Controlling ratchet transport via a finite kicked environment

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We study the effects of a finite kicked environment (bath) composed of N harmonic oscillators on the particle transport in a weakly dissipative quasisymmetric potential system. The small spatial asymmetry is responsible for the appearance of directed particle transport without a net bias, known as the ratchet transport. The whole dynamics is governed by a generalized map where dissipation in the system emerges due to its interaction with the kicked environment. Distinct spectral densities are imposed to the bath oscillators and play an essential role in such models. By changing the functional form of the spectral density, we observe that the transport can be optimized or even suppressed. We show evidences that the transport optimization is related to stability properties of periodic points of the ratchet system and depends on the bath temperature. In a Markovian approach, transport can be increased or suppressed depending on the bath influence.

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I. INTRODUCTION

A better understanding of the nonequilibrium thermodynamics is a challenging problem in statistical mechanics [1–9]. While most of these works are concerned with fluctuation theorems for systems in equilibrium, in this work we are interested in the dynamics and transport of particles in nonequilibrium situations arising from kicked environments. Consider a system composed of a free particle receiving kicks from the environment at periodic times. Such a system will be out of equilibrium if the time between the kicks is not large enough. In general, any particle coupled to a finite kicked environment is driven out of equilibrium by the kicks. In distinction to systems coupled to usual infinite and nonkicked environments, where the environment effects on the system can be described by the generalized Langevin equation [10], finite kicked environments can be described by a generalized finite map and unusual fluctuation-dissipation relations are observed due to the kicked nature of the environment [11].

More specifically, the present work is concerned with the effect of kicked environments on the directed transport of particles in the absence of a net bias, the so-called ratchet effect. These systems are genuine candidates to describe transport since their spatial and time asymmetry, combined with a periodic time-dependent force, induce the net motion of particles, named ratchet current. Such a current has relevant applications, for example, in cells built capable to control metastasis [12], cold atoms [13], ordered pedestrian motion [14], microfluidics [15], self-propelled leidenfrost droplets [16], coherent transport of ultracold atoms [17], transport in a graphene lattice [18], and the drift ratchet current in conformal crystal pinning arrays [19], among many others. The direction and intensity of the ratchet current strongly depends on the appropriate combination of the involved physical parameters: dissipation, intensity of the time-dependent force, and the spatial asymmetry. It has been shown that optimized ratchet currents occur along generic isoperiodic stable structures, which live in the parameter space [20] and tend to survive for reasonable temperature intervals [21]. It has been shown in these works that when parameters are chosen inside the generic structures, the regular motion leads to optimized ratchet currents and may resist to temperature changes. Another way to obtain larger ratchet transport is by using properties of mixed phase spaces of conservative systems. Almost invisible islands embedded in the chaotic sea can be responsible for the generation of large ratchet currents at least in two cases: small dissipation and large spatial asymmetry [22], small dissipation and tiny spatial asymmetry [23]. Here, we show the effect of the finite kicked environment on the ratchet transport in the regime of tiny asymmetry.

The paper is presented as follows. In Sec. II we present the model, which consists of a Markovian generalized map capable to mimic a particle subject to a general potential. Next, in Sec. III we discuss the ratchet current effect for systems with and without spatial symmetry and how the ratchet current can be influenced by the properties of the kicked environment. In Sec. IV we show how it is possible to control the ratchet transport by changing the bath parameters. Finally, in Sec. V we present the conclusions of our work.

II. THE MODEL

In this work we use the generalized map derived in Ref. [11], which consists of a generic system bilinearly coupled to a bath composed of N free particles kicked harmonically. For consistency, we summarize here the main steps of the derivation of the generalized map. The original problem is a system plus bath model and is described by the total Hamiltonian:

$$H = H_S + H_B + H_I, \tag{1}$$

being H_S the system particle Hamiltonian given by

$$H_S = \frac{P^2}{2M} + V_{\rm rat}(X) \sum_{n=-\infty}^{\infty} \delta\left(\frac{t}{\tau} - n\right), \tag{2}$$

with the ratchet kick $V_{\rm rat}(X) \sum_{n=-\infty}^{\infty} \delta(\frac{t}{\tau} - n)$ [see Eq. (18)]. P and X are, respectively, the momentum and position of the system particle, and M is its mass. H_B is the Hamiltonian from

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the bath composed of N free particles kicked harmonically and is given by

$$H_B = \sum_{i=1}^{N}\left[\frac{p_i^2}{2m_i} + \frac{m_i\omega_i^2}{2}x_i^2\sum_{n=-\infty}^{\infty}\delta\left(\frac{t}{\tau}-n\right)\right], \quad (3)$$

with p_i and x_i being, respectively, the momentum and position of the ith oscillator with mass m_i and frequency ω_i. The interaction Hamiltonian H_I is given by

$$H_I = -\sum_{i=1}^{N}\gamma_i x_i X\sum_{n=-\infty}^{\infty}\delta\left(\frac{t}{\tau}-n\right). \quad (4)$$

Uppercase letters refer to the system particle, while lowercase letters refer to the environmental degrees of freedom. The model is quite general since kicked systems, kicked interactions, and kicked baths can be nowadays implemented in many experiments (see Ref. [11]).

Since the equations of motion from the bath are linear, which allows for analytical solutions, *effective* equations of motion can be obtained *solely* for the system. As shown in details in Ref. [11], this leads to the effective generalized dimensionless map

$$P_{n+1} = (1-\lambda)P_n - \frac{\partial V_{\rm rat}(X_n)}{\partial X_n} + F_n - \sum_{n'=1}^{n-1}\kappa_{n,n'}P_{n'}, \quad (5)$$

$$X_{n+1} = X_n + P_{n+1}, \quad (6)$$

where λ is the dissipation given by $\sum_{i=1}^{N}\gamma_i^2/k_i^2$, and γ_i is the coupling between the system and the ith oscillator. The dimensionless frequency parameter of each oscillator, $k_i \equiv \omega_i\tau$, is constructed such that it is dependent on the kicking time. There is no special reason to choose the same period τ for the kicks in system, bath, and interaction. One can choose different periods for each one of the kicks, given by the sum of δ's in Eqs. (2), (3), and (4). Physically speaking, due to the scaling $k_i = \omega_i\tau$, one can change τ for each oscillator of the bath and keeping ω constant. Hence, the effect in change τ translates in change its natural frequency. The sum term in Eq. (5) is the discrete analog of the Langevin memory kernel in the generalized Langevin equation (GLE) [10] and is given by

$$\kappa_{n,n'} = \sum_{i=1}^{N}\left(\frac{\gamma_i}{k_i}\right)^2 g_{n-n'-1}^{(i)}. \quad (7)$$

The discrete fluctuating force F_n, which also contains information coming from the bath, writes

$$F_n = \sum_{i=1}^{N}\gamma_i\left[g_{n-1}^{(i)}x_0^{(i)} + k_i f_{n-1}^{(i)}p_0^{(i)}\right], \quad (8)$$

with properties

$$\langle F_n\rangle = 0, \quad (9)$$

$$\langle F_nF_m\rangle = \sum_{i=1}^{N}\frac{\gamma_i^2}{\beta k_i^2(1-k_i^2/4)}g_{n-m-1}^{(i)}, \quad (10)$$

where $\langle\cdots\rangle$ means ensemble average.

The set $(x_0^{(i)}, p_0^{(i)})$ corresponds to the initial condition of the ith bath oscillator, whose dynamics is described through the dimensionless map:

$$p_{n+1}^{(i)} = p_n^{(i)} - k_i x_n^{(i)}, \quad (11)$$

$$x_{n+1}^{(i)} = x_n^{(i)} + k_i p_{n+1}^{(i)}. \quad (12)$$

As in the GLE from a microscopic derivation [10,11,24,25], it is possible to assign a temperature to the bath through a canonical distribution $\rho(x_0,p_0) \sim e^{-\beta H_B(x_0,p_0)}$, $H_B(x_0,p_0)$ being the bath Hamiltonian. In this case, H_B is not constant due to the periodic kicks. However, an average (over one kicking time) constant of motion for each of the N kicked harmonic oscillators can be obtained and is given by

$$h(x_0,p_0) = \frac{x_0^2}{2} + \frac{p_0^2}{2} - \frac{k}{2}x_0p_0, \quad (13)$$

where we omit the i index at each p_0 and x_0 variable and also in k, meaning that the constant of motion has the same form for all the N bath oscillators.

The coefficients $\{g_n\}_{n=-1}^{n-1}$ and $\{f_n\}_{n=-1}^{n-1}$ are evolved according to the maps:

$$g_{n+1}^{(i)} = g_n^{(i)} - k_i^2 f_{n+1}^{(i)}, \quad (14)$$

$$f_{n+1}^{(i)} = f_n^{(i)} + g_n^{(i)}, \quad (15)$$

with initial conditions satisfying necessarily $g_0^{(i)} = 1 - k_i^2$ and $f_0^{(i)} = 1$ for $i = 1,2,\ldots,N$, and $k_i \leqslant 2$. For $k_i > 2$, and any $i = 1,2,\ldots,N$ the bath dynamics diverge.

Similar to the GLE, our generalized map Eq. (5) describes the evolution of the momentum of a particle coupled to a non-Markovian bath. In order to consider a bath with Markovian dynamics, the coefficients $g_{n-n'-1}$ from Eq. (7) must be chosen in such a way that we obtain $\kappa_{n,n'} \to 0$. In this case the fluctuating force properties obey

$$\langle F_n\rangle = 0, \quad (16)$$

$$\langle F_n^2\rangle = \sum_{i=1}^{N}\frac{\gamma_i^2}{\beta k_i^2(1-k_i^2/4)}. \quad (17)$$

It is well known that transport of particles without external bias can be studied using ratchet systems. Here we assume the ratchet potential for the system as

$$V_{\rm rat}(X) = K\left[\cos(X) + \frac{a}{2}\cos\left(2X+\frac{\pi}{2}\right)\right], \quad (18)$$

where K is related to kick intensity and a is the asymmetry parameter. If $a = 0$, the potential is completely symmetric and no ratchet effect is expected. In this symmetric case, with no coupling with the bath, the problem reduces to the standard map [26]. Without bath and for $a \neq 0$ the particle dynamics is completely described by the ratchet map studied before [20,22,23]. The ratchet potential from Eq. (18) is plotted in Fig. 1 for two values of a.

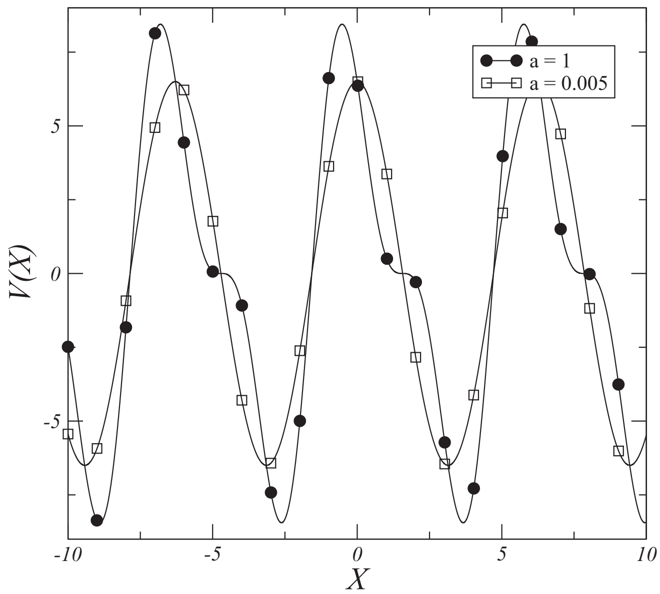

FIG. 1. Form of the ratchet potential for the system particle given by Eq. (18) for the case where $K = 6.5$ and two values of asymmetry parameter a specified in the figure.

Finally, the generalized map becomes

$$P_{n+1} = (1-\lambda)P_n + K\left[\sin(x_n) + a\sin\left(2x_n + \frac{\pi}{2}\right)\right] + F_n, \quad (19)$$

$$X_{n+1} = X_n + P_{n+1}, \quad (20)$$

and describes a particle subject to a ratchet potential coupled to a finite kicked Markovian environment. As we will see, the statistical properties of F_n strongly depend on the statistical properties of the bath harmonic oscillators frequencies.

III. THE RATCHET CURRENT

In the absence of the stochastic force [F_n in the model Eq. (19)], the ratchet current has been analyzed carefully in the following parameters interval: for fixed $K = 6.5$, $a = 0.25$ and dissipation inside $0 \leqslant \lambda < 1$ in Ref. [22]; for fixed λ and $5 \leqslant K \leqslant 10$, $a = 0.005$ in Ref. [23]; in the parameter space (K,λ) with $0 \leqslant \lambda < 1$ and $2 \leqslant K \leqslant 15$ with fixed $a = 0.5$ in Ref. [20]; and for all parameter combinations in Ref. [27]. The dynamics as a function of the parameters is very complex and can be summarized as if parameters are chosen inside the generic isoperiodic stable structures (regular motion), the ratchet current can be very large and efficient [20]. In general, temperature effects tend to destroy the current but remains persistent for reasonable temperatures when the parameters are chosen inside the stable structures [21]. However, temperature-induced currents were observed in the classical case and also vacuum fluctuations induced quantum currents [28].

The transport property of such systems can be quantified through the ratchet current, defined as

$$\langle P\rangle = \frac{1}{\mathcal{N}}\sum_{c=1}^{\mathcal{N}} P_m^{(c)}, \quad (21)$$

$\mathcal{N}$ being the number of initial conditions, and $m = 1000$ is the length of the trajectory used to compute the time average particle momentum P.

If the system particle is governed by a symmetric potential, like the standard map, the island structures responsible for the anomalous transport presented by the map are symmetric, generated in pairs, and the bifurcations occur at the same control parameter. Thus, we expect a zero value for the transport since the effort of each island to promote positive or negative transport is the same. On the other hand, a particle subject to an asymmetric potential will present distinct bifurcation parameters for pairs of islands, where a nonzero transport will take place. Another possibility was reported recently in Ref. [23] showing that taking advantage of the hyperbolic and nonhyperbolic characteristics displayed by the phase space of weakly dissipative systems, even for the case of slightly asymmetric potentials, it is possible to obtain large ratchet currents. In fact, in these cases, the presence of ratchet currents is influenced not so much by the potential asymmetry but rather by the existence of such strongly nonhyperbolic regions in the phase space of weakly dissipative systems.

The effect mentioned above can be seen in the phase space of the conservative ($\lambda = 0$) and dissipative ($0 < \lambda < 1$) ratchet map, given by Eq. (19) without bath ($F_n = 0, \forall n$). In Fig. 2 we depicted the evolution of 10 initial conditions equally distributed for 2×10^3 times for $\lambda = 0$ in (a) and $\lambda = 0.02$ in (b). While at $\lambda = 0$ the system displays two symmetric islands (one before $X = \pi$ in line $P = 0$ and the other after $X = \pi$), both islands become attractors with asymmetric basin

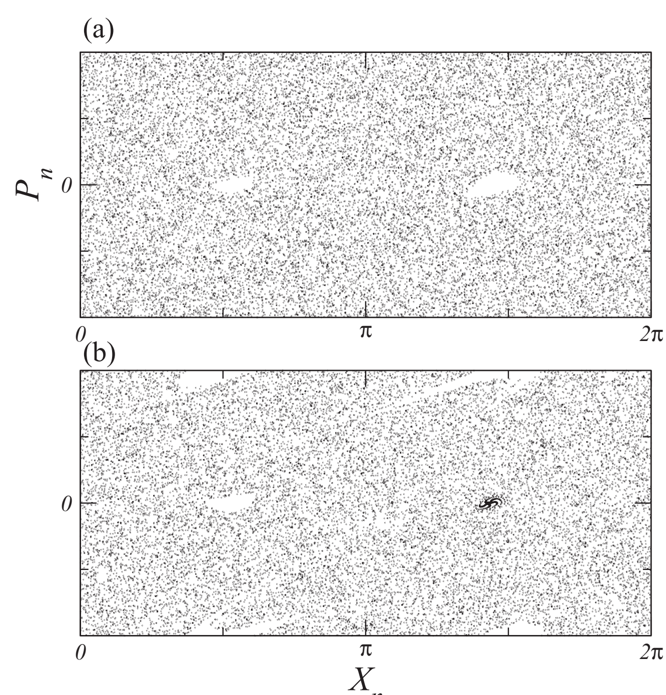

FIG. 2. Phase space numerically generated by following 10 trajectories initialized by equally distributed initial conditions inside the square $[0,2\pi)$ for X and $(-\pi,\pi)$ for P in 2×10^3 times illustrating the dissipation effects. For this figure, $a = 0.005$ and (a) $\lambda = 0$ and (b) $\lambda = 0.02$.

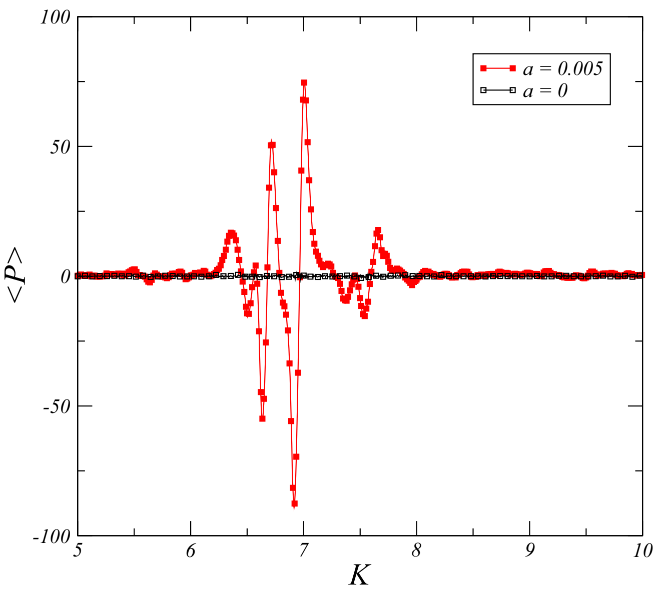

FIG. 3. Ratchet current as a function of nonlinearity parameter K evaluated for $a = 0$ (red line with filled squares) and $a = 0.005$ (black line with empty squares).

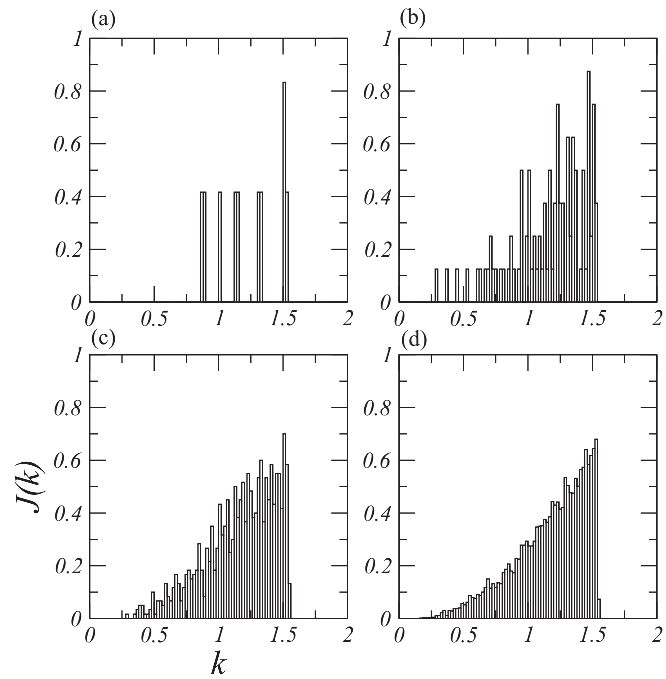

FIG. 4. Frequencies of the bath oscillators chosen according to a quadratic distribution law ($\alpha = 2$) for (a) $N = 10$, (b) $N = 100$, (c) $N = 1000$, and (d) $N = 10\,000$ oscillators.

boundaries for $\lambda \neq 0$. This kind of asymmetry is responsible for the transport measured.

For $a \neq 0$ the spatial asymmetry is relevant and nonzero currents are obtained even without external bias. In Fig. 3 we depicted the ratchet current for the standard map (back, gray line) and compare it with the current from the ratchet map (no bath) with a small asymmetry $a = 0.005$ (red, light gray line). We clearly observe that, while the standard map does not generate finite currents, the ratchet asymmetry induces large positive and negative currents for values of K in the interval [6,8].

In the next section we will show how a kicked bath can affect the ratchet current in an asymmetric kicked potential.

A. The current dependency of the spectral density

The results for our model are strongly dependent on the choice of the bath frequencies. It is not possible to choose any frequency for the kicked harmonic oscillators of the bath, since their trajectories become all unstable for $k_i > 2$ with $i = 1,2,\ldots,N$. Consequently, the frequencies k_i must obey $0 < k_i < 2$. With this restriction in mind, we are free to choose the frequencies according to some distribution. For finite values of N, the frequencies are chosen accordingly to a given law, so the distribution becomes a continuous curve in the limit $N \to \infty$. For example, calling $J(k)$ the spectral density, we show in Fig. 4 how the spectral density evolves to a quadratic distribution when N grows monotonically.

Now we analyze the ratchet current obtained from Eq. (21) as a function of the number N of bath oscillators for the quadratic frequency distribution. This is shown in Fig. 5 for 10^6 initial conditions, and 10^3 iterations and temperature $\beta = 1$. We choose $\gamma = 0.01/\sqrt{N}$ for the coupling constant in all the simulations.

As we can see from Fig. 5 the number of oscillators does not affect very much the ratchet current. For the same temperature $\beta = 1$, there are no observable relevant differences in the current when changing N by two orders of magnitude.

Next we compare the effect of two distinct distributions on the ratchet current. We consider $J(k) = 0.1 + k^{\alpha}$, with $\alpha = 0$ and $\alpha = 1$. The corresponding ratchet current is plotted in Fig. 6 for $\beta = 1$ and same parameters from Fig. 5, with $N =$

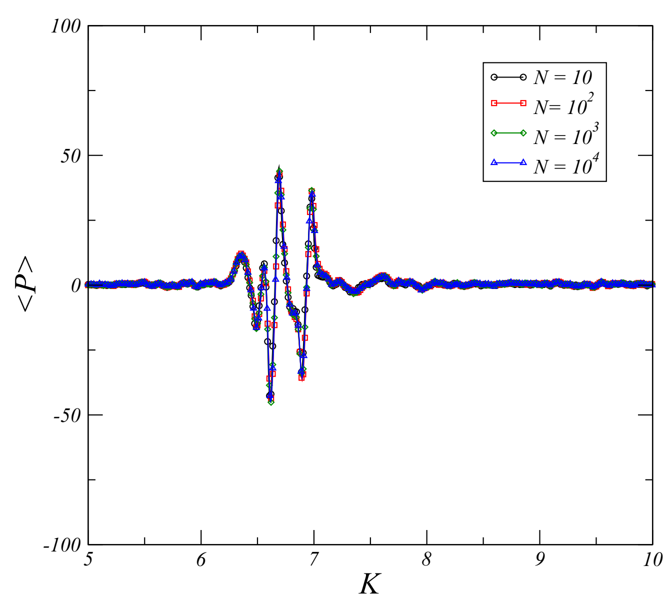

FIG. 5. Ratchet current evaluated for $a = 0.005$ for various values of the number of oscillators N.

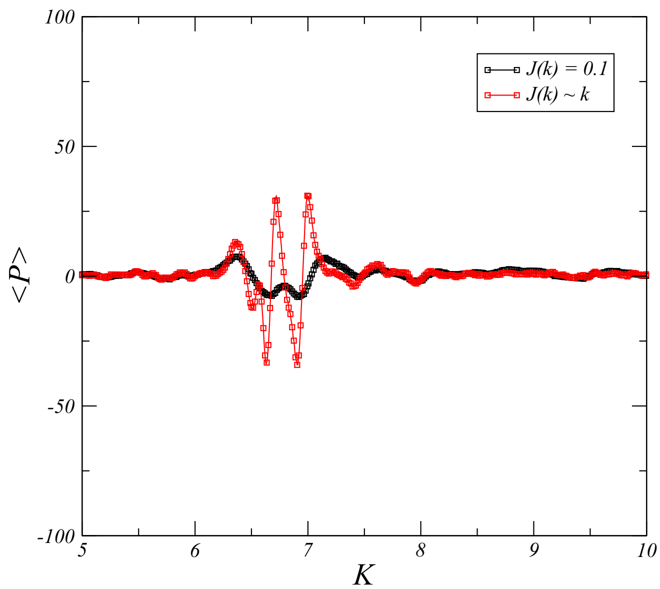

FIG. 6. Ratchet current as function of nonlinearity parameter for $J(k) = 0.1$ (black line) and for $J(k) = 0.1 + k$ (red line).

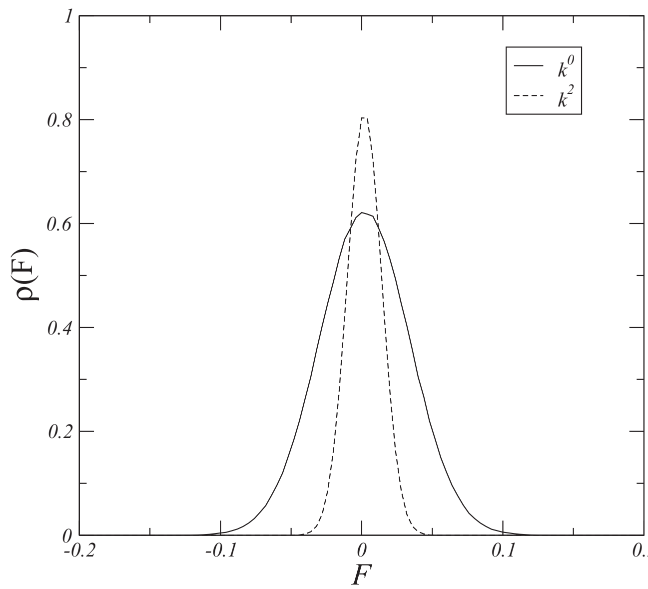

FIG. 7. Discrete fluctuating force time series histogram for $J(k) \sim k^2$ (solid line) and for $J(k)$ constant (dash line). Here the frequencies are limited in the range [0.1,1.5] with $N = 100$ oscillators for 10^6 realizations.

100. We see that a linear distribution ($\alpha = 1$) tends to increase the current.

Due to the weak asymmetry and the topological structure around islands, a large transport was observed between the bifurcation points $6 \leqslant K \leqslant 7$ and can be seen in Fig. 3. However, even in this scenario we can obtain even larger transport or almost no transport depending on the properties of the spectral density of the kicked bath. This point we will address in details in the next section.

In order to explain the above results, we mention that in our Markovian assumption the discrete fluctuating force F_n is given in terms of the Gaussian distribution with mean $\langle F \rangle = 0$ for all times and variance $\langle F^2 \rangle = \sum_{i=1}^{N} \frac{\gamma_i^2}{\beta k_i^2 (1-k_i^2/4)}$. Due to the $(1 - k_i^2/4)$ term in the denominator of the sum contained in $\langle F^2 \rangle$, the variance has a dependence on bath properties, like the spectral density $J(k)$. It is possible to relate numerically the fluctuating force F_n with $J(k)$. This is shown numerically by constructing a histogram of the time evolution of F_n in Fig. 7 for the case where $J(k)$ is a quadratic function in k and for a constant distribution.

IV. INCREASING AND SUPPRESSING THE RATCHET TRANSPORT

The temperature plays an essential role in order to decrease or increase the ratchet transport in the case of usual baths. Generally speaking, thermal fluctuations tend to destroy the ordered motion decreasing the current [21]. However, such fluctuations may also break the symmetry of the attractors in the dissipative case, generating the temperature-induced classical [21] current and the vacuum activation of the quantum current [28].

We can ask now which bath parameter really influences the transport properties in the case of kicked baths. It is important to remark that the ratchet current is evaluated in the map defined in a cylinder and not in a torus. In other words the map variables are defined such that $P \in (-\infty,\infty)$ and $X \in [-\pi,\pi]$ in the map given by Eq. (19). Using $J(k) \sim k^2$ we depicted in Fig. 8 the ratchet current (see color bar) in the parameter space (K,a) for $N = 100$ and λ given by $\sum_{i=1}^{N} \gamma_i^2/k_i^2$. Clearly we observe an alternation between positive [blue (dark gray)] and negative [green (light gray)] currents depending on the values of the pair (K,a). In the case of no environments, stable isoperiodic structures appear in the parameter space (K,a) where optimal ratchet currents occur [27].

We argue that environment effects are responsible for transport suppression and it depends on temperature. We can show this statement from the stability analysis of the fixed points of the map. To do this we use a slightly different procedure than the usual: we include the random variable F_n in the determination of the fixed points and calculate their stability dependence on F_n. In other words, for the moment

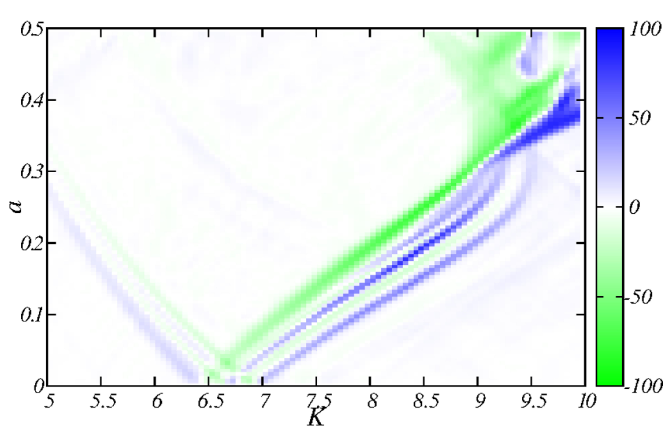

FIG. 8. Parameter space (K,a) where the ratchet current $\langle P \rangle$ is showed in color for a bath with spectral density given by $J(k) \sim k^2$.

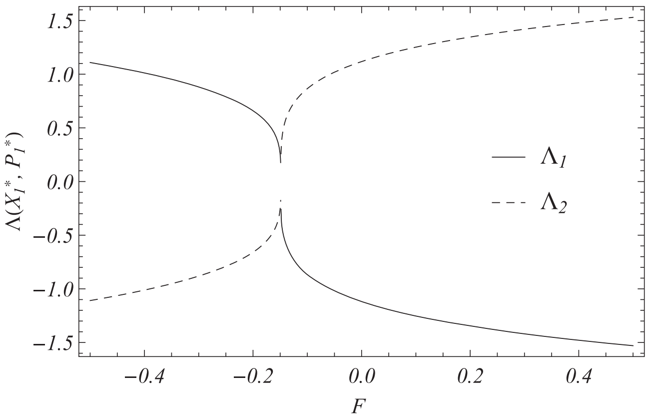

FIG. 9. Logarithm of the first and second eigenvalues for the first fixed point (X_+^*, P^*) given by Eqs. (22) with $L = 1$.

we assume that $F_1 = F_2 = \ldots = F$ is a parameter in the map Eq. (19), and we want to determine the stability of the fixed point as a function of F. Consequently, the fixed points for map Eq. (19) are

$$P^* = 2\pi L$$
$$X_\pm^* = \sin^{-1}\left[\frac{1 - \sqrt{1 + 8a^2 \pm \frac{16\pi a L}{K} - \frac{8aF}{K}}}{4a}\right], \quad (22)$$

with $L \in \mathbb{Z}$. We note here the dependence of fixed points with the intensity of the "constant" fluctuating force F, the asymmetry parameter a and the nonlinearity parameter K.

By solving the characteristic equation one can obtain the eigenvalues of the stability matrix evaluated at the fixed points Eq. (22). We show in Figs. 9 and 10 the logarithm of the absolute values of eigenvalues for each point, where $L = 1$. Here $\Lambda(X_\pm^*, P^*) = \ln|\sigma(X_\pm^*, P^*)|$, $\sigma(X_\pm^*, P^*)$ being the eigenvalues evaluated at each fixed point (X_-^*, P^*) and (X_+^*, P^*). These figures show that the larger the fluctuating force intensity F, the larger will be the unstable eigenvalue for both fixed points. As we already know that large ratchet currents come from stable orbits and unstable orbits are

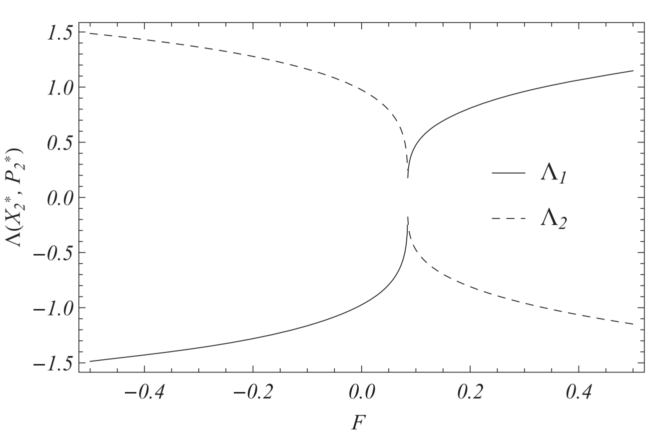

FIG. 10. Logarithm of the first and second eigenvalues for the second fixed point (X_-^*, P^*) given by Eqs. (22) with $L = 1$.

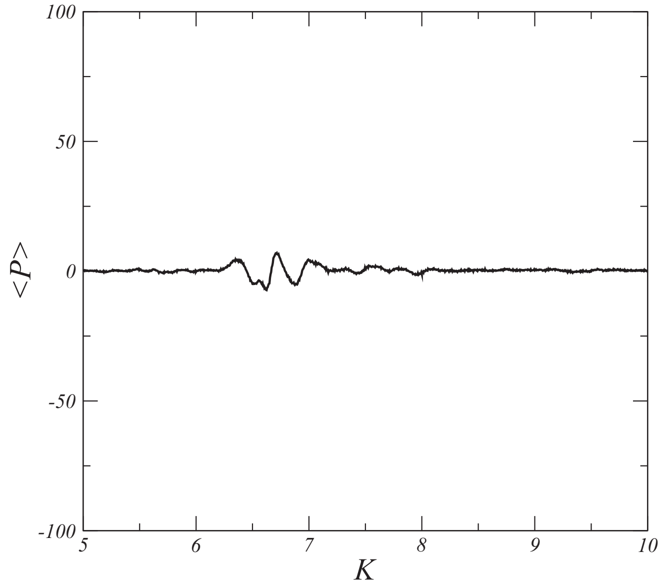

FIG. 11. Ratchet current $\langle P\rangle$ as function of nonlinearity parameter K for a particle subjected to a ratchet potential and a finite kicked thermal bath composed of $N = 100$ oscillators. Here, $\beta = 0.1$ and we can see the suppression of transport even for $J(k) \sim k^2$.

associated with suppression of ratchet current, larger F values should induce larger instabilities, as we will observe later.

The above analysis can nicely be interpreted as follows: imagine that F_n is not large enough so that on average the trajectory remains close to the fixed point. Since the random variable varies in the interval $-1 < F_n < 1$, it is a fixed point inside a circle of radius 1. If $\langle F_n^2\rangle$ is large we expect many values of $|F_n|$ close to 1 and a more unstable average fixed point. On the other hand, if $\langle F_n^2\rangle$ is small, almost all values of $|F_n|$ are close to 0 and a more stable average fixed point is expected. Since $\langle F_n^2\rangle$ is proportional to the temperature, i.e., $1/\beta$, this explains the suppression of the ratchet currents under larger temperatures. While this statement is valid for the fixed point discussed above, another possibility comes to attention. From the general point of view, in generic nonlinear systems, other periodic orbits may become more stable under larger values of $\langle F_n^2\rangle$, leading to a larger current. This could explain a new mechanism of thermal induced ratchet current.

In order to check our analysis, Fig. 11 shows the ratchet current as a function of K for larger $\langle F_n^2\rangle$ ($\beta = 0.1$, which means higher temperature than used before) and $J(k) \sim k^2$ for $N = 100$ bath oscillators. In this situation, where we could expect larger currents, we obtain almost zero currents for small asymmetry parameter and quadratic spectral density. In fact, our analysis indicates that the functional form of the spectral density is less important for the ratchet current than the intensity of $\langle F_n^2\rangle$.

V. CONCLUSIONS

Systems coupled to finite kicked environments belong to the actual and interesting subject of nonequilibrium thermodynamics. While the finite nature of the bath breaks the thermodynamics limit, the kicked property induces a periodic injection of energy into the bath and, therefore, nonequilibrium if the time between successive kicks is not too large. However, an average energy for the bath can be defined and a generalized map can be used to describe the main system [11]. The effect of the kicked environment on the main system is concentrated in the stochastic quantity F_n, with properties Eqs. (16) and (17).

In this work we use the generalized map to describe the transport in a ratchet system coupled to a finite kicked Markovian environment. While the number of oscillators from the environment affects the intensity of the dissipation, their frequencies distribution are observed to suppress or increase the ratchet currents. Basically, it is not the quadratic or linear behavior that decides the suppression or increasing of the current, but the width of the distribution of the stochastic quantity F_n with zero mean. In other words, the suppression of the current decreases with increasing temperatures. This behavior was nicely explained by using analytical results for the stability of the fixed points. As the bath temperature increases, the width of the frequency distribution increases and the fixed points become more unstable. It is worth mentioning that our results are only valid for discrete dynamical systems coupled bilinearly to harmonically time periodic functions describing the bath. Nevertheless, considering those cases, the results are quite general, since it explains the dynamical effect of the temperature in suppressing the ratchet current.

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