Q-voter model with nonconformity in freely forming groups: Does the size distribution matter?

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We study a q-voter model with stochastic driving on a complete graph with q being a random variable described by probability density function $P(q)$, instead of a constant value. We investigate two types of $P(q)$: (1) artificial with the fixed expected value $\langle q \rangle$, but a changing variance and (2) empirical of freely forming groups in informal places. We investigate also two types of stochasticity that can be interpreted as different kinds of nonconformity (anticonformity or independence) to answer the question about differences observed at the macroscopic level between these two types of nonconformity in real social systems. Moreover, we ask the question if the behavior of a system depends on the average value of the group size q or rather on probability distribution function $P(q)$.

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I. INTRODUCTION

Although the origins of agent-based models (ABM) in sociology can be traced back to the 1960s, it was only from the 1990s that ABM applications reached a critical mass [1,2]. Almost simultaneously, yet somehow independently, a new field of sociophysics emerged [3]. There are many reasons for parallel but independent development of agent-based social simulations and sociophysics, such as rare scientific contacts between statistical physicists and sociologists or lack of the common terminology. However, it seems that the fundamental problem is differences between social scientists and physicists in their approach to modeling. Interdisciplinary research, particularly on the border between disciplines which significantly different from each other, always involves certain danger. The models analyzed within the framework of statistical physics may be too simplistic to have any utility in social sciences. On the other hand, the desire to describe the accurately social phenomena can lead to an excessive complexity of the models, which are too difficult for theoretical analysis and therefore in general not of interest from the point of view of physics. For these reasons we believe that searching for models and questions that may be interesting for both social scientists and physicists are particularly important.

Such an interesting question, related to the theory of social response, has been asked within the q-voter model [4,5]. A basic goal of researchers, developing the theory of social response, is to identify the minimum number of variables that is needed to distinguish between responses such as conformity, independence, and anticonformity [6]. Conformity, defined as a change in behavior to match that of others refereed, leads to the consensus (ferromagnetic order). Independence, i.e., resisting social influence, and anticonformity, i.e., rebelling against influence, are recognized as two types of nonconformity and can destroy consensus. The question asked in [4,5], namely "Do differences between two types of nonconformity, that are recognized by social psychologists on the individual (microscopic) level, manifest on the society (macroscopic) level?" occurred to be interesting not only for the theory of social response but also from the physical perspective [4–6].

The q-voter model, proposed as a generalization of two earlier models of opinion dynamics—linear voter and Sznajd model [7]—itself seems to be merely yet another oversimplified model of opinion dynamics. However, it has been shown that its generalized version is suitable to describe many social phenomena [8–12]. Moreover, the q-voter model occurred to be also very interesting from theoretical point of view and gained considerable interest in physical literature [13–23].

In the original q-voter model all individuals are homogeneous, which means that we cannot speak about leaders, authorities, etc. The only trait that characterizes an agent is a dichotomous opinion, i.e., dynamical variable $S_i(t) = \pm 1$, which is reminiscent of the spin in the Ising model. Therefore, such a particularly simple agent was named a spinson (=spin+person) and graphically represented by the combination of a man and an arrow [5]; see Fig. 1. Spinsons, alternately called agents, individuals, voters, or just spins, are placed in the nodes of a given network and potentially can interact with all other individuals to whom they are linked. A characteristic feature of the q-voter model is that in a single time step each agent can interact only with q spinsons randomly chosen from its neighborhood. Besides, voter is influenced by its neighbors only if a chosen group is unanimous (i.e., all q individuals have the same opinion). The validity of this assumption has been broader discussed in [9]. In the original model [7], and in most of the later works, the size q of the influence group (so called q panel) has been a parameter of the model, which means that q has been a constant value [8–11,13–17,17,18,21–23]. Only recently this model's assumption has been modified by introducing zealots (inflexible voters that never change their opinion) and two types of susceptible voters (type q_1 or q_2): at each time step, a q_i-susceptible voter ($i = 1,2$) consults a group of q_i neighbors and adopts their opinion if all group members agree [19,20]. Such a modification introduces heterogeneity to the model giving voters personal traits and is compatible with a personality oriented approach [24].

On the contrary, within the situation-oriented approach all agents are homogeneous and each of them can behave with some probability differently. For example, in [4] it has been proposed that each agent with probability p behaves like a nonconformist and with probability $1 - p$ as a conformist.

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Similarly, we could assume that q is a random variable given by a certain probability distribution $P(q)$. Within such an approach each agent at every time could be influenced by a group of a different size. An idea to introduce probability distribution for gathering sizes into the opinion dynamics model was introduced already in 2002 by Serge Galam within a simple diffusion reaction model [25] and here we will use it within the q-voter model with nonconformity.

This is not entirely clear which approach, personality oriented (PO) or situation oriented (SO), is more suitable to describe social phenomena and there has been a long-lasting person-situation debate related to this issue [24]. If we study different sizes of groups then the PO approach can describe two types of personality—loners and gregarious. On the other hand, SO allows one to investigate opinion dynamics in freely forming groups [26,27]. Both approaches are interesting from a physical point of view: PO can be viewed as the quenched approach and SO as the annealed one.

In this paper we will continue studies initiated in [4] and therefore we will use mainly the annealed (situation-oriented) approach. Nonetheless, we will also check how results would change if we use the quenched approach. Once again we will ask the same question if the difference between two types of nonconformity (anticonformity or independence), which are different behaviors at the individual level, would lead to a different behavior of the system on the macroscopic level. In other words, is it really necessary to introduce two types of nonconformity to the theory of social response from the point of view of sociology? Previously, we have shown that both types of nonconformity lead to the order-disorder phase transition, but for the model with anticonformity the transition is continuous for all $q \geqslant 2$ (there is no phase transition for $q = 1$), whereas for the model with independence the transition is continuous only for $q \in [2,5]$ and for $q > 5$ it switches to discontinuous. Here we check how results would change if one replaces a constant value of q (parameter) by a random variable with certain probability density distribution $P(q)$.

Until now it has been claimed that the constant value of q could be interpreted as a selection of an optimal or an average group size [9,28–31]. Here we check if indeed the behavior of the model depends on the expected value itself or rather probability density distribution (PDF). Therefore, we start with investigating artificial PDF's with the same average value $\langle q \rangle$. Next, we consider empirical human group size distributions $P(q)$ collected by other researchers during various social studies, including observations of pedestrians, shopping groups (observed in department stores and public markets), play groups (public playgrounds), public gathering (public beach swimming pool, public parks, etc.), students (observed in coffee shop in a student union, dining hall, undergraduate library, etc.), and football fans [32–34]. This part will allow one to answer the question about the difference between anticonformity and independence in real freely forming groups.

Summarizing, in this paper we will try to answer the following questions.

(i) Does the behavior of the system depend on the average value of the group size q or rather on probability distribution function $P(q)$?

(ii) Is it possible to observe qualitative differences at the macroscopic level between the system with independence and the system with anticonformity for real (measured empirically) $P(q)$?

(iii) Does it influence results if we use a personality-oriented (quenched) instead of situation-oriented (annealed) approach to model distribution of group sizes?

II. MODEL

We consider a system of N agents (voters, spinsons), each of which is described by a single dynamical variable (opinion) $S_i(t) = \pm 1$, where $i = 1, \dots, N$ and t denotes time. Voters are placed in the nodes of a given network. Originally only one type of the social response, namely conformity, has been taken into account [7]. Within this type of response a voter adopts an opinion of its q randomly chosen neighbors (so-called q panel) if all q neighbors share the same opinion. Later on we have proposed to introduce also nonconformity—with probability p, an agent acting as nonconformist and with the complementary probability $1 - p$, an agent conforming to the q panel [4]. In general, conformity is a force which tries to order the system (i.e., leads to the consensus), whereas nonconformity acts against consensus and tries to destroy the order. As a result of competition between conformity and nonconformity, a phase transition should appear. Indeed it has been found that there is a critical value of nonconformity $p = p^*$, below which there is a majority of one opinion and above which there is a stalemate state, i.e., there is an equal number of individuals with positive and negative opinion. However, the character of this phase transition depends on the type of nonconformity.

Social psychologists distinguish between two basic types of nonconformity: anticonformity (rebelling against influence) and independence (resisting influence) [6]. In [4,5] we have asked the following question: do differences between two types of nonconformity, that are recognized by social psychologists on the individual (microscopic) level, manifest on the society (macroscopic) level, at least within the q-voter model? To answer this question, we have considered two special cases, each with conformity and one type of nonconformity: model A (conformity+anticonformity) and model I (conformity+independence). Here we will again consider model A and model I, but the size q of the influence group is no longer a parameter, as in the original q-voter model, but a random variable described by a certain PDF $P(q)$ (see left panel in Fig. 1).

This assumption has been motivated by the empirical studies on the freely forming groups [32–34]. It has been shown that typically q varies between 1 and 7 and the size distribution depends on the situation (place of the meeting, type of the event, etc.). Later in this paper we will present results for several empirically measured $P(q)$, but we start with asking the following question: would results on the macroscopic level depend on the size distribution or maybe rather on the average value of q? Therefore, we compare results for different $P(q)$ but with the same average value $\langle q \rangle$, given by

$$P(q = Q) = \begin{cases} \epsilon & \text{for } Q = \langle q \rangle - \delta, \\ 1 - 2\epsilon & \text{for } Q = \langle q \rangle, \\ \epsilon & \text{for } Q = \langle q \rangle + \delta, \end{cases} \quad (1)$$

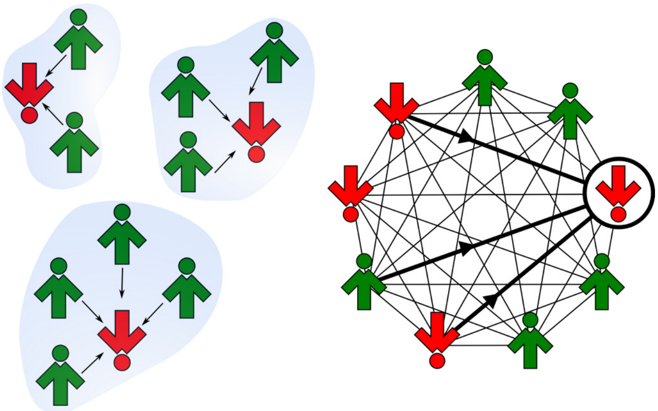

FIG. 1. Schematic illustration of the model's assumptions. Left panel: the size q of the influence group is not longer a parameter, as in the original q-voter model, but is given by a certain density probability distribution $P(q)$. This means that the q panel (green up spinsons in this illustration), which influences a voter (red down spinson), can change size in each update. Right panel: social network is described here by the complete graph (clique) and therefore each agent is connected with any other agent. Yet, in a single update an agent is influenced only by a group of q spinsons (here $q = 3$).

where ϵ and δ are parameters of the model. This means that $P(q)$ consists of three peaks: the middle one at the expected value $\langle q \rangle$ and two side peaks equally spaced by a distance δ from the central one. This model is a generalization of the q-voter model with the constant value of q and reduces to this model for $\epsilon = 0$ or $\delta = 0$. Certainly also other PDF's could be tested but $P(q)$ given by Eq. (1) is particularly useful in answering the question posed in this paper.

For clarity in further calculations let us introduce

$$\vec{w} = (w_1, w_2, w_3) = (\epsilon, 1 - 2\epsilon, \epsilon) \tag{2}$$

and

$$\vec{q} = (q_1, q_2, q_3) = (\langle q \rangle - \delta, \langle q \rangle, \langle q \rangle + \delta). \tag{3}$$

Then Eq. (1) can be rewritten in the following form:

$$P(q = Q) = \begin{cases} w_1 & \text{for } Q = q_1, \\ w_2 & \text{for } Q = q_2, \\ w_3 & \text{for } Q = q_3, \end{cases} \tag{4}$$

which could be easily generalized to the simple form that allows for arbitrary probability distribution:

$$P(q = q_i) = w_i, \quad \text{where } \sum w_i = 1. \tag{5}$$

Such a general form will be useful later on, when we use the empirical distributions for freely forming groups.

Similarly, as in [4], we consider the model on a complete graph (see right panel in Fig. 1), which means that every agent is linked to all other agents. To distinguish between continuous and discontinuous phase transitions, we consider two types of initial conditions: (i) ordered, i.e., all agents have positive opinions [$S_i(t = 0) = 1$ for all $i = 1, \ldots, N$], and (ii) random, i.e., each agent at $t = 0$ is positive or negative with equal probability.

After initialization the system evolves according to the algorithm, which depends on the model (A or I).

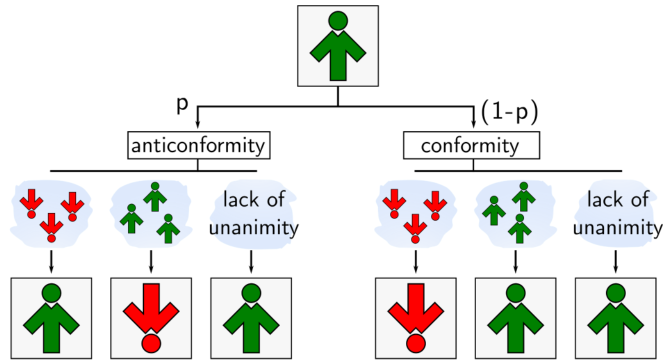

FIG. 2. Example of a single update for the q voter with anticonformity (model A). In this example $q = 3$ and a chosen target spinson (in the square) has initially positive opinion.

Algorithm for model A (see also Fig. 2).

(1) Choose randomly target agent i, i.e., choose a random number i from the uniform probability distribution $U(1,N)$.

(2) Choose the size of the influence group q from a given PDF $P(q)$.

(3) Choose randomly q agents from the whole system (without repetitions)—they will form a q panel.

(4) If the q panel is unanimous, i.e., all q agents have the same opinion, then the q panel influences voter i: with probability p voter i takes the opinion opposite to the q panel, otherwise voter i takes the opinion of the q-panel; go to (1).

Algorithm for model I (see also Fig. 3).

(1) Choose randomly target agent i, i.e., choose a random number i from the uniform probability distribution $U(1,N)$.

(2) Decide if a target agent is independent, i.e., choose a random number r from the uniform probability distribution $U(0,1)$ and if $r < p$ then an agent acts independently, i.e., go to (3), otherwise go to (4).

(3) A target agent changes opinion to the opposite with probability f, i.e., choose a random number r from the uniform probability distribution $U(0,1)$ and if $r < f$ then agent $S_i(t + \Delta t) = -S_i(t)$, otherwise nothing happens; go to (1).

(4) Choose the size of the influence group q from a given PDF $P(q)$.

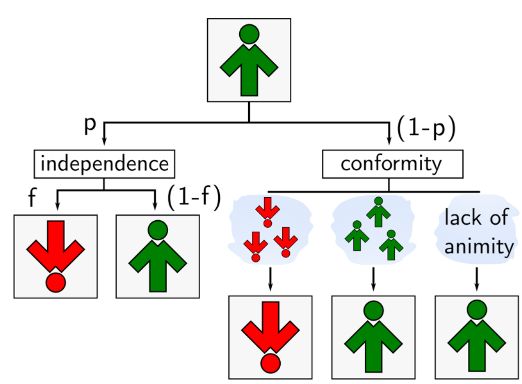

FIG. 3. Example of a single update for the q voter with independence (model I). In this example $q = 3$ and a target spinson (in the square) has initially positive opinion. In this paper we will use $f = 1/2$, analogous as in [4].

(5) Choose randomly q agents (without repetitions)—they will form a q panel.

(6) If the q panel is unanimous, i.e., all q agents have the same opinion then a target voter i takes the opinion of the q panel, otherwise nothing happens; go to (1).

We are aware that after reading this section, one could get the impression that there is not much novelty in this paper with respect to the previous one [4]. However, we believe that the purpose of this work is meaningful because it brings us closer to answering the question about qualitative differences between two types of nonconformity in real social systems. In the previous work, we have claimed that the significant distinction between independence and anticonformity is given by the type of the phase transition. Independence has led to much richer behavior displaying tricriticality, resulting from the switch from continuous to discontinuous phase transition at $q = 6$. Nevertheless, it may occur that our previous findings are interesting only from a theoretical point of view, because in reality groups are not always of the same size q. Of course, one could claim that parameter q plays the role of the average value, as proposed in [9]. However, in that case, the question arises if results will indeed depend only on the average value of q or rather on the probability distribution function $P(q)$. This latter question is important not only from the social point of view but also in respect to the field of sociophysics or agent-based modeling in general.

A. Results

In a single time step Δt, three events are possible—the number of "up" spins $N_{\uparrow}$ increases or decreases by 1 or remains constant. Of course all three events can be rewritten for the number of "down" spins $N_{\downarrow}$ as $N_{\uparrow} + N_{\downarrow} = N$. Also concentration $c = N_{\uparrow}/N$ of spins up increases or decreases by $1/N$ or remains constant:

$$\gamma^+ = \text{Prob}\left\{c(t+\Delta t) = c(t) + \frac{1}{N}\right\},$$
$$\gamma^- = \text{Prob}\left\{c(t+\Delta t) = c(t) - \frac{1}{N}\right\},$$
$$\gamma^0 = \text{Prob}\{c(t+\Delta t) = c(t)\} = 1 - \gamma^+ - \gamma^-. \tag{6}$$

In the original model with a constant value of q, the transition probabilities $\gamma^+,\gamma^-,\gamma^0$ depend on the size of the influence group q [4]. Here q is given by the probability distribution function (5), and therefore the total transition probability reads $\gamma^{\pm} = \sum_i w_i \gamma^{\pm}_{q_i}$, where $\gamma^{\pm}_{q_i}$ is a transition probability for a given constant value q_i of the influence group, as derived in [4]. If we assume that $N \to \infty$ then we can write the following.

(i) Model A (conformity + anticonformity):

$$\gamma^+ = \sum_i w_i[(1-p)(1-c)c^{q_i} + p(1-c)^{q_i+1}],$$
$$\gamma^- = \sum_i w_i[(1-p)c(1-c)^{q_i} + pc^{q_i+1}]. \tag{7}$$

The terms in square brackets [...] describe the probability of increment or decay of c for a fixed value of the influence group q_i. The first term in [...] corresponds to the probability that a spin will flip due to the conformity and the second term describes the probability of a flip due to the anticonformity. This can be easily understood by tracing back all processes that are needed for a given change. Let us take as an example the first term in [...] for γ^+. First of all, conformity takes place with probability $1 - p$. Concentration of spins up will increase if a spin chosen randomly to be changed is down, which takes place with probability $1 - c$. Finally, in order to flip a spin down to the up position according to conformity all q_i neighbors have to be up and the probability of this is equal to c^{q_i}. Therefore, the total probability of this event is a product $(1-p)(1-c)c^{q_i}$. Analogously we can consider all other processes that change c.

(ii) Model I (conformity+independence):

$$\gamma^+ = \sum_i w_i[(1-p)(1-c)c^{q_i} + fp(1-c)],$$
$$\gamma^- = \sum_i w_i[(1-p)c(1-c)^{q_i} + fpc]. \tag{8}$$

Again, the term in the square brackets [...] describes the probability of increment or decay of c for a fixed value of the influence group q_i. The first term in [...] corresponds to the probability that a spin will flip due to the conformity and the second term describes the probability of a flip due to the independence.

Generally $f \in [0,1]$, but in this paper we will use $f = 1/2$, analogously as in [4]. Yet, for any other value of $f > 0$ results can be rescaled [35].

Evolution of the expected value of concentration is given by the rate equation:

$$\langle c(t+1)\rangle = \langle c(t)\rangle + (\gamma^+ - \gamma^-), \tag{9}$$

and thus the stationary state from the condition

$$\gamma^+ - \gamma^- = 0. \tag{10}$$

Solving analytically Eq. (10), i.e., finding c_{st} as a function of p is impossible, but we can easily derive the opposite relations satisfying Eq. (10) as follows. (i) Model A (conformity+anticonformity):

$$p = \sum_i w_i \left[\frac{c_{st}(1-c_{st})^{q_i} - (1-c_{st})c_{st}^{q_i}}{(1-c_{st})^{q_i+1} + c_{st}(1-c_{st})^{q_i} - (1-c_{st})c_{st}^{q_i} - c_{st}^{q_i+1}}\right]. \tag{11}$$

(ii) Model I (conformity+independence):

$$p = \sum_i w_i \left[\frac{c_{st}(1-c_{st})^{q_i} - (1-c_{st})c_{st}^{q_i}}{(1-c_{st})/2 + c_{st}(1-c_{st})^{q_i} - (1-c_{st})c_{st}^{q_i} - c_{st}/2}\right]. \tag{12}$$

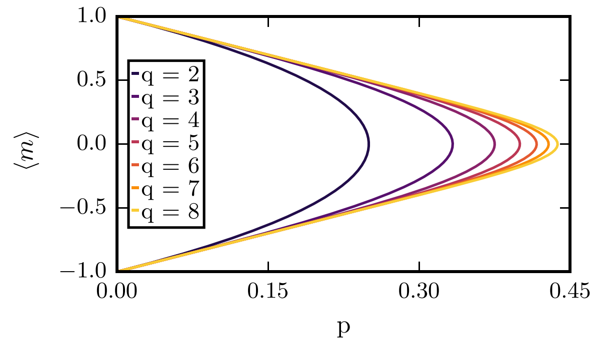

FIG. 4. Dependencies between the average public opinion $\langle m\rangle$ in the steady state and the level of nonconformity p for the model with conformity+anticonformity (model A). In this case the size of the q panel is constant [$\epsilon = 0$ in Eq. (1)] and q increases from left to right. Results have been obtained from Eq. (11). It is seen that phase transition is continuous for all values of q and critical temperature increases with q. The same results have been obtained within less general model with constant q in [4].

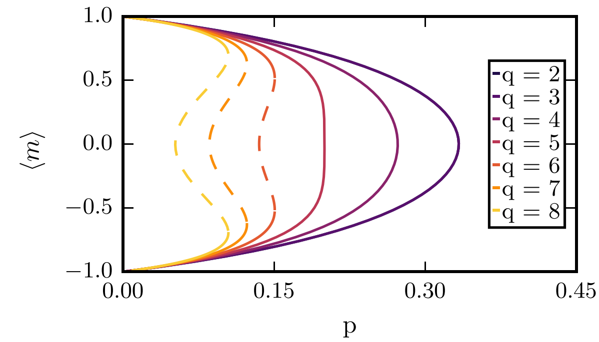

FIG. 5. Dependencies between the average public opinion $\langle m\rangle$ in the steady state and the level of nonconformity p for the model with conformity+independence (model I). In this case the size of the q panel is constant [$\epsilon = 0$ in Eq. (1)] and q decreases from left to right. Results have been obtained from Eq. (12). Dotted lines have been used to mark instability of the solution, whereas solid lines denote stable solution. This shows that for model I the phase transition is continuous only $q \in [2,5]$, whereas for $q \geqslant 6$ the phase transition is discontinuous. Moreover, the critical value of p decreases with q, contrary to model A. The same results have been obtained within a less general model with constant q in [4].

For clarity of discussion, we will present all results in the language of public opinion (magnetization):

$$m(t) = \frac{1}{N}\sum_{i=1}^{N} S_i(t), \tag{13}$$

which is related to the concentration c of spinsons up by the simple formula

$$m(t) = \frac{N_\uparrow - N_\downarrow}{N} = 2c(t) - 1. \tag{14}$$

We will use formulas (11) and (12) to plot the dependence between the steady value of public opinion and the level of nonconformity p. Although only the relation $p(c_{st})$ is calculated analytically and the opposite relation is unknown, we plot $m_{st}(p)$ by simply rotating the figure with the relation $p(c_{st})$ and then applying formula (14); see Figs. 4 and 5. Results presented in Figs. 4 and 5 are nothing new, because in these figures we show the case with $\epsilon = 0$, which reduces the model to the original one with a constant value of q. They are presented here for two reasons: (1) to recall differences between models A and I; (2) to confirm that our generalized formulas reduce to the known results from [4].

Before we discuss results for different $P(q)$, let us compare obtained analytical results with Monte Carlo simulations. Besides, we compare Monte Carlo results for the annealed (situation-oriented) and quenched (personality-oriented) approach. In this case we investigate the system of size 10^4 and average results over 10^2 samples after 10^3 Monte Carlo steps of "thermalization." It is shown in Fig. 6 that all methods give the same results for both types of nonconformity (anticonformity and independence). In this example, we use the probability density function of q given by Eq. (1) with $\langle q\rangle = 6, \delta = 1$ and $\epsilon = 0.1$, which means that $\vec{q} = (5,6,7)$ with probabilities $\vec{w} = (0.1,0.8,0.1)$. This particular form of PDF has been chosen just as an example, to show that Monte Carlo results agree very well with analytical ones. Monte Carlo simulations from two types of initial conditions (ordered and random) give exactly the same results for model A, which confirm that transition is continuous, whereas for model I hysteresis is observed, which indicates discontinuous phase transition. Moreover, it is seen that annealed and quenched approaches give the same results. We would like to stress here that the annealed versus quenched approach relates here only to the distribution of q. In the annealed approach q is a random variable and each agent at every time can be influenced by a group of different size. On the contrary within the quenched approach, q is a trait that characterizes an agent, which means that each agent has fixed value of q. We recall it here because the situation is completely different if we investigate the annealed versus quenched approach in respect to the level of nonconformity p. In the latter issue, an annealed approach means that each agent acts as nonconformists with probability p and follows the q-panel with probability $1 - p$. On the other hand, within a quenched approach pN individuals are nonconformist for ever [24]. In such a case, the quenched approach gives qualitatively different results than the annealed one.

Because Monte Carlo results agree with our analytical formulas (11) and (12), which is not surprising as far as we consider a system on a complete graph, we will use only these formulas to discuss two problems posed in the introduction, namely as follows.

(i) Does the behavior of the system depend on the average value of q or rather $P(q)$?

(ii) Will one observe qualitative differences at the macroscopic level between the system with independence and the system with anticonformity for real (measured empirically) $P(q)$?

To answer the former question we consider probability distribution function $P(q)$ given by Eq. (1). We keep constant expected value $\langle q\rangle$ but change distribution itself using different values of δ and ϵ. A particularly interesting value of an average group size is $\langle q\rangle = 6$, because for $q = 6$ a switch from continuous to discontinuous phase transition has been

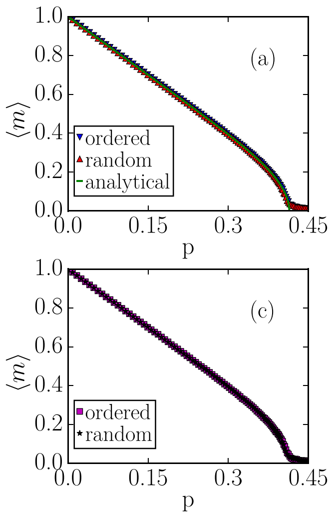

FIG. 6. Dependencies between the average public opinion $\langle m \rangle$ in the steady state and the level of nonconformity p for the probability density function of q is given by Eq. (1) with $\langle q \rangle = 6$, $\delta = 1$, and $\epsilon = 0.1$. This means that $\vec{q} = (5,6,7)$ with probabilities $\vec{w} = (0.1,0.8,0.1)$. Results for the annealed approach are presented in the upper panels, whereas in the bottom panels results for the quenched approach are shown. Besides, results for the model with anticonformity (model A) are presented in the left column and for the model with independence (model I) in the right column: (a) model A with the annealed approach; (b) model I with the annealed approach; (c) model A with the quenched approach; (d) model I with the quenched approach. The simulations were performed for the system consisting of 10^4 spinsons. The results were averaged over 10^2 realizations after 10^3 Monte Carlo steps.

observed for the model with independence (model I) [4]. Results for $\langle q \rangle = 6$ are presented in Fig. 7. It is seen that for the model with anticonformity (model A) phase transition is continuous for any distribution $P(q)$. On the other hand, for the model with independence results depend on $P(q)$, not only on $\langle q \rangle$. For the same average value $\langle q \rangle = 6$ transition can be continuous or discontinuous, which is seen particularly well for $P(q)$ with the higher variance (e.g., $\delta = 3$; left panels in Fig. 7). For $\epsilon = 0$ the transition is clearly discontinuous. With increasing ϵ the jump in order parameter and hysteresis decreases monotonically to zero and for $\epsilon \in (0.2,0.4)$ the transition becomes continuous.

Calculating the jump of the order parameter at the transition point for different values of δ and ϵ allows one to construct the phase diagram for model I. Such a phase diagram for $P(q)$ given by Eq. (1) with $\langle q \rangle = 6$ is presented in Fig. 8. For small values of δ and ϵ the system undergoes discontinuous phase transition and below the critical line $(\delta^*,\epsilon^*) = (\delta^*,\epsilon(\delta^*))$ there is a continuous phase transition.

Our results clearly show that the average value of the group size is not enough to describe opinion dynamics in social systems, especially if we take into account the possibility of independent behavior. Now we turn to the second question posed here, namely if qualitative differences between anticonformity and independence would be seen in real societies, i.e., if we consider empirical $P(q)$.

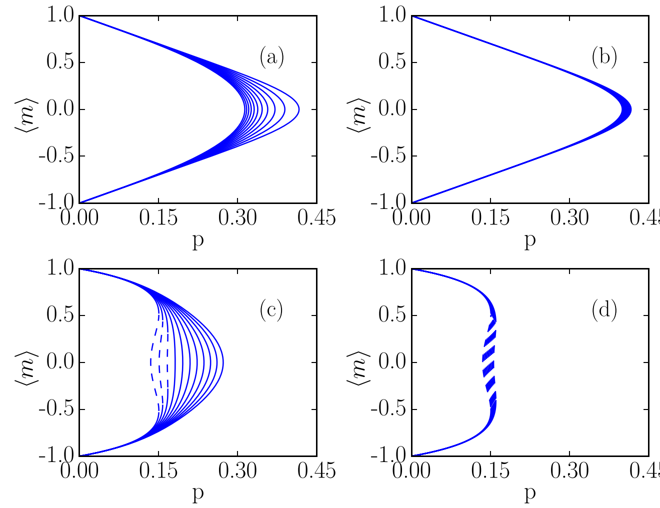

FIG. 7. Dependencies between the average public opinion $\langle m \rangle$ in the steady state and the level of nonconformity p for the models with anticonformity (model A; upper panels) and independence (model I; bottom panels) for $P(q)$ given by Eq. (1) with $\langle q \rangle = 6$ and two values of δ: left panels (a),(c) $\delta = 3$; right panels (b),(d) $\delta = 1$. In each panel results for several values of ϵ are shown: ϵ changes with a step $\Delta\epsilon = 0.1$ from zero to 1 (from right to left for model A and from left to right for model I).

For over 60 years, social scientists have examined the size of naturally occurring groups in different informal locations like sidewalks, stores, playgrounds, carnivals, receptions, swimming pools, basketball game intermissions, church socials, and train depots. It has been shown that groups ranged in size from two to seven [26,32–34]. Already this information suggests that differences between anticonformity and independence may be not visible in real societies, since qualitative differences between these two types of social response are visible on the macroscopic level only for $q \geqslant 6$. Nevertheless, we have decided to examine several empirical $P(q)$ and see phase diagrams of models A and I. In Fig. 9 we present results for two empirical $P(q)$: distribution for fans gathered to watch a college football game [34] and pedestrians on the

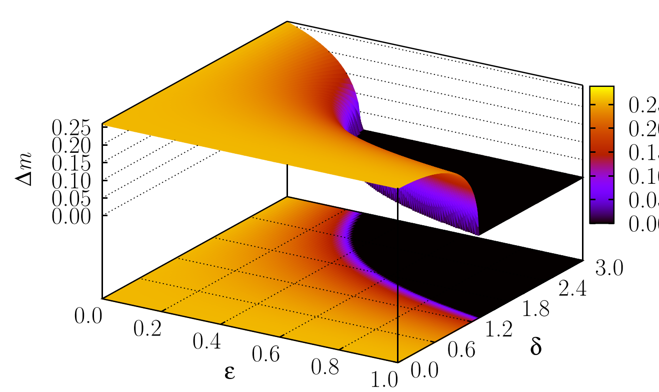

FIG. 8. Phase diagram for model I (conformity+independence) with $P(q)$ given by Eq. (1) for $\langle q \rangle = 6$ and ϵ,δ being two model's parameters. In the black region the system undergoes continuous phase transition (there is no jump in order parameter, i.e., $\Delta m = 0$) and in the brighter area there is discontinuous phase transition.

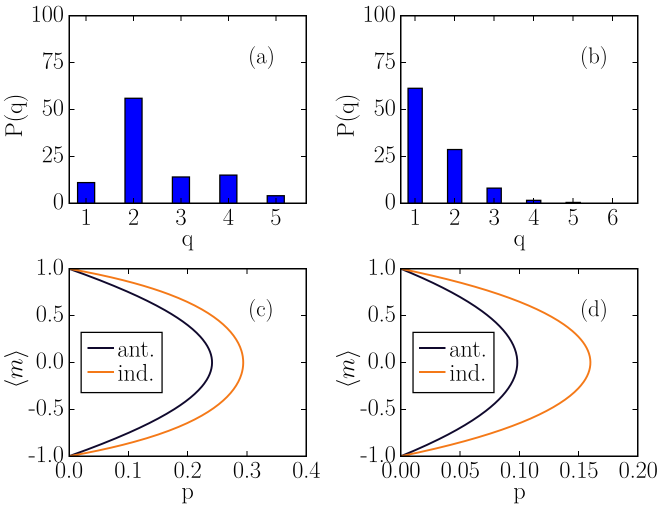

FIG. 9. Empirical probability density functions of $P(q)$ (upper panels) for fans gathered to watch a college football game [34] (a) and pedestrians on the streets in Eugene (Oregon) [33] (b). Corresponding dependencies between the average public opinion $\langle m \rangle$ in the steady state and the level of nonconformity are presented in bottom panels.

streets in Eugene (Oregon) [33]. It is seen that in both cases, and for many other empirical PDF's not shown here, phase transition is continuous for both models. Lower critical level of anticonformity than independence is sufficient to destroy an order in the system for all investigated empirical $P(q)$. Besides, for most empirical data the level of nonconformity needed for the stable disordered (stalemate) state is surprisingly low, i.e., $p_c \in (0.05,0.2)$. It is worth mentioning here the results obtained for U.S. presidential elections, where the best agreement for the voter model with noise, which can be interpreted as independence, has been obtained for the noise level 0.03 [36]. Results obtained here could also explain voting at fifty-fifty in political elections, which has been discussed also by Galam [37].

III. CONCLUSIONS

It is obvious that competition between two opposing forces such as conformity and nonconformity should lead to the order-disorder phase transition. Nonetheless, it is far less obvious what type of the transition will appear. For the q-voter model with independence it was shown that the transition is continuous for smaller group sizes ($q < 6$) and discontinuous for larger. For the q-voter model with anticonformity the phase transition is always continuous. It should be recalled here that in other opinion dynamic models, with different types of nonconformity, only continuous phase transitions have been observed [38–40]. Therefore, it seems that the q-voter model with independence is quite unique. Yet, the question arises if in the real social systems this discontinuity will be seen. Furthermore, we have asked the following: is the average value of the group size itself responsible for the type of the phase transition or rather the probability distribution $P(q)$? To answer these questions we have studied two types of PDF's: (1) artificial with the fixed expected value $\langle q \rangle$, but a changing variance, and (2) empirical of freely forming groups in informal places. It has occurred that the average value $\langle q \rangle$ itself does not determine the type of the phase transition. Moreover, it seems that in real systems indeed only continuous phase transitions would be observed and the order would be more easily destroyed by anticonformity than independence. Surprisingly the level of nonconformity sufficient to destroy the order and reach the stalemate situation is very low, at least within the q-voter model. This could be an alternative explanation for the often fifty-fifty result of recent political elections [37,41].

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