Kramers problem for a dimer: Effect of noise correlations

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The Kramers problem for a dimer in a bistable piecewise linear potential is studied in the presence of correlated noise processes. The effect of such a correlation is to redistribute the thermal power between the dynamical degrees of freedom, and this leads to significant deviations in the dynamics of the system from the case of independent noise processes. The distribution of first passage times from one minima to the basin of attraction of the other minima is found to have exponentially decaying tails with the parameter dependent on the amount of correlation and the coupling between the particles. The strong coupling limit of the problem is analyzed using adiabatic elimination, where it is found that the initial probability density relaxes towards a stationary value on the same time scale as the mean escape time when the noise intensity of the system is low. For higher noise fluctuations, the relaxation towards the stationary state is slower in comparison to escape times. In the extreme limit of perfect anticorrelation, the random dynamical system behaves as a deterministic system in a steady state in which the center of mass starting from the unstable maxima moves down the hill and gets trapped in the potential minima. The implications for polymer dynamics in a potential are discussed.

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I. INTRODUCTION

Escape of a particle confined in a metastable state is a ubiquitous problem arising in domains varying from chemical kinetics to transport theory. The theory of Brownian motion provides one of the most elegant approaches to studying the problem by identifying the additional degrees of freedom as noise and friction [1]. This approach towards the escape problem was grounded in the seminal work of Kramers [2,3], who provided theoretical estimates for the rate of escape for a particle trapped in a metastable state in the limits of low and high friction.

The single-particle problem defined above naturally generalizes to the dynamics of two coupled particles: dimers, in potential wells. The escape of such dimers across a barrier has been investigated in both deterministic [4] and stochastic limits [5], where it was found that coupling between the particles plays an important role in driving the escape process. The effects of interparticle interactions have also been reported to significantly affect current reversals in noise-induced transport in ratchet potentials [6,7]. These studies, however, have generally focused on uncorrelated noise processes, whereas it is known that noises of identical origin are generally correlated [8–10] and have significant effects on the dynamics of both single- [11–15] and multiple-particle systems [16,17]. Physically such a correlation provides a new bifurcation branch in the dynamics of the system; e.g., a random dynamical system affected by two additive but correlated noise sources can behave deterministically for a perfect anticorrelation, a nontrivial change in the dynamics of such a system. The origin of such a deterministic dynamics is consequent of Eq. (0) in Ref. [9] and implies that the correlation between the noise processes serves as a bifurcation parameter of the system dynamics, controlling the power transferred to the system.

Motivated by the above observations, in this paper we study the dynamics of two harmonically coupled Brownian particles in a piecewise linear bistable potential. Additional thermal degrees of freedom are Gaussian white and correlated with each other. Existence of such a correlation between the two noises is natural because the two particles are thermally coupled to one and the same heat bath. Consequent of such a coupling between the dynamical and thermal degrees of freedom, the independence of the noise processes becomes a special case where the two particles exhibit coupled dynamics but carry separate heat baths. In the general case of correlated noises, it is found that positive correlations facilitate barrier crossing whereas negative correlations tend to diminish the effect of thermal degrees of freedom. The structure of the paper is as follows: in the next section the effect of coupling and correlation is studied on the motion of the dimer. Following it the strong coupling limit of the dynamics is analyzed using the method of adiabatic elimination. The results are generalized to the dynamics of a polymer in a potential field with conclusions in the final section.

II. DYNAMICAL SYSTEM

Let us start with the dynamical equations for a dimer in a bistable potential U:

$$\dot{x}_1 = -U'(x_1) + F_{12}(x_1,x_2) + \eta_1(t), \tag{1a}$$

$$\dot{x}_2 = -U'(x_2) + F_{21}(x_1,x_2) + \eta_2(t), \tag{1b}$$

where η_1 and η_2 are Gaussian white noises of mean zero and correlations:

$$\langle \eta_1(t)\eta_1(t')\rangle = \langle \eta_2(t')\eta_2(t)\rangle = 2D\delta(t-t'), \tag{2a}$$

$$\langle \eta_1(t)\eta_2(t')\rangle = \langle \eta_1(t')\eta_2(t)\rangle = 2D\rho\delta(t-t'), \tag{2b}$$

with D being the noise intensity and $\rho \in [-1,1]$ the measure of correlation. The noise intensity is a measure of the dimensionless temperature of the associated heat bath. Consequently, the existence of a correlation between the two noise processes is natural as η_1 and η_2 have the same thermal origin. The

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potential U in Eq. (1) is a piecewise linear function defined as

$$U(x) = \begin{cases} -x - 1 & \text{for } x < -1, \\ x + 1 & \text{for } -1 \leqslant x \leqslant 0, \\ -x + 1 & \text{for } 0 \leqslant x \leqslant 1, \\ x - 1 & \text{for } x > 1, \end{cases} \tag{3}$$

having global minima at $x = \pm 1$ and a local maxima at $x = 0$. Components of the dimer interact via a harmonic potential $U_{\rm sh} = \frac{k}{2}(x_1 - x_2)^2$, with the corresponding forces $F_{ij}(x_1,x_2) = -\frac{\partial}{\partial x_i} U_{\rm sh}(x_1,x_2)$ with $i \in \{1,2\}$ and $i \neq j$, and k being the spring constant. It is noted that the natural length of the spring is chosen to be negligibly small as compared to the separation of the global minima of the potential U and hence is ignored in the definition of the interaction potential $U_{\rm sh}$.

In order to diagonalize the correlation matrix in (2), let us transform the dynamical equations to $x_c = \frac{x_1+x_2}{2}$ and $x_r = \frac{x_1-x_2}{2}$, which are respectively the coordinates of the center of mass and relative separation between the two particles. In terms of the variables x_c and x_r, the dynamical equations in (1) are transformed as

$$\dot{x}_c = -\frac{U'(x_c + x_r) + U'(x_c - x_r)}{2} + \zeta_c(t), \tag{4a}$$

$$\dot{x}_r = -\frac{U'(x_c + x_r) - U'(x_c - x_r)}{2} - 2kx_r + \zeta_r(t), \tag{4b}$$

where $\zeta_c = \frac{\eta_1+\eta_2}{2}$ and $\zeta_r = \frac{\eta_1-\eta_2}{2}$ are independent noise processes with mean zero and correlations

$$\langle \zeta_c(t)\zeta_c(t')\rangle = D(1+\rho)\delta(t-t'), \tag{5a}$$

$$\langle \zeta_r(t)\zeta_r(t')\rangle = D(1-\rho)\delta(t-t'). \tag{5b}$$

The stochastic differential equations in (4) and (5) are solved numerically using Heun's method [18] with the initial conditions $(x_c,x_r) = (-1.0,0.02)$.

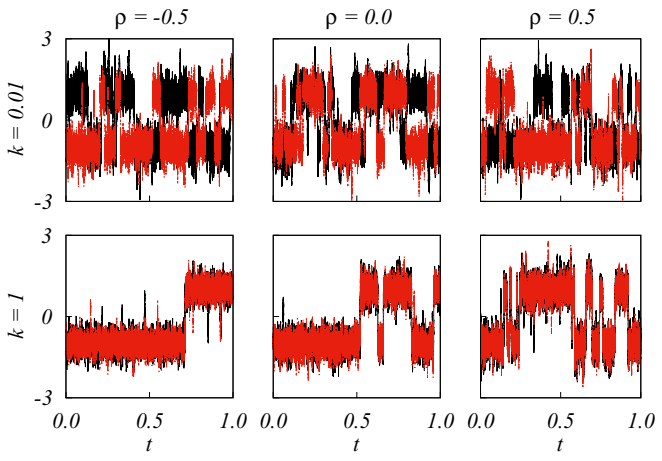

FIG. 1. Sample trajectories of the dimer x_1 (black solid) and x_2 (red dash) in the piecewise linear bistable potential U, with the particles interacting simple harmonically for the initial conditions $(x_c,x_r) = (-1.0,0.02)$ for different values of spring constant k and noise correlation ρ for noise intensity $D = 0.25$. The time t in the figure is in multiples of 10^3.

Figure 1 shows the trajectories of the dimer in the bistable potential U for varying correlations ρ and spring constant k for noise intensity $D = 0.25$. The dependence of the nature of trajectories on the spring constant k is evident from the figure. For low k the two particles move nearly independently of each other, but for high k the dimer moves as an effective single particle with the two particles fluctuating about the mean position independent of the value of noise correlation ρ. However, ρ plays a decisive role in the dimer crossing the potential barrier when the coupling between the monomers is high, with positive correlation aiding in the back-and-forth hoping between the two minima and the negative ρ confining the monomers in the stable position. To quantify the above observations let us study the residence time statistics of the center of mass in the potential wells, which identifies with the statistics of escape times [19].

With the initial condition $x_c = -1$, let us look at the time it takes for the center of mass to reach the basin of attraction of the minima at $x_c = 1$. Figure 2 shows the distribution of first passage times τ for different values of noise correlation ρ and spring constant k for noise intensity $D = 0.25$. The distribution shows exponentially decaying tails with parameter $\langle\tau\rangle$, the mean first passage time. However, when the coupling between the monomers is low ($k = 0.01$), $\langle\tau\rangle$ is nearly independent of the correlation ρ. This is because for such low values of spring constant k, the particles move nearly independently of each other, and hence the correlation between the thermal degrees of freedom does not have any significant impact on the rate of barrier crossing of the nearly independent particles. On the other hand, with increasing values of the spring constant, e.g., $k = 0.1$ and 1, it is observed that $\langle\tau\rangle$ decreases with increasing correlation ρ. The reason for such behavior follows from the dynamical equations in (4) and (5), which imply that the noise intensity affecting the dynamics of the center of mass is $D(1+\rho)$. It is to be noted that the presence of correlation ρ makes the power transferred to the center of mass x_c and relative coordinates x_r different. Hence, for negative values of ρ the center of mass does not feel the additive perturbations to the extent as felt in the absence of any correlations. As a result, for $\rho < 0$, the relative fluctuations of the two monomers are enhanced due to the increased magnitude of the noise process $D(1-\rho)$ affecting the dynamics of the relative coordinate x_r. Consequently, even though the two particles move relative to each other, the center of mass x_c remains relatively static, making the escape of the dimer difficult for negatively correlated noises. On the other hand, $\rho > 0$ reduces the effect of the thermal degrees of freedom on the relative coordinate x_r due to decreased magnitude of the intensity $D(1-\rho)$, making the relative fluctuations of the two monomers freeze. However, the intensity of fluctuations $D(1+\rho)$ is enhanced for the center-of-mass motion for positive correlations, which make the escape of the dimer across the barrier easier in comparison to independent noises ($\rho = 0$). The physical origin of such behavior is easily understood. We know that the temperature D of the associated heat reservoir is a measure of the power transferred to the system, i.e., to the relative and center-of-mass coordinates, which is respectively $D(1-\rho)$ and $D(1+\rho)$. The total power transferred is $2D$, which is independent of the noise correlation ρ. This implies that the effect of correlation is

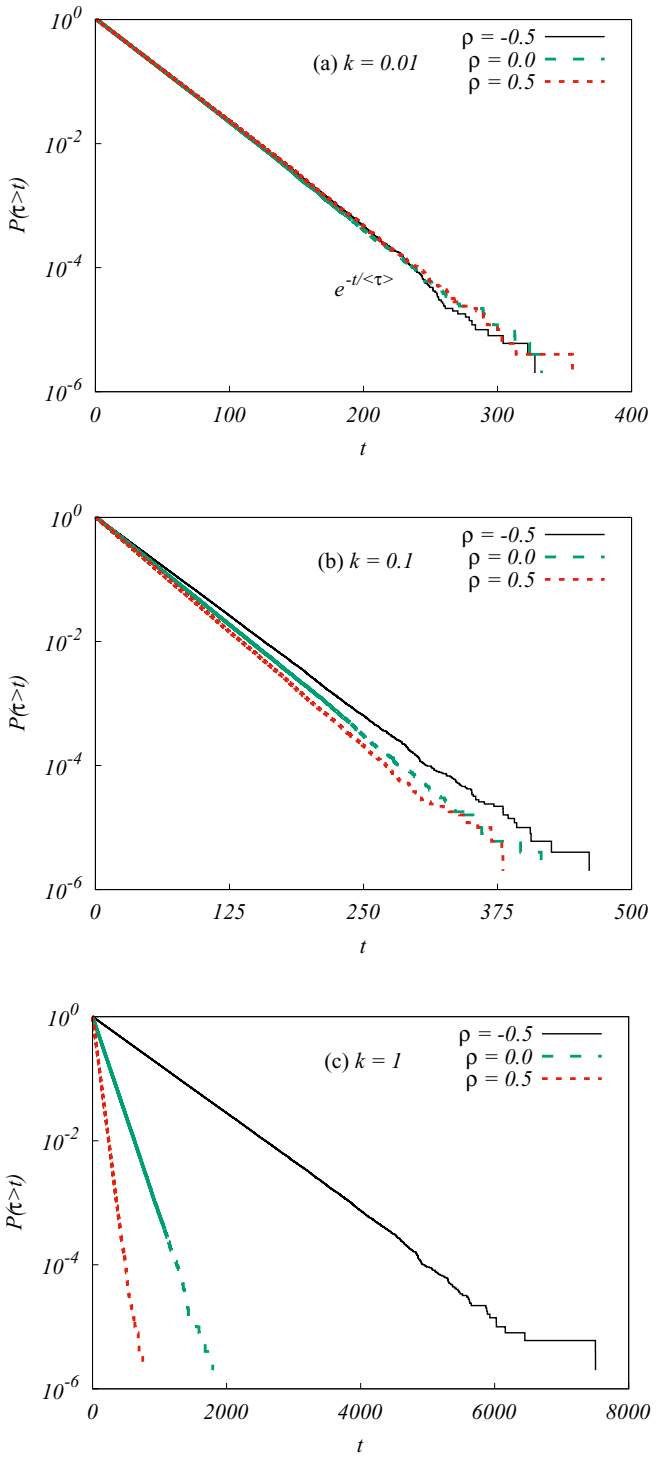

FIG. 2. Cumulative distribution of the first passage times τ of the center of mass starting in the left well to the absorbing boundary at $x_c = 0$. The distribution has exponentially decaying tails with parameter $\langle\tau\rangle$. The effect of the coupling between the two monomers is evident on the nature of the distribution: for nearly independent movement of the particles ($k = 0.01$) (a) the distributions trace each other for different values of ρ. When the coupling between the monomers increases, the mean first passage time $\langle\tau\rangle$ is found to decrease with increasing ρ as observed for $k = 0.1$ (b) and $k = 1$ (c). The distributions are calculated using 500 000 data points for noise intensity $D = 0.25$.

only to redistribute the thermal power between the two degrees of freedom x_c and x_r, keeping the total amount of power transferred constant. The results supply us with the dynamical properties of coupled Brownian particles for different values of the spring constant k and noise correlation ρ. It is also inferred from Fig. 2 that the mean escape time $\langle\tau\rangle$ is lowest when the two monomers move relatively independently of each other, i.e., for low values of the spring constant k. The overall effect of coupling is to slow the escape process, and it is in this limit that the noise correlations play a significant role. Consequently, it becomes interesting to study the limit of a large coupling constant in which the dimer moves effectively as a single particle at its center of mass, and we take up this analysis in the next section using the method of adiabatic elimination of the fast degrees of freedom [20].

III. ADIABATIC ELIMINATION

The adiabatic elimination of the fast variable requires marginalization of the probability distribution $p(x_c,x_r,t)$ via the stationary solution of the Fokker-Planck operator for the fast variable x_r. The Fokker-Planck equation associated with the dynamical equations (4) and (5) is

$$\frac{\partial}{\partial t}p(x_c,x_r,t) = \left(L^c_{\rm FP} + L^r_{\rm FP}\right)p(x_c,x_r,t), \tag{6}$$

where $L^c_{\rm FP}$ and $L^r_{\rm FP}$ are the Fokker-Planck operators associated with the slow x_c and fast x_r degrees of freedom, respectively. In the limit of the large spring constant k, the two harmonically coupled particles experience the same potential, hence $U(x_c + x_r) \approx U(x_c - x_r)$. As a result, $L^r_{\rm FP} = \frac{\partial}{\partial x_r}(2kx_r + \frac{D(1-\rho)}{2}\frac{\partial}{\partial x_r})$, which admits the Gaussian distribution of mean zero and variance $\sigma^2(x_c,x_r) = \frac{D(1-\rho)}{4k}$ as its stationary solution $\psi_0(x_c,x_r)$. Marginalization of p using ψ_0 leads to the effective drift term for the center-of-mass motion in the large k limit and is given by

$$\begin{aligned} V'(x_c) &= \int dx_r \frac{U'(x_c+x_r)+U'(x_c-x_r)}{2}\psi_0(x_c,x_r) \\ &= \int dy U'(y)\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{(y-x_c)^2}{2\sigma^2}\right] \\ &= 1 - 2\Phi(-1|x_c,\sigma) + 2\Phi(0|x_c,\sigma) - 2\Phi(1|x_c,\sigma), \end{aligned}$$

where $\Phi(x|x_c,\sigma) = \frac{1}{2}[1 + \mathrm{erf}(\frac{x-x_c}{\sigma\sqrt{2}})]$ is the cumulative distribution function associated with the normal distribution of mean x_c and variance σ^2, with erf being the error function. Evaluating $\Phi(x|x_c,\sigma)$ at the turning points of the potential U defined by Eq. (3), we obtain the effective drift term for the center-of-mass motion:

$$V'(x_c) = \mathrm{erf}\left(\frac{x_c+1}{\sigma\sqrt{2}}\right) - \mathrm{erf}\left(\frac{x_c}{\sigma\sqrt{2}}\right) + \mathrm{erf}\left(\frac{x_c-1}{\sigma\sqrt{2}}\right), \tag{7}$$

which approaches $U'(x_c)$ due to the smallness of the variance σ^2. Hence, in the limit of the large spring constant the center-of-mass motion is equivalent to the motion of a single particle in the potential given by Eq. (3) and with the noise intensity modified to $D(1+\rho)$. Such modification of the noise intensity has strong implications on the dynamics of the coupled Brownian particles as shown below.

The effect of noise correlation on the dynamics of the center of mass can be studied using the above result. To investigate the effect, let us calculate the mean first passage time $\langle\tau\rangle$ of the center of mass starting at $x_c = -1$ to the absorbing boundary at $x_c = 0$. Using the backward Fokker-Planck operator, the expression for the mean first passage time reads

$$\begin{aligned}\langle\tau\rangle &= (2/D_\rho)\int_{-1}^{0} dz e^{2U(z)/D_\rho}\int_{-\infty}^{z} dx e^{-2U(x)/D_\rho}\\ &= (2/D_\rho)\int_{-1}^{0} dz e^{2U(z)/D_\rho}\int_{-\infty}^{-1} dx e^{-2U(x)/D_\rho}\\ &\quad + (2/D_\rho)\int_{-1}^{0} dz e^{2U(z)/D_\rho}\int_{-\infty}^{z} dx e^{-2U(x)/D_\rho},\end{aligned}$$

where $D_\rho = D(1+\rho)$. The first term in the above sum of double integrals leads to a contribution $\frac{D_\rho}{2}[\exp(2/D_\rho) - 1]$, and the second term evaluates to $\frac{D_\rho}{2}[\exp(2/D_\rho) - 1] - 1$. Combining the two terms gives the expression for the mean first passage time $\langle\tau\rangle$ as

$$\langle\tau\rangle = D_\rho(e^{2/D_\rho} - 1) - 1 \approx D_\rho e^{2/D_\rho}, \tag{8}$$

where the approximation holds only in the limit of small noise intensity, i.e., either when D is low or when ρ is sufficiently negative such that the overall noise intensity $D(1+\rho)$ is low enough for the approximation to be valid. Hence, the rate of escape of the center of mass from the minima of the potential well to the absorbing boundary is $R = 1/\langle\tau\rangle$ and reduces to $e^{-2/D_\rho}/D_\rho$, in the limit of low noise intensity, which is of the same form as proposed originally by Kramers. Consequently it becomes nearly impossible for the dimer to escape the potential well for strongly anticorrelated noise processes when the coupling between the two monomers is high.

The calculation of the mean first passage time $\langle\tau\rangle$ above shows the dependence of the transient properties of the coupled particle system on correlation ρ, in the limit of strong coupling. Hence, it is interesting to investigate the steady state properties of the center-of-mass motion which follows

$$\dot{x}_c = -U'(x_c) + \zeta_c(t), \tag{9}$$

with $\langle\zeta_c(t)\rangle = 0$ and $\langle\zeta_c(t)\zeta_c(t')\rangle = D_\rho\delta(t-t')$. With the Fokker-Planck equation corresponding to Eq. (9) derived above, it follows that the steady-state density of the center-of-mass motion is $p(x_c) = N\exp[-U(x_c)/D_\rho]$, where $1/N = 2D_\rho(2 - e^{-1/D_\rho})$ is the normalization constant. This implies that the noise correlations affect not only the transient properties but also steady-state properties of the system. To further understand the properties of the steady state and its dependence on ρ, let us calculate the mean and variance of the center of mass coordinate x_c. Now, by symmetry of the pdf, $p(x_c) = p(-x_c)$, which leads to $\langle x_c\rangle = 0$. The variance is given by

$$\begin{aligned}\langle x_c^2\rangle &= \int dx_c x_c^2 p(x_c) = N\int dx_c x_c^2 \exp\left[-\frac{U(x_c)}{D_\rho}\right]\\ &= 4ND_\rho\left[1 + D_\rho^2(2 - e^{-1/D_\rho})\right] = 2D_\rho^2 + \frac{2}{2 - e^{-1/D_\rho}},\end{aligned} \tag{10}$$

where in the last equality we have used the expression for the normalization constant N. It is clear from the above equation that variance of the center-of-mass motion increases monotonically with correlation ρ at a given temperature D. In particular, using Eq. (10), we find that $\lim_{\rho\to-1^+}\langle x_c^2\rangle = 1$. This is an interesting result, saying that even in the limit of strong anticorrelations when the transient motion of the center of mass is nearly stagnant, by the time steady state is reached, the center of mass has moved a finite distance. This follows from the observation that the average position of the center of mass is at the origin, as a result of which even the slightest perturbations can drive it down the hill due to the unstable nature of the position $x_c = 0$, after which it remains generally trapped in either of the two potential minima, which is reflected in the two peaks of the bimodal probability distribution $p(x_c)$. The above results imply that even when the temperature of the system is fixed (constant D), different values of correlation ρ can lead to different steady states. And in the extreme limit when the noise processes are in perfect anticorrelation, the random dynamical system given in Eq. (1) behaves deterministically in the steady state at any temperature D in the limit of strong coupling. The dynamics of such a deterministic system is, however, simple and outlined above.

The strong coupling limit of the dimer motion also allows us to calculate the relaxation time of the initial probability density to its steady state. We have seen previously that the correlation ρ affects both transient and steady-state properties of the system. Hence, it becomes interesting to know the time scale on which the initial density $\delta(x_c + 1)$ relaxes towards the steady state. In order to calculate the relaxation time T, define $Q(t) = \int_0^\infty dx_c\, p(x_c,t)$ where $p(x_c,t)$ is the probability distribution associated with the center-of-mass motion and $Q(t)$ is the density of the center of mass being found in the basin of attraction of the minima at $x_c = 1$. Using the results in Ref. [21] it is found that $Q(t) - \frac{1}{2} \approx -\frac{1}{2}e^{-\frac{-e^{-2/D_\rho}}{D_\rho}t} - \sqrt{\frac{D_\rho}{2\pi}}\frac{e^{-1/D_\rho}}{(2/D_\rho - 1)^2}t^{-3/2}e^{-t/D_\rho}$, where the first term is the contribution of the pole of the Laplace transform of $p(x_c,t)$, and the second term, which is valid only in the limit of long times, is the contribution of the branch cut associated with $\hat{p}(x_c,s)$, the Laplace transform of $p(x_c,t)$. Using Ref. [22], the relaxation time T is given by

$$T = \frac{\int_0^\infty dt[Q(\infty) - Q(t)]}{Q(\infty) - Q(0)} \approx D_\rho e^{2/D_\rho}, \tag{11}$$

which is the same as the mean first passage time of the center of mass to the absorbing boundary at the peak. It is to be noted that the contribution of the branch cut has been ignored in the calculation of the relaxation time T, as it is valid only in the long-time limit. The dependence of T on noise correlation ρ implies that the time to approach stationarity can also be controlled by the correlation. Figure 3 shows the variation of the rate of escape R and the rate of relaxation $1/T$ of the dimer as a function of the noise correlation ρ for different values of noise intensity D when the coupling between the monomers is very strong. The analytical results for the rate of escape are also compared with numerical calculations of the mean first passage time, and a good agreement is observed. The monotonic variation of R with ρ in the strong coupling limit also conforms with

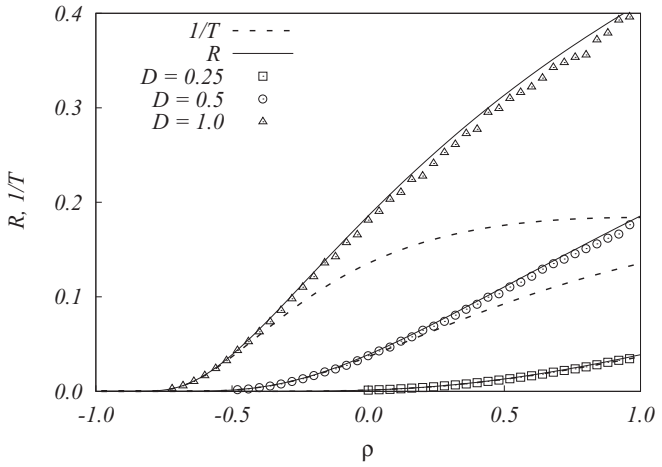

FIG. 3. Rate of escape R(solid lines) and the rate of relaxation $1/T$(dashed lines) against the correlation ρ. The numerical results for the rate of escape are calculated from the mean first passage time of the dimer from the potential minima at $x_c = -1$ to the basin of attraction of the other minima. The numerical values are obtained by the solution of Eq. (9) by averaging over 10 000 ensembles. The formula for the rate of escape R is used from Eq. (8), with $1/R = D_\rho(e^{2/D_\rho} - 1) - 1$, as the approximate form by Kramers holds only in the limit of small noise intensity and is also reflected in the deviation between the solid and dashed lines.

the numerically observed results for relatively weaker values of k. The results in Fig. 3 imply that the initial probability density relaxes towards its stationary value at the same rate as the escape rate, in the limit of low noise. For higher noise intensity, the relaxation is slower as compared to the escape rate. This is not surprising because of the rate at which the system relaxes, and during that period the center of mass makes many to-and-fro jumps between the two potential wells due to enhanced fluctuations. The results in Fig. 3 also imply that the system may take a very long time to relax to its stationary state when the noise processes are strongly anticorrelated. This can leave the dimer confined in the potential minima for longer times as compared to independent noise sources and can be employed as a mechanism for confinement. With the dynamical properties of the dimer motion understood, in the next section let us generalize to the dynamics of a polymer in a confining potential.

IV. GENERALIZATION TO POLYMER DYNAMICS

The dynamics of a polymer chain in some potential $U = U(x_1, \ldots, x_N)$ is given by the Langevin equation:

$$\dot{x}_i = k(x_{i-1} - 2x_i + x_{i+1}) - \frac{\partial}{\partial x_i} U(\mathbf{x}) + \eta_i(t), \qquad (12)$$

where $i = 1, \ldots, N$, and the noise processes η_i are Gaussian white with mean zero and correlations $\langle \eta_i(t)\eta_j(t')\rangle = 2D\rho_{ij}\delta(t - t')$. The diagonal elements of the correlation matrix are unity by definition, and the off-diagonal elements are symmetric and take values from the interval $[-1, 1]$, which generalizes the dynamics of a polymer chain with independent noise processes [23]. The dynamical equations in (12) can be transformed to the equation for the center-of-mass motion and the motion of monomers relative to the center of mass. The equation of motion for the center of mass of the polymer is

$$\dot{x}_c = -\sum_i \frac{\partial}{\partial x_i} U(\mathbf{x}) + \zeta_c(t), \qquad (13)$$

where $\zeta_c = \sum_i \eta_i/N$. The noise process ζ_c has mean zero and correlation $\langle \zeta_c(t)\zeta_c(t')\rangle = \frac{2D}{N^2}(N + 2\sum_{i<j}\rho_{ij})\delta(t - t')$. Now, if the correlations ρ_{ij} are chosen such that the term in the brackets becomes small, this can make the polymer to be trapped in a metastable state for longer times as compared to the uncorrelated noise processes. Positively correlated noise processes on the other hand aid in the escape with respect to uncorrelated noises. This can be easily understood in the limit when the coupling between the monomers is chosen to be very strong, adiabatic elimination of the relative coordinates rendering the equation of motion of the center of mass: $\dot{x}_c = -\tilde{U}(x_c) + \zeta_c(t)$, with $\tilde{U}$ being the effective potential. This is equivalent to the dynamics of a single particle in the potential $\tilde{U}$ and the thermal degrees of freedom controlled by the parameters D and ρ_{ij}. As a result, the Kramers formula can be used to calculate the rate of escape from a potential minima: $R \approx \exp[-N\Delta\tilde{U}/D(1 + 2\sum_{i<j}\rho_{ij}/N)]$, where $\Delta\tilde{U}$ is the height of the potential barrier. The expression generalizes the previously known results for R for uncorrelated noise processes [23–26] by incorporating noise correlations. Now, for a given value of noise intensity D, the correlations ρ_{ij} can always be chosen such that the term in the brackets, $1 + 2\sum_{i<j}\rho_{ij}/N$, becomes small enough to drastically reduce the magnitude of thermal fluctuations preventing the polymer from crossing the barrier even when the assigned value of D is strong enough to drive the barrier crossing process in the absence of noise correlations. On the other hand, if the correlations are chosen such that $\rho_{ij} > 0$ for all i, j, then these enhance the magnitude of the thermal fluctuations, thereby making the barrier crossing of the polymer more likely in comparison to the case with uncorrelated noises. This generalizes the results of the previous sections for dimers with correlated noises and has implications for controlling the rates of chemical reactions involving polymers by varying the correlation between the noise processes.

V. CONCLUSIONS

In summary, the paper discusses the dynamics of harmonically coupled Brownian particles in a symmetric, piecewise linear bistable potential under the effect of correlated noise processes. When the coupling between the particles is strong enough, the effect of such a correlation is to redistribute the power of the thermal degrees of freedom among the dynamical variables. Such a redistribution leads to nontrivial deviations in the dynamical properties of the coupled particle system as compared to when the noise sources are independent. It is found that for fixed temperature D of the coupled particle system, the correlation between the two noise processes affects both the transient and steady-state properties. The calculations show that for a fixed value of noise intensity, positively correlated noise processes aid in the escape of the dimer from the metastable state, whereas anticorrelated noises tend towards confinement provided the particles are not moving completely

independent of each other. In addition, in the strong coupling limit it is shown that the steady-state distribution of the system depends on noise correlation ρ, and in the limit $\rho \to -1^+$, the random system behaves deterministically independent of the temperature D. These results have significant implications towards the dynamics of polymers in potential fields, e.g., the rates of chemical reactions involving polymers can be controlled by varying the noise correlations, and if very strongly anticorrelated noises are used, then the polymer can be confined in metastable states for longer periods of time. Alternatively, correlated noise sources can be employed to confine polymers in a metastable state with the amounts of correlation controlling the residence times in the confinement.

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