Electromagnetic instability in plasmas heated by a laser field

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Electromagnetic instability is investigated in homogeneous plasmas heated by a laser wave in the range $\alpha = v_0^2/v_t^2 \leqslant 2$, where v_0 is the electron quiver velocity and v_t is the thermal velocity. The anisotropic electron distribution function that drives unstable quasistatic electromagnetic modes is calculated numerically with the Vlasov-Landau equation in the high ion charge number approximation. A dispersion relation of electromagnetic waves which accounts for further nonlinear terms on v_0^2 from previous results is derived. In typical simulation with ion charge number $Z = 13$, a temperature $T = 5$ keV, a density $n = 9.8 \times 10^{20}\,\text{cm}^{-3}$, and a laser wavelength $\lambda_{\text{laser}} = 1.06\,\mu\text{m}$, growth rates larger than $10^{12}\,\text{s}^{-1}$ in the quasicollisionless wave-number range were found for $\alpha \geqslant 1$. In the same physical conditions and in the mildly collisional range a growth rate about $10^{11}\,\text{s}^{-1}$ was also obtained. The extent of the growth wave-number region increases significantly with increasing α .

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I. INTRODUCTION

Self-generated magnetic fields in plasmas are one of the most active research topics in the literature [1–11]. The central effect of the magnetic field on particles is their trapping around the B-field lines. This effect has considerable impact on many plasma research fields; in particular the plasma confinement with magnetic field structures is used in many schemes in magnetic confinement fusion such as tokamaks, etc. In addition the presence of intense magnetic fields in plasmas leads to the modification of their physical properties as they induce anisotropy owing to the privileged direction of B. In particular the magnetic field greatly alters the transport coefficients, e.g., the inhibition of the thermal heat flux, the generation of the transverse Righi-Leduc heat flux, etc. [12,13], and it is responsible also for the generation of transverse waves inherent to magnetized plasmas.

Several sources of magnetic fields are reported in the literature. Among these sources we mention the Weibel instability [1] which generates quasistatic magnetic modes in two-temperature plasmas. This instability requires a kinetic treatment in the velocity space, and the electron distribution function (EDF) must involve the second anisotropic distribution function [5,6]. Various sources for this instability are investigated in the literature such as thermal transport [3,8], plasma expansion [8,11], and collisional absorption of electromagnetic (EM) waves by the inverse bremsstrahlung mechanism [7,8]. This instability is produced through a scheme where the current density which creates the magnetic field is supplied in turn by the same magnetic field, leading thus to an unstable mechanism. Thereby the magnetic field grows at the expense of the plasma free energy.

For the EM instability source driven by the collisional absorption of a laser wave [7] the relevant parameter is the ratio of the square of the quiver velocity to the square of the thermal velocity $\alpha = \frac{v_0^2}{v_t^2}$, where $v_0 = \frac{eE_0}{m\omega_0}$ is the quiver velocity; $v_t = \sqrt{T/m}$ is the electron thermal velocity; e is the elementary charge; m is the electron mass; ω_0 and E_0 are, respectively, the frequency and the amplitude of the laser wave; and T is the electron temperature in energy units. For typical values of laser and plasma parameters considered in this work, the ponderomotive energy $\frac{1}{2}mv_0^2$ could be of the order of a few keV. In a previous work reported in Ref. [7] the stability analysis of magnetic modes is investigated in laser heated plasmas in physical conditions such that $\alpha \ll 1$ and the results obtained have shown that moderate quasistatic magnetic fields can be self-generated in such plasmas. In a recent work [14] devoted to the calculation of the EDF in homogeneous plasmas in the presence of a laser wave, the authors calculated the EDF for moderate values $\alpha < 0.5$. The kinetic model accounts for the anisotropies of the EDF but uses for the isotropic component a numerical fit of Matte *et al.* [15] within the condition $\alpha \ll 1$. An application to the stability analysis of the EM modes was performed using the dispersion relation (DR) derived in Ref. [7] (valid also only for $\alpha \ll 1$) with, in addition, a significant restrictive condition; the collisions were neglected. Thus in this weakly nonlinear regime, the result obtained in Ref. [14] provides a crude order of magnitude for the growth rate.

The question which then arises is how this instability evolves when the relevant parameter α is greater than the unity. Such physical conditions can be met if the plasma temperature is rather low, if the laser wavelength is large, and if the intensity of the laser wave is high. Regarding the latter condition it is well known that very high laser intensities are now valuable, up to $10^{15}\,\text{W/cm}^2$ for long laser pulse of about a nanosecond. We expect that for increasing α the nonlinear stabilizing terms increase but the source term of the instability increases too. It is therefore crucial to investigate the behavior of this instability in this regime in order to estimate its growth rate and the range of the wave-number spectrum. In this work we address this issue investigating the EM instability in a nonlinear regime where the quiver velocity is comparable in magnitude to the electron thermal velocity. This requires that we first revisit the DR of quasistatic EM modes [7] in order to take into account these nonlinear terms on α, neglected in previous works. Second, it is mandatory to account for the electron-electron collisions in

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the kinetic model and therefore the isotropic part of the EDF is calculated on the same footing as the anisotropic one. This second improvement needs to solve numerically nonlinear PDE's involving integrodifferential operators instead of linear ODE's as in Ref. [14].

It is important to note that the collisional absorption by inverse bremsstrahlung represents in this work the mechanism responsible for the excitation of EM instabilities. It competes with anomalous heating and resonant absorption but generally it represents the principal mechanism in laser-plasma interaction. The most important works focused on nonlinear absorption where the energy absorbed by electrons is not proportional to the wave intensity but instead it undergoes a saturation regime. The pioneer works of Dawson and Oberman [16] and those of Silin [17] allowed the highlighting of the reduction in absorption rate for significant values of α. Comparatively to the linear regime an effective electron-ion collision frequency dependent on α is proposed. The second important contribution in this area is that of Langdon [18] who demonstrated that absorption can also be reduced significantly (up to a factor 2) for $\alpha \ll 1$ and $Z\alpha > 1$, where Z is the ion charge number. This is due to the significant deformation of the isotropic component of the EDF by the EM field, as in this scheme electron-electron collisions are not efficient to Maxwellianize the electrons. All these studies were conducted in the nonrelativistic regime. Avetissian *et al.* [19,20] completed these studies by extending their range of validity for strong fields and high temperatures in the relativistic range. With the use of quantum mechanics treatment they calculated numerically and analytically (in asymptotic limits) the absorption rate as a function of mv_0^2 normalized to the electron rest energy or photon energy. In Ref. [19], the low-frequency regime is investigated and in Ref. [20], the inverse bremsstrahlung absorption of an intense x-ray laser field was studied. Both contributions complement each other and thus cover a wide range of applications in classical and quantum plasmas. In particular they could be very significant in currents experiments using ultrashort and ultraintense laser pulses and in future inertial fusion experiments where the temperature can be about 10–15 keV which corresponds to mildly relativistic temperatures. Let us specify the validity of our kinetic model. It is based on the Fokker-Planck equation defined by the Landau collision operator and this restricts its validity to kinetic and nondegenerate plasmas. In addition, we assumed implicitly that the Landau collision operator is valid in the high-frequency spectrum [4]. We note that in dense and cold plasmas, correlations between particles as well as plasma degeneracy should be taken into account. In these physical conditions the Fokker-Planck equation is no longer valid and further models are proposed in the literature to study the collisional absorption [21].

This work is organized as follows. Section II presents the physical problem addressed in this work by specifying the model equations and the approximations used. The Vlasov-Landau equation is solved numerically, and the isotropic and the relevant second anisotropic components of the EDF are reported. Section III deals with the analytical derivation of the DR for quasistatic EM modes in an arbitrary regime of α and the numerical calculation of the instability growth rate. Finally, we briefly summarize our results in the last section.

II. MODEL EQUATIONS FOR THE BACKGROUND ELECTRON DISTRIBUTION FUNCTION

The largest contributions on the interaction of a laser wave with plasmas are those performed within the condition $\alpha \ll 1$. In homogeneous plasmas, numerous works were reported in the framework of this approximation as the reduction of the inverse bremsstrahlung absorption [18] and the modification of photoionization mechanisms [15]. Under these physical conditions, the anisotropic part of the EDF is a small part regarding the isotropic one. As a consequence physical phenomena driven by mechanisms which involve the plasma anisotropy should be weak. As far as we know, studies on the quasistatic EM instabilities are restricted to the range $\alpha \ll 1$ and this work constitutes an investigation of these instabilities in a strongly nonlinear regime $\alpha > 1$. The model equation used in this work is similar to that of Ref. [14]. It is based on the Fokker-Planck equation in homogeneous plasmas in the presence of a laser wave assumed in the dipole approximation. In addition, as in Ref. [14] collisions are modeled by the Landau collision operators and to solve the kinetic equation, the orthogonal polynomials expansion was used.

Thus the evolution of the EDF $g(\vec{\mathrm{v}},t)$ in the velocity space as a function of time t in the presence of electron-ion and electron-electron collisions is described by

$$\frac{\partial g}{\partial t} - \frac{e}{m}(\vec{E}_h + \vec{\mathrm{v}} \times \vec{B}_h) \cdot \frac{\partial g}{\partial \vec{\mathrm{v}}} = C_{ei}(g) + C_{ee}(g), \quad (1)$$

where the variables used have their common meaning. We restricted our analysis to the interaction of oscillating electric field with homogeneous plasmas in nonrelativistic range and typically this corresponds to laser intensity $I < 10^{18}\ \mathrm{W/cm^2}$ and relativistic parameters $mc^2/T > 100$. By respecting these two limits, it is obvious that α can take values well above 1 without thereby that the plasma is relativistic. Such plasmas can be found in many experimental and theoretical applications (see Ref. [15] and references therein).

The relevant mechanisms underlying this interaction are therefore electron-ion and electron-electron collisions, and the three-body photon-ion-electron interaction. It is supposed that the laser electric field is linearly polarized, $\vec{E}_h = \mathrm{Re}[E_0 \exp(-i\omega_0 t)\vec{e}_x]$, where ω_0 is the laser wave frequency, $\vec{e}_x$ is a unit vector along the x axis, and ions are assumed immobile because of their inertia. To solve Eq. (1) one assumes that the EDF can be split into a slowly varying part and a high-frequency part, respectively,

$$g(\vec{\mathrm{v}},t) = F(\vec{\mathrm{v}},t) + f_h(\vec{\mathrm{v}},t), \quad (2)$$

where $F(\vec{\mathrm{v}},t)$ is the homogeneous background EDF which evolves in time on the hydrodynamic time scale, and $f_h(\vec{\mathrm{v}},t)$ is the high-frequency EDF part induced by the laser field with the same laser time dependence on $\exp(-i\omega_0 t)$. Thus the kinetic equation can be also separated into these two time scales as it is corroborated by numerous simulations that have shown that the EDF is stationary on the velocity frame oscillating with the frequency ω_0. The high-frequency equation can be easily deduced from Eqs. (1) and (2),

$$\frac{\partial f_h}{\partial t} - C_{ei}(f_h) = \frac{e}{m}\vec{E}_h \cdot \frac{\partial F}{\partial \vec{\mathrm{v}}}. \quad (3)$$

To derive Eq. (3) we have used some approximations. The high-frequency magnetic field was neglected since $E_h/B_h \sim c$, and in the high-Z limit used in this work, electron-electron collisions smaller than the electron-ion collisions are not considered. The right-hand side of Eq. (3) is the source term for f_h and on the left-hand side, the analysis of the strength of each term gives $\omega_0 f_h \gg \nu_{ei} f_h$ where ν_{ei} is the electron-ion collision frequency. Within this ordering we can use the iterative method to solve Eq. (3), considering that the first term is the leading one and we get at the first iteration,

$$f_h = \frac{ie}{m\omega_0} E_h \frac{\partial F}{\partial v_x} - \frac{e}{m\omega_0^2} E_h C_{ei}\left(\frac{\partial F}{\partial v_x}\right). \tag{4}$$

The low-frequency kinetic equation is deduced from Eq. (1) taking its time average over the laser cycle $2\pi/\omega_0$,

$$\frac{\partial F}{\partial t} + \frac{v_0^2}{2}\frac{\partial}{\partial v_x} C_{ei}\left(\frac{\partial F}{\partial v_x}\right) = C_{ei}(F) + C_{ee}(F). \tag{5}$$

The second term on the left-hand side of Eq. (5) accounts for the heating term $\langle \frac{e}{m}\vec{E}_h \cdot \frac{\partial f_h}{\partial \vec{v}}\rangle$. Equation (5) is a nonlinear PDE depending on the variables $(\frac{v_x}{v}, v, t)$ that we will transform into an infinite set of equations by performing the expansion of the EDF on the Legendre polynomials basis $P_n(v_x/v)$, i.e., $F(\overrightarrow{v},t) = \sum_{n=0}^{\infty} F_n(v,t) P_n(v_x/v)$, and we get

$$\frac{\partial F_0}{\partial t} - C_{ee}(F_0) = \frac{\mathrm{v}_t^6}{6\lambda_{ei}} \frac{\alpha}{\mathrm{v}^2}\frac{\partial}{\partial \mathrm{v}}\left\{\frac{1}{\mathrm{v}}\left[\frac{\partial F_0}{\partial \mathrm{v}} + \frac{2}{5\mathrm{v}^3}\frac{\partial(\mathrm{v}^3 F_2)}{\partial \mathrm{v}}\right]\right\}, \tag{6}$$

$$\frac{\partial F_l}{\partial t} - C_{ee}(F_l) + \frac{l(l+1)}{\mathrm{v}^3}F_l = \frac{\mathrm{v}_0^2}{2}\left[\frac{l^2(l-1)}{2l-1}\mathrm{v}^{l-1}\frac{\partial}{\partial \mathrm{v}}\left(\frac{l-1}{2l-3}\frac{1}{\mathrm{v}^4}\frac{\partial}{\partial \mathrm{v}}\frac{F_{l-2}}{\mathrm{v}^{l-2}} + \frac{l}{2l+1}\frac{1}{\mathrm{v}^{2l+3}}\frac{\partial}{\partial \mathrm{v}}\mathrm{v}^{l+1}F_l\right)\right]$$
$$+ \frac{\mathrm{v}_0^2}{2}\left[\frac{(l+1)^2(l+2)}{2l+3}\frac{1}{\mathrm{v}^{l+2}}\frac{\partial}{\partial \mathrm{v}}\left(\frac{l+1}{2l+1}\mathrm{v}^{2l-1}\frac{\partial}{\partial \mathrm{v}}\frac{F_l}{\mathrm{v}^l} + \frac{l+2}{2l+5}\frac{1}{\mathrm{v}^4}\frac{\partial}{\partial \mathrm{v}}\mathrm{v}^{l+3}F_{l+2}\right)\right], \tag{7}$$

where

$$C_{ee}(F_0) = \frac{4\pi}{Z}\frac{v_t^4}{\lambda_{ei}}\left(\frac{F_0}{v^2}\int_0^v v^2 F_0 dv + \frac{1}{3v^3}\frac{\partial F_0}{\partial v}\int_0^v v^4 F_0 dv - \frac{1}{3}\frac{\partial F_0}{\partial v}\int_\infty^v v F_0 dv\right). \tag{8}$$

The subscript l is an even number ($l = 2,4,\ldots$), $\lambda_{ei} = \frac{\mathrm{v}_t}{\nu_{ei}} = \frac{4\pi\varepsilon_0^2 T^2}{ne^4 Z ln\Lambda}$ is the electron mean free path (see Ref. [17] and references therein), n is the electron density, ε_0 is the permittivity in vacuum, and $\ln\Lambda$ is the Coulomb logarithm. We assume that the stationary and the high-Z limit approximations are fulfilled thus, $C_{ei}(F_{l>0}) \gg [\frac{\partial F_{l>0}}{\partial t}, C_{ee}(F_{l>0})]$, and we can drop the first and the second terms in Eq. (7); however, they must be kept in Eq. (6). Equations (6) and (7) are an infinite set of coupled integrodifferential equations for the EDF components $f_{2n}(n = 0,1,\ldots)$ that we solved numerically with the standard finite difference scheme up to the order $n = 20$. One has checked that this order of truncation is sufficient for the accuracy required in this work. In addition the simulation accounts for the conservative properties of the lower moments of the EDF; i.e., $\int_{-\infty}^{\infty} F_0 d^3v = n$ and $\int_{-\infty}^{\infty}\frac{1}{2}mv^2 F_0 d^3v = \frac{3}{2}nT$. For the stability analysis of the EM modes, which constitutes the aim of this work, the second anisotropy $F_2(y)$ is the relevant component, where $y = v^2/2v_t^2$ is the normalized square velocity. We give in Fig. 1 the numerical results for $Z = 13$ and we can see that the maximum of $F_2(y)$ increases significantly with increasing α; e.g., for $\alpha = 0.1$ and $\alpha = 2$ we get $\frac{F_2(v)}{F_0(v=0)} = 0.018$ and $\frac{F_2(v)}{F_0(v=0)} = 0.24$, respectively. These results significantly improve previous results [14] limited to the range $\alpha < 0.5$. It is important to note that the inclusion of electron-electron collisions significantly alters the result of the anisotropic part of the EDF. To emphasize this point, we give in Fig. 2 the symmetric function $F_0(v)$ calculated by the present numerical simulation and the one provided by the numerical fit of Ref. [15] for $\alpha = 2$. We can see the large difference between the two results in particular for velocities smaller than the thermal velocity v_t. As a result the present simulation shows that the low-energy-electron population is more important. These findings play a central role in the stability analysis of the quasistatic EM modes addressed in the next section.

III. DISPERSION RELATION OF QUASISTATIC EM WAVES

Let us now consider the laser heated plasmas studied in Sec. II in the presence of a small low-frequency EM perturbation defined by the electric field $\overrightarrow{\delta E} = \delta E\vec{e}_x$ and $\overrightarrow{\delta B} = \delta B\vec{e}_y$ involving the geometry of the modes considered in this work.

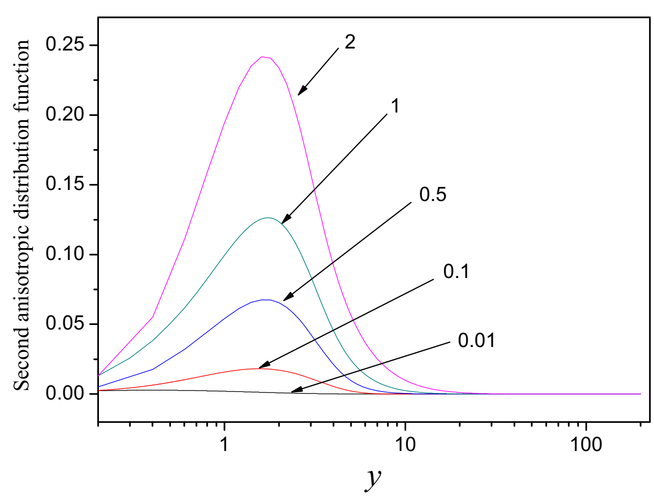

FIG. 1. Normalized second anisotropic distribution function $F_2(y)/F_0(y=0)$ as a function of the normalized square velocity $y = v^2/2v_t^2$ for different values of $\alpha = v_0^2/v_t^2$.

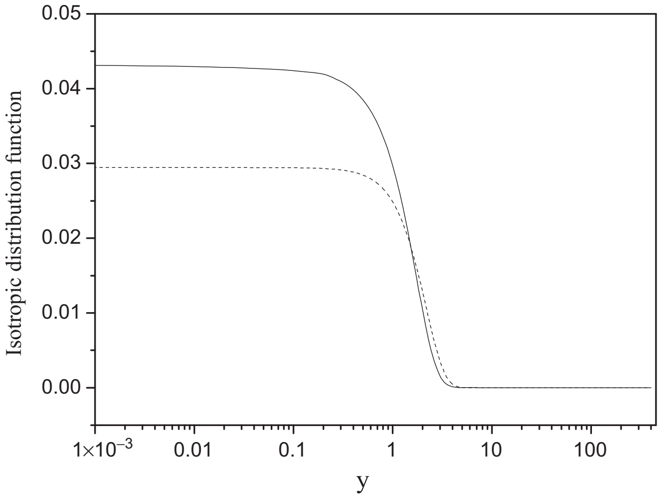

FIG. 2. Normalized isotropic electron distribution function $F_{0\text{norm}} = (2\pi)^{3/2} F_0(y) v_t^3/n$ as a function of the normalized square velocity $y = v^2/2v_t^2$ for $\alpha = \frac{v_0^2}{v_t^2} = 2$. The solid line is obtained from the present numerical simulation and the dashed line corresponds to the numerical fit of Ref. [15].

The plasma can be represented by a homogeneous unperturbed state represented as shown in Sec. II by the EDF $g(\overrightarrow{\mathrm{v}},t) = F(\overrightarrow{\mathrm{v}},t) + f_h(\vec{\mathrm{v}},t)$ and a time-dependent and inhomogeneous state represented by the perturbed EDF $\delta f(\vec{\mathrm{v}},\vec{r},t)$. We assume that $[\overrightarrow{\delta E},\overrightarrow{\delta B},\delta f(\vec{\mathrm{v}},\vec{r},t)] \propto \exp(-i\omega t + i\vec{k}\cdot\vec{r})$, where ω and $\vec{k}$ are the low frequency and the wave vector directed along the direction Oz, of the EM mode. The total EDF therefore can be written as $h(\vec{r},\vec{\mathrm{v}},t) = F(\vec{\mathrm{v}},t) + f_h(\vec{\mathrm{v}},t) + \delta f(\vec{r},\vec{\mathrm{v}},t)$ and it obeys the following Vlasov-Landau equation:

$$\frac{\partial h}{\partial t} + \vec{\mathrm{v}}\cdot\nabla h - \frac{e}{m}(\overrightarrow{\delta E} + \vec{v}\times\overrightarrow{\delta B})\cdot\frac{\partial h}{\partial \vec{\mathrm{v}}} - \frac{e}{m}\vec{E}_h\cdot\frac{\partial h}{\partial \vec{\mathrm{v}}} = C_{ei}(h) + C_{ee}(h). \tag{9}$$

Keeping the high-frequency terms in Eq. (9) we obtain

$$\frac{\partial f_h}{\partial t} - \frac{e}{m}(\overrightarrow{\delta E} + \vec{v}\times\overrightarrow{\delta B})\cdot\frac{\partial f_h}{\partial \vec{\mathrm{v}}} - C_{ei}(f_h) = \frac{e}{m}\vec{E}_h\cdot\frac{\partial F}{\partial \vec{\mathrm{v}}}. \tag{10}$$

The low-frequency part is split into a leading order given by Eq. (5) and the following perturbed equation:

$$-i\omega\delta f + ikv_z\delta f - C_{ei}(\delta f) - \frac{e}{m}\delta E\frac{\partial F}{\partial v_x} - \frac{e}{m}\delta B\left(v_x\frac{\partial F}{\partial v_z} - v_z\frac{\partial F}{\partial v_x}\right) = S_{\mathrm{IB}}, \tag{11}$$

where $S_{\mathrm{IB}} = \delta\langle\frac{e}{m}\vec{E}_h\cdot\frac{\partial f_h}{\partial \vec{\mathrm{v}}}\rangle$ is the inverse bremsstrahlung term.

To get the explicit expression of S_{IB} we solve Eq. (10) iteratively with the use of the ordering $\omega_0 f_h \gg (\nu_{ei} f_h, \omega_c f_h)$ where $\omega_c = eB/m$ is the electron cyclotron frequency,

$$f_h = \frac{ie}{m\omega_0}E_h\frac{\partial F}{\partial v_x} - \frac{e^2}{m^2\omega_0^2}\delta E\frac{\partial}{\partial v_x}\left(E_h\frac{\partial F}{\partial v_x}\right) - \frac{e^2}{m^2\omega_0^2}\left[v_x\delta B\frac{\partial}{\partial v_z}\left(E_h\frac{\partial F}{\partial v_x}\right) - v_z\delta B\frac{\partial}{\partial v_x}\left(E_h\frac{\partial F}{\partial v_x}\right)\right] - \frac{e}{m\omega_0^2}E_h C_{ei}\left(\frac{\partial F}{\partial v_x}\right), \tag{12}$$

Then we deduce the inverse bremsstrahlung term taking the time average over a laser cycle,

$$S_{\mathrm{IB}} = -\frac{v_0^2}{2}\frac{\partial}{\partial v_x}C_{ei}\left(\frac{\partial\delta f}{\partial v_x}\right) - \frac{ev_0^2}{2m}\delta E\frac{\partial^3 F}{\partial v_x^3} - \frac{ev_0^2}{2m}\delta B\frac{\partial}{\partial v_x}\left(v_x\frac{\partial^2 F}{\partial v_x\partial v_z} - v_z\frac{\partial^2 F}{\partial v_x^2}\right), \tag{13}$$

and finally, it results in the desired perturbed low-frequency equation,

$$-i\omega\delta f + ikv_z\delta f + \frac{v_0^2}{2}\frac{\partial}{\partial v_x}C_{ei}\left(\frac{\partial\delta f}{\partial v_x}\right) - C_{ei}(\delta f) = S, \tag{14}$$

where the source term stands for

$$S = \frac{e}{m}\delta E\frac{\partial F}{\partial v_x} + \frac{e}{m}\delta B\left(v_x\frac{\partial F}{\partial v_z} - v_z\frac{\partial F}{\partial v_x}\right) - \frac{ev_0^2}{2m}\delta E\frac{\partial^3 F}{\partial v_x^3} - \frac{ev_0^2}{2m}\delta B\frac{\partial}{\partial v_x}\left(v_x\frac{\partial^2 F}{\partial v_x\partial v_z} - v_z\frac{\partial^2 F}{\partial v_x^2}\right). \tag{15}$$

We can see that the third term on the left-hand side of Eq. (14) involves a differential operator of δf. This term considerably enhances the mathematical difficulty for analytically solving this equation. Even though its role in the Weibel analysis is marginal since the more unstable modes are mainly collisionless, we first solved Eq. (14) neglecting this differential term, and then we iteratively calculated its strength, finding as expected a negligible contribution. In Ref. [7], the terms proportional to v_0^2 in Eq. (15) were neglected while in Ref. [22] the lower order on α, included in the last term, was kept as a stabilizing term. In this work we consider the full expression (15) and make use of the results derived in Ref. [7] to get the following growth rate:

$$\gamma(k) = \frac{\frac{3k^2c^2n_0}{64\pi\omega_p^2\lambda_{ei}\mathrm{v}_t{}^2} - \frac{12}{5}\sqrt{2}\lambda_{ei}\mathrm{v}_t k^2\int_0^\infty y^{9/2}G\,W(y)dy}{\int_0^\infty y^3 H\,P(y)dy}, \tag{16}$$

where the functions $G(y)$ and $H(y)$ are continued fractions defined in Ref. [5] and their numerical fits read

$$F = \frac{(1 + 2.9k^2\lambda_{ei}{}^2y^4)^{-1/2}}{2},$$

and

$$G = \frac{1 + 865k^2\lambda_{ei}^2y^4}{6\sqrt{1 + 2.9k^2\lambda_{ei}^2y^4}\left(1 + \sqrt{1 + 2.9k^2\lambda_{ei}^2y^4}\right)\left(1 + 640k^2\lambda_{ei}^2y^4\right)}.$$

The new source term of the EM instability and the terms generated by the electric field are derived in the Appendix where we found

$$W(y) = F_2 - \frac{1}{3}\alpha y \frac{\partial^2 F_0}{\partial y^2} + \alpha\left(-\frac{5}{7}\frac{\partial F_2}{\partial y} + \frac{5}{7}\frac{F_2}{y} - \frac{10}{21} y \frac{\partial^2 F_2}{\partial y^2}\right) + \alpha\left(-\frac{80}{63}\frac{\partial F_4}{\partial y} - \frac{20}{21}\frac{F_4}{y} - \frac{16}{63} y \frac{\partial^2 F_4}{\partial y^2}\right), \tag{17}$$

and

$$P(y) = \frac{\partial F_0}{\partial y} + \left(\frac{3}{5}\frac{F_2}{y} + \frac{2}{5}\frac{\partial F_2}{\partial y}\right) - \alpha\left(\frac{1}{7}\frac{F_4}{y^2} + \frac{76}{105}\frac{1}{y}\frac{\partial F_4}{\partial y} + \frac{4}{21}\frac{\partial^2 F_4}{\partial y^2} - \frac{9}{35}\frac{F_2}{y^2} + \frac{9}{35}\frac{1}{y}\frac{\partial F_2}{\partial y} + \frac{48}{35}\frac{\partial^2 F_2}{\partial y^2} + \frac{12}{35} y \frac{\partial^3 F_2}{\partial y^3} + \frac{3}{2}\frac{\partial^2 F_0}{\partial y^2} + \frac{3}{5} y \frac{\partial^3 F_0}{\partial y^3}\right). \tag{18}$$

Thus the expression of the growth rate (16) accounts for all the contributions generated by the electric $\delta\overrightarrow{E}$ and the magnetic field $\delta\overrightarrow{B}$. In addition, unlike previous works, the anisotropy of order four is involved in the expression of the DR. One has found also that all the other components $F_{n>4}$ do not contribute to the growth rate.

In Fig. 3 we display the growth rate as a function of the collisionality parameter $k\lambda_{ei}$. We can see that for $\alpha = 1$ and $\alpha = 2$, the quasistatic EM modes are strongly unstable. In particular for $\alpha = 1$ we obtain $\gamma_{\max} = 5.5 \times 10^{12}\,\mathrm{s}^{-1}$ and for $\alpha = 2$, the growth rate reaches a very high value, about $\gamma_{\max} = 1.6 \times 10^{13}\,\mathrm{s}^{-1}$. In both cases the most unstable modes are within the quasicollisionless range since $k\lambda_{ei} = 17$ and $k\lambda_{ei} = 27$, respectively. We should remark, however, that significant unstable modes are also driven in the collisional part of the spectrum; e.g., for $\alpha = 2$ and $k\lambda_{ei} = 0.2$, growth rates of about $1.4 \times 10^{11}\,\mathrm{s}^{-1}$ can be observed. For increasing α, the general behavior of this instability can be summarized as follows: the maximum growth rate increases significantly with α, the spectrum of unstable modes becomes wider toward both collisional and collisionless range, and the most unstable modes are almost collisionless. It may be noted that nonlinear stabilizing terms significantly reduce the growth rates as shown by the dashed curves in Fig. 3.

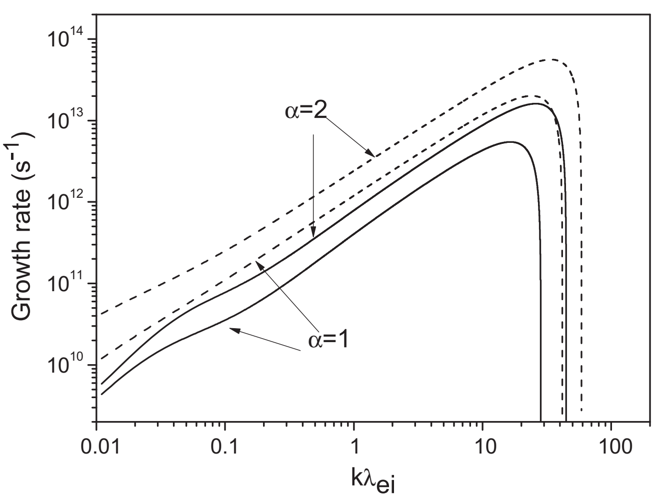

FIG. 3. Growth rate as a function of the dimensionless parameter $k\lambda_{ei}$. The solid lines correspond to the results with the full expressions given by Eqs. (17) and (18); the dashed lines correspond to the results obtained with only the first terms in Eqs. (17) and (18). The simulation parameters are $Z = 13$, $T = 5$ keV, an electron density $n = 9.8 \times 10^{20}\,\mathrm{cm}^{-3}$, and a laser wavelength $\lambda_{\mathrm{laser}} = 1.06\,\mu$m.

It was numerically checked that for $\alpha \geqslant 1$, the new stabilizing terms in Eqs. (17) and (18) are comparable in magnitude to the terms used previously [22]; therefore this justifies taking them into account in the stability analysis of the EM modes. For $\alpha \ll 1$ the stabilizing term derived in Ref. [22] [the second term in Eq. (17)] is dominant; the other terms proportional to α^n with $n > 2$ are still negligible. By increasing α, the temperature anisotropy becomes larger and the instability driven term is, hence, more important. However, in the same time the stabilizing terms increase but less significantly than the source term; as a result the growth rate finally enhances with increasing α. For high values of α typically greater than 0.5, oscillatory electric fields may drive strong quasistatic EM growing modes and magnetic fluctuations present in the plasma could be amplified to the megagauss range. In Ref. [14] only one numerical value for the growth rate was presented. Taking the same physical conditions ($\alpha = 0.3$, $T = 4$ keV, and $n = 9.8 \times 10^{20}\,\mathrm{cm}^{-3}$) we obtain a growth rate about $10^{11}\,\mathrm{s}^{-1}$ which is one order of magnitude lower than the result obtained in Ref. [14]. This can be explained by the fact that in Ref. [14], the kinetic model does not include the stabilizing terms and uses a numerical fit for the isotropic EDF F_0 valid only in the range $\alpha \ll 1$.

We can mention other saturation mechanisms of this instability such as those displayed in Eqs. (17) and (18). First, generally the feedback effects of the magnetic field on the plasma should lead to a saturation of the B field when it reaches very high intensities. In addition unstable EM modes are generated in a wide extent of wave numbers. For $\alpha = 2$, a typical mean free path about $\lambda_{ei} \sim 1\,\mu$m, and growth rates greater than $10^9\,\mathrm{s}^{-1}$, the wave numbers range from 10^4 to $4.4 \times 10^7\,\mathrm{m}^{-1}$. For a typical Larmor radius of about $r_L \sim 1\,\mu$m, the quasilinear regime is met if roughly the condition $\Delta k r_L < 1$ is verified [23]. It should therefore be expected that the EM modes in turn influence the EDF. Owing to the quasilinear diffusion, the EDF will evolve slowly with the time, resulting in saturated growing modes. Likewise the energy of a strongly unstable mode can be transferred from a part of the spectrum to another one by the mode coupling mechanisms which have the effect of reducing the instability efficiency. In laser-created plasmas the effects of the magnetic field can also be moderated by the plasma inhomogeneity responsible for the convection of the magnetic modes.

IV. SUMMARY

In this paper the stability analysis of EM modes in plasmas heated by the inverse bremsstrahlung mechanism is presented in the physical conditions as the electron quiver velocity can be comparable in magnitude to the thermal velocity ($\alpha \leqslant 2$). A DR was analytically derived that takes into account the contribution of all nonlinear terms on α into the expression of the growth rate. Applications of our results to typical inertial fusion plasmas have shown that EM modes with growth rate larger than $10^{12}\ \mathrm{s}^{-1}$ are driven by the temperature anisotropy induced by the collisional absorption. This result complements that of Ref. [7] where moderate growing modes were found in the range $\alpha \ll 1$. Although quasilinear or nonlinear mechanisms should lead to a saturation of the instability, strongly growing EM modes that reach the megagauss range are expected in plasmas heated by a laser wave in the regime $\alpha \geqslant 1$. We note that in inhomogeneous plasmas this magnetic field source must be added to other magnetic sources (thermoelectric source, etc.) and as a result the total self-generated magnetic field is the sum of all these contributions. This should produce a B field that has a large spatial scale extent ranging from the collisional to collisionless limits. The relativistic regime as studied by Avetissian *et al.* for nonlinear collisional absorption [19,20] remains an open problem for this instability and it will be addressed in a future work.

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APPENDIX: SOURCE AND STABILIZING TERMS IN THE PERTURBED VLASOV-LANDAU EQUATION

In this Appendix we give the expansion of the source term given by Eq. (15) on the Legendre polynomial basis. For the sake of clarity we rewrite this equation below:

$$S = \frac{e}{m}\delta B\left(v_x\frac{\partial F}{\partial v_z} - v_z\frac{\partial F}{\partial v_x}\right) + \frac{e}{m}\delta E\frac{\partial F}{\partial v_x} - \frac{ev_0^2}{2m}\delta E\frac{\partial^3 F}{\partial v_x^3} - \frac{ev_0^2}{2m}\delta B\frac{\partial}{\partial v_x}\left(v_x\frac{\partial^2 F}{\partial v_x \partial v_z} - v_z\frac{\partial^2 F}{\partial v_x^2}\right), \tag{A1}$$

where the EDF is expanded as

$$F(\vec{v},t) = \sum_{n=0}^{\infty} P_n(\mu)F_n(v,t),$$

and where

$$P_n(\mu) = \frac{1}{2^n n!}\frac{d^n}{d\mu^n}[(\mu^2-1)^n]$$

is the Legendre polynomial of order n. To calculate the different terms in Eq. (A1) we will use throughout this Appendix the standard recursive relations

$$(1-\mu^2)P' = \frac{n(n+1)}{2n+1}(P_{n-1} - P_{n+1}), \tag{A2}$$

$$\mu P_n = \frac{n+1}{2n+1}P_{n+1} + \frac{n}{2n+1}P_{n-1}\theta. \tag{A3}$$

In the first step let us develop the first order derivative of the EDF,

$$\frac{\partial F}{\partial v_z} = \sum_{n=0}^{\infty}\frac{v_z}{v}\left(-\mu P'_n\frac{F_n}{v} + P_n\frac{dF_n}{dv}\right),$$

$$\frac{\partial F}{\partial v_x} = \sum_{n=0}^{\infty}\left[P'_n(1-\mu^2)\frac{F_n}{v} + \mu P_n\frac{dF_n}{dv}\right].$$

We can easily express the two first terms in (A1) as

$$\left(v_x\frac{\partial F}{\partial v_z} - v_z\frac{\partial F}{\partial v_x}\right) = \sum_{n=0}^{\infty} -\frac{v_z}{v}P'_n F_n, \tag{A4}$$

and

$$\frac{\partial F}{\partial \mathrm{v}_x} = \sum_{n=0}^{\infty}\left[\frac{(n+1)(n+2)}{2n+3}\frac{F_{n+1}}{v} + \frac{n+1}{2n+3}\frac{dF_{n+1}}{dv} - \frac{n(n-1)}{2n-1}\frac{F_{n-1}}{v} + \frac{n}{2n-1}\frac{dF_{n-1}}{dv}\right]P_n. \tag{A5}$$

Then to calculate the third term in (A1), we calculate the second order derivative,

$$\frac{\partial^2 F}{\partial v_x^2} = \sum_{n=0}^{\infty} C_n P_n,$$

where

$$C_n = \left[\frac{(n+1)(n+2)}{2n+3}\frac{B_{n+1}}{v} + \frac{n+1}{2n+3}\frac{dB_{n+1}}{dv} - \frac{n(n-1)}{2n-1}\frac{B_{n-1}}{v} + \frac{n}{2n-1}\frac{dB_{n-1}}{dv}\right]$$

and

$$B_n = \left[\frac{(n+1)(n+2)}{2n+3}\frac{F_{n+1}}{v} + \frac{n+1}{2n+3}\frac{dF_{n+1}}{dv} - \frac{n(n-1)}{2n-1}\frac{F_{n-1}}{v} + \frac{n}{2n-1}\frac{dF_{n-1}}{dv}\right].$$

In addition we calculate the third order derivative, obtaining

$$\frac{\partial^3 F}{\partial \mathrm{v}_x^3} = \sum_{n=0}^{\infty}\left[\frac{(n+1)(n+2)}{2n+3}\frac{C_{n+1}}{v} + \frac{n+1}{2n+3}\frac{dC_{n+1}}{dv} - \frac{n(n-1)}{2n-1}\frac{C_{n-1}}{v} + \frac{n}{2n-1}\frac{dC_{n-1}}{dv}\right]P_n. \tag{A6}$$

The last term in Eq. (A1) involves two second order derivatives of the EDF that we can express in the form

$$\frac{\partial}{\partial v_x}\left(v_x\frac{\partial^2 F}{\partial v_x \partial v_z} - v_z\frac{\partial^2 F}{\partial v_x^2}\right) = v_z\left(\{(\mu^2-2)P'_n + [2n(n+1)+1]\mu P_n\}\frac{1}{v^2}\frac{dF_n}{dv} + n(n+1)[(1-\mu^2)P'_n - 2\mu P_n]\frac{F_n}{v^3} - (\mu P_n + \mu^2 P'_n)\frac{1}{v}\frac{d^2F_n}{dv^2}\right). \tag{A7}$$

The geometry of the EM modes studied in this work is summarized by $\frac{v_z}{v} = \cos\theta$ and $\mu = \sin\theta\cos\varphi$. Thus the perturbed EDF and the right-hand side of Eq. (14) are expanded on the spherical harmonics basis $Y_l^m(\theta,\varphi)$ and the relevant component is $\delta f_2^{\mp 1}$. This component is required to deduce the low-frequency current density in the Ampere-Maxwell equation. The result is that only the components proportional to $Y_2^{\mp 1}(\theta,\varphi)$ in the expressions W and P in Eqs. (17) and (18) are relevant. More precisely we keep the components proportional to μ in Eqs. (A5) and (A6) and the components proportional to $\mu\cos\theta$ in Eqs. (A4) and (A7). The relevant terms in Eqs. (A5) and (A6) are obvious and can be readily obtained. To determine explicit terms from Eqs. (A4) and (A7) we integrate these equations over the solid angle $d\Omega = \sin(\theta)d\theta d\varphi$ and use the orthonormalization condition for the spherical harmonics. We get from (A4) the first term in Eq. (17) and from (A7) the three terms proportional to α. The other components provided a vanishing contribution.

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