

Gap solitons in Rabi lattices

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We introduce a two-component one-dimensional system, which is based on two nonlinear Schrödinger or Gross-Pitaevskii equations (GPEs) with spatially periodic modulation of linear coupling (“Rabi lattice”) and self-repulsive nonlinearity. The system may be realized in a binary Bose-Einstein condensate, whose components are resonantly coupled by a standing optical wave, as well as in terms of the bimodal light propagation in periodically twisted waveguides. The system supports various types of gap solitons (GSs), which are constructed, and their stability is investigated, in the first two finite bandgaps of the underlying spectrum. These include on- and off-site-centered solitons (the GSs of the off-site type are additionally categorized as spatially even and odd ones), which may be symmetric or antisymmetric, with respect to the coupled components. The GSs are chiefly stable in the first finite bandgap and unstable in the second one. In addition to that, there are narrow regions near the right edge of the first bandgap, and in the second one, which feature intricate alternation of stability and instability. Unstable solitons evolve into robust breathers or spatially confined turbulent modes. On-site-centered GSs are also considered in a version of the system that is made asymmetric by the Zeeman effect, or by birefringence of the optical waveguide. A region of alternate stability is found in the latter case too. In the limit of strong asymmetry, GSs are obtained in a semianalytical approximation, which reduces two coupled GPEs to a single one with an effective lattice potential.

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I. INTRODUCTION

Solitons in lattice potentials have drawn a great deal of interest in recent decades, as they occur in diverse physical settings, and exist in many different varieties [1–4]. Periodic potentials, added to the underlying nonlinear Schrödinger or Gross-Pitaevskii equations (GPEs), help to create and stabilize solitons, which do not exist or are unstable in free space. In optics, one source of the spatial periodicity is provided by Bragg gratings [5]. Effective periodic potentials can also be readily induced by photonic crystals, which are built as permanent structures by means of various techniques [6–8], or as virtual lattices formed by interfering laser beam in photorefractive crystals [6,9]. For matter waves in atomic Bose-Einstein condensates (BECs), perfect periodic potentials are imposed by optical lattices, i.e., interference patterns constructed by counterpropagating coherent optical beams [1,10]. In the presence of the self-repulsive nonlinearity, localized modes that self-trap in periodic potentials are usually called *gap solitons* (GSs), as they exist in bandgaps of the underlying Bloch spectrum induced by the potential in the linear approximation [11–15]. Different families of GSs are distinguished by the number of the bandgaps in which they reside.

In the presence of an appropriate periodic potential, binary BECs [16] and binary photonic systems [17–19] can host two-component GSs, which have been theoretically elaborated in various settings [17–25]. The use of the Feshbach-resonance technique [26], which switches the repulsion between atoms into attraction, makes it possible to create two-component *symbiotic solitons* in binary BEC, which are supported by attraction between the two components, while each of them is subject to self-repulsion [25]. This concept was extended to symbiotic GSs in a system of two mutually repelling components loaded into a common lattice potential [23].

Another extension of this concept was elaborated for dark solitons in spinor systems [24].

An essential ingredient of many two-component systems is linear interconversion (Rabi coupling) between the components. In binary BEC, the interconversion is driven by a resonant electromagnetic field, which couples different atomic states representing the components [27,28]. Two-component GSs coupled by the linear interconversion were studied too [22,29]. In optics, the Rabi coupling is emulated by the linear coupling between copropagating waves in dual-core wave guides [30]. In particular, GSs in a dual-core Bragg grating were studied in Ref. [17]. Similarly, a dual-core BEC trap may hold two matter-wave fields with an effective Rabi coupling between them, provided by tunneling of atoms across a gap separating the two cores [31]. In this connection, it is relevant to mention that GSs may be supported by *Zeeman lattices*, i.e., spatially periodic modulation of the difference in the chemical potential between two BEC components, which are linearly coupled by spin-orbit coupling [32].

In the present work, our objective is to propose a different mechanism of the creation of two-component GSs, without the use of any lattice potential, but rather making use of the linear coupling between two components of the wave field in the form of a standing wave, which may be called a “Rabi lattice” (cf. “Rabi management”, i.e., the linear coupling with a time-periodic coefficient, introduced in Ref. [28]), assuming intrinsic self-repulsion in each component. This setting may be realized in a binary BEC illuminated by a pair of counterpropagating resonantly coupling waves, the interference of which builds the standing wave. In this connection, it is relevant to mention recent work [18], in which it was demonstrated that a spatially localized (rather than periodically modulated) linear coupling between the components may play a role in the soliton dynamics similar to that of localized attractive potentials. We here focus on the basic case of the linear coupling, neglecting

nonlinear cross-repulsion between the components (it can be suppressed by means of the Feshbach resonance [33]). An extended version of the system includes asymmetry between the two components, which may be imposed by the Zeeman splitting between them. As shown in the next section, a similar model can be implemented in nonlinear optics, considering the copropagation of two polarizations in a periodically twisted waveguide, while the asymmetry may be imposed by the waveguide's intrinsic birefringence.

In the symmetric system, the equations for the two components merge into a single one, if solutions with equal components are looked for. In this case, the standing-wave-shaped linear coupling turns into an effective lattice potential. Although shapes of the corresponding GSs are known, a new problem is their stability in the framework of the two-component system, as well as constructing GSs in the asymmetric one. In this work, we concentrate on the 1D setting, while the 2D version, which is possible in BEC, will be considered elsewhere.

The rest of the paper is structured as follows. In Sec. II, we introduce the model and present a method for the study of stability of the GSs. Several families of GSs and results for their stability are produced in Sec. III. The solitons are classified as on- and off-site-centered ones and symmetric or antisymmetric, with respect to the two component. The off-site-centered GSs are additionally categorized as spatially even or odd modes. In Sec. IV, we address on-site-centered GSs in an asymmetric version of the system, with the objective to identify their existence and stability areas. For a strongly asymmetric system, analytical approximation is developed too. The paper is concluded by Sec. V.

II. THE MODEL

The two-component system with the linear-coupling coefficient spatially shaped as the standing wave (Rabi lattice) is based on the system of scaled GPEs for two components of the mean-field wave function, $u(x,t)$ and $v(x,t)$:

$$\left(i \frac{\partial}{\partial t} + \frac{1}{2} \frac{\partial^2}{\partial x^2} - \sigma |u|^2\right)u + \epsilon \cos(2x)v + bu = 0, \quad (1)$$

$$\left(i \frac{\partial}{\partial t} + \frac{1}{2} \frac{\partial^2}{\partial x^2} - \sigma |v|^2\right)v + \epsilon \cos(2x)u - bv = 0, \quad (2)$$

where ϵ is the amplitude of the Rabi lattice, whose period is fixed to be π by making use of obvious rescaling, and real coefficient b accounts for possible asymmetry introduced by the Zeeman splitting. Results are reported below for $\epsilon = 6$ in Eqs. (1) and (2), which adequately represents the generic situation.

Further, $\sigma = +1$ and -1 correspond to the self-defocusing and focusing signs of the contact nonlinearity. Being interested in GSs, we focus on the case of self-defocusing, $\sigma = +1$, which cannot create regular solitons in free space. Effects of the nonlinear interaction between the two components, with relative strength g (the use of the Feshbach resonance makes it relevant to consider all the cases of $g > 0$, $g = 0$, and $g < 0$ [33]), are accounted for by the addition of cross-phase-modulation (XPM) terms to the nonlinearity in these

equations:

$$|u|^2 u \rightarrow (|u|^2 + g|v|^2)u, |v|^2 v \rightarrow (|v|^2 + g|u|^2)v. \quad (3)$$

We chiefly disregard the XPM terms here (as mentioned above, in BEC this interaction may be eliminated with the help of the Feshbach-resonance method), except for the limit case of strong asymmetry [large b in Eqs. (1) and (2)], in which the XPM can be easily taken into account in the framework of the semianalytical approximation, see Eqs. (12)–(14) below. In a systematic form, XPM effects will be considered elsewhere.

Unlike the BEC system, introduction of a similar model in terms of optical dual-core waveguides is problematic, as the coefficient of the intercore coupling cannot, normally, change its sign. On the other hand, the same linear coupling as defined in Eqs. (1) and (2) may naturally appear in the model of the copropagation of two linear polarizations of light in a “rocking” optical waveguide, subject to a periodically modulated twist [34] (with evolution variable t replaced by the propagation distance, z), while parameter b represents the phase-velocity birefringence of the waveguide. In the latter case, however, the XPM terms with $g = 2/3$ in Eq. (3) should be taken into account, therefore this case too will be considered in detail elsewhere. In principle, more freedom in the choice of the XPM coefficient is offered by photonic-crystal waveguides, which may also carry periodic twist [35]. Last, the optical model may include a group-velocity birefringence too, although this effect is usually much weaker than the phase-velocity birefringence.

Stationary solutions of Eqs. (1) and (2) are looked for as usual,

$$u(x,t) = e^{-i\mu t} U(x), v(x,t) = e^{-i\mu t} V(x), \quad (4)$$

where μ is a real chemical potential, and real wave functions U and V obey the stationary equations, as said above:

$$\mu U + \frac{1}{2} U'' - U^3 + \epsilon \cos(2x)V + bU = 0, \quad (5)$$

$$\mu V + \frac{1}{2} V'' - V^3 + \epsilon \cos(2x)U - bV = 0, \quad (6)$$

where the prime stands for d/dx (hereafter, we fix the defocusing sign of the nonlinearity, $\sigma = +1$, as said above). Numerical solutions of Eqs. (5) and (6) was produced by means of the Newton's method. The bandgap spectrum generated by the solution of the linearized version of the equations is shown in Fig. 1.

Equations (1) and (2) conserve the total norm, i.e., scaled number of atoms, in terms of BEC,

$$N = \int_{-\infty}^{+\infty} (|u|^2 + |v|^2) dx \equiv N_u + N_v, \quad (7)$$

and the Hamiltonian,

$$H = \int_{-\infty}^{+\infty} \left[\frac{1}{2} \left(\left| \frac{\partial u}{\partial x} \right|^2 + \left| \frac{\partial v}{\partial x} \right|^2 + |u|^4 + |v|^4 \right) - \epsilon(u^* v + uv^*) \cos(2x) + b(|v|^2 - |u|^2) \right] dx, \quad (8)$$

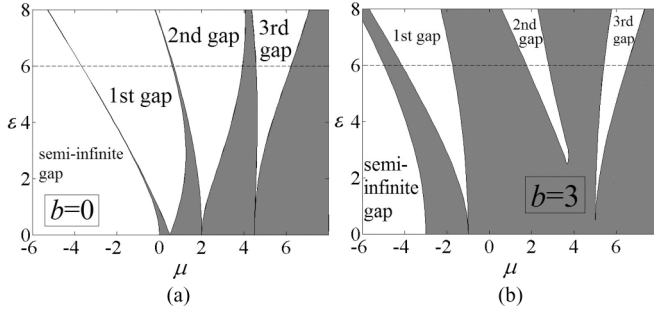


FIG. 1. The bandgap spectrum of the linearized version of Eqs. (1) and (2) with $\epsilon = 6$, for $b = 0$ (a) and $b = 3$ (b), which correspond, respectively, to the symmetric and asymmetric system. Shaded areas designate Bloch bands where gap solitons do not exist. The semi-infinite gap and lowest finite ones are labeled.

where $*$ stands for the complex conjugate, while the conservation of the momentum is destroyed by the presence of the Rabi lattice. Asymmetry of two-component solitons is determined by the respective ratio,

$$R = (N_u - N_v)/(N_u + N_v). \quad (9)$$

For families of GSs, N and H , as well as R (if $R \neq 0$), may be naturally considered as functions of chemical potential μ .

Stability of stationary solutions can be investigated by means of the linearization with respect to small complex perturbations $\phi_{1,2}(x, z)$ and $\psi_{1,2}(x, z)$ added to solution Eq. (4):

$$\begin{aligned} u(x, t) &= e^{-i\mu t} [U(x) + \phi_1(x)e^{-i\lambda t} + \phi_2^*(x)e^{i\lambda^* t}], \\ v(x, t) &= e^{-i\mu t} [V(x) + \psi_1(x)e^{-i\lambda t} + \psi_2^*(x)e^{i\lambda^* t}], \end{aligned} \quad (10)$$

where λ is the perturbation eigenfrequency, which may be complex. Instability takes place if there is at least one eigenvalue with $\text{Im}(\lambda) > 0$. Oscillatory instabilities correspond to complex λ , with both real and imaginary parts different from zero, which happens in previously studied related systems [14,23]. The substitution of Eqs. (10) into Eqs. (1) and (2) and subsequent linearization leads to the eigenvalue problem for λ , based on the following system of equations:

$$\begin{aligned} -\frac{1}{2}\phi_1'' + U^2(2\phi_1 + \phi_2) - (b + \mu)\phi_1 - \epsilon \cos(2x)\psi_1 &= \lambda\phi_1, \\ \frac{1}{2}\phi_2'' - U^2(2\phi_2 + \phi_1) + (b + \mu)\phi_2 + \epsilon \cos(2x)\psi_2 &= \lambda\phi_2, \\ -\frac{1}{2}\psi_1'' + V^2(2\psi_1 + \psi_2) + (b - \mu)\psi_1 - \epsilon \cos(2x)\phi_1 &= \lambda\psi_1, \\ \frac{1}{2}\psi_2'' - V^2(2\psi_2 + \psi_1) - (b - \mu)\psi_2 + \epsilon \cos(2x)\phi_2 &= \lambda\psi_2. \end{aligned} \quad (11)$$

These equations can be rewritten in the matrix form, $\hat{M}(\phi_1, \phi_2, \psi_1, \psi_2)^T = \lambda(\phi_1, \phi_2, \psi_1, \psi_2)^T$, where operator \hat{M} corresponds to the matrix in the left-hand side of Eqs. (11). For the numerical solution of the stability problem, we discretize functional expressions in matrix elements by means of the center-difference numerical scheme, and then calculate the eigenvalue spectrum of the matrix, truncated to a sufficiently large finite size, for stationary solutions.

III. GAP SOLITONS

The numerical solution of the symmetric version of Eqs. (5) and (6), with $b = 0$, shows that the system produces several basic types of GSs. With respect to their spatial structure, they can be classified as on- and off-site-centered localized modes, which feature, respectively, a single density maximum coinciding with a local minimum of the modulation function, $-\epsilon \cos(2x)$, i.e., $x = 0$, or two density maxima placed at adjacent modulation maxima, $x = \pm\pi/2$. Further, the off-site-centered modes, with the pair of density maxima, may be spatially even or odd as functions of x (all on-site-centered modes are, obviously, of the even type). Then, in the absence of the Zeeman splitting ($b = 0$), the two-component GSs are categorized as “symmetric” or “antisymmetric”, if their two components are, respectively, identical, or differ by opposite signs. In the following analysis, we first focus on on-site-centered symmetric, off-site-centered even antisymmetric, and off-sited odd antisymmetric GS species. We also consider on-site-centered antisymmetric, off-site-centered even symmetric, and off-site-centered odd symmetric modes, whose stability is, severally, the same as that of the three above-mentioned species. The analysis is performed for the GSs residing in the first and second finite bandgaps; see Fig. 1.

A. On-site-centered symmetric gap solitons

It is obvious that on-site-centered symmetric GSs [see typical examples in Figs. 2(a) and 2(b), 3(a) and 3(b)], with equal components, $U(x) = V(x)$ (in the system with $b = 0$),

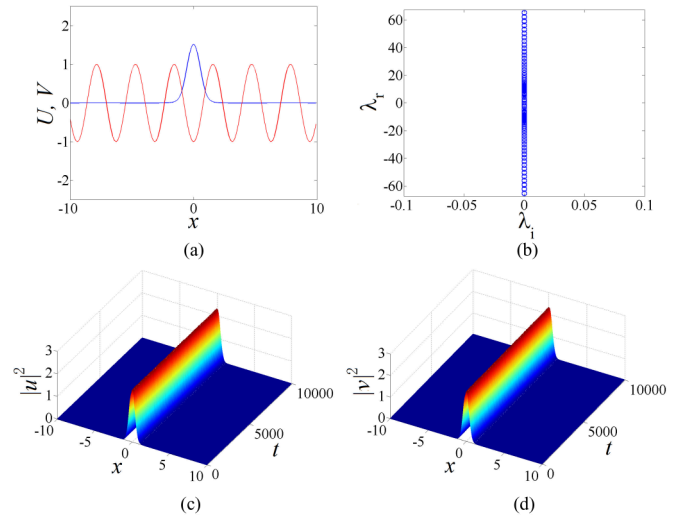


FIG. 2. (a) A typical example of stable on-site-centered symmetric GSs, found in the first finite bandgap, at $\mu = -2$, $b = 0$, and $\epsilon = 6$. Here, and in similar figures below, the background pattern (red sinusoid) represents the scaled underlying Rabi lattice [periodic modulation of the coupling constant, $-(\epsilon/6) \cos(2x)$, with scaling factor $1/6$ added to keep the sinusoid within boundaries of the panel]. (b) The eigenvalue spectrum for small perturbations around the soliton, which confirms its stability. It is further corroborated by simulations of the evolution of the soliton initially perturbed by random noise at the level of 5% of the soliton’s amplitude, which are displayed in panels (c) and (d).

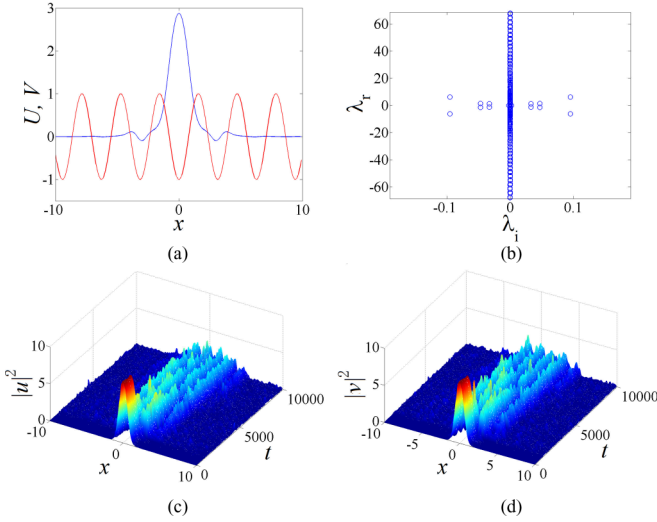


FIG. 3. The same as described in the caption of Fig. 2, but for a typical unstable on-site-centered symmetric soliton in the second finite bandgap, at $\mu = 3$ and $\epsilon = 6$. As seen in panels (c) and (d), development of the oscillatory instability replaces the GS by a “turbulent” pattern, which remains spatially confined.

are identical, in their shape, to the GSs produced by the single-component GPE, the Rabi lattice becoming equivalent to a single-component lattice potential, $-\epsilon \cos(2x)$. Accordingly, the density maximum placed at $x = 0$ in Figs. 2(a) and 3(a) tends to minimize the effective potential. However, the difference in the stability between the single-component model and the present two-component system is essential, as shown in Figs. 4(a) and 4(b). Recall that the GS family tends to be stable in the first two finite bandgaps of the single-component model, except for a weak oscillatory instability, caused by the appearance of complex eigenvalues, close to the right edge of the first bandgap, and in the second one [36]. In addition, dashed magenta lines display, in Figs. 4(a) and 4(b), analytical predictions based on the Thomas-Fermi approximation (see details in the figure caption), which was elaborated for single-component GSs in Ref. [23]

Figures 4(a) and 4(b) demonstrate that the on-site-centered GSs are stable in the first finite bandgap of the two-component system (see a typical example in Fig. 2), except for a small region near the right edge of the bandgap, which roughly resembles the above-mentioned weak oscillatory instability of single-component GSs near the edge of the first bandgap [36]. In this small region, stable solitons alternate with unstable ones, which are subject to a weak oscillatory instability caused by complex eigenvalues. A detailed structure of this region is displayed in Fig. 4(d), and a typical example of a weakly unstable GS, which keeps a nearly undisturbed shape, is presented in Fig. 5. For $\epsilon = 6$, the stable part of the first finite bandgap is $-3.70 < \mu < 0.14$ and $0 < N < 11.11$ in terms of the chemical potential and total norm, respectively.

In the present system the GSs are primarily unstable in the second finite bandgap, as is indicated in Fig. 4(b), and illustrated by a typical example in Fig. 3, which shows that the unstable solitons evolve into *spatially confined* chaotic modes (“solitons of conservative turbulence”). An exception is the

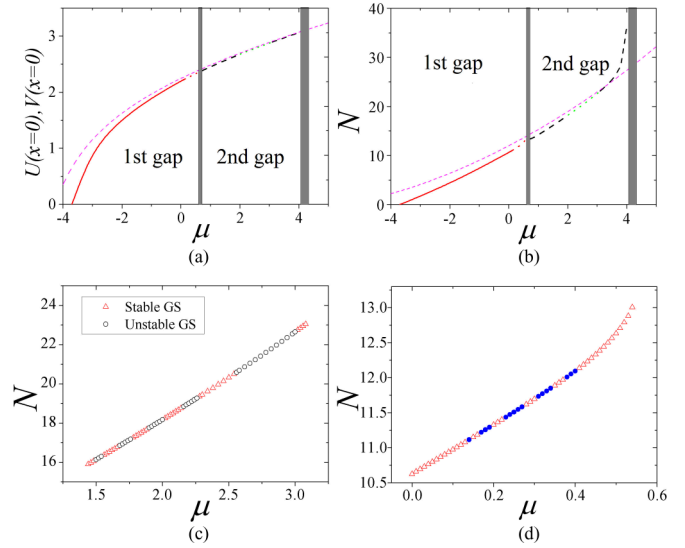


FIG. 4. (a) The numerically found amplitude of fundamental on-site-centered symmetric GSs versus chemical potential μ , for $\epsilon = 6$ and $b = 0$. The dashed magenta curve is produced by the Thomas-Fermi approximation (TFA) for the single-component GSs: $U(x = 0) = V(x = 0) = \sqrt{6 + \mu} - 3(6 + \mu)^{-3/2}$. Panel (b) shows the total norm N versus μ , along with its dashed-magenta TFA counterpart, $N = 2[\sqrt{36 - \mu^2} + \mu \cos^{-1}(-\mu/6)]$. In these panels, as well as in Fig. 7(a) below, red solid and black dashed segments represent stable and unstable GSs in the first and second bandgaps, respectively, while the green dotted segment designates a region of alternate stability and instability in the second bandgap. The red dotted segment near the right edge of the first bandgap designates a region of alternating stable GSs and ones weakly unstable against oscillatory perturbations. (c) The alternation of the stability and instability in the green dotted segment of (b). Here, red triangles and black circles represent stable and unstable GSs, respectively. (d) The alternation of the stability and weak oscillatory instability in the red dotted segment of (b). Here, and also in Fig. 7(c), red triangles and blue filled circles represent stable GSs and ones subject to the weak instability, respectively.

green dotted segment, which contains alternating stable and unstable solitons, as shown in detail in Fig. 4(c).

B. Off-site-centered spatially even antisymmetric gap solitons

For antisymmetric states, with $U(x) = -V(x)$ (in the system with $b = 0$), the above-mentioned effective single-component lattice potential, generated by the Rabi lattice, inverts its sign, taking the form of $\epsilon \cos(2x)$. Accordingly, off-site-centered antisymmetric GSs tend to minimize their energy by placing two density maxima at potential-minima points, $x = \pm\pi/2$, which gives such solitons a chance to be stable. A typical example of a stable antisymmetric GS with the off-site-centered spatially even shape is shown in Fig. 6. The stability of this GS species is summarized in Fig. 7(a), which demonstrates that they are stable solely in the first finite bandgap [the instability in the second finite bandgap is illustrated by Fig. 6(d)]. Similar to the situation shown for the on-site-centered symmetric GSs in Fig. 4(b), at the right edge of the bandgap there is a narrow segment of alternating stability and weak oscillatory instability, whose structure is displayed in detail in Fig. 7(c).

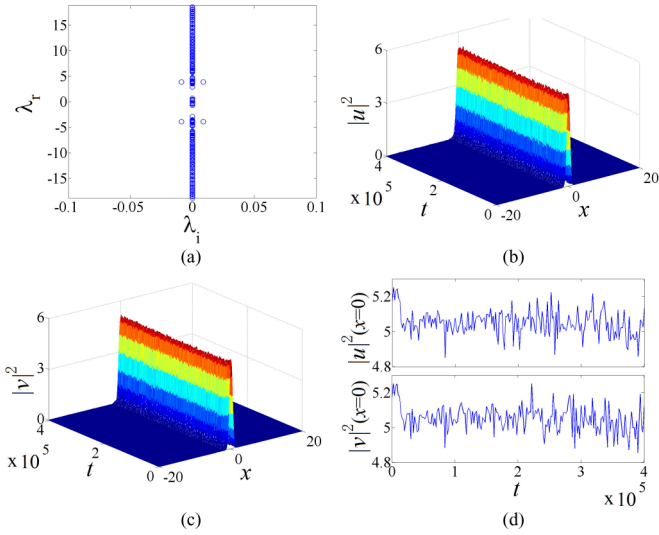


FIG. 5. An example of a weakly unstable GS with $\mu = 0.32$, belonging to the red dotted segment near the right edge of the first finite bandgap in Fig. 4. (a) Unstable eigenvalues of small perturbations are complex with small imaginary parts. (b, c) Direct simulations corroborate the weak oscillatory instability of the soliton, which keeps its localized shape. (d) The weak instability is additionally illustrated by the time dependence of squared amplitudes of both components of the same solutions.

C. Other types of symmetric and antisymmetric gap solitons

The system with $b = 0$ supports off-site-centered antisymmetric GSs with the spatially odd shape, in addition to their even counterparts considered above. An example, and the family of such solitons in the (μ, N) plane, are displayed in

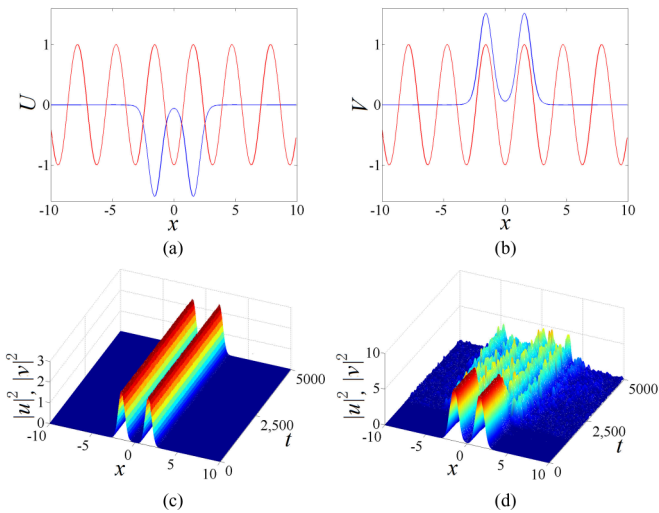


FIG. 6. Panels (a) and (b) display a typical stable off-site-centered antisymmetric spatially even GS for $\mu = -2$, which falls into the first finite bandgap. (c) Direct simulations (with the initial random-amplitude perturbation at the 5% level) of the evolution of the same soliton, which corroborate its stability. (d) Simulations of the perturbed evolution of the GS of the same type, but belonging to the second finite bandgap, with $\mu = 2$, demonstrate that this unstable soliton is finally transformed into a spatially confined turbulent state.

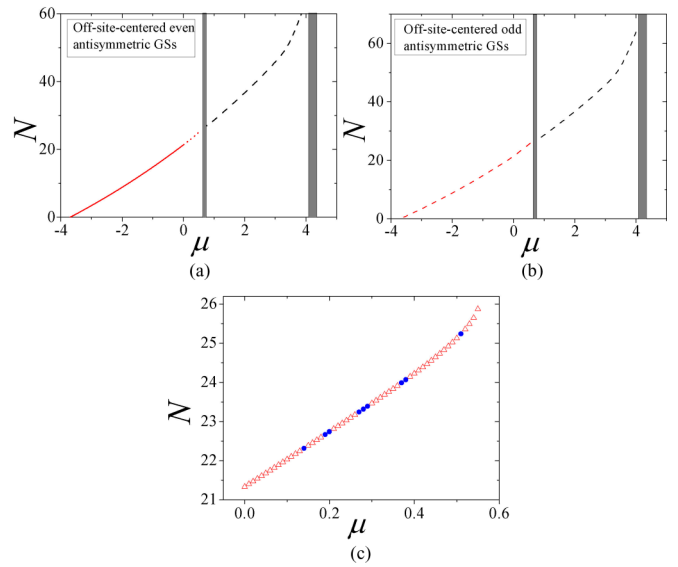


FIG. 7. (a) Total norm N versus chemical potential μ for off-site-centered antisymmetric spatially even GSs, cf. Fig. 4(b) for the on-site centered symmetric solitons. (b) The $N(\mu)$ curve for the family of off-site-centered antisymmetric spatially-odd GSs. Here, the red and black dashed curves refer to unstable solitons which evolve, respectively, into robust breathers [see Fig. 8(c)], or into a confined turbulent state shown in Fig. 8(d). (c) The detailed structure of the red-dotted segment with alternate stability and instability in panel (a). The red triangles and blue filled circles label stable and weakly unstable solutions, respectively.

Figs. 8 and 7(b), respectively. The conclusion of the analysis is that the family is completely unstable in both first and second finite bandgaps. Further, direct simulations demonstrate that,

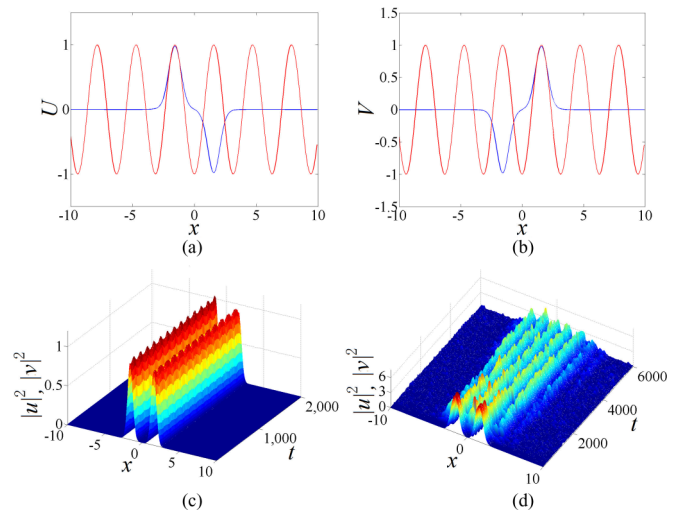


FIG. 8. (a, b) An example of an unstable off-site-centered spatially-odd antisymmetric GS for $\mu = -3$ (which belongs to the first finite bandgap) with $b = 0$ and $\epsilon = 6$. (c) The perturbed evolution (with initial random perturbations at the 5% level) of the same soliton shows its transformation into a persistent breather. (d) Simulations of the perturbed evolution of the GS of the same type, but with $\mu = 2$ (which falls into the second finite bandgap) show its transformation into a confined turbulent mode.

in the first bandgap, the instability transforms the stationary spatially odd GSs into persistent breathers, see Fig. 8(c), while in the second bandgap, the unstable GSs evolve into apparently turbulent spatially confined modes, as shown in Fig. 8(d). In either case, the dynamical states produced by the instability keep the spatially odd shape, as indicated by the persistence of zero amplitude at the midpoint in Figs. 8(c) and 8(d).

Additional types of solitons have been found too: on-site-centered antisymmetric, off-site-centered spatially even symmetric, and off-site-centered spatially odd symmetric GSs. Due to the nature of the present system, in which the GSs are supported by the Rabi lattice, these additional species are actually tantamount to the three species considered above. Indeed, the on-site-centered symmetric GS is obviously equivalent to an antisymmetric one which is centered at the site shifted by half a spatial period, their stability being identical too. The same pertains to off-site-centered spatially even and odd symmetric GSs, which may be easily converted into their antisymmetric counterparts.

IV. ON-SITE-CENTERED GAP SOLITONS IN THE ASYMMETRIC SYSTEM

A. Numerical results

It is relevant to stress that, while GSs in symmetric dual-core systems with the usual lattice potential and constant intercore coupling readily feature spontaneous breaking of the (anti-)symmetry between their components, followed by generation of asymmetric solitons, provided that the nonlinearity strength exceeds a certain critical value [20,21], this effect is not observed in the present system, i.e., all the GSs, both stable and unstable ones, are either symmetric or antisymmetric with respect to the two components. In this section, we report results

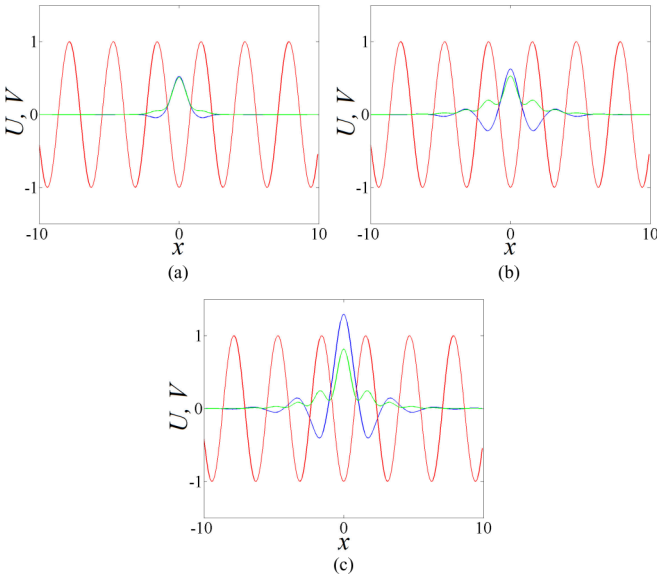


FIG. 9. Examples of stable on-site-centered asymmetric GSs with $\epsilon = 6$, and fixed chemical potential $\mu = -3.5$: (a) $b = 0.2$; (b) $b = 1$; (c) $b = 3$. Fields $U(x)$ and $V(x)$ with larger and smaller amplitudes are shown by blue and green lines, respectively.

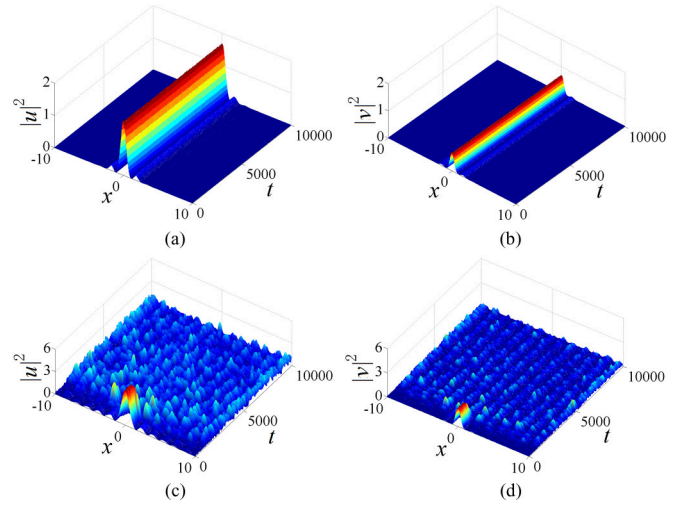


FIG. 10. (a, b) Direct simulations of the perturbed evolution of a stable asymmetric on-site-centered GSs with $(b, \mu) = (3, -3.5)$. (c, d) The same for an unstable asymmetric soliton, with $(b, \mu) = (3, -1.5)$.

for the most fundamental on-site-centered asymmetric GSs, which are naturally produced by the asymmetric system, based on Eqs. (1) and (2) with $b \neq 0$ (obviously, it is sufficient to consider $b > 0$). We do not aim here to consider other GS species in the asymmetric system.

Examples of stable asymmetric GSs are presented in Fig. 9. With the increase of b , the shapes of the $U(x)$ and $V(x)$ components become less localized and develop undulations in their tails. Examples of the perturbed evolution of stable and unstable asymmetric GSs are further shown in Fig. 10. Different from the case of symmetric on-site-centered GSs, asymmetric ones which are unstable tend to develop a chaotic state expanding to the entire spatial domain, cf. Figs. 3(c) and 3(d).

Results obtained for families of asymmetric GSs and their stability are summarized in Fig. 11, by means of $N(\mu)$ and $R(N)$ curves [recall R is the asymmetry ratio defined in

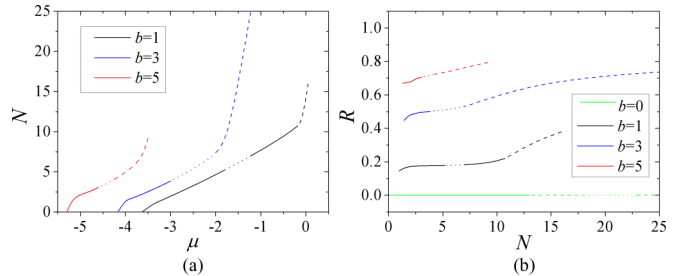


FIG. 11. (a) Total norm N of asymmetric on-site-centered GSs versus chemical potential μ , at $\epsilon = 6$ and fixed values of the asymmetry coefficient: $b = 1, b = 3$, and $b = 5$. (b) Asymmetry ratio, R , defined as per Eq. (9), versus N , for the same soliton families. The family with $b = 0$, which has $R \equiv 0$, is included too, for the completeness' sake. In these panels, solid and dashed lines represent stable and unstable GSs, respectively, while the dotted segments designate regions of alternate stability and instability.

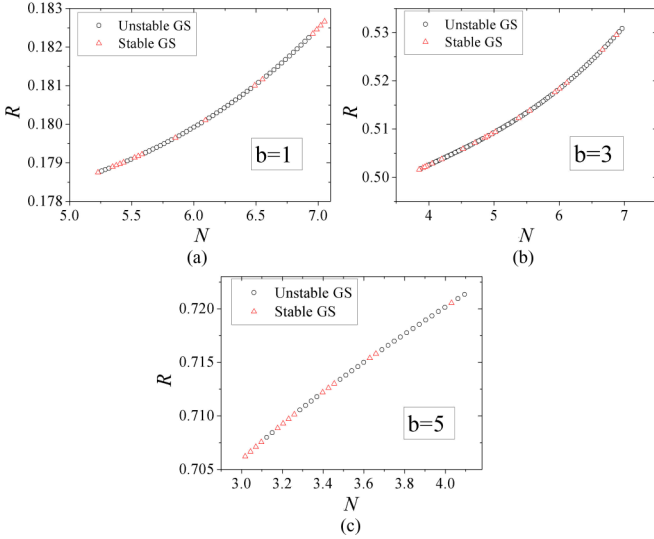


FIG. 12. Panels (a), (b), and (c) show the detailed structure of regions with alternating stable and unstable on-site-centered asymmetric GSs in the regions shown by the dotted line in Fig. 11(a), for $b = 1, 3$, and 5 , respectively. Red triangles and black circles represent, severally, stable and unstable solitons.

Eq. (9)]. These dependencies suggest that the asymmetric GSs are more stable at lower values of the total norm, N , and stability regions shrink with the increase of asymmetry coefficient b and asymmetry ratio R . Similar to the symmetric system ($b = 0$), here there also exist regions of alternate stability and instability, which are designated by dotted segments in Fig. 11. The detailed structure of these segments is rendered in Fig. 12.

In Fig. 13, we have collected results produced by the stability analysis for the on-site asymmetric GSs in the planes of (b, μ) and (N, b) . Figures 13(a) and 13(b) demonstrate that the stability area originally shrinks with the increase of the asymmetry coefficient, b , and then stays narrow but nearly constant. In the (N, b) plane, the stability region also narrows at first with the increase of b , but then it broadens at still larger b .

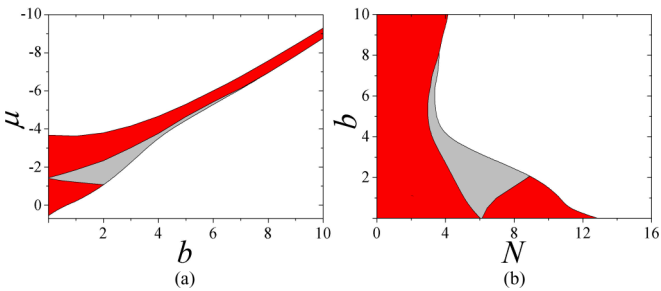


FIG. 13. Stability diagram for on-site-centered asymmetric GSs in the planes of (b, μ) (a) and (N, b) (b). The red and gray colors designate, respectively, the stability area, and the one of alternate stability and instability. GSs do not exist in the top blank areas in panels (a). In the bottom blank area in (a) and blank area in (b), there exist completely unstable GSs.

B. A semianalytical approximation

In the limit of $b \gg 1$, the two-component system, based on Eqs. (1) and (2) can be easily reduced to a single equation with the usual lattice potential, by means of an approximation similar to that recently elaborated for the two-component BEC under the action of strong Zeeman splitting in Ref. [37]. The approximation is based on the fact that large b gives rise to solutions with chemical potential $\mu = -b + \delta\mu$, which implies $|\delta\mu| \ll b$. Accordingly, the solutions are looked for in the form of $\{u(x, t), v(x, t)\} = \exp(i b t) \{\tilde{u}(x, t), \tilde{v}(x, t)\}$, where the remaining t -dependence in \tilde{u} and \tilde{v} is slow, in comparison with $\exp(i b t)$. Then Eq. (2), in which the the XPM terms may be restored, as per Eq. (3), readily yields an approximate expression for the small v component:

$$\tilde{v} \approx \frac{\epsilon \cos(2x)}{2b + \sigma g |u|^2} \tilde{u} \approx \epsilon \left(\frac{1}{2b} - \frac{\sigma g}{4b^2} |u|^2 \right) \cos(2x) \tilde{u}. \quad (12)$$

To justify the expansion of the the fraction in this expression, it is assumed that the soliton’s peak density is not too large, viz.,

$$(|u|^2)_{\max} \ll b/|g| \quad (13)$$

(this condition is easily satisfied for large b). Then, the substitution of this approximation in Eq. (1) leads to a single

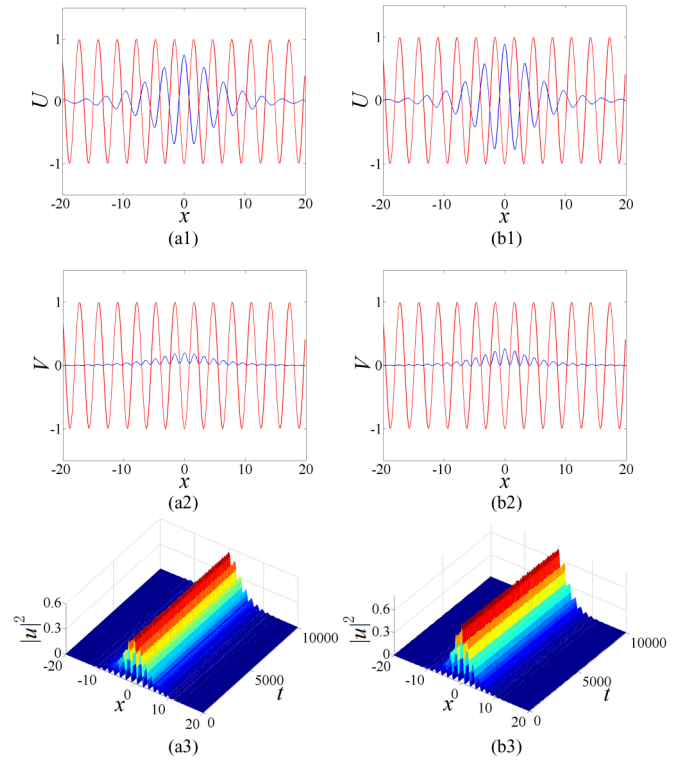


FIG. 14. Panels (a1, a2) and (b1, b2) present typical examples of stable GSs produced, respectively, by the full system of Eqs. (1) and (2), and by the semianalytical approximation which amounts to the single Eq. (14), for $(b, \mu) = (10, -9.1)$. Results of direct simulations, initiated by the full two-component solution, and by the single-component approximation, are displayed, for the u component, in panels (a3) and (b3), respectively.

equation for $\tilde{u}(x,t)$, with an effective lattice potential:

$$\left[i \frac{\partial}{\partial t} + \frac{1}{2} \frac{\partial^2}{\partial x^2} - \sigma \left(1 + \frac{\epsilon^2 g}{8b^2} \right) |\tilde{u}|^2 + \frac{\epsilon^2}{2b} \cos^2(2x) \right] \tilde{u} = 0 \quad (14)$$

[it is easy to see that, to obtain the nonlinearity coefficient in Eq. (14) under the above condition (13), one may substitute $\cos^2(2x)$ by its average value, $1/2$, while this substitution is not relevant in the effective lattice potential]. In turn, Eq. (14) with $\sigma(1 + \epsilon^2 g/8b^2) > 0$ is the standard equation which gives rise to the usual GSs [2,3]. Note that the renormalization of the nonlinearity coefficient in Eq. (14), represented by the term proportion to the XPM coefficient, g , is essential if the Rabi lattice is strong enough, namely, $|g|\epsilon^2 \sim 8b^2$, which is compatible with the underlying condition $b \gg 1$.

This approximation may be naturally named semianalytical, as its analytical part replaces the underlying system of two Gross-Pitaevskii equations, coupled by the Rabi lattice, by the single standard Eq. (14), whose solution is not, generally, available analytically, but is known very well in the numerical form [2,3,36]. Comparison between a typical stationary GS predicted by this approximation and its counterpart produced by the numerical solution of Eqs. (5) and (6) is presented in Fig. 14. Obviously, the semianalytical and numerical profiles are close to each other, and both are stable. In fact, the broad modes displayed in this figure resemble the known nonlinear states generalizing GSs in the usual single-component model, in the form of “truncated Bloch waves”, which were demonstrated experimentally in Ref. [38] and explained theoretically in Ref. [39].

V. CONCLUSION

The objective of this work is to introduce the model of two-component gap solitons, based on two GPEs (Gross-Pitaevskii equations) with self-repulsive nonlinearity, coupled by linear terms that are subject to the spatially periodic cosinusoidal modulation (*Rabi lattice*). The system can be implemented in binary BEC with the superimposed standing wave of a resonantly-coupling electromagnetic field, and in the bimodal light propagation in twisted waveguides. We have demonstrated that this setting gives rise to stable two-component GSs (gap solitons), in the absence of periodic potentials which

are necessary for the existence of GSs in usual models. Several types of the GSs have been found, including on-site-centered symmetric and antisymmetric modes, and spatially even and odd off-site-centered symmetric and antisymmetric ones. Both symmetric and antisymmetric GSs are stable chiefly in the first finite bandgap, as well as in a small segment of the second bandgap. These findings are, roughly, similar to what was recently found for the stability of usual GSs in the single-component model with a periodic potential [36]. A noteworthy finding is the alternation of stable and unstable on-site-centered symmetric GSs in the latter segment. For on-site-centered symmetric and off-site-centered spatially even symmetric and antisymmetric GSs, there also exists a narrow segment of alternating stability and weak oscillatory instability near the right edge of first finite bandgap. Unstable GSs spontaneously transform into robust breathers or spatially confined turbulent states. On-site-centered GSs were found in the asymmetric system too, where segments featuring the alternate stability exist as well. Thus, the alternation of stability and instability of GSs, which was not reported in previously studied models, is a characteristic generic feature of the present system. It is worthy to note that the stability area of the on-site-centered asymmetric GSs originally shrinks with the increase of the asymmetry coefficient, b , but then it expands with the further increase of b , in terms of the total power of the solitons. In the limit of $b \gg 1$, an analytical approximation makes it possible to transform the system into a single GPE with an effective periodic potential and respective GS solutions.

A natural extension of the present analysis should produce a detailed analysis of the system including the XPM interaction between the two components. A challenging direction for further work is a two-dimensional version of the present system. In that case, two-component solitary vortices may be looked for, in addition to fundamental GSs. In fact, off-site-centered spatially odd GSs, which are considered above, are one-dimensional counterparts of the two-dimensional vortices.

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