Transition of multidiffusive states in a biased periodic potential

Jia-Ming Zhang and Jing-Dong Bao*
Department of Physics, Beijing Normal University, Beijing 100875, People's Republic of China
(Received 17 December 2016; revised manuscript received 13 February 2017; published 6 March 2017)

We study a frequency-dependent damping model of hyperdiffusion within the generalized Langevin equation. The model allows for the colored noise defined by its spectral density, assumed to be proportional to $\omega^{\delta-1}$ at low frequencies with $0 < \delta < 1$ (sub-Ohmic damping) or $1 < \delta < 2$ (super-Ohmic damping), where the frequency-dependent damping is deduced from the noise by means of the fluctuation-dissipation theorem. It is shown that for super-Ohmic damping and certain parameters, the diffusive process of the particle in a titled periodic potential undergos sequentially four time regimes: thermalization, hyperdiffusion, collapse, and asymptotical restoration. For analyzing transition phenomenon of multidiffusive states, we demonstrate that the first exist time of the particle escaping from the locked state into the running state abides by an exponential distribution. The concept of an equivalent velocity trap is introduced in the present model; moreover, reformation of ballistic diffusive system is also considered as a marginal situation but does not exhibit the collapsed state of diffusion.

DOI: 10.1103/PhysRevE.95.032107

I. INTRODUCTION

Diffusion in a periodic structure is of great interest, because it is simple to formulate and can be used to describe a surprising range of systems, including Josephson junctions [1], charge-density waves [2], superionic conductors [3], rotation of dipoles in an external field [4], phase-locking loops [5], diffusion on surfaces [6], and separation of particles by electrophoresis [7,8]. The monumental work on this topic is to be found in Risken's textbook [9], where the diffusion in a washboard potential is analyzed within the framework of the Fokker-Planck equation. Lately, with the term Brownian motion on a periodic substrate, Marchesoni, Hänggi, and collaborators have studied systemically Gaussian noise-driven diffusion and transport in such a potential [10]. Understanding particle diffusion in a one-dimensional system has been recognized as a key issue in transport control, but then the authors did not refer necessarily to a pointlike object diffusing in a periodic potential [11]. When pumping a dilute mixture of interacting particles through a narrow channel [12], either by applying external (dc or ac) gradients [13] or by rectifying ambient fluctuations, the efficiency of the transport mechanism is largely influenced by the diffusion of the pumped particles. In particular, the notions of a hysteresis mechanism, multiple jumps, jump reversal, and backward-to-forward rates have been discussed in detail [14,15]. Those models may explain various instances of low-frequency excess damping in material science.

The forced Brownian motion on periodic substrates also provides an archetypal model of transport in a condensed phase. Analytical calculations and a huge enhancement for effective diffusion coefficient relative to the force-free case have been addressed in the overdamped [16,17] and underdamped [18–22] cases. In fact, the diffusion coefficient is measured by the envelope width of the spatial distribution of the particle. This is indeed realized through the phenomenon of "diffusion helped by transport." The problem is whether or not one can change the scaling index of the coordinate variance increasing with time. Recent studies on anomalous diffusion found across many different branches of physics are mostly on the absence of potential or the constant force case of the mean square displacement of the particle expressed as $\langle x^2(t)\rangle \sim t^\delta$ with $0 < \delta < 2$. An important aspect in regard to understanding a random process, in particular an anomalous diffusive system, is its behavior in external fields. To the best of our knowledge, however, unexpected properties due to "transport changing diffusive scaling law" in a corrugated plane need to be investigated in detail.

Clearly, test particles in the unbiased periodic potential are distributed over many spatial wells at long times, so the diffusion changes eventually into a normal one [9,23]. Remarkably, our previous works showed that there exhibit two motion modes: the running state and the oscillating state for a super-Ohmic damping particle moving in a tilted periodic potential [24,25]. The particle coordinate variance (CV) was approximately written as a power function of time, i.e., $\langle \Delta x^2(t)\rangle \sim t^{\delta_{\rm eff}}$, where the index $\delta_{\rm eff}$ can be enhanced twice related to that of the force-free diffusive case. A minimal non-Markovian embedding model of ballistic diffusion was developed to interpret the hyperdiffusion phenomenon and demonstrated that the transient hyperdiffusion will end up in the long-time limit [26,27]. Nevertheless, there exists a concern for transport that pervious numerical simulations may be not efficient to achieve asymptotical results; moreover, for a more general non-Markovian dynamics subjected to no-Ohmic damping, the phenomenon of diffusion in a washboard potential remains unclear.

This paper is organized as follows. In Sec. II, we give numerical evidence for the existence of four regimes for diffusion of the super-Ohmic damping particle in a biased periodic potential. In Sec. III, we demonstrate that transformation from the locked state to the running state obeys an exponential law and analyze the complicated diffusive behavior in the velocity space. Reformation of ballistic diffusion is discussed in Sec. IV. A summary is drawn in Sec. V.

II. THE MODEL AND DIFFUSIVE PROPERTIES

We start by considering a generalized Langevin equation (GLE) of a particle in a potential $V(x)$ and subjected to a

*jdbao@bnu.edu.cn

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memory damping and a thermal colored noise,

$$m\ddot{x} + m\int_0^t \gamma(t-t')\dot{x}(t')\,dt' + V'(x) = \xi(t), \quad (1)$$

where the noise of the zero mean obeys both the Gaussian distribution and the fluctuation-dissipation theorem $\langle\xi(t)\xi(t')\rangle = mk_BT\gamma(|t-t'|)$, k_B is the Boltzmann constant, T denotes the temperature, $\gamma(t) = \frac{2}{m\pi}\int_0^\infty d\omega \frac{J(\omega)}{\omega}\cos(\omega t)$, and $J(\omega)$ is the bath spectral density corresponding to the system coupled bilinearly to a heat bath consisting of infinite harmonic oscillators [28–31]. The GLE (1) under a constant biased force F, thus where the potential is $V(x) = -Fx$, can be solved analytically [31–34] for reference. The mean displacement (MD) and the mean velocity (MV) of the particle are written as

$$\langle x(t)\rangle = \{x(0)\} + \frac{F}{m}\int_0^t H(\tau)\,d\tau,$$

$$\langle v(t)\rangle = \{v(0)\} + \frac{F}{m}H(t). \quad (2)$$

Herein we indicate by $\{\cdots\}$ the average with respect to the initial values of the state variables, and by $\langle\cdots\rangle$ we denote the average over the noise $\xi(t)$, respectively. The CV and the velocity variance (VV) are given by

$$\langle\Delta x^2(t)\rangle = \langle x^2(t)\rangle - \langle x(t)\rangle^2 = \frac{2k_BT}{m}\int_0^t H(\tau)\,d\tau + \left[\{v^2(0)\} - \frac{k_BT}{m}\right]H^2(t),$$

$$\langle\Delta v^2(t)\rangle = \langle v^2(t)\rangle - \langle v(t)\rangle^2 = \frac{k_BT}{m} + \left[\{v^2(0)\} - \frac{k_BT}{m}\right]h^2(t), \quad (3)$$

where $H(t)$ and $h(t) = \dot{H}(t)$ are two response functions and the Laplace transform of $H(t)$ is $\tilde{H}(s) = [s^2 + s\tilde{\gamma}(s)]^{-1}$ [35].

Within the non-Ohmic damping (friction) model [30], as a widely applied one of non-Markovian dynamics, we have $J(\omega) = m\gamma_\delta\omega^\delta\exp(-\omega/\omega_c)$. Consequently, the index in the frequency-dependent damping appears to be linked, not only to δ as would be the case in a heat bath, but also to characterize the color of noise. The Laplace transform of memory damping function $\gamma(t)$ is given by $\tilde{\gamma}(s) = \gamma_\delta/\sin(\delta\pi/2)s^{\delta-1}$ when $s \ll \omega_c$. The low-frequency part of the spectral density governs long-time dynamics of the particle. Herein the response function in Eqs. (2) and (3) is expressed as $H(t) \sim \sin(\delta\pi/2)[\gamma_\delta\Gamma(\delta)]^{-1}t^{\delta-1}$ at long times [30], and, thus the asymptotical CV of the particle subjected to a constant force reads $\langle\Delta x^2(t)\rangle \sim 2D_\delta t^\delta/\Gamma(\delta+1)$, where D_δ is the δ-dependent diffusion constant. In this work, we pay attention to diffusion of a particle in the following tilted washboard potential:

$$V(x) = -V_0\cos(2\pi x) - Fx, \quad (4)$$

where V_0 is the amplitude strength of periodic potential and F is equal to a constant. The potential $V(x)$ has local minima if the driving force F is smaller than the critical value, $F_c = V_0 2\pi$, and when $F > F_c$, the minima vanish.

Unfortunately, the colored noise with non-Ohmic spectrum cannot be simulated directly, or there is not a set of Markovian embedding equations through introducing additional variables for GLE (1). In order to simulate transport process of a non-Ohmic damping particle in a nonlinear potential, we develop an efficient method by using the spectral approach [36–39] to generate the required colored noise and the Runge-Kutta algorithm to solve numerically the GLE whole. In the calculations, the natural units $m = 1$ and $k_B = 1$, the dimensionless parameters $V_0 = 1.0$ and $\gamma_\delta = 1.0$; the cutoff frequency $\omega_c = 10.0$, the time step $\Delta t = 0.01$, and 5×10^3 test particles are used. All the test particles start from a minimum of the titled washboard potential at $x(0) = (2\pi)^{-1}\arcsin[F/(2\pi V_0)]$, and their velocities are sampled from the Gaussian distribution with zero mean and width $\{v^2(0)\} = k_BT/m$.

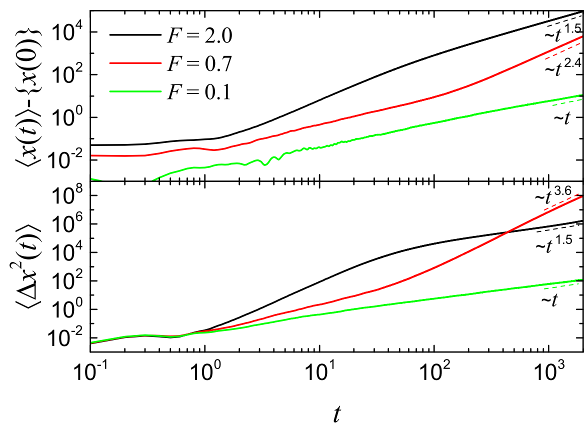

FIG. 1. Time-dependent MD and CV of the particle for various biased forces $F = 2.0, 0.7, 0.1$ from top to bottom.

In Fig. 1 we reveal both supercurrent and superdiffusion of the particle for $\delta = 1.5$. For a large titled force $F = 2.0$ but being less than the critical value F_c, during a long-time window, superdiffusion appears; however, for a very small force $F = 0.1$, there exhibits normal diffusion. For a moderate force $F = 0.7$, after a short initial period processing normal diffusion, both MD and CV increase with time faster than $t^{1.5}$ as $t^{\delta_{\rm eff}}$ with effective power indices $\delta_{\rm eff} = 2.4$ and $\delta_{\rm eff} = 3.6$, respectively. Nevertheless, the hyperdiffusion of this kind [23,25,26] is only a transient behavior. If the tilted force is very large, for example, $F = 2.0$, the time regime of hyperdiffusion extends approximately over one decade from $t = 1$ to $t = 10$ with $\delta_{\rm eff} > 1.5$, but then turns into the superdiffusion of $\delta_{\rm eff} = 1.5$.

In Fig. 2 we plot time-dependent CV and VV of the particle. It is seen that the diffusive process can be distinguished into four time regimes: thermalization, hyperdiffusion, collapse, and asymptotical restoration. Snapshots of the particle coordinate and velocity distributions at $t = 0.1, 10, 200, 2000$ are presented for various regimes. In the thermalized regime, both CV and VV have small changes related to their initial values, and thus, the distributions of coordinate and velocity of the particle are always Gaussian forms. Then diffusive process evolves into the hyperdiffusive regime. The particle gains enough energy to overcome dissipation loss and thus moves along the direction of biased force. This results in a long tail in front of the peak of coordinate distribution at time $t = 10$.

The above phenomena can be understood well in the velocity space. Up inspection, we find a prominent result: The VV of the particle becomes rapidly large, and the velocity distribution exhibits a bimodal structure. Namely, with time

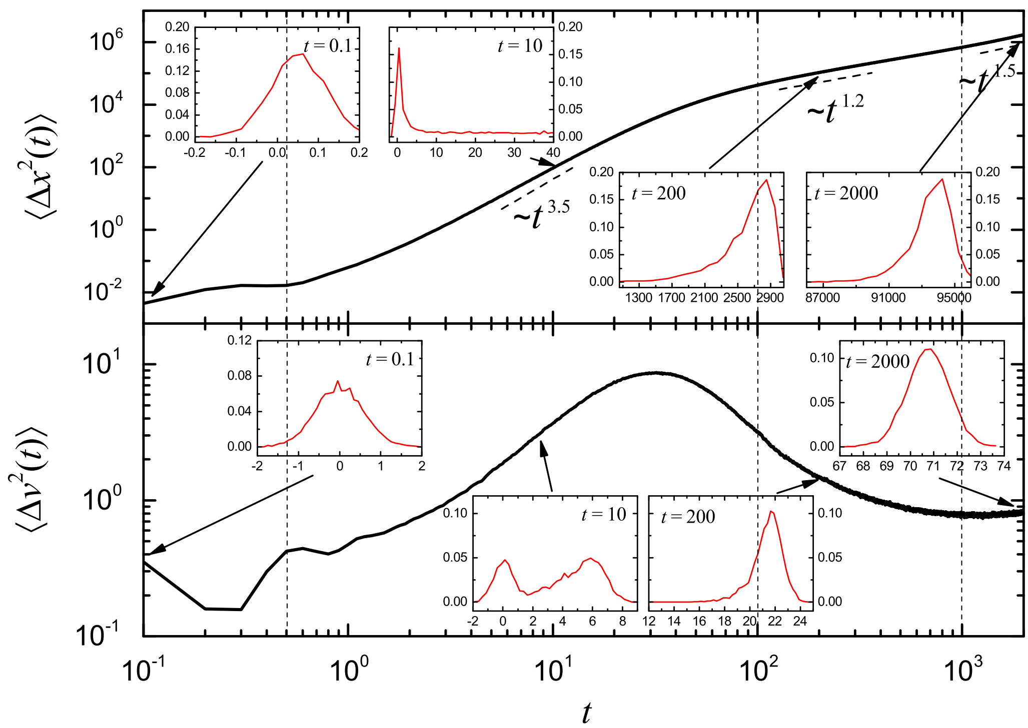

FIG. 2. Time-dependent CV and VV of the particle. Four regimes are divided with dashed vertical lines: $\langle \Delta x^2(t) \rangle \sim t^{\delta_{\mathrm{eff}}}$, the thermalized regime $\delta_{\mathrm{eff}} \sim 0$, the hyperdiffusive regime $\delta_{\mathrm{eff}} > \delta = 1.5$, the collapsed regime $\delta_{\mathrm{eff}} < \delta$, and the asymptotical regime $\delta_{\mathrm{eff}} = \delta$. The parameters used are $F = 2.0$, $T = 0.5$, and $\delta = 1.5$.

increasing, the test particles give gradually rise from the initial Gaussian velocity distribution, which is regarded as the locked state. For the running state, consequently, the peak of the coordinate distribution moves with a certain acceleration. With test particles escaping continuously from the locked state into the running state, the VV arrives gradually at its maximum and then begins declining. The CV increases with time slower than before but follows an effective diffusive index δ_{eff}, which is still larger than 1.5. This implies that the hyperdiffusive regime may last a long time for smaller biased forces until the numerical simulation finishes.

The criterion for the end of the second regime is that all the test particles escape from the initial locked state to the running state. Next, the third regime, i.e., the collapsed one, rises to appear. All the test particles move toward to the direction of biased force, and none are trapped by the potential well again. The test particles with lower velocities will get the energies larger than that of the faster ones, so that the CV of the particle collapses with an effective index $\delta_{\mathrm{eff}} = 1.2$ in comparison with $\delta = 1.5$. In the velocity space, the tail behind the peak of velocity distribution shortens gradually with the increase of time. Finally, the asymptotical regime comes in turn. The velocity distribution of the particle returns to a Gaussian form, which gives rise to the diffusion returning to the superdiffusive state with $\delta_{\mathrm{eff}} = 1.5$.

III. TRANSITION OF VELOCITY FROM LOCKED STATE TO RUNNING STATE

As demonstrated herein, due to double modes of the velocity distribution, we observe the strong amplification of diffusive behavior in the underdamped case [21,22,40]. We also find from Fig. 2 that both the locked state and the running state of the particle velocity exist and expect to interpret the complicated phenomena of hyperdiffusion. It is worth noticing that for the super-Ohmic damping particle, its average velocity is about equal to zero in the locked state, and the running state is "running" with a velocity increasing with time rather than a constant value. We admit that the velocity of the particle in the running state emerges as $v(t) \approx F \sin(\delta\pi/2)t^{\delta-1}/[m\gamma_\delta \Gamma(\delta)]$ according to Eq. (2). Once test particles enter the running state, the average kinetic energy of the particle gained from the external driving is larger than the dissipated energy due to the memory damping effect, so these test particles will never return to the locked state. A burning question is how the particle transforms from the locked state to the running state.

In Fig. 3 we count negative velocity of 5×10^3 test particles and take twice the value as the number of test particles remaining in the locked state for various values of T, F, and δ. The exponential characteristic of proportion decay in the locked state is very like the escape process from a metastable potential, thus we put forward a concept of equivalent velocity "trap". The locked state is considered as the one where test particles are confined in the velocity trap, and the running state is recognized as the one where test particles are driven by an effective force $f(t) \approx F \sin(\pi\alpha/2)t^{\alpha-2}/[m\gamma_\alpha \Gamma(\alpha)]$. This is similar to the heuristic treatment of Ref. [19]. In the hyperdiffusive time regime, the escape of the particle from a spatial well can represent the effect of multiple scattering on the barriers of the periodic potential.

Up observing Fig. 3, we assume that the first exit time t_e of the particle escaping from the locked state follows an

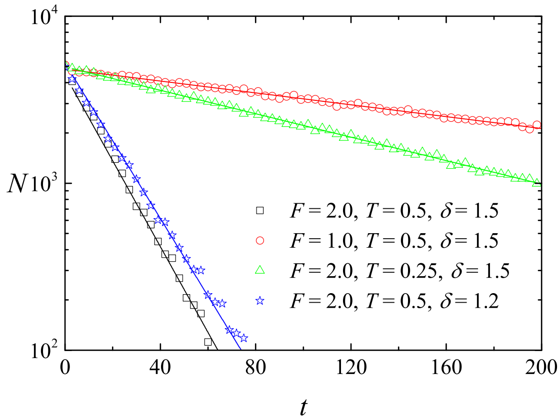

FIG. 3. Log-linear plotting of number of test particles remaining in the locked state for various values of external force, temperature, and power index.

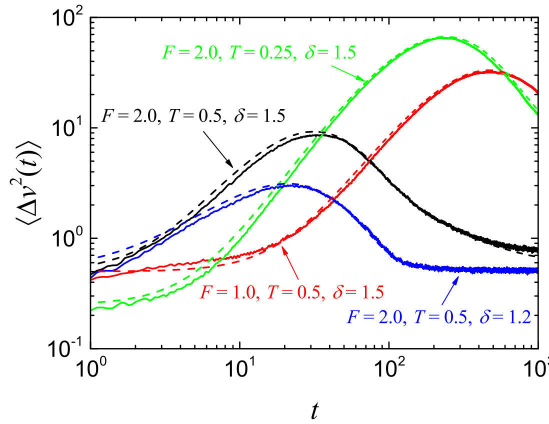

FIG. 4. The VV of 5×10^3 test particles simulated (solid line) and theoretical (dashed line) results for comparison. Theoretical mean exit time t_0 is taken from Fig. 3, and the parameters used are the same as Fig. 3.

exponential distribution:

$$p(t_e) = \frac{1}{t_0} \exp\left(-\frac{t_e}{t_0}\right). \tag{5}$$

The mean exit time t_0 can be extracted from the numerical results. It is seen from Fig. 3 that $t_0 = 16.3$ under the following parameters: $F = 2.0, T = 0.5, \delta = 1.5$; $t_0 = 244.0$ under $F = 1.0, T = 0.5, \delta = 1.5$; $t_0 = 124.1$ under $F = 1.0, T = 0.5, \delta = 1.5$; and $t_0 = 19.4$ under $F = 1.0, T = 0.5, \delta = 1.5$. At time t, the distribution of running time t_r of the particle after escaping over the locked state is defined by $p_r(t_r) = t_0^{-1} \exp[-(t - t_r)/t_0]$ with $t_e + t_r = t$.

The temporal distribution of the first exit time can be transformed into a normalized velocity distribution via $v = F \sin(\delta\pi/2) t_r^{\delta-1}/[m\gamma_\delta \Gamma(\delta)]$; we obtain

$$p_r(v) = \frac{1}{\delta - 1} v_0^{-\frac{1}{\delta-1}} v^{\frac{2-\delta}{\delta-1}} \times \frac{\exp[(v/v_0)^{1/(\delta-1)} - (v_t/v_0)^{1/(\delta-1)}]}{1 - \exp[-(v_t/v_0)^{1/(\delta-1)}]}, \tag{6}$$

where $v_0 = F \sin(\delta\pi/2) t_0^{\delta-1}/[m\gamma_\delta \Gamma(\delta)]$ and $v_t = F \sin(\delta\pi/2) t^{\delta-1}/[m\gamma_\delta \Gamma(\delta)]$. In fact, there are two contributions to the width of $p_r(v)$. One is the expositional distribution due to dispersion of emergence time from the velocity trap, and the other comes from the thermal fluctuation. Considering both of them, the modified velocity distribution of the particle in the running state is written as

$$p_r^m(v) = \int_0^{v_t} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(v - v')^2}{2\sigma^2}\right] p_r(v')\, dv', \tag{7}$$

where $\sigma^2 = k_B T/m$.

In the spirit of relation between the group diffusion and the phase diffusion [41], the total probability density function (PDF) $p(v)$ is composed of two sub-PDFs: $p_l(v)$ (the locked state) and $p_r(v)$ (the running state). We yield $p(v) = a(t) p_l(v) + [1 - a(t)] p_r(v)$, where $a(t) = \exp(-t/t_0)$ is a proportion of test particles in the locked state. Then the total MV and VV are given by $\langle v \rangle = a(t) \langle v \rangle_l + [1 - a(t)] \langle v \rangle_r$ and $\langle \Delta v^2 \rangle = a(t) \langle \Delta v^2 \rangle_l + [1 - a(t)] \langle \Delta v^2 \rangle_r + a(t)[1 - a(t)][\langle v \rangle_r - \langle v \rangle_l]^2$. Here $\Delta v = v - \langle v \rangle$, $\langle \cdots \rangle_l$ and $\langle \cdots \rangle_r$ denote statistical averages of test particles in the locked and running states, respectively. The particle velocity distribution in the locked state is a Gaussian function, and its stationary form reads $p_l(v) = \exp[-v^2/(2\sigma_l^2)]/(\sqrt{2\pi}\sigma_l)$. It has zero mean and variance $\langle \Delta v^2 \rangle_l = \sigma_l^2 \approx k_B T/m$. In the running state, the particle velocity distribution $p_r(v)$ has been determined by Eq. (6) for simplicity, where the MV is given by

$$\langle v \rangle_r = \int_0^{v_t} v p(v)\, dv = v_0 \exp(-i\pi\delta) \gamma[\delta, -(v_t/v_0)^{1/(\delta-1)}] \times \frac{\exp[-(v_t/v_0)^{1/(\delta-1)}]}{1 - \exp[-(v_t/v_0)^{1/(\delta-1)}]}, \tag{8}$$

and the mean square velocity emerges as

$$\langle v^2 \rangle_r = \int_0^{v_t} v p_r(v)\, dv = -v_0^2 \exp(-2i\pi\delta) \gamma[2\delta - 1, -(v_t/v_0)^{1/(\delta-1)}] \times \frac{\exp[-(v_t/v_0)^{1/(\delta-1)}]}{1 - \exp[-(v_t/v_0)^{1/(\delta-1)}]}, \tag{9}$$

where $\gamma[\cdot,\cdot]$ is the incomplete gamma function. Notice that $\langle \Delta v^2 \rangle_r = k_B T/m + \langle v^2 \rangle_r - \langle v \rangle_r^2$, which comes simply from the sum of thermal fluctuation and dispersion effect. In this way, we obtain the total VV, reading

$$\langle \Delta v^2(t) \rangle = \frac{k_B T}{m} - \frac{F^2 t_0^{2\delta-2} \sin\frac{\pi\delta}{2}}{m^2 \gamma_\delta^2 \Gamma^2(\delta)} e^{-2i\pi\delta} \times [\gamma^2(\delta, -t/t_0) e^{-\frac{2t}{t_0}} + \gamma(2\delta - 1, -t/t_0) e^{-\frac{t}{t_0}}], \tag{10}$$

which will be used to analyze the hyperdiffusion.

In Fig. 4 we depict the theoretical VV for various parameters and compare with numerical results. It is seen that our analytical results are in good agreement with the simulations. At the beginning, the total VV approaches σ_l^2. When the test particles enter the hyperdiffusive regime, Eq. (10) can be locally expanded as $\langle \Delta v^2(t) \rangle \propto t^\lambda$. This result shows evidence that the CV of the particle emerges as $\langle \Delta x^2(t) \rangle \propto t^{2+\lambda}$, and the hyperdiffusion appears. At enough long times, all test particles enter into the running state, and the particle velocity distribution due to dispersion will gradually collapse to a δ-peaked distribution. Finally, the VV returns to the thermal equilibrium value $\lim_{t\to\infty}\langle\Delta v^2(t)\rangle \approx k_BT/m$. Based on the above facts, we conclude that although superdiffusion has a complicated intermediate transient process, the effect of dispersion by the periodic potential vanishes at long times.

IV. REFORMATION OF BALLISTIC DIFFUSIVE SYSTEM

The ballistic diffusion of a force-free particle is indeed the limitation of thermal diffusion described by GLE. Differing from superdiffusion in terms of the non-Ohmic damping model with $1 < \delta < 2$, the memory to the initial velocity preparation does not vanish, and consequently the velocity variable is a nonergodic one because of vanishing Markovian low-frequency friction [23], which is regarded as a marginal case of our model. A simple but physically reasonable model for the memory damping function is chosen to be $\gamma(t) = \gamma[2\delta(t) - \nu e^{-\nu t}]$ [26,42], which corresponds to the spectral density of the heat bath given by $J(\omega)$ is cubic when $\omega \ll \nu$. The non-Markovian GLE of this kind can allow for a three-dimensional Markovian LE embedding [26,42]:

$$\begin{aligned}\dot{x}(t) &= v(t),\\ m\dot{v}(t) &= -V'(x) - u(t) - m\gamma v(t) + \sqrt{2mk_BT\gamma}\,\zeta(t),\\ \dot{u}(t) &= -m\nu\gamma v(t) - \nu u(t) + \nu\sqrt{2mk_BT\gamma}\,\zeta(t),\end{aligned} \tag{11}$$

where $\zeta(t)$ is a zero-mean Gaussian white noise and $\langle\zeta(t)\zeta(t')\rangle = \delta(t-t')$. The initial value of the auxiliary variable $u(0)$ obeys the Gaussian distribution with zero mean and variance $\langle u^2(0)\rangle = mk_BT\nu\gamma$.

Similar to analysis in the above section, we yield all the test particles with the velocity $v \approx F\nu t/[m(\gamma+\nu)]$ at long times after they escape from the initial velocity trap. The distribution of the first exit time from the well is still $p(t_e) = t_0^{-1}\exp(-t_e/t_0)$, and it can be transformed into the normalized velocity distribution: $p_r(v) = v_0^{-1}\exp[(v - v_t)/v_0]/[1-\exp(-v_t/v_0)]$ $(0 \leqslant v \leqslant v_t)$ for the test particles in the running state. The particle velocity distribution in the locked state still remains a Gaussian form, and we get the total VV given by

$$\langle\Delta v^2(t)\rangle = \frac{k_BT}{m} + \frac{F^2\nu^2t_0^2}{m^2(\gamma+\nu)^2} \times\left[1 - \exp\left(-\frac{t}{t_0}\right) - 2\exp\left(-\frac{t}{t_0}\right)\frac{t}{t_0}\right]. \tag{12}$$

In Fig. 5(a) we compare the simulation result with the theoretical one with the parameters used in Ref. [26], i.e., $\gamma = 1.0$, $\nu = 0.25$, $V_0 = 1.0$, and $\Delta t = 10^{-4}$. Likewise, the mean exit time t_0 is also extracted by numerical statistics. The results are $t_0 = 615.13$ $(F = 0.75, T = 0.5)$, $t_0 = 33.48$ $(F = 1.5, T = 0.5)$, and $t_0 = 348.64$ $(F = 1.5, T = 0.25)$. The mean square velocity rises gradually to the stationary value, and no collapsed process exists in comparison with the superdiffusion for $1 < \delta < 2$.

The "kinetic temperature" notion $T_{\rm kin}$ was used to characterize the width of a nonequilibrium velocity distribution

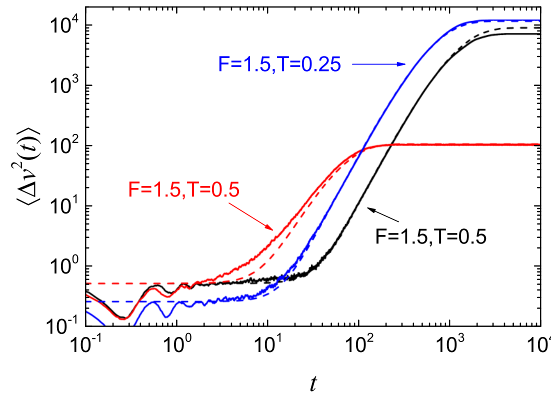

FIG. 5. The VV of 10^4 test particles simulated (solid line) [26] and theoretical Eq. (12) (dashed line) for different values of F and T.

[26]. Here the average kinetic energy of the particle can be decomposed as $K = K_m + K_T$, where K_m represents the kinetic energy of the particle under a constant force and K_T characterizes multiple scattering on the barriers of periodic potential. Our theoretical result is associated with the effective kinetic temperature by $K_m = k_BT/2$ and $K_T = k_BT_{\rm kin}/2 = F^2\nu^2t_0^2[1-\exp(-t/t_0) - 2t\exp(-t/t_0)/t_0]/[2m(\gamma+\nu)^2]$. If all the particles escape from the initial velocity trap, they arrive at the maximal kinetic temperature $T_{\max}$. In our analyses, the mean exit time t_0 of the particle from the velocity trap determines the crossover time from the locked state to the running state, and the VV depends primarily on it. It is an important parameter and depends more strongly than exponential form on both the temperature and the biased force, similar to the kinetic temperature. The long-time limitation of Eq. (12) yields $\lim_{t\to\infty}\langle\Delta v^2(t)\rangle \approx k_BT/m + F^2\nu^2t_0^2/[m(\gamma+\nu)]^2$. This result is composed of thermal fluctuation and multiple scattering on the barriers of periodic potential, and it differs markedly with that of the usual superdiffusion because the latter contains only the thermal fluctuation. This implies that the dispersion effect will not collapse for a ballistic diffusion system, so there is no collapsed regime. Therefore, the CV of the particle will turn to that of the ballistic diffusion from the hyperdiffusion situation.

V. SUMMARY

We have researched within the GLE formalism diffusive dynamics of a non-Ohmic damping particle in a titled periodic potential. The emerging nonequilibrium features are manifested by the model parameters and process-dependent diffusive scaling law. It has found that the diffusion of the super-Ohmic damping particle might undergo four time regimes: thermalization, hyperdiffusion, collapse, and asymptotical restoration. In order to understand rich properties of hyperdiffusion, we propose a velocity trap where the motion is composed of two parts: (1) the locked state, where the particle in captured in a potential well, and (2) the running state, where the particle gains enough energy to escape the potential well. In particular, the latter seems as if the particle is driven by a time-dependent force. We have demonstrated that the escaping time of the particle from the locked state follows an exponential distribution law. This leads to the dispersion behavior and induces the hyperdiffusion. Moreover, the ballistic diffusion found in previous works is also considered a marginal case of our model, where the velocity variance of the particle contains only the thermal fluctuation and no dispersion. It has shown that the dispersion effect will not collapse, so there is no the collapsed regime.

The present study may be related to self-propelled motion or active Brownian motion. This is one of the key features of life appearing on levels ranging from flocks of animals to single-cell motility and intracellular transport by molecular motors [43–46]. We are also confident that our results for the biased periodic potential induced hyperdiffusion will serviceably impact other properties of the washboard-potential device and quantum diffusion. Thus this field is open for future research that in turn may reveal surprising findings.

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China under Grant No. 11575024.

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