PHYSICAL REVIEW E **95**, 043211 (2017)

Influence of collective nonideal shielding on fusion reaction in partially ionized classical nonideal plasmas

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(Received 12 January 2017; revised manuscript received 28 March 2017; published 28 April 2017)

The collective nonideal effects on the nuclear fusion reaction process are investigated in partially ionized classical nonideal hydrogen plasmas. The effective pseudopotential model taking into account the collective and plasma shielding effects is applied to describe the interaction potential in nonideal plasmas. The analytic expressions of the Sommerfeld parameter, the fusion penetration factor, and the cross section for the nuclear fusion reaction in nonideal plasmas are obtained as functions of the nonideality parameter, Debye length, and relative kinetic energy. It is found that the Sommerfeld parameter is suppressed due to the influence of collective nonideal shielding. However, the collective nonideal shielding is found to enhance the fusion penetration factor in partially ionized classical nonideal plasmas. It is also found that the fusion penetration factors in nonideal plasmas represented by the pseudopotential model are always greater than those in ideal plasmas represented by the Debye-Hückel model. In addition, it is shown that the collective nonideal shielding effect on the fusion penetration factor decreases with an increase of the kinetic energy.

DOI: 10.1103/PhysRevE.95.043211

I. INTRODUCTION

The atomic and nuclear collisions in plasmas have been of great interest since these processes are the most fundamental processes in many areas of physics such as astrophysics, atomic and molecular physics, chemical physics, nuclear physics, and plasma physics, and also provide useful information on the collision systems as well as their physical environments [1–8]. This collision process in plasmas has been extensively investigated due to its wide applications in plasma diagnostics in order to obtain the physical information on various plasma parameters [9–14]. The nuclear fusion reaction process in plasmas has received considerable attention in astrophysics, atomic physics, nuclear physics, and plasma physics since the fusion reaction processes take place in various astrophysical and laboratory plasmas usually using the Debye-Hückel model of the plasma [15–21]. It has been also shown that the Yukawa-type Debye-Hückel screening model describes the properties of classical low-density plasmas and corresponds to pair correlation approximation ideal plasmas since the average interaction energy between particles is smaller than the average kinetic energy of a particle [7]. However, it has been shown that the multiparticle correlation effects caused by simultaneous interactions of many charged particles should be taken into account with an increase of the plasma density since it is necessary to contemplate not only short-range collective effects but also long-range screening effects in the case of a dense plasma [22–29]. Hence the screened interaction in these nonideal plasmas would not be described by the conventional Yukawa-type Debye-Hückel interaction model because of the strong collective effects of nonideal particle interactions [22,29]. It has been shown that high-density plasmas with nonideality effects can be found in numerous astrophysical objects and modern technical devices such as white dwarfs, the Sun, giant planets, liquid metals, and semiconductor plasmas [29]. Hence it can be then expected that the fusion reaction process in nonideal plasmas is quite different from those in ideal plasmas due to the influence of collective nonideal shielding on the fusion penetration factor. Thus, in this paper, we investigate the collective nonideal shielding effects on the fusion reaction process in partially ionized classical nonideal plasmas. By using the Wentzel-Kramers-Brillouin (WKB) method with the pseudopotential model [22], the analytic expression of the tunneling radius and the closed form of the Sommerfeld parameter and the fusion penetration factor in nonideal plasmas are obtained as functions of the nonideality parameter, Debye length, and relative kinetic energy. In addition, the variation of the fusion penetration factor due to the change of the collective nonideal shielding is also discussed.

II. THEORY AND CALCULATIONS

In a recent paper by Baimbetov *et al.* [22], the integrodifferential equation for the effective potential of particle interactions taking into account the simultaneous collective correlations of many charged particles has been obtained on the basis of a sequential solution of the Bogoliubov chain equations for the equilibrium distribution function in nonideal plasmas. In addition, the analytic expression for the pseudopotential of the particle interaction in classical nonideal plasmas has been obtained by application of the spline

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approximation [22]. Using the pseudopotential model [22] taking into account the collective nonideal shielding effects, the interaction energy $V_{\rm eff}(r)$ between two reacting nuclei in nonideal plasmas is then represented by

$$V_{\rm eff}(r) = \frac{Z_1 Z_2 e^2}{r} e^{-r/\lambda_D} \frac{1+\gamma f(r)/2}{1+c(\gamma)}, \quad (1)$$

where r is the internuclear distance, $Z_1 e$ and $Z_2 e$ are charges of two nuclei, λ_D is the Debye length, $f(r) = (e^{-\sqrt{\gamma}\, r/\lambda_D} - 1)(1 - e^{-2r/\lambda_D})/5$, $\gamma(\equiv e^2/\lambda_D k_B T_e)$ is the nonideality plasma parameter, $c(\gamma) \cong -0.008\,617 + 0.455\,861\gamma - 0.108\,389\gamma^2 + 0.009\,377\gamma^3$ is the correlation coefficient for different values of γ, k_B is the Boltzmann constant, and T_e is the electron temperature. The validity range of the nonideality plasma parameter γ is known as $0 < \gamma < 4$ [22]. This nonideality parameter stands for the ratio of the collective interaction energy in a Debye sphere to the thermal energy. In ideal plasmas, i.e., when $\gamma \to 0$, the pseudopotential [Eq. (1)] goes over into the Yukawa-type Debye-Hückel potential $V_{\rm DH}(r) \to (Z_1 Z_2 e^2/r)e^{-r/\lambda_D}$. Since the WKB approximation [18,30,31] has been known to be quite accurate to evaluate the screening corrections to the fusion reaction cross section, the cross section $\sigma_{NF}(E)$ for the nuclear fusion reaction using the WKB analysis can be written as

$$\sigma_{NF}(E) = \frac{S_{\alpha\beta}(E)}{E} \Gamma_{\rm FP}(E), \quad (2)$$

where $S_{\alpha\beta}(E)$ is the nuclear cross section factor, $E(=\mu_{\alpha\beta} v^2/2)$ is the energy of the reactive motion, $\mu_{\alpha\beta}[=m_\alpha m_\beta/(m_\alpha + m_\beta)]$ is the reduced mass of the reacting two nuclei, v is the relative collision velocity, and E^{-1} factor represents the geometrical factor associated with the wavelength of the incoming nucleus since the WKB forms of the wave functions $u_{\rm WKB}(r \gg \varphi_T)$ and $u_{\rm WKB}(r \ll \varphi_T)$ are represented in the following expressions:

$$u_{\rm WKB}(r \gg \varphi_T) \propto [E - V_{\rm eff}(r)]^{-1/4} \times \exp\left\{ i\sqrt{\frac{2\mu_{\alpha\beta}}{\hbar^2}} \int_{\varphi_T}^{r} dr [E - V_{\rm eff}(r)]^{1/2} \right\}, \quad (3)$$

and

$$u_{\rm WKB}(r \ll \varphi_T) \propto e^{i\pi/4}[V_{\rm eff}(r) - E]^{-1/4} \times \exp\left\{ i\sqrt{\frac{2\mu_{\alpha\beta}}{\hbar^2}} \int_{r}^{\varphi_T} dr [V_{\rm eff}(r) - E]^{1/2} \right\}, \quad (4)$$

where φ_T is the classical turning point satisfying the relation $V_{\rm eff}(\varphi_T) = E$. For $E > V_{\rm eff}(r)$, the validity of the WKB method is given by $|(d^2k/dr^2)/(dk/dr)| \ll k$ and $|(dk/dr)/k| \ll k$, where $k(r) = \sqrt{2\mu_{\alpha\beta}/\hbar^2}[E - V_{\rm eff}(r)]^{1/2}$ [31]. For $V_{\rm eff}(r) > E$, the validity of the WKB method is represented by $|(d^2\kappa/dr^2)/(d\kappa/dr)| \ll \kappa$ and $|(d\kappa/dr)/\kappa| \ll \kappa$, where $\kappa(r) = \sqrt{2\mu_{\alpha\beta}/\hbar^2}[V_{\rm eff}(r) - E]^{1/2}$ [31]. In Eq. (2), $\Gamma_{\rm FP}(E)$ is the transmission coefficient or known as the fusion penetration factor [17,19] for the effective interaction potential $V_{\rm eff}(r)$ between two reacting nuclei:

$$\Gamma_{\rm FP}(E) = \exp\left\{ -2\sqrt{\frac{2\mu_{\alpha\beta}}{\hbar^2}} \int_0^{\varphi_T} dr [V_{\rm eff}(r) - E]^{1/2} \right\}, \quad (5)$$

since the penetration probability is determined by the ratio of the absolute square of the wave functions $|u_{\rm WKB}(\infty)|^2/|u_{\rm WKB}(0)|^2$. It would be expected that the WKB wave functions and corresponding energy eigenvalues would be different from those of a free atom because the nucleus is shielded by the surrounding dense nonideal plasma electrons. Using the effective interaction potential $V_{\rm eff}(r)$, the classical turning point φ_T for the fusion penetration in classical nonideal plasmas is then obtained by

$$\frac{Z_1 Z_2 e^2}{\varphi_T} e^{-\varphi_T/\lambda_D} \frac{1 + (\gamma/10)(e^{-\sqrt{\gamma}\,\varphi_T/\lambda_D} - 1)(1 - e^{-2\varphi_T/\lambda_D})}{1 + c(\gamma)} = E. \quad (6)$$

In a pure Debye plasma, i.e., without the influence of collective nonideal plasma screening, the classical turning point $\varphi'_T(v, \lambda_D)$ would be determined by

$$\frac{Z_1 Z_2 e^2}{\varphi'_T} e^{-\varphi'_T/\lambda_D} = E. \quad (7)$$

Hence the classical turning point $\varphi'_T(v, \lambda_D)$ in classical ideal plasmas is then given by

$$\varphi'_T(v, \lambda_D) = \lambda_D W\left(\frac{Z_1 Z_2 e^2}{r_D E} \right), \quad (8)$$

where the special function $W(z)$ is known as the Lambert W function [32]. However, the nonlinear equation (6) cannot be represented in terms of the Lambert W function. Hence, after some mathematical manipulations using the perturbation calculation [33] since the classical turning point $\varphi_T(v, \gamma, \lambda_D)$ is usually smaller than the Debye length λ_D, i.e., $\varphi_T/\lambda_D \ll 1$ and $e^{-\varphi_T/\lambda_D} \approx 1 - \varphi_T/\lambda_D + (1/2)(\varphi_T/\lambda_D)^2 - (1/6)(\varphi_T/\lambda_D)^3$, the classical turning point $\varphi_T(v, \gamma, \lambda_D)$ in nonideal plasmas including the influence of collective nonideal shielding is then found to be

$$\varphi_T(v, \gamma, \lambda_D) \cong \frac{\varphi_0(v)}{1 + \varphi_0(v)/\lambda_D + c(\gamma)} + \frac{\varphi_0^3(v)}{\lambda_D^2} \frac{(1/2 - \gamma^{3/2}/5)}{[1 + \varphi_0(v)/\lambda_D + c(\gamma)]^3}, \quad (9)$$

where $\varphi_0(v) \equiv 2Z_1 Z_2 e^2/\mu_{\alpha\beta} v^2$. After some mathematical manipulations using Eqs. (5)–(7), the closed expression of the fusion penetration factor $\Gamma_{\rm NP}(\bar{E}, \gamma, \bar{\lambda}_D)\{= \exp[-\xi_{\rm NP}(\bar{E}, \gamma, \bar{\lambda}_D)]\}$ in classical nonideal plasmas is then given by

$$\Gamma_{\rm NP}(\bar{E}, \gamma, \bar{\lambda}_D) = \exp\left\{ -2\left(\frac{\mu}{m_e}\right)^{1/2} \bar{E}^{1/2} \int_0^{\bar{\varphi}_T(\bar{E}, \gamma, \bar{\lambda}_D)} d\bar{r} \left[\frac{2Z_1 Z_2}{\bar{E}} \frac{1}{\bar{r}} e^{-\bar{r}/\bar{\lambda}_D} \times \frac{1 + (\gamma/10)(e^{-\sqrt{\gamma}\,\bar{r}/\bar{\lambda}_D} - 1)(1 - e^{-2\bar{r}/\bar{\lambda}_D})}{1 + c(\gamma)} - 1 \right]^{1/2} \right\}, \quad (10)$$

where $\bar{E}(\equiv \mu_{\alpha\beta}v^2/2\text{Ry})$ is the scaled energy of the reactive motion, $\text{Ry}(= m_e e^4/2\hbar^2 \approx 13.6\,\text{eV})$ is the Rydberg constant, m_e is the mass of the electron, e is the magnitude of electron charge, $\hbar$ is the rationalized Planck constant, $\bar{\lambda}_D(\equiv \lambda_D/a_0)$ is the scaled Debye length, $a_0(= \hbar^2/m_e e^2)$ is the Bohr radius of the hydrogen atom, $\bar{r}(\equiv r/a_0)$ is the scaled internuclear distance, and $\bar{\varphi}_T(\bar{E},\gamma,\bar{\lambda}_D)(\equiv \varphi_T/a_0)$ is the scaled classical turning point in units of a_0:

$$\bar{\varphi}_T(\bar{E},\gamma,\bar{\lambda}_D) \cong \frac{\bar{\varphi}_0(\bar{E})}{1+\bar{\varphi}_0(\bar{E})/\bar{\lambda}_D + c(\gamma)} + \frac{\bar{\varphi}_0^3(\bar{E})}{\bar{\lambda}_D^2}\frac{(1/2-\gamma^{3/2}/5)}{[1+\bar{\varphi}_0(\bar{E})/\bar{\lambda}_D + c(\gamma)]^3}, \quad (11)$$

with $\bar{\varphi}_0(\bar{E}) = 2Z_1Z_2/\bar{E}$. Hence the Sommerfeld parameter $\xi_{\text{NP}}(\bar{E},\gamma,\bar{\lambda}_D)$ for nuclear reaction in classical nonideal plasmas is then found to be

$$\xi_{\text{NP}}(\bar{E},\gamma,\bar{\lambda}_D) = 2\left(\frac{\mu}{m_e}\right)^{1/2}\bar{E}^{1/2}\int_0^{\bar{\varphi}_T(\bar{E},\gamma,\bar{\lambda}_D)} d\bar{r}\left[\frac{2Z_1Z_2}{\bar{E}}\frac{1}{\bar{r}}e^{-\bar{r}/\bar{\lambda}_D} \times \frac{1+(\gamma/10)(e^{-\sqrt{\gamma}\,\bar{r}/\bar{\lambda}_D}-1)(1-e^{-2\bar{r}/\bar{\lambda}_D})}{1+c(\gamma)} - 1\right]^{1/2}. \quad (12)$$

Hence the fusion penetration factor $\Gamma_C(\bar{E})$ in Coulomb plasmas ($\gamma \to 0$, $\bar{\lambda}_D \to \infty$), i.e., $V_C(r) = Z_1Z_2e^2/r$, is given by

$$\Gamma_C(\bar{E}) = \exp\left[-2\pi\left(\frac{\mu}{m_e}\right)^{1/2}\frac{Z_1Z_2}{\bar{E}^{1/2}}\right], \quad (13)$$

since the scaled classical turning point $\bar{\varphi}_T(\bar{E})$ in a pure Coulomb field becomes $\bar{\varphi}_T(\bar{E}) = 2Z_1Z_2/\bar{E}$.

III. RESULTS AND DISCUSSION

The physical characteristic function $F_{\text{NP}}(\bar{E},\gamma,\bar{\lambda}_D)$ of the collective nonideal shielding effect on the fusion penetration factor in classical nonideal plasmas can be represented by

$$F_{\text{NP}}(\bar{E},\gamma,\bar{\lambda}_D) = \frac{\Gamma_{\text{NP}}(\bar{E},\gamma,\bar{\lambda}_D)}{\Gamma_C(\bar{E})} = \exp\left\{-2\left(\frac{\mu}{m_e}\right)^{1/2}\left[\bar{E}^{1/2}\int_0^{\bar{\varphi}_T(\bar{E},\gamma,\bar{\lambda}_D)} d\bar{r}\left[\frac{2Z_1Z_2}{\bar{E}}\frac{1}{\bar{r}}e^{-\bar{r}/\bar{\lambda}_D} \times \frac{1+(\gamma/10)(e^{-\sqrt{\gamma}\,\bar{r}/\bar{\lambda}_D}-1)(1-e^{-2\bar{r}/\bar{\lambda}_D})}{1+c(\gamma)} - 1\right]^{1/2} - \frac{\pi Z_1Z_2}{\bar{E}^{1/2}}\right]\right\}. \quad (14)$$

As shown in Eqs. (10)–(12) and (14), the influence of collective nonideal shielding on the nuclear fusion reaction process in classical nonideal plasmas is explicitly included through the nonideality plasma parameter γ and the correlation coefficient $c(\gamma)$. In order to specifically investigate the influence of collective nonideal shielding on the nuclear fusion reaction process, we consider the proton-proton reaction case, i.e., $Z_1 = Z_2 = 1$, in nonideal plasmas. Recently, the $^{12}C + {}^{12}C$ fusion reaction by γ ray originating from the α, p, and n evaporation has been measured by using the stable-beam accelerator [34]. However, the fusion reaction process in dense plasmas has not been measured in laboratory experiments. Hence it would be expected that the current results of the fusion penetration factor $\Gamma_{\text{NP}}(\bar{E},\gamma,\bar{\lambda}_D)$, the classical turning point $\bar{\varphi}_T(\bar{E},\gamma,\bar{\lambda}_D)$, and the Sommerfeld parameter $\xi_{\text{NP}}(\bar{E},\gamma,\bar{\lambda}_D)$, including the influence of plasma shielding in fusion reaction process in nonideal plasmas, would provide useful input information for future laboratory measurements of fusion reaction cross sections. Actually, the screening effects would soften the Coulomb potential or decrease the penetration barrier, which should increase the probabilities of fusion penetration. Very recently, an excellent work has provided the dynamical ion structure factor and ion stopping power including the complex nature of charge screening in quantum plasmas [35]. Hence the investigation of the nuclear fusion reaction process in a dense quantum plasma will be treated elsewhere since a linearized viscoelastic quantum hydrodynamical model [35] is known to be valid for a wide range of the ion coupling parameter and plasma densities.

Figure 1 shows the surface plot of the Sommerfeld parameter ξ_{NP} for the nuclear reaction in partially ionized classical nonideal plasmas as a function of the scaled energy of the reactive motion $\bar{E}$ and the nonideality parameter γ. As it is seen, the Sommerfeld parameter ξ_{NP} decreases with an increase of the nonideality parameter γ. It is then found that the influence of collective nonideal shielding suppresses the Sommerfeld parameter ξ_{NP} in partially ionized nonideal plasmas. It is also found that the energy dependence on the Sommerfeld parameter ξ_{NP} is more significant for small values of the nonideality parameter γ. Figure 2 represents the fusion penetration factor Γ_{NP} in nonideal plasmas as a function of the nonideality parameter γ for various values of the scaled energy of the reactive motion $\bar{E}$. Figure 3 represents the surface plot of the fusion penetration factor Γ_{NP} in nonideal plasmas as a function of the nonideality parameter γ and the scaled energy of the reactive motion $\bar{E}$. As shown in these figures, the fusion penetration factor Γ_{NP} increases with an increase of the nonideality parameter γ. Hence we have found that the fusion penetration factors in nonideal plasmas represented by the pseudopotential model are always greater than those in ideal plasmas represented by the conventional Debye-Hückel model. As expected, the

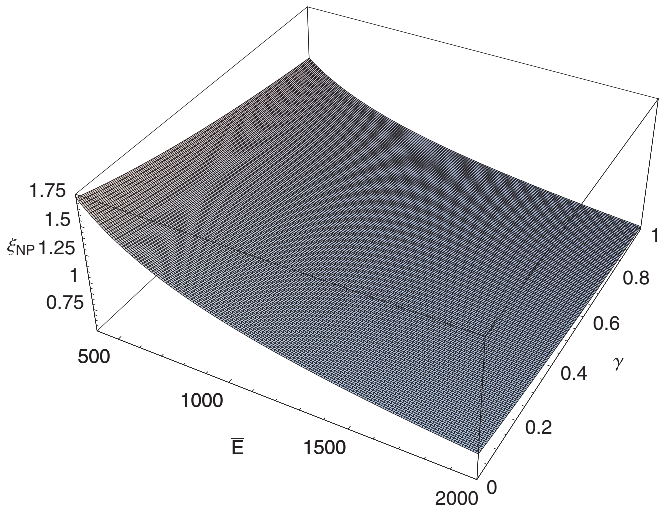

FIG. 1. The surface plot of the Sommerfeld parameter ξ_{NP} for the proton-proton reaction in partially ionized nonideal plasmas as a function of the scaled energy of the reactive motion $\bar{E}$ and the nonideality parameter γ when $\bar{\lambda}_D = 10$.

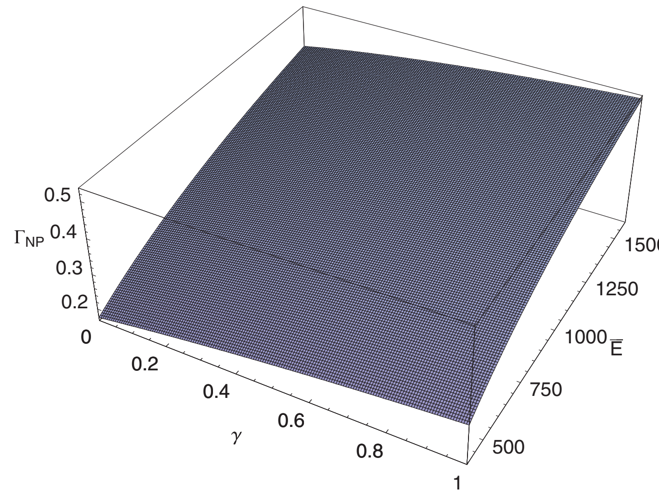

FIG. 3. The surface plot of the fusion penetration factor Γ_{NP} in nonideal plasmas as a function of the nonideality parameter γ and the scaled energy of the reactive motion $\bar{E}$ when $\bar{\lambda}_D = 10$.

fusion penetration factor Γ_{NP} increases with an increase of the scaled energy of the reactive motion $\bar{E}$. It is then expected that the fusion penetration reaction is more significant in strongly nonideal plasmas with high reactive energies. Figure 4 shows the fusion characteristic function F_{NP} of the collective nonideal shielding effect on the fusion penetration factor in partially ionized nonideal plasmas as a function of the scaled energy of the reactive motion $\bar{E}$ for various values of the nonideality parameter γ. From this figure, it is shown that the collective nonideal shielding effect on the fusion penetration factor decreases with an increase of the scaled energy of the reactive motion $\bar{E}$. Figure 5 represents the surface plot of the fusion characteristic function F_{NP} as a function of the scaled energy of the reactive motion $\bar{E}$ and the nonideality parameter γ. As it is seen, the collective nonideal shielding effects enhance the fusion penetration factor about 40% when $\gamma = 1$ and $\bar{E} = 500$, and 20% when $\gamma = 1$ and $\bar{E} = 1600$. In addition, the influence of collective nonideal shielding is found to be more significant for low kinetic energies. It is known that the energy generation rate by the fusion reaction is proportional to the nuclear reaction cross section $\sigma_{\mathrm{NF}}(E)$ [36]. From our results in this work, it would be expected that the nonideal shielding effect of the surrounding plasma enhances the fusion energy generation rate in stellar interiors. In stellar burning, the deuterium ^{2}H can be produced by the proton-proton reaction (pp chain) such as $^1\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^2\mathrm{H} + e^+ + \nu$, where e^+ is the positron and ν is the neutrino. Hence the reaction to form a deuterium ^{2}H would be faster due to the enhancement of the fusion reaction cross section by the influence of collective nonideal shielding. It can be also expected that the nuclear reaction for the formation of a helium nucleus ^3He by the reaction $^1\mathrm{H} + {}^2\mathrm{H} \rightarrow {}^3\mathrm{He} + \gamma$ would be increased due to the nonideal shielding effect of the nonideal plasma, where γ is

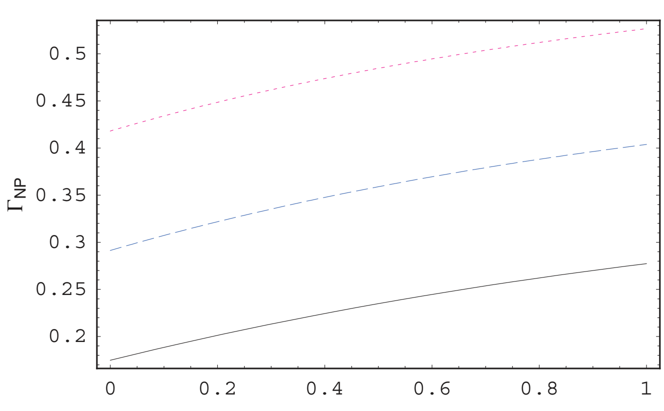

FIG. 2. The fusion penetration factor Γ_{NP} in nonideal plasmas as a function of the nonideality parameter γ when $\bar{\lambda}_D = 10$. The solid line represents the case of $\bar{E} = 400$. The dashed line represents the case of $\bar{E} = 800$. The dotted line represents the case of $\bar{E} = 1600$.

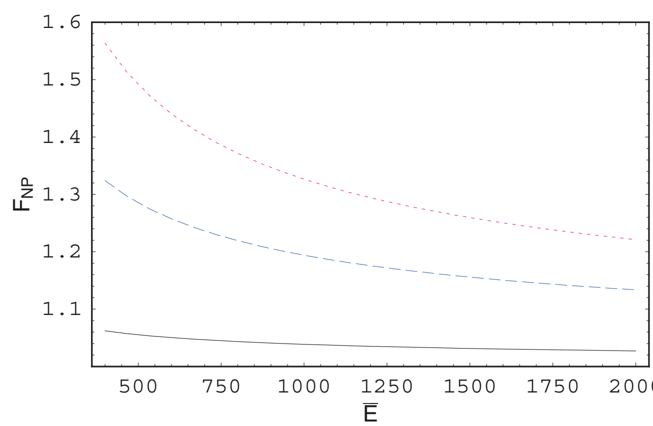

FIG. 4. The fusion characteristic function F_{NP} of the collective nonideal shielding effect on the fusion penetration factor in partially ionized nonideal plasmas as a function of the scaled energy of the reactive motion $\bar{E}$. The solid line represents the case of $\gamma = 0.1$. The dashed line represents the case of $\gamma = 0.5$. The dotted line represents the case of $\gamma = 1$.

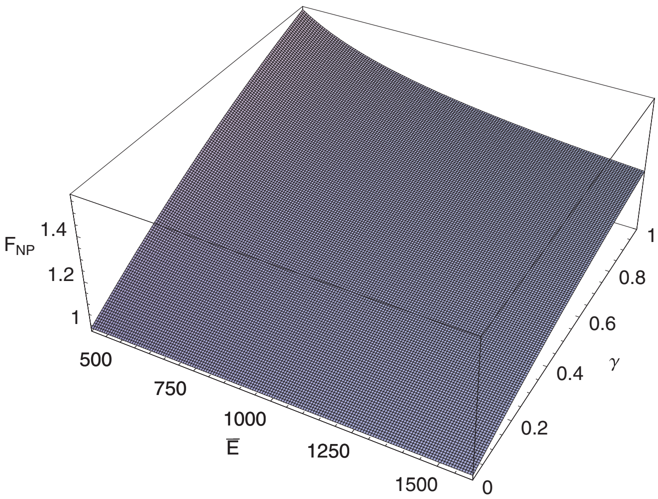

FIG. 5. The surface plot of the fusion characteristic function F_{NP} as a function of the scaled energy of the reactive motion $\bar{E}$ and the nonideality parameter γ when $\bar{\lambda}_D = 10$.

the photon radiation. Hence the nuclear fusion reaction rates of the whole *pp* chain reactions can be increased by the enhancement of the nuclear reaction cross sections in nonideal astrophysical plasmas. Hence we see that the collective nonideal shielding effects play significant roles in the fusion reaction process in partially ionized nonideal plasmas. These results would provide useful information on the fusion reaction processes in nonideal plasmas.

ACKNOWLEDGMENTS

One of the authors (Y.-D.J.) gratefully acknowledges Professor W. Roberge for useful discussions and warm hospitality while visiting the Department of Physics, Applied Physics, and Astronomy at Rensselaer Polytechnic Institute. This research was initiated while one of the authors (Y.-D.J.) was affiliated with Rensselaer as a visiting professor. The work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korean Government (MISP) (Grant No. NRF-2016R1A2B4011356).

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