langle H \rangle = \omega\left(n + \frac{1}{2}\right) = \frac{\omega}{2}\coth\left(\frac{\beta\omega}{2}\right), \tag{4}$$

where $n = \langle \hat{N} \rangle = \frac{1}{e^{\beta\omega}-1}$ is the mean population. Given a harmonic oscillator, its time-dependent frequency $\omega(t)$ determined by the external field is the control variable. We note that, for a spin and a harmonic system [46], the expectation Hamiltonian $\langle H \rangle$ is a function of its temperature β and external field ω: $\langle H \rangle = \langle H(\beta,\omega) \rangle$.

An exact treatment of the dynamics for an open quantum system in contact with a heat reservoir is extremely complicated. The dynamics of an open quantum system away from thermal equilibrium, however, can be modeled by the semigroup formalism [47], with the time evolution of the density operator $\hat{\rho}$, $d\hat{\rho}/dt = -i[H,\hat{\rho}] + \mathcal{L}_D(\hat{\rho})$, where $\mathcal{L}_D$ is the dissipative generator that can drive the system to thermal equilibrium for a static Hamiltonian $H = H(\omega)$ with time-independent ω. The dissipator $\mathcal{L}_D$, conforming to Lindblad's form for a Markovian evolution in which the positivity of the density matrix $\hat{\rho}$ is preserved, can be given by $\mathcal{L}_D(\hat{\rho}) = \sum_\alpha k_\alpha(\hat{V}_\alpha\hat{\rho}\hat{V}_\alpha^\dagger - \frac{1}{2}\hat{\rho}\hat{V}_\alpha^\dagger\hat{V}_\alpha - \frac{1}{2}\hat{V}_\alpha^\dagger\hat{V}_\alpha\hat{\rho})$. Here $\hat{V}_\alpha^\dagger$ and $\hat{V}_\alpha$ are referred to as Lindblad operators in the Hilbert space of the system and Hermitian conjugates, and the k_α are positive coefficients that could be determined from the first principles calculation [47]. An operator can be written as a set of quantum expectations $\langle \hat{X} \rangle = \mathrm{Tr}(\hat{\rho}\hat{X})$. It follows that for a thermodynamic process the time evolution of the operator $\hat{X}$ in the Heisenberg picture is [29,31–33]

$$\frac{d\hat{X}}{dt} = i[H,\hat{X}] + \frac{\partial \hat{X}}{\partial t} + \mathcal{L}_D(\hat{X}), \tag{5}$$

where $\mathcal{L}_D(\hat{X}) = \sum_\alpha k_\alpha(\hat{V}_\alpha^\dagger[\hat{X},\hat{V}_\alpha] + [\hat{V}_\alpha^\dagger,\hat{X}]\hat{V}_\alpha)$. We set $\hat{X} = H$, where $E = \langle H \rangle$, and then substitute H into Eq. (5), obtaining

$$\frac{dE}{dt} = \frac{dW}{dt} + \frac{dQ}{dt} = \left\langle \frac{\partial H}{\partial t} \right\rangle + \langle \mathcal{L}_D(\hat{H}) \rangle, \tag{6}$$

where $\mathcal{P} = \frac{dW}{dt} = \langle \frac{\partial H}{\partial t} \rangle$ and $\frac{dQ}{dt} = \langle \mathcal{L}_D(H) \rangle$ are the power and the instantaneous heat flow, respectively. For a harmonic (spin-$\frac{1}{2}$) system, the operators $\hat{V}^\dagger$ and $\hat{V}$ become the spin (harmonic) creation $\hat{a}^\dagger$ ($\hat{S}^\dagger = \hat{S}_x + i\hat{S}_y$) and annihilation operators $\hat{a}$ ($\hat{S} = \hat{S}_x - i\hat{S}_y$). Setting $\hat{X} = H = \omega(\hat{a}^\dagger\hat{a} + \frac{1}{2})$ ($\hat{X} = H = \omega\hat{S}_z$) in Eq. (5) and using Eq. (6), where $\mathcal{L}_D(\hat{X}) = k_+(\hat{a}[\hat{X},\hat{a}^\dagger] + [\hat{a},\hat{X}]\hat{a}^\dagger) + k_-(\hat{a}^\dagger[\hat{X},\hat{a}] + [\hat{a}^\dagger,\hat{X}]\hat{a})$ or $\mathcal{L}_D(\hat{X}) = k_+(\hat{S}[\hat{X},\hat{S}^\dagger] + [\hat{S},\hat{X}]\hat{S}^\dagger) + k_-(\hat{S}^\dagger[\hat{X},\hat{S}] + [\hat{S}^\dagger,\hat{X}]\hat{S})$, the instantaneous heat current is written as

$$\dot{Q} = -K(\langle H \rangle_t - \langle H \rangle_{eq}), \tag{7}$$

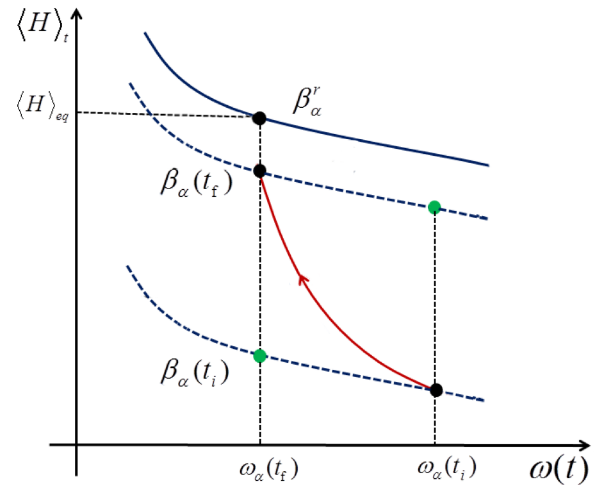

FIG. 1. The $\langle H \rangle_t$-$\omega(t)$ diagram of an isothermal expansion where heat is injected into the system from the heat reservoir. The isothermal process indicated by the red line begins and ends at the instants $[\omega(t_i),\beta(t_i)]$ and $[\omega(t_f),\beta(t_f)]$, respectively. The expectation values of the system Hamiltonian with varying control variable $\omega(t)$ but constant temperatures $\beta(t) = \beta_\alpha(t_f),\beta_\alpha(t_i)$ and $\beta(t) = \beta_\alpha^r$ are indicated by the dashed and solid blue lines, respectively. Here $\langle H \rangle_{eq}$ is the expectation of the system Hamiltonian at thermal equilibrium with the heat reservoir of constant temperature β_α^r.

where $K = k_- - k_+$ ($K = k_- + k_+$) is the heat conductivity for the harmonic (spin) system and $k_+/k_- = e^{-\beta\omega}$ obeys the detailed balance ensuring that the system evolves asymptotically in a specific way to the thermal equilibrium state. Here $n_{eq} = \frac{k_+}{k_- - k_+} + \frac{1}{2}$ ($S_{eq} = -\frac{1}{2}\frac{k_- - k_+}{k_- + k_+}$), the asymptotic population (polarization), corresponds to the value at thermal equilibrium: $n = \frac{1}{2}\coth(\beta^r\omega)$ [$S = -\frac{1}{2}\tanh(\beta^r\omega)$]. Here and hereafter β^r is referred to as the temperature of the reservoir and it is assumed to be positive. We also assume that the energy of the system under consideration is not unbounded upward and its temperature is then positive.

We first consider an isothermal process during which the system is interacting with an idealized heat resource α at constant temperature β_α^r ($=1/T_\alpha^r$). Throughout the paper the word "isothermal" is adopt to merely mean that the system is coupled to a heat reservoir whose temperature is constant. The idealized constant temperature can be realized by assuming that the reservoir relaxes infinitely fast to its equilibrium when compared to the time scale of the system dynamics, which means that such a heat reservoir is exactly the thermodynamic thermal reservoir. The process starts with the initial time $t = t_i$ and ends at the final time $t = t_f$, during which the corresponding control variable varies from $\omega(t_i)$ to $\omega(t_f)$ and the expectation value of the Hamiltonian $\langle H \rangle_t = \langle H[\beta_\alpha(t),\omega_\alpha(t)] \rangle$ with $t_i \leqslant t \leqslant t_f$ is time dependent. Figure 1 schematically depicts an isothermal process in which the system absorbs heat from the heat reservoir α with temperature β_α^r. (A similar schematic diagram for an isothermal compression, along which the system is kept in contact with a cold heat reservoir, is not plotted here.) At the instant $t = t_f$, the control variable $\omega(t)$ is frozen and becomes time independent by $\omega(t_f) = \omega$. Under such a fixed value of ω, the system kept in contact with the heat reservoir undergoes

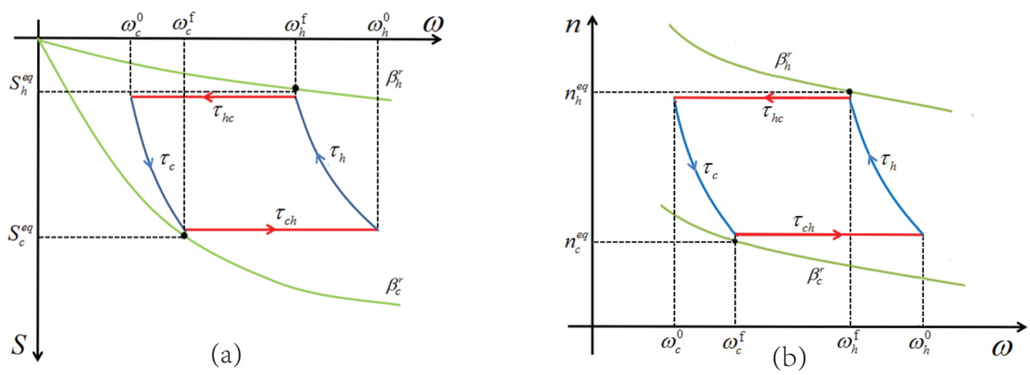

FIG. 2. Schematic diagram of a heat engine cycle based on (a) a spin system in the (ω,S) plane and (b) a harmonic system in the (ω,n) plane. Two isothermal processes are indicated by two blue lines while two adiabatic processes are denoted by two red lines. The values of average populations (n or S) with varying control variable $\omega(t)$ but constant temperatures β_h^r and β_c^r are shown by the two green lines, respectively. Here S_h^{eq} (n_h^{eq}) and S_c^{eq} (n_h^{eq}) are the average spin polarizations (populations) at thermal equilibrium with two heat reservoirs at temperatures β_h^r and β_c^r, respectively.

thermalization, the process of the system approaching thermal equilibrium through mutual interaction between the system and its environment. Specifically, along this thermalization, where the control parameter $\omega(t)=\omega(t_f)$ is constant but the temperature is varied from $\beta_\alpha(t_f)$ to β_α^r with $\beta_\alpha^r=\beta_\alpha(t\to\infty)$, the system exchanges heat with the reservoir and can reach thermal equilibrium after relaxation with infinitely long time and the expectation of the Hamiltonian approaches its time-independent asymptotic value $\langle H\rangle_{t\to\infty}=\langle H\rangle_{eq}=\langle H[\beta_\alpha^r,\omega_\alpha(t_f)]\rangle$.

We approximate the instantaneous expectation of the Hamiltonian $\langle H[\beta_\alpha(t),\omega_\alpha(t)]\rangle$ around the thermal equilibrium point $\beta_\alpha(t)=\beta_\alpha^r$ and $\omega_\alpha(t)=\omega_\alpha(t_f)$, retaining only the first nonzero term

$$\langle H\rangle_t=\langle H\rangle_{eq}+\frac{1}{K_\alpha}\langle H\rangle_{eq}^{\beta_\alpha}\left[\beta_\alpha(t)-\beta_\alpha^r\right]+\frac{1}{K_\alpha}\langle H\rangle_{eq}^{\omega_\alpha}[\omega_\alpha(t)-\omega_\alpha(t_f)], \tag{8}$$

where we have defined $H_{eq}^{\beta_\alpha}\equiv K_\alpha\frac{\partial\langle H\rangle_t}{\partial\beta_\alpha}|_{\beta_\alpha(t)=\beta_\alpha^r,\omega_\alpha(t)=\omega_\alpha(t_f)}$ and $H_{eq}^{\omega_\alpha}\equiv K_\alpha\frac{\partial\langle H\rangle_t}{\partial\omega_\alpha}|_{\beta_\alpha(t)=\beta_\alpha^r,\omega_\alpha(t)=\omega_\alpha(t_f)}$ [48]. Combining Eqs. (7) and (8), we obtain

$$\dot{Q}_\alpha=H_{eq}^{\beta_\alpha}\left[\beta_\alpha^r-\beta_\alpha(t)\right]+H_{eq}^{\omega_\alpha}[\omega_\alpha(t_f)-\omega_\alpha(t)]. \tag{9}$$

Here $\beta_\alpha(t)$ and $\omega_\alpha(t)$ can be expressed as follows:

$$\beta_\alpha(t)=\beta_\alpha(t_i)+\gamma(t)\left[\beta_\alpha^r-\beta_\alpha(t_i)\right], \tag{10}$$

$$\omega_\alpha(t)=\omega_\alpha(t_i)+g(t)[\omega_\alpha(t_f)-\omega_\alpha(t_i)], \tag{11}$$

where $\gamma(t)$ and $g(t)$, as functions of time t, satisfy the boundary conditions $\gamma(t_i)=g(t_i)=0$ and $g(t_f)=\gamma(t\to\infty)=1$. The duration of the isothermal process in contact with a heat reservoir is finite, which means that the thermodynamic state of the working substance evolves along the curves of nonquasistatic isothermal processes. Considering Eqs. (9)–(11), the instantaneous heat current $\dot{Q}_\alpha$ can be approximated by

$$\dot{Q}_\alpha=H_{eq}^{\beta_\alpha}\tilde{\gamma}_\alpha(t)\left[\beta_\alpha(t_i)-\beta_\alpha^r\right]+H_{eq}^{\omega_\alpha}\tilde{g}_\alpha(t)[\omega_\alpha(t_i)-\omega_\alpha(t_f)], \tag{12}$$

where we have used $\tilde{\gamma}_\alpha(t)\equiv\gamma_\alpha(t)-1$ and $\tilde{g}_\alpha(t)\equiv g_\alpha(t)-1$.

III. OPTIMIZING THE OPERATION OF A CYCLIC QUANTUM HEAT ENGINE MODEL

The cyclic heat engine model consisting of two adiabatic and two isothermal processes is sketched in Fig. 2. The present engine may be a Carnot cycle, a Brayton cycle, or an Otto cycle [49–51] (the control variable ω is kept constant in an isochoric process of the Otto cycle) and its working substance is composed of a quantum system, envisioned as an ensemble of noninteracting spin-$\frac{1}{2}$ subsystems or harmonic oscillators [46]. Such an engine model, for which the temperatures of the hot and cold (thermodynamic thermal) reservoirs are constant, is now briefly described. (i) For the isothermal expansion, we set the initial and final values of time to be $t_i=0$ and $t_f=\tau_h$. Along this process the working substance is kept in contact with the hot reservoir of temperature β_h, while the control variable $\omega(t)$ is varied from ω_h^0 to ω_h^f. A result of the variation of the control variable is that the effective temperature of the working substance $\beta(t)$ is changing from β_h^0 to β_h^f. (ii) In the adiabatic expansion, the system is decoupled from the heat reservoir for time τ_{hc} and the control variable is varied from ω_h^f to ω_c^0, without heat transfer between the system and its surroundings. (iii) The time required to complete the isothermal compression is τ_c. In this step, the system is coupled to the cold reservoir of temperature β_c, while the control variable changes from ω_c^0 to ω_c^f, causing the effective temperature $\beta_c(t)$ to change from β_c^0 to β_c^f. (iv) The adiabatic compression, similar to the adiabatic expansion, closes the full cycle, after which the system is back in its original state. During this process, the system is isolated from the heat reservoir and the control variable changes back to its initial value ω_h^0 in a time interval τ_{ch}.

The cycle period for the heat engine is then given by $\tau_{cyc} = \tau_h + \tau_{hc} + \tau_c + \tau_{ch}$. For the cyclic heat engine, we consider the quantity $\sum_\alpha \int_0^{\tau_{cyc}} \beta_\alpha(t)\dot{Q}_\alpha$ in the linear response regime. Since no heat is transferred during the adiabatic processes, we find from Eqs. (10) and (11) that

$$\begin{aligned}\sum_\alpha \int_{\tau_\alpha^i}^{\tau_\alpha^f} \beta_\alpha(t)\dot{Q}_\alpha dt &= \int_{\tau_h^0}^{\tau_h^f} \left[H_{eq}^{\beta_h}\tilde{\gamma}_h(t)(\beta_h^0-\beta_h^r) + H_{eq}^{\omega_h}\tilde{g}_h(t)(\omega_h^0-\omega_h^f)\right]dt \\ &\quad + \int_{\tau_c^0}^{\tau_c^f} \left[H_{eq}^{\beta_c}\tilde{\gamma}_c(t)(\beta_c^0-\beta_c^r) \right.\\ &\quad \left. + H_{eq}^{\omega_c}\tilde{g}_c(t)(\omega_c^0-\omega_c^f)\right]dt + \mathcal{O}(\Delta^2) \\ &= \beta_h^0\langle Q_h\rangle + \beta_c^0\langle Q_c\rangle + \mathcal{O}(\Delta^2), \end{aligned} \tag{13}$$

where $\langle Q_\alpha\rangle = \int_{\tau_\alpha^i}^{\tau_\alpha^f}\dot{Q}_\alpha dt$, with τ_α^0 and τ_α^f $(\alpha = h,c)$ denoting the initial and final times for the hot or cold isothermal processes, is the average heat exchanged between the system and the heat reservoir.

We assume that the quantum heat engine satisfies the endoreversible condition, under which the irreversibility hindering the performance of the ideal quantum heat engines is exclusively caused by imperfect thermal interaction between the working substance and the heat reservoirs. Based on this condition, the relaxation time scale of the working substance can be assumed to be much smaller than that of heat exchange in the isothermal processes and the working substance can thus relax to internal equilibrium at any instant along these processes [30], implying that the instantaneous temperature of the working substance can indeed be determined from the equilibriumlike forms (2) and (4) for the endoreversible cycles in the linear response regime [52]. It is also indicated that the entropy of the working substance at any infinitesimal difference can be defined by $d\mathcal{S}_{\rm entropy} = \beta(t)dQ$. As the entropy is the state variable, there is no net change in the entropy of the system for a single cycle, i.e., $\oint \dot{Q}(t)\beta(t) = 0$, with β being the time-dependent temperature of the working substance [25,34]. This condition where the temperature of the working substance is time dependent, unlike the conventional endoreversible assumption [10] under which the temperature of the working substance is presumed to be constant, was called the weak endoreversible assumption by Wang and Tu [30]. In the linear response regime, we have the restriction from Eq. (13),

$$\beta_h^0\langle Q_h\rangle + \beta_c^0\langle Q_c\rangle + \mathcal{O}(\Delta^2) = 0, \tag{14}$$

which is exactly the conventional endoreversible assumption [10], where the effective temperature of the working substance is assumed to be kept constant along an isothermal process. That is, the weak endoreversible assumption reduces to the conventional one. Alternatively, the conventional endoreversible assumption, in which the temperature of the working substance is constant [i.e., $\gamma(t) = \gamma(t_i) = 0$ in Eq. (10)], automatically holds well under the weak endoreversible condition, where the temperature of the working substance is not necessarily assumed to be fixed, provided that the heat engine works in the linear response regime. For simplicity, we set $\bar{\beta}^0 = \frac{\beta_h^0+\beta_c^0}{2}$ and $\bar{\omega}^0 = \frac{\omega_h^0+\omega_c^0}{2}$, and $\bar{\beta}^r = \frac{\beta_h^r+\beta_c^r}{2}$ and $\bar{\omega}^f = \frac{\omega_h^f+\omega_c^f}{2}$. Keeping the first order, we have

$$\bar{\beta}^0 = c_0 + c_1\Delta\beta + c_2\Delta\beta^r, \quad \bar{\omega}^0 = d_0 + d_1\Delta\omega + d_2\Delta\omega^f, \tag{15}$$

where we used $\Delta\beta \equiv \beta_h^0 - \beta_c^0$, $\Delta\beta^r \equiv \beta_h^r - \beta_c^r$, $\Delta\omega \equiv \omega_h^0 - \omega_c^0$, and $\Delta\omega^f \equiv \omega_h^f - \omega_c^f$. Substituting the expressions of $\bar{\beta}^0$ and $\bar{\omega}^0$ into Eq. (14), we find

$$c_0 = \bar{\beta}^r, \tag{16}$$

$$c_1 = \frac{H_{eq}^{\beta_c}\Gamma_c - H_{eq}^{\beta_h}\Gamma_h}{2(H_{eq}^{\beta_c}\Gamma_c + H_{eq}^{\beta_h}\Gamma_h)}, \tag{17}$$

$$c_2 = -\frac{H_{eq}^{\beta_c}\Gamma_c - H_{eq}^{\beta_h}\Gamma_h}{2(H_{eq}^{\beta_c}\Gamma_c + H_{eq}^{\beta_h}\Gamma_h)}, \tag{18}$$

$$d_0 = \bar{\omega}^f, \tag{19}$$

$$d_1 = \frac{H_{eq}^{\omega_c}G_c - H_{eq}^{\omega_h}G_h}{2(H_{eq}^{\omega_c}G_c + H_{eq}^{\omega_h}G_h)}, \tag{20}$$

$$d_2 = -\frac{H_{eq}^{\omega_c}G_c - H_{eq}^{\omega_h}G_h}{2(H_{eq}^{\omega_c}G_c + H_{eq}^{\omega_h}G_h)}, \tag{21}$$

where we have used $\Gamma_h\tau_{cyc} = \int_{\tau_h^0}^{\tau_h^f}\tilde{\gamma}_h(t)dt$, $\Gamma_c\tau_{cyc} = \int_{\tau_c^0}^{\tau_c^f}\tilde{\gamma}_c(t)dt$, $G_h\tau_{cyc} = \int_{\tau_h^0}^{\tau_h^f}\tilde{g}_h(t)dt$, and $G_c\tau_{cyc} = \int_{\tau_c^0}^{\tau_c^f}\tilde{g}_c(t)dt$, with Γ_α $(0 < \Gamma_\alpha < 1)$ and G_α $(0 < G_\alpha < 1)$ being dimensionless parameters. Employing Eq. (15), $\bar{\beta}^0$ and $\bar{\omega}^0$ become

$$\begin{aligned}\bar{\beta}^0 &= \bar{\beta}^r + \frac{H_{eq}^{\beta_c}\Gamma_c - H_{eq}^{\beta_h}\Gamma_h}{2(H_{eq}^{\beta_c}\Gamma_c + H_{eq}^{\beta_h}\Gamma_h)}\Delta\beta \\ &\quad - \frac{H_{eq}^{\beta_c}\Gamma_c - H_{eq}^{\beta_h}\Gamma_h}{2(H_{eq}^{\beta_c}\Gamma_c + H_{eq}^{\beta_h}\Gamma_h)}\Delta\beta^r, \end{aligned} \tag{22}$$

$$\begin{aligned}\bar{\omega}^0 &= \bar{\omega}^f + \frac{H_{eq}^{\omega_c}G_c - H_{eq}^{\omega_h}G_h}{2(H_{eq}^{\omega_c}G_c + H_{eq}^{\omega_h}G_h)}\Delta\omega \\ &\quad - \frac{H_{eq}^{\omega_c}G_c - H_{eq}^{\omega_h}G_h}{2(H_{eq}^{\omega_c}G_c + H_{eq}^{\omega_h}G_h)}\Delta\omega^f. \end{aligned} \tag{23}$$

Note from Eq. (14) that the efficiency of these engines, $\eta = \langle\mathcal{W}\rangle/\langle Q_h\rangle$ with $\langle\mathcal{W}\rangle = \langle Q_h\rangle + \langle Q_c\rangle$ being the average work output per cycle, takes the form

$$\eta = 1 - \frac{\beta_h^0}{\beta_c^0} \simeq -\frac{\Delta\beta}{\bar{\beta}^r}. \tag{24}$$

The power output $\mathcal{P} = (\int_0^{\tau_{cyc}}\eta\dot{Q}_h dt)/\tau_{cyc}$ reads, from Eqs. (12) and (24),

$$\mathcal{P} = -\frac{\Delta\beta}{\bar{\beta}^r}\left[H_{eq}^{\beta_h}\Gamma_h(\beta_h^0-\beta_h^r) + H_{eq}^{\omega_h}G_h(\omega_h^0-\omega_h^f)\right]. \tag{25}$$

It follows, substituting Eqs. (22) and (23) into Eqs. (12) and (25), that the power output and the average heat absorbed by the system per cycle become

$$\mathcal{P} = -\frac{\Delta\beta}{\beta^r}\left[\frac{H_{eq}^{\beta_c}H_{eq}^{\beta_h}\Gamma_h\Gamma_c}{H_{eq}^{\beta_c}\Gamma_c + H_{eq}^{\beta_h}\Gamma_h}(\Delta\beta - \Delta\beta^r) + \frac{H_{eq}^{\omega_c}H_{eq}^{\omega_h}G_hG_c}{H_{eq}^{\omega_c}G_h + H_{eq}^{\omega_h}G_h}(\Delta\omega - \Delta\omega^f)\right] \quad (26)$$

and

$$\langle Q_h\rangle = \left[\frac{H_{eq}^{\beta_c}H_{eq}^{\beta_h}\Gamma_h\Gamma_c}{H_{eq}^{\beta_c}\Gamma_c + H_{eq}^{\beta_h}\Gamma_h}(\Delta\beta - \Delta\beta^r) + \frac{H_{eq}^{\omega_c}H_{eq}^{\omega_h}G_hG_c}{H_{eq}^{\omega_c}G_h + H_{eq}^{\omega_h}G_h}(\Delta\omega - \Delta\omega^f)\right]\tau_{cyc}, \quad (27)$$

respectively. In deriving Eqs. (26) and (27) we have used $\beta_h^0 = \bar{\beta}^0 + \Delta\beta/2$, $\beta_h^r = \bar{\beta}^r + \Delta\beta^r/2$, $\omega_h^0 = \bar{\omega}^0 + \Delta\omega/2$, and $\omega_h^f = \bar{\omega}^f + \Delta\omega^f/2$.

To proceed with our analysis, we now reveal the relationship between the temperature difference and the difference of the control variable. For our purpose it suffices to consider the density operators for the quantum system $\rho(\beta,\omega) = \frac{\exp(-\beta\langle H\rangle)}{\mathrm{Tr}\exp(-\beta\langle H\rangle)}$, where the expectation of the Hamiltonian was given by Eq. (2) for the spin system or Eq. (4) for the harmonic system. Without going through the details of the calculation, we note that the entropy $\mathcal{S}_{\text{entropy}} = -\rho\mathrm{Tr}\ln\rho$ for the spin or harmonic system takes the explicit form $\mathcal{S}_{\text{entropy}} = \mathcal{S}_{\text{entropy}}(\beta\langle H\rangle) = \mathcal{S}_{\text{entropy}}(\beta\omega)$, which means that $\mathcal{S}_{\text{entropy}}$ is a function of the "parameter" $\beta\omega$ only. Specifically, during an isoentropic adiabatic process, one has the following relations: $\mathcal{S}_{\text{entropy}} = \text{const} \Leftrightarrow \rho = \text{const} \Leftrightarrow \beta\langle H\rangle = \text{const} \Leftrightarrow \beta\omega = \text{const}$. For the adiabatic compression and expansion, we then have $\beta_h^0\omega_h^0 = \beta_c^f\omega_c^f$ and $\beta_h^f\omega_h^f = \beta_c^0\omega_c^0$. In the linear response regime where the temperature gradient $\Delta\beta^r$ $(=\beta_c^r - \beta_h^r)$ is very small, the temperature of the working substance β_h^f (β_c^f) at the final state during the hot (cold) isothermal process must be very close to its asymptotic value β_h^r (β_c^r), the temperature of the heat reservoir. Specifically, although the system requires an infinitely long time to achieve its (global) equilibrium state during an isothermal process, its value at the final state must be very close to the value of the equilibrium state. That is, $\beta_\alpha^f \simeq \beta_\alpha^r$ and $\beta_c^f \simeq \beta_c^r$, leading to the approximation

$$\beta_h^0\omega_h^0 \simeq \beta_c^r\omega_c^f, \quad \beta_c^0\omega_c^0 \simeq \beta_h^r\omega_h^f, \quad (28)$$

which, together with Eq. (13), gives rise to

$$\Delta\omega - \Delta\omega^f = \frac{2(\beta_c^0\beta_h^0 - \beta_c^r\beta_h^r)(\Gamma_hH_{eq}^{w_h} + \Gamma_cH_{eq}^{w_c})\bar{\omega}^f}{-\beta_h^0\beta_h^rG_cH_{eq}^{w_c} + \beta_c^0(-\beta_h^0G_cH_{eq}^{w_c} + \beta_c^rG_hH_{eq}^{w_h} + \beta_c^0G_hH_{eq}^{w_h})}, \quad (29)$$

where we have used $\omega_h^0 = \bar{\omega}^0 + \Delta\omega/2$ and $\omega_h^f = \bar{\omega}^f + \Delta\omega^f/2$. Substituting $\beta_h^0 = \bar{\beta}^0 + \Delta\beta/2$ and $\beta_h^r = \bar{\beta}^r + \Delta\beta^r/2$ into Eq. (29) and expanding that with respect to $\Delta\beta$ and $\Delta\beta^r$ as well as $\Delta\omega$ and $\Delta\omega^f$, we have

$$\Delta\omega - \Delta\omega^f \simeq \frac{\bar{\omega}^f}{\bar{\beta}^r}(\Delta\beta^r - \Delta\beta), \quad (30)$$

which, together with Eq. (26), leads to the final function of power

$$\mathcal{P} = -\left(\frac{H_{eq}^{\beta_c}H_{eq}^{\beta_h}\Gamma_h\Gamma_c}{H_{eq}^{\beta_c}\Gamma_c + \Gamma_hH_{eq}^{\beta_h}} - \frac{H_{eq}^{\omega_c}H_{eq}^{\omega_h}G_hG_c}{H_{eq}^{\omega_c}G_h + G_hH_{eq}^{\omega_h}}\frac{\omega^f}{\bar{\beta}^r}\right) \times \left(\frac{\Delta\beta}{\bar{\beta}^r}\right)(\Delta\beta - \Delta\beta^r). \quad (31)$$

Here the power $\mathcal{P}$, a quadratic function of the temperature difference $\Delta\beta$, can thus be optimized with respect to $\Delta\beta$. The power becomes vanishing either in the quasistatic limit $\Delta\beta = \Delta\beta^r$, when the heat engine runs at an infinitely slow speed, or in the extreme case $\Delta\beta = 0$, where the heat flux from the hot heat reservoir is completely injected into the cold heat reservoir without producing any work. From Eq. (31), maximizing the power output for the heat engine in a finite time is equivalent to setting $\partial\mathcal{P}/\partial\Delta\beta = 0$, yielding

$$\Delta\beta = \frac{\Delta\beta^r}{2}. \quad (32)$$

With consideration of Eqs. (24) and (32), we obtain the efficiency at maximum power as

$$\eta^* = -\frac{\Delta\beta^r}{2\bar{\beta}^r} = \eta_{\text{CA}} + \mathcal{O}[(\Delta\bar{\beta}^r)^2]. \quad (33)$$

We reproduce a reported universal upper bound of the maximum-power efficiency obtained from a strong-coupling Carnot heat engine [24,27]. Notice, however, that Eq. (33) was obtained for models of quantum heat engines, which work either a spin (Fermi) or a harmonic (Bose) system and are not restricted to the Carnot cycle model. In the special case when the heat engine is the quantum version of the Otto cycle, the second term in Eq. (26) or (27) is vanishing and one can easily reproduce Eq. (33). There is no clear physical reason why the efficiency at maximum power achieves the upper bound η_{CA}, which is realized only for the heat engines under the tight-coupling (no-heat-leakage) condition. Upon thought, however, the result obtained may not be surprising, provided the heat engines satisfy the strong-coupling condition. That is, such ideal energy conversion mechanisms, in which the energy fluxes are proportional to each other at all times, may be tagged tightly coupled, which will be discussed in the following.

IV. OPTIMAL ANALYSIS BASED ON LINEAR IRREVERSIBLE THERMODYNAMICS

The entropy production rate of the system $\dot{\sigma}$ can be expressed as the sum of the entropy increase rates of the two heat reservoirs

$$\dot{\sigma} = -\frac{1}{\tau_{cyc}}\sum_\alpha\int_{\tau_\alpha^0}^{\tau_\alpha^f}\beta_\alpha^r\dot{Q}_\alpha dt = -[(\langle\dot{Q}_h\rangle + \langle\dot{Q}_c\rangle)\beta_c^r + \langle\dot{Q}_h\rangle(\beta_h^r - \beta_c^r)]. \quad (34)$$

Considering Eq. (24), Eq. (34) can be approximated by using $\beta_c^r \simeq \bar{\beta}^r$ as

$$\dot{\sigma} \simeq \bar{\beta}^r \langle \dot{Q}_h \rangle \frac{\Delta\beta}{\bar{\beta}^r} + \langle \dot{Q}_h \rangle (-\Delta\beta^r) = J_e X_e + J_t X_t, \quad (35)$$

where $\langle \dot{Q}_h \rangle = (\int_0^{\tau_{cyc}} \dot{Q}_h dt)/\tau_{cyc}$ is the average heat current per cycle. Here the entropy and thermal fluxes are identified by using Eq. (27),

$$J_e = \bar{\beta}^r \langle \dot{Q}_h \rangle = \bar{\beta}^r \left(\frac{H_{eq}^{\beta_c} H_{eq}^{\beta_h} \Gamma_h \Gamma_c}{H_{eq}^{\beta_c} \Gamma_c + \Gamma_h H_{eq}^{\beta_h}} - \frac{H_{eq}^{\omega_c} H_{eq}^{\omega_h} G_h G_c}{H_{eq}^{\omega_c} G_h + G_h H_{eq}^{\omega_h}} \frac{\omega^f}{\bar{\beta}^r} \right) (\Delta\beta - \Delta\beta^r) \quad (36)$$

and

$$J_t = \langle \dot{Q}_h \rangle = \left(\frac{H_{eq}^{\beta_c} H_{eq}^{\beta_h} \Gamma_h \Gamma_c}{H_{eq}^{\beta_c} \Gamma_c + \Gamma_h H_{eq}^{\beta_h}} - \frac{H_{eq}^{\omega_c} H_{eq}^{\omega_h} G_h G_c}{H_{eq}^{\omega_c} G_h + G_h H_{eq}^{\omega_h}} \frac{\omega^f}{\bar{\beta}^r} \right) (\Delta\beta - \Delta\beta^r), \quad (37)$$

respectively, and their conjugate thermodynamic forces are

$$X_e = \frac{\Delta\beta}{\bar{\beta}^r} = \frac{\beta_h^0 - \beta_c^0}{\bar{\beta}_r}, \quad X_t = -\Delta\beta^r = \beta_c^r - \beta_h^r. \quad (38)$$

In the linear response regime, there is a linear relation between fluxes $J_{e,t}$ and forces $X_{e,t}$ such that

$$J_e = L_{ee} X_e + L_{et} X_t, \quad J_t = L_{te} X_e + L_{tt} X_t, \quad (39)$$

where the Onsager coefficients are required to satisfy $L_{et} = L_{te}, L_{tt}, L_{ee} \geqslant 0$ and $L_{tt} L_{ee} \geqslant L_{et} L_{te}$. Using Eqs. (36)–(39), the Onsager coefficients are obtained as

$$L_{ee} = \left(\frac{H_{eq}^{\beta_c} H_{eq}^{\beta_h} \Gamma_h \Gamma_c}{H_{eq}^{\beta_c} \Gamma_c + \Gamma_h H_{eq}^{\beta_h}} - \frac{H_{eq}^{\omega_c} H_{eq}^{\omega_h} G_h G_c}{H_{eq}^{\omega_c} G_c + G_h H_{eq}^{\omega_h}} \frac{\bar{\omega}^f}{\bar{\beta}^r} \right) (\bar{\beta}^r)^2, \quad (40)$$

$$L_{et} = \left(\frac{H_{eq}^{\beta_c} H_{eq}^{\beta_h} \Gamma_h \Gamma_c}{H_{eq}^{\beta_c} \Gamma_c + \Gamma_h H_{eq}^{\beta_h}} - \frac{H_{eq}^{\omega_c} H_{eq}^{\omega_h} G_h G_c}{H_{eq}^{\omega_c} G_c + G_h H_{eq}^{\omega_h}} \frac{\bar{\omega}^f}{\bar{\beta}^r} \right) \bar{\beta}^r, \quad (41)$$

$$L_{te} = \left(\frac{H_{eq}^{\beta_c} H_{eq}^{\beta_h} \Gamma_h \Gamma_c}{H_{eq}^{\beta_c} \Gamma_c + \Gamma_h H_{eq}^{\beta_h}} - \frac{H_{eq}^{\omega_c} H_{eq}^{\omega_h} G_h G_c}{H_{eq}^{\omega_c} G_c + G_h H_{eq}^{\omega_h}} \frac{\bar{\omega}^f}{\bar{\beta}^r} \right) \bar{\beta}^r, \quad (42)$$

$$L_{tt} = \frac{H_{eq}^{\beta_c} H_{eq}^{\beta_h} \Gamma_h \Gamma_c}{H_{eq}^{\beta_c} \Gamma_c + \Gamma_h H_{eq}^{\beta_h}} - \frac{H_{eq}^{\omega_c} H_{eq}^{\omega_h} G_h G_c}{H_{eq}^{\omega_c} G_c + G_h H_{eq}^{\omega_h}} \frac{\bar{\omega}^f}{\bar{\beta}^r}, \quad (43)$$

which confirm that the Onsager reciprocity $L_{et} = L_{te}$ holds and the tight-coupling (no-heat-leakage) condition

$$q \equiv \frac{L_{et}}{\sqrt{L_{tt} L_{ee}}} = 1 \quad (44)$$

is fulfilled. Based on Eqs. (31), (38), (40), and (41), the power output $\mathcal{P}$ can be expressed in terms of the Onsager coefficients:

$$\mathcal{P} = J_t \frac{\Delta\beta}{\bar{\beta}^r} = (L_{ee} X_e + L_{et} X_t) X_e. \quad (45)$$

In then follows, using the condition $\partial\mathcal{P}/\partial X_e = 0$, that the maximum power is realized at the optimal point $X_e^* = \Delta\beta^r \bar{\beta}^r/2$ and the corresponding efficiency η^* is still given by Eq. (33). We therefore show that the heat engine is tightly coupled through appropriate identification of thermodynamic fluxes and forces and that the efficiency at maximum power (when accurate to the first order of η_C) attains the upper bound $\eta_{\rm CA}$, with no use of any particular working substance or heat conductance.

As a final brief remark, we note that Eq. (34) can be reexpressed as $\dot{\sigma} = -(\beta_h^r \langle \dot{Q}_h \rangle + \beta_c^r \langle \dot{Q}_c \rangle) \simeq -\mathcal{P}\bar{\beta}^r + \langle \dot{Q}_h \rangle \Delta\beta^r = J_m X_m + J_t X_t$ [24]. Here J_m and J_t, the mechanical and thermal fluxes, are identified as $J_m = 1/\tau_{cyc}$ and $J_t = \langle \dot{Q}_h \rangle$, with the affinities $X_m = -\bar{\beta}^r W$ and $X_t = -\Delta\beta^r$, and they have linear constitutive relations $J_m = L_{mm} X_m + L_{mt} X_t$ and $J_t = L_{tm} X_m + L_{tt} X_t$, where the Onsager coefficients satisfy $L_{mt} = L_{tm}, L_{tt}, L_{mm} \geqslant 0$ and $L_{tt} L_{mm} \geqslant L_{mt} L_{tm}$. However, these kinetic coefficients, $L_{\mu\nu} = (\partial J_\mu/\partial X_\nu)_{X=0}$ for $\mu,\nu = m,t$, cannot be expressed explicitly in such a case when $J_m = 1/\tau_{cyc}$. We note that the power and the efficiency can be written as $\mathcal{P} = -J_m X_m/\bar{\beta}^r$ and $\eta = -\bar{\beta}^r J_t/J_m X_m$, respectively. We then find, setting $\partial\mathcal{P}/\partial X_m = 0$, that the efficiency at maximum power becomes $\eta^* = \frac{\Delta\beta^r}{2}\frac{q^2}{1-q^2}$, where the definition of the coupling strength parameter $q = L_{mt}/\sqrt{L_{mm} L_{tt}}$ has been used, with $-1 \leqslant q \leqslant 1$. Again, the upper bound of the efficiency at maximum power is achieved only when the tight-coupling condition $|q| = 1$ is satisfied.

V. CONCLUSION

We have studied the finite-power performance of endoreversible quantum heat engines, whose working substance obeys one of the two typical quantum statistics (Fermi-Dirac and Bose-Einstein) and which do not employ any specific law(s) of thermal conduction. Based on the quantum master equation in the Lindblad form, we have calculated the instantaneous heat flux between the working substance and heat reservoir of constant temperature along any isothermal process. The power and efficiency are expressed in terms of the thermodynamic variables of the working substance. Optimizing directly the power with respect to the temperature of the working substance, the efficiency at maximum power was found to be the CA efficiency $\eta_{\rm CA}$, the known upper bound for the heat engines in the linear response regime. This result was then further confirmed and these engines were proved to be tightly coupled through appropriate identification of thermodynamic fluxes and forces, within the context of linear irreversible thermodynamics.

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