Metastable memristive lines for signal transmission and information processing applications

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Traditional studies of memristive devices have mainly focused on their applications in nonvolatile information storage and information processing. Here, we demonstrate that the third fundamental component of information technologies—the transfer of information—can also be employed with memristive devices. For this purpose, we introduce a metastable memristive circuit. Combining metastable memristive circuits into a line, one obtains an architecture capable of transferring a signal edge from one space location to another. We emphasize that the suggested metastable memristive lines employ only resistive circuit components. Moreover, their networks (for example, Y-connected lines) have an information processing capability.

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I. INTRODUCTION

Currently, the term memristive device [1] (memristor) is primarily used to denote resistive switching memories that have been considered the most promising candidates for replacing the state-of-the-art memory technology. Moreover, it has been established that memristive networks (the networks of memristive devices) are useful to implement neuromorphic [2,3], digital [4,5], and some unconventional [6] computing architectures. The main advantages of computing with memristors (as well as with memcapacitors [7–9]) are related, in particular, to their ability to store and process information on the same physical platform, massively parallel dynamics of memristors in networks [6], subnanosecond computing times [10], and low power consumption. Computing with memory circuit elements [7] (*memcomputing* [11]) is thus a promising alternative to the conventional von Neumann computing [12].

The information transfer is another important aspect of modern information technologies. Typically, the signal transmission is considered in the framework of transmission line models having a wide applicability range [13–16]. The conventional transmission line models employ reactive components—capacitors and inductors—for signal transmission. The transmission line losses are taken into account by resistors. Recently, reconfigurable transmission lines utilizing memcapacitors [7] instead of capacitors were suggested [17]. Transmission characteristics of such lines and thus their functionality can be preprogrammed on demand [17].

The present paper introduces a different approach to signal transmission uniquely based on the resistive devices. Figure 1(a) presents its basic block—a metastable memristive circuit—combining a resistor R and memristor M. This circuit employs the most common type of memristors characterized by the bipolar threshold-type switching [18]. According to the selected connection polarity of M in Fig. 1(a), R_M increases at positive voltages across M, $V_M > V_t$. Here, R_M is the memristance (memory resistance) of M changing between $R_{\rm on}$ and $R_{\rm off}$ (the low- and high-resistance states of the memristor), and V_t is the threshold voltage. Moreover, R [the resistance of R in the Fig. 1(a) circuit] is selected such that at $R_M = R_{\rm on}$, V_M is slightly below V_t [see Fig. 1(b)]. This circuit configuration can be referred to as a *metastable state*. The circuit can spend an extended time in this state prior to being driven out by an input signal or its fluctuation triggering an abrupt (accelerated [19]) switching of M. The final state of the circuit [see Fig. 1(c)] is perfectly stable and thus can be referred to as the *ground state*.

A metastable memristive line is a set of in-series connected metastable memristive circuits. This paper establishes the information transfer and processing capabilities of metastable memristive lines. While these lines are composed of only resistive components, their operation requires a power source and periodic reset. In this regard, the metastable memristive lines are different from the traditional transmission lines.

This paper is organized as follows. The mathematical model of metastable memristive lines is formulated in Sec. II. In Sec. III, we investigate numerically the dynamics of pulse edge propagation along the line and develop a theory of this phenomenon. In particular, we formulate a time-nonlocal equation describing an infinite line and find its solution in a certain limit. This analytical model allows finding a closed form expression for the signal propagation time per metastable memristive circuit. Information processing applications based on few coupled lines are explored in Sec. IV. Concluding remarks are given in Sec. V.

II. MODEL

Next, let us consider a chain (line) of metastable memristive circuits (see Fig. 2). We argue that under appropriate conditions, this line can transfer a signal edge from one space location to another. Indeed, as it is shown below, there is a certain parameter space where the switching of M_1 (from $R_{\rm on}$ to $R_{\rm off}$) initiates the switching of M_2, the switching of M_2 initiates the switching of M_3, and so forth. The applied signal (see Fig. 2), thus, can set off a chain of switching events propagating along the line. We note that since our approach relies on metastable states of memristive devices, such states should be prepared in advance and periodically refreshed (similarly to the laser pumping [20] in the area of lasers).

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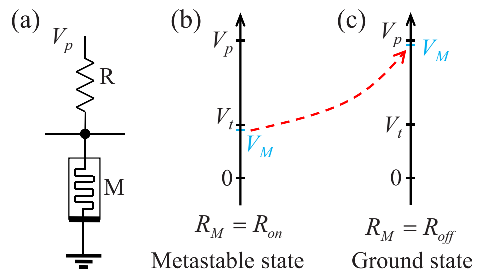

FIG. 1. (a) Metastable memristive circuit. Here, V_p is a power supply voltage. (b) Metastable state of the circuit in (a) is realized when the voltage across M is slightly below its threshold voltage V_t. (c) The stable (ground) state corresponds to $R_M = R_{\rm off}$. The red (dashed) line represents the transition from the metastable to the ground state.

In our calculations, we use the following model of first-order [1] bipolar memristive system with threshold [21]:

$$I = \frac{V}{R_M}, \quad \frac{dR_M}{dt} = f(V,R_M), \tag{1}$$

$$f(V,R_M) = \begin{cases} \mathrm{sgn}(V)\beta(|V| - V_t) & \text{if } |V| \geqslant V_t \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

where I is the current, V is the voltage, R_M is the memristance changing continuously between $R_{\rm on}$ and $R_{\rm off}$, β is the constant defining the rate of change of R_M, and sgn$\{\cdot\}$ is the sign of the argument. According to the above equations, the memristance R_M changes only when $|V| > V_t$, and the direction of change is defined by the device connection and applied voltage polarities.

The model of Eqs. (1) and (2) is applicable to many experimental memristive systems. Examples of those include silicon-based devices, where the resistance of amorphous-Si changes due to the formation of conductive filaments out of electrode atoms [22–24]. Some of these devices demonstrate symmetric switching thresholds, such as an Ag/a-Si/Ni structure [24], where the switching occurs at about ±3.5 V. In the case of Cu/SiO_2/Pt electrochemical metallization cells, it was observed that the switching thresholds are asymmetric (about 1.5 V for the off-to-on switching and −0.4 V for the on-to-off one) [25]. This feature, being unimportant for our proposal, can be easily incorporated into the model of Eqs. (1) and (2). The memristive devices exemplified in this paragraph and similar ones can be used to experimentally implement the metastable memristive states and lines proposed in this work. To keep the discussion general, below we do not focus on any specific experimental realization of memristors.

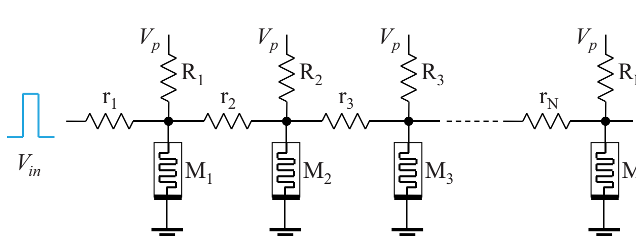

FIG. 2. Metastable memristive line consisting of N metastable R-M circuits connected via coupling resistors r_i, $i = 1,\ldots,N$. The input signal $V_{\rm in}$ is applied to r_1. Note that all the memristors are initialized into the $R_{\rm on}$ state.

To describe the line dynamics, we set up a system of equations based on Kirchoff's rules supplemented by Eq. (1) for the evolution of memristive components. The voltages across memristive systems M_i are chosen as unknown variables. The equation for the ith metastable circuit ($i = 1,2,\ldots,N$) reads

$$\left(\frac{1}{r_i} + \frac{1}{r_{i+1}} + \frac{1}{R_i} + \frac{1}{R_{M,i}}\right)V_i - \frac{V_{i-1}}{r_i} - \frac{V_{i+1}}{r_{i+1}} = \frac{V_p}{R_i}. \tag{3}$$

The boundary conditions for the first and Nth circuits are selected as follows: $V_0 = V_{\rm in}$ is the input voltage (see Fig. 2) and $r_{N+1} = \infty$. Generally, $2N$ equations (1) and (3) for $2N$ variables $V_1,V_2,\ldots,V_N$ and $R_{M,1},R_{M,2},\ldots,R_{M,N}$ supplemented with initial conditions (specifically, the initial memristances) fully define the memristive line dynamics.

III. SIGNAL TRANSMISSION

A. Numerical results

In what follows, we consider a homogeneous metastable memristive line with $r_i = r$, $R_i = R$, and $R_{M,i}(t=0) = R_{\rm on}$ for $i = 1,\ldots,N$. Let us take a closer look at the memristive line dynamics triggered by a rectangular voltage pulse shown in Fig. 3(b). Figure 3 presents a numerical solution of the line equations found with a set of parameters specified in the figure caption. In particular, Fig. 3(a) demonstrates that the switchings of memristors occur sequentially with almost the same time interval between adjacent switchings. The time dependencies of voltages [see Fig. 3(b)] are similarly shifted with respect to each other. Their wave forms (neglecting the boundary effects noticeable in V_1 and V_2 lines) are essentially the same. Moreover, taking a closer look at any of these voltages, say V_i, one can notice that a slow increase of V_i changes to a fast increase followed by a slow increase. These stages of voltage growth are mainly associated with the switchings of $i-1$, i and $i+1$ memristors, respectively.

B. Analytical solution

It is amply clear that the dynamics in the central part of the line is determined solely by the line properties but not by the boundary conditions (for example, the input pulse wave form or coupling to the external circuit). Next, we consider the dynamics of pulse edge propagation in the limit of an infinite line as we are not interested in the boundary effects. It is evident that we can safely assume that V_i and V_{i+1} are simply time shifted with respect to each other, namely, $V_{i+1}(t) = V_i(t-\tau)$, where τ is the pulse edge propagation time per metastable circuit (time interval between adjacent switchings). Then, the system of coupled equations (3) reduces to a single time-nonlocal equation of the form

$$\left(\frac{2}{r} + \frac{1}{R} + \frac{1}{R_{M,i}}\right)V_i(t) - \frac{V_i(t-\tau)}{r} - \frac{V_i(t+\tau)}{r} = \frac{V_p}{R}. \tag{4}$$

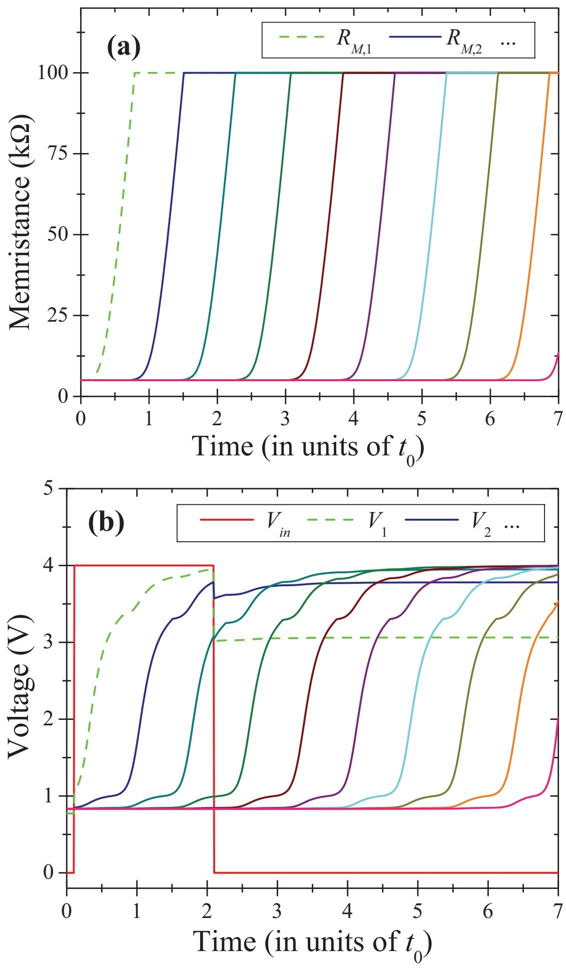

FIG. 3. Dynamics of pulse edge propagation in Fig. 2 memristive line found as a numerical solution of Eqs. (1) and (3). The curves were obtained using the following parameter values: $N = 10$, $V_p = 5$ V, $r = 50$ kΩ, $R = 25$ kΩ, $R_{\rm on} = 5$ kΩ, $R_{\rm off} = 100$ kΩ, $\beta t_0 = 10^2$ kΩ/(V), $V_t = 1$ V, and $R_{M,i}(t = 0) = R_{\rm on}$.

Equation (4) together with Eq. (1) thus describe the pulse edge propagation in an infinite line. Unfortunately, even the set of Eqs. (1) and (4) is a rather complicated one, not only because Eq. (4) includes retarded and advanced times, but also because it is nonlinear. Even for one of the simplest possible $f(V,M)$ given by Eq. (2), we cannot solve the system (1) and (4) analytically.

In fact, an approximate solution to the problem can be obtained in the limit of independent dynamics. In this limit, we assume that at each instant of time, only one memristor is changing its state (switching). To proceed, let us focus on Eqs. (3) for $(i-1)$, ith, and $(i+1)$ metastable circuits:

$$\left(\frac{2}{r}+\frac{1}{R}+\frac{1}{R_{M,i-1}}\right)V_{i-1}-\frac{V_{i-2}}{r}-\frac{V_i}{r}=\frac{V_p}{R}, \quad (5)$$

$$\left(\frac{2}{r}+\frac{1}{R}+\frac{1}{R_{M,i}}\right)V_{i}-\frac{V_{i-1}}{r}-\frac{V_{i+1}}{r}=\frac{V_p}{R}, \quad (6)$$

$$\left(\frac{2}{r}+\frac{1}{R}+\frac{1}{R_{M,i+1}}\right)V_{i+1}-\frac{V_{i}}{r}-\frac{V_{i+2}}{r}=\frac{V_p}{R}. \quad (7)$$

Next, the following approximations are made: (i) the voltages at $(i-2)$ and $(i+2)$ nodes are replaced by some constant values, $V_{i-2} = V_{\rm off}$, $V_{i+2} = V_{\rm on}$, and (ii) it is assumed that $(i-1)$ and $(i+1)$ memristors are in the $R_{\rm off}$ and $R_{\rm on}$ states, respectively, that is, $R_{M,i-1} = R_{\rm off}$ and $R_{M,i+1} = R_{\rm on}$. Here, $V_{\rm on}$ and $V_{\rm off}$ are given by

$$V_{\rm on(off)} = \frac{R_{\rm on(off)}}{R_{\rm on(off)}+R}V_p \quad (8)$$

representing the voltages in the line with all memristors in either the $R_{\rm on}$ or $R_{\rm off}$ state. The above approximations make it possible to truncate the system of Eqs. (3).

From the truncated system of equations (5)–(7), we find

$$V_i(R_{M,i}) = V_p\frac{Y_1R_{M,i}}{Y_2R_{M,i}+1}, \quad (9)$$

$$V_{i+1}(R_{M,i}) = \frac{V_i(R_{M,i})}{rY_{\rm on}}+\gamma_{\rm on}V_p, \quad (10)$$

$$V_{i-1}(R_{M,i}) = \frac{V_i(R_{M,i})}{rY_{\rm off}}+\gamma_{\rm off}V_p, \quad (11)$$

where

$$Y_1 = \frac{1}{R}+\frac{r(R+R_{\rm off})+RR_{\rm off}}{r^2R(R+R_{\rm off})Y_{\rm off}} + \frac{r(R+R_{\rm on})+RR_{\rm on}}{r^2R(R+R_{\rm on})Y_{\rm on}}, \quad (12)$$

$$Y_2 = \frac{2}{r}+\frac{1}{R}-\frac{1}{r^2}\left(\frac{1}{Y_{\rm on}}+\frac{1}{Y_{\rm off}}\right), \quad (13)$$

$$Y_{\rm on(off)} = \frac{2}{r}+\frac{1}{R}+\frac{1}{R_{\rm on(off)}}, \quad (14)$$

and

$$\gamma_{\rm on(off)} = \frac{r(R+R_{\rm on(off)})+RR_{\rm on(off)}}{rR(R+R_{\rm on(off)})Y_{\rm on(off)}}. \quad (15)$$

Now taking into account Eq. (9) and using the initial condition $M_i(t=0) = R_{\rm on}$, we integrate Eq. (1) and obtain implicitly the time dependence of $R_{M,i}(t)$:

$$t = \frac{1}{\beta}\left\{\frac{Y_2(R_{M,i}-R_{\rm on})}{Y_1V_p-Y_2V_t} + \frac{Y_1V_p}{(Y_1V_p-Y_2V_t)^2}\ln\frac{(Y_1V_p-Y_2V_t)R_{M,i}-V_t}{(Y_1V_p-Y_2V_t)R_{\rm on}-V_t}\right\}. \quad (16)$$

We note that Eq. (16) can be used to find the switching time T of every memristor in the line. For this purpose, one should just substitute $R_{M,i} = R_{\rm off}$ in the right-hand side of Eq. (16). We have found a very good agreement between the results of exact numerical calculations and analytical solution (9) and (16). Figure 4 presents $R_{M,i}(t)$ and $V_i(t)$ found using the analytical model. Equations (9) and (16) determine the time dependence of voltage across the switching memristor. The pairs of Eqs. (10) and (16) and Eqs. (11) and (16) can be used to obtain the voltage across the ith memristor in the situation when the $(i-1)$ memristor is switching and, correspondingly, when the $(i+1)$ memristor is switching.

In the above consideration, the ith memristor starts switching at $t = 0$. Therefore, the time τ it takes for the switching edge to move from the ith to $(i+1)$ metastable circuit along

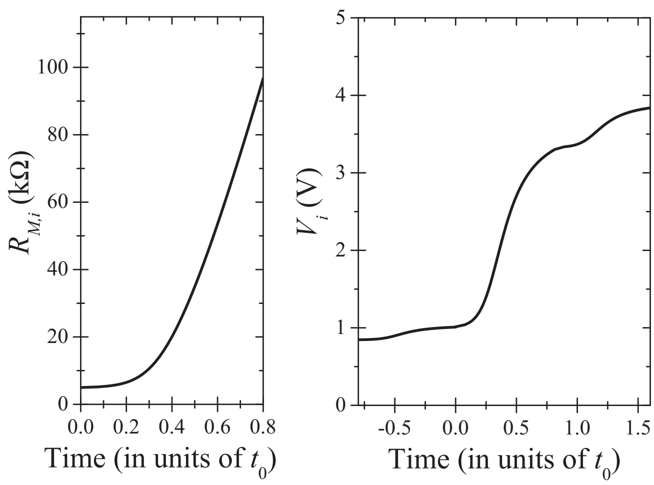

FIG. 4. Time dependencies of the memristance, $R_{M,i}$, and voltage across the memristor, V_i, found using the analytical model described in the text. This plot was obtained using the same set of parameters as the set detailed in Fig. 3.

the line can be found as the time when a suitable condition to start the switching of the $(i+1)$ memristor is established, namely, when

$$V_{i+1}\left(R^{\tau}_{M,i}\right) = V_t. \tag{17}$$

By using Eqs. (17), (10), and (9), we determine the value of $R^{\tau}_{M,i}$ such that the condition (17) is satisfied,

$$R^{\tau}_{M,i} = \frac{rY_{\rm on}(V_t - \gamma_{\rm on}V_p)}{Y_2 rY_{\rm on}(V_t - \gamma_{\rm on}V_p) - Y_1 V_p}. \tag{18}$$

Plugging this value of $R_{M,i}$ into the right-hand side of Eq. (16), we get τ. Note that the switching time T is always longer than the pulse edge propagation time per metastable circuit, τ.

IV. INFORMATION PROCESSING

Metastable memristive lines can also find applications in the area of information processing. For example, the time delays introduced by these lines could be of use in the development of race logic architectures [26]. Moreover, capacitively and resistively Y-connected lines (see Fig. 5 for an example) are capable to implement some Boolean logic operations, such as AND and OR. In order to further explore this idea, we numerically simulate the dynamics of capacitively and

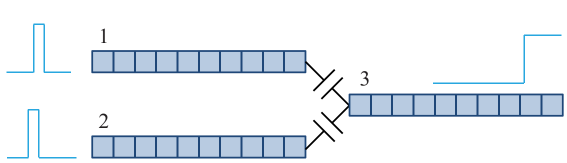

FIG. 5. Boolean logic with metastable memristive lines. This plot conceptually presents two input (1,2) and one output (3) lines coupled capacitively. Depending on the coupling, either one or two simultaneously propagating pulse edges in the input lines are required to induce the switching in the output line. One can interpret such a functionality as an OR or AND gate, respectively (more details are given in the text). Note that a resistive coupling between the lines can also result in a similar functionality.

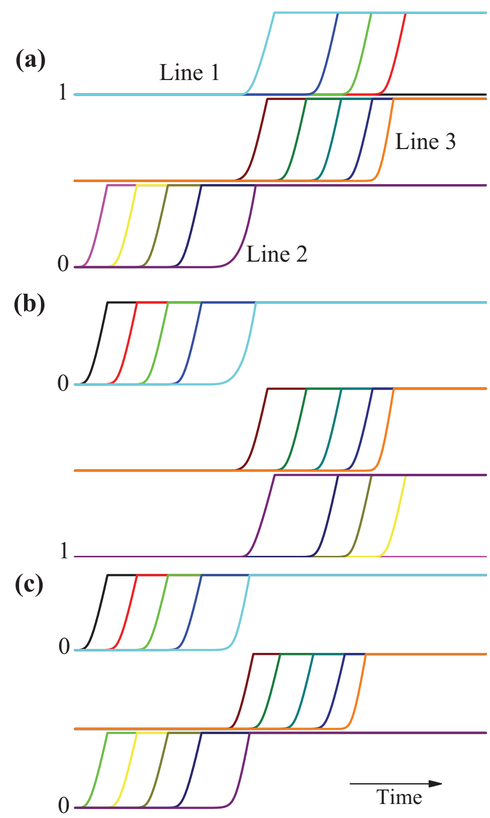

FIG. 6. Implementation of the AND operation using capacitively coupled metastable memristive lines (see Fig. 5 for schematics). (a)–(c) Time dependence of memristances in the input (1,2) and output (3) lines for various combinations of input signals. The switching in the output line occurs if at least one input 0 is applied. The parameters of memristive lines are the same as in Fig. 3, except for $N = 5$ for each line. The coupling capacitances are $C/t_0 = 50$ μF/s. The order of lines in (b) and (c) is the same as in (a).

resistively coupled metastable memristive lines in the Fig. 5 circuit configuration (for the case of resistively coupled lines, resistors were used instead of capacitors in the Fig. 5 circuit).

For the sake of definiteness, we associate the $R_{\rm on}$ state of memristors with logic 1, and $R_{\rm off}$ with logic 0 (this type of notation is frequently adopted in the literature [4]). It is assumed that at the initial moment of time, all memristors are initialized into the $R_{\rm on}$ state. The logic 0's are applied to the lines 1,2 in terms of positive pulses, similar to the pulse $V_{\rm in}$ depicted in Fig. 3(b). The absence of pulse corresponds to input 1. Consequently, the result of the input combination (1,1) is always 1 as there is no switching dynamics involved. Keeping this trivial case in mind, we do not show it in Figs. 6 and 7, where we present our modeling results.

Let us first consider the dynamics of capacitively connected metastable memristive lines, each consisting of five metastable circuits. The coupling capacitors were connected directly between the last node of an input line and first node of the output line without any additional resistors. The results of our numerical simulations shown in Fig. 6 demonstrate that this circuit configuration realizes the AND operation. Indeed, according to Fig. 6, (0,1), (1,0), and (0,0) input combinations result in logic 0 output, while (1,1) input leads

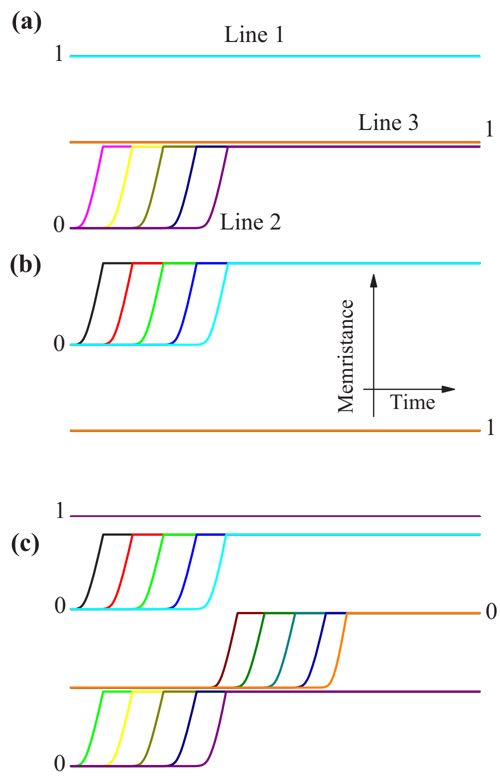

FIG. 7. Implementation of the OR operation using resistively coupled metastable memristive lines. (a)–(c) Time dependence of memristances in the input (1,2) and output (3) lines for various combinations of input signals. The switching in the output line (line 3) occurs only for the (0,0) input combination. The parameters of memristive lines are the same as in Fig. 3, except for $N = 5$ for each line. The coupling resistances are 50 kΩ. The order of lines in (b) and (c) is the same as in (a).

to 1. Importantly, the calculation result is stored in the states of line 3 memristors and thus can be potentially used in subsequent calculation steps.

Taking a closer look at, for example, Fig. 6(a) curves, one can notice that the logic 0 arriving at the common point of Y-connected lines splits between lines 1 and 3. The similar feature can be also observed in Fig. 6(b), where the logic 0 is applied to line 1. It is interesting that after the splitting, the signal propagation along different lines [for instance, in Fig. 6(a) in the backward direction of line 1 and forward direction of line 3] occurs without attenuation. In this regard, it would be more precise to say that the signal is amplified as, after the splitting, the switching occurs in a “double amount”.

Figure 7 presents simulation results of metastable memristive lines coupled by resistors. Similarly to the previous case, each line consisted of five metastable circuits. Moreover, the coupling resistors were connected directly between the last node of an input line and first node of the output line. Our simulations show that this circuit configuration realizes the OR operation. It appears that with selected values of parameters, the switching in a single input line does not provide suitable conditions to initiate the switching in the output line (line 3). This is also why the signal splitting is not observed in this situation.

V. CONCLUSION

In conclusion, in this paper we introduced metastable memristive states, circuits, and lines. The signal transmission through metastable memristive lines was investigated using both numerical and analytical approaches. An approximate analytical solution was found in the framework of a single memristor switching approximation. It was shown that capacitively and resistively coupled metastable memristive lines can implement logic operations. Moreover, at the point of lines connection, the signal can be split without attenuation (with an effective amplification).

Thus, we have established an innovative approach to signal transmission, which is unique in being based on only resistive components. Moreover, one can also envisage purely capacitive metastable lines, where the capacitive components replace the corresponding resistive ones. However, this idea needs further investigation.

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[1] L. O. Chua and S. M. Kang, Proc. IEEE **64**, 209 (1976).
[2] Y. V. Pershin and M. Di Ventra, Neural Netw. **23**, 881 (2010).
[3] M. Prezioso, F. Merrikh-Bayat, B. D. Hoskins, G. C. Adam, K. K. Likharev, and D. B. Strukov, Nature (London) **521**, 61 (2015).
[4] J. Borghetti, G. S. Snider, P. J. Kuekes, J. J. Yang, D. R. Stewart, and R. S. Williams, Nature (London) **464**, 873 (2010).
[5] Y. V. Pershin, L. K. Castelano, F. Hartmann, V. Lopez-Richard, and M. D. Ventra, IEEE Trans. Circuits Syst. II **63**, 558 (2016).
[6] Y. V. Pershin and M. Di Ventra, Phys. Rev. E **84**, 046703 (2011).
[7] M. Di Ventra, Y. V. Pershin, and L. O. Chua, Proc. IEEE **97**, 1717 (2009).
[8] F. L. Traversa, F. Bonani, Y. V. Pershin, and M. D. Ventra, Nanotechnology **25**, 285201 (2014).
[9] Y. V. Pershin, F. L. Traversa, and M. D. Ventra, Nanotechnology **26**, 225201 (2015).
[10] A. C. Torrezan, J. P. Strachan, G. Medeiros-Ribeiro, and R. S. Williams, Nanotechnology **22**, 485203 (2011).
[11] M. Di Ventra and Y. V. Pershin, Nat. Phys. **9**, 200 (2013).

[12] J. Backus, *Communications of the ACM* (ACM, New York, 1978), Vol. 21, pp. 613–641.

[13] L. Goleniewski and K. W. Jarrett, *Telecommunications Essentials, The Complete Global Source*, 2nd ed. (Addison-Wesley, Reading, MA, 2006).

[14] F. Martín, *Artificial Transmission Lines for RF and Microwave Applications*, Wiley Series in Microwave and Optical Engineering (Wiley, New York, 2015).

[15] T. I. Christophe Caloz, *Electromagnetic Metamaterials: Transmission Line Theory and Microwave Applications*, 1st ed. (Wiley-IEEE, New York, 2005).

[16] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Kumar, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **431**, 162 (2004).

[17] Y. V. Pershin, V. A. Slipko, and M. D. Ventra, Appl. Phys. Lett. **107**, 253101 (2015).

[18] Y. V. Pershin and M. Di Ventra, Adv. Phys. **60**, 145 (2011).

[19] Y. V. Pershin, V. A. Slipko, and M. Di Ventra, Phys. Rev. E **87**, 022116 (2013).

[20] O. Svelto, *Principles of Lasers*, 5th ed. (Springer, New York, 2015).

[21] Y. V. Pershin, S. La Fontaine, and M. Di Ventra, Phys. Rev. E **80**, 021926 (2009).

[22] S. H. Jo and W. Lu, Nano Lett. **8**, 392 (2008).

[23] Y. Dong, G. Yu, M. C. McAlpine, W. Lu, and C. M. Lieber, Nano Lett. **8**, 386 (2008).

[24] S. H. Jo, K.-H. Kim, and W. Lu, Nano Lett. **9**, 870 (2009).

[25] C. Schindler, G. Staikov, and R. Waser, Appl. Phys. Lett. **94**, 072109 (2009).

[26] A. Madhavan, T. Sherwood, and D. Strukov, IEEE Micro **35**, 48 (2015).