Stochastic resonance subject to multiplicative and additive noise: The influence of potential asymmetries

Zijian Qiao, Yaguo Lei,* Jing Lin, and Shantao Niu
State Key Laboratory for Manufacturing Systems Engineering, Xi'an Jiaotong University, Xi'an 710049, China
(Received 2 August 2016; published 15 November 2016)

The influence of potential asymmetries on stochastic resonance (SR) subject to both multiplicative and additive noise is studied by using two-state theory, where three types of asymmetries are introduced in double-well potential by varying the depth, the width, and both the depth and the width of the left well alone. The characteristics of SR in the asymmetric cases are different from symmetric ones, where asymmetry has a strong influence on output signal-to-noise ratio (SNR) and optimal noise intensity. Even optimal noise intensity is also associated with the steepness of the potential-barrier wall, which is generally ignored. Moreover, the largest SNR in asymmetric SR is found to be relatively larger than the symmetric one, which also closely depends on noise intensity ratio. In addition, a moderate cross-correlation intensity between two noises is good for improving the output SNR. More interestingly, a double SR phenomenon is observed in certain cases for two correlated noises, whereas it disappears for two independent noises. The above clues are helpful in achieving weak signal detection under heavy background noise.

DOI: 10.1103/PhysRevE.94.052214

I. INTRODUCTION

Stochastic resonance (SR) was initially coined to explain the periodic recurrences of Earth's ice ages [1,2]. Since SR is a counterintuitive behavior that weak input information is able to be amplified and optimized by the addition of moderate noise [3], such a behavior has attracted considerable attention from theoretical research [4,5] to engineering applications [6–9]. Gammaitoni *et al.* [10] also has shown the large number of SR applications in science and technology to enhance and detect weak signals, ranging from paleoclimatology to electronic circuits, lasers, and chemical systems to the connection with some situations of biological interest.

Up to now, there are many theoretical studies on SR in conventional bistable systems [11–13]. On the one hand, since many physical systems need to consider various noise sources, some studies pay attention to the symmetric bistable SR driven by different noise sources. For example, McNamara, Wiesenfeld [14], and Roy [15] derived the output signal-to-noise ratio (SNR) to quantify SR in a symmetric bistable system with additive Gaussian noise by using two-state theory, where the SNR undergoes a resonancelike curve as a function of the noise intensity. Duan *et al.* not only studied the output SNR gain of a parallel array of SR [16] but also investigated the vibrational resonance effect under the action of additive noise [17]. It is found that the SNR gain in a parallel array exhibits two maxima under different internal noise intensity or sinusoidal vibration amplitude. Moreover, the vibrational resonance effect is discovered in the parallel array. These results suggest that vibrational devices characterize the potential applications in array signal processing. Galdi *et al.* [18] evaluated the performance of a signal detector based on symmetric bistable SR driven by additive Gaussian white noise, which shows that the SR detector is superior to the linear one for extracting weak time-harmonic signals overwhelmed in Gaussian white noise. Later, Barzykin and Seki [19] observed SR phenomenon in a linear system induced by multiplicative noise with exponential correlation rather than additive noise. It is found that a maximum of SNR is observed as a function of both the noise level and autocorrelation time. However, such a behavior disappears as the correlation increases. Li and Han [20] also discovered the SR phenomenon in a linear system subject to multiplicative dichotomous noise, whereas it disappears for additive Gaussian white noise [21]. Even Berdichevsky and Gitterman [22] studied the effect of both multiplicative and additive noise on SR in an overdamped linear system. The results show that the SR is absent for Gaussian white noise, while it occurs for asymmetric dichotomous noise and moreover strongly depends on the cross-correlation intensity between multiplicative and additive noise. Jia *et al.* [23,24] also examined the SR in a symmetric bistable system jointly subject to multiplicative and additive noise. It is observed that for the two noises without correlation the SNR is independent of the initial condition of the system, while for two correlated noises the SNR is not only dependent on the cross-correlation intensity but also on the initial condition. Guo *et al.* [25] further explored SR behavior in a symmetrically piecewise bistable system excited by both multiplicative and additive noise. It is discovered that the SNR vs noise intensity curve has a single peak, indicating a traditional SR phenomenon. Moreover, the height of the peak increases as the multiplicative noise intensity increases.

Obviously, the above-mentioned literature mainly focuses on the influence of different noise sources on symmetric bistable SR, and some interesting phenomena are also discovered for different noise sources. On the other hand, however, since the symmetry in real physical or natural systems is difficult to preserve, asymmetry has been introduced to fluxgate magnetometers and superconducting quantum interference devices (SQUIDs) to detect weak signals [26,27]. Originally, Wio and Bouzat [28] extended the two-state theory to the bistable systems with asymmetric potentials induced by additive noise, where the transition rates between two wells are evaluated as the inverse of the mean first passage time

*yaguolei@mail.xjtu.edu.cn

(MFPT) through the widely used Kramers approximation, and therefore an analytic expression of output SNR is able to be obtained under the adiabatic approximation. The result shows that the SNR increases as the symmetry of the potential increases. Afterwards, Jin *et al.* [29] and Li [30] investigated the SR phenomenon in an asymmetric bistable system subject to additive noise, where the potential is composed of a symmetric bistable potential plus an asymmetric term. For an asymmetric bistable potential, the asymmetry can weaken the SR driven by additive noise, which indicates that the symmetric SR under the action of additive noise is superior to asymmetric one. Bouzat and Wio [31] further studied the effect of potential symmetry on SR in a three-field reaction-diffusion system subject to additive Gaussian white noise, where the asymmetry also weakens the SR behavior. Moreover, Arathi and Rajasekar [32] also discovered the characteristics of SR in an asymmetric Duffing oscillator under the action of additive noise by using numerical analysis. In addition, Li *et al.* [33] analyzed the influence of asymmetry on the noise enhanced stability, where the asymmetric potential is obtained by tilting the symmetric bistable potential. Finally, Borromeo and Marchesoni [34] explored the characteristics of SR with a deformable asymmetric double-well potential in their optomechanical read-out device under the action of additive noise, where a spectral signature of the double SR effect exists.

Though there are many studies on SR in asymmetric bistable systems by tilting an otherwise symmetric double-well potential, i.e., by adding a constant bias which changes both depth and location of the two wells, rather than adding a typically force excitation including a periodic tilt, the influences of potential asymmetries have not been explored completely, especially the difference between well depth and well width under the action of noise and a harmonic excitation. Moreover, most of studies on asymmetric SR focus on only one noise source. In practice, it is inevitable to encounter various noise sources, especially the synchronous action of both additive and multiplicative noise, e.g., the external and internal noise in electronic devices. There even exists a correlation between two noises under certain situations. Therefore, the present paper attempts to study the influence of certain forms of potential asymmetries on bistable SR under the synchronous action of both a harmonic excitation and multiplicative and additive noise with or without the correlation between them. For this purpose, the bistable potentials with three different types of asymmetries are initially constructed in Sec. II, which include a well-depth asymmetry, a well-width one, and a both well-depth and well-width one, respectively. Then Sec. III explores the influence of asymmetries on SR under the simultaneous action of both additive and multiplicative noise with and without correlation between the two noises, where the analytic expression of output SNR is derived to evaluate the influences of potential asymmetries and cross-correlation intensity, etc., on SR. Finally, a discussion is given in Sec. IV and conclusions are drawn in Sec. V.

II. ASYMMETRIC BISTABLE POTENTIALS

Though asymmetric SR has been studied by tilting an otherwise symmetric double-well potential, the influence of well-depth and well-width asymmetries alone on output SNR has not been explored. In order to explore the difference between them, the conventional bistable potential is modified as a new potential with three different types of asymmetries, which is described as

$$U_i(x) = \begin{cases} -ax^2/2 + bx^4/4 & x \geqslant 0, \\ -aA_i x^2/2 + bB_i x^4/4 & x < 0, \end{cases} \tag{1}$$

where $i = 1,2,3$, $A_1 = B_1 = \alpha$, $A_2 = 1/\alpha^2$, $B_2 = 1/\alpha^4$, $A_3 = 1$, $B_3 = 1/\alpha^2$, and $a,b,\alpha > 0$. α is named as an asymmetric ratio and stands for the degree of asymmetry, where a larger α means a higher degree of asymmetry for a certain range of α. It can be noticed from Eq. (1) that the shape of the right well located at $x > 0$ is unaffected by α in all three potentials. In potential $U_1(x)$, α just controls the depth of the left well, i.e., $\alpha a^2/(4b)$. Two stable and one unstable states are at $x_\pm = \pm\sqrt{a/b}$ and $x_u = 0$, respectively. Obviously, the well width of potential $U_1(x)$ remains as $2\sqrt{a/b}$, which no longer depends on α. Therefore, potential $U_1(x)$ only reflects the influence of asymmetric well depth on bistable SR. In potential $U_2(x)$, however, the depths of the two wells remain as $a^2/(4b)$, whereas two stable states locate at $x_- = -\alpha\sqrt{a/b}$ and $x_+ = \sqrt{a/b}$, respectively. Therefore, potential $U_2(x)$ attempts to explore the influence of well-width asymmetry on SR alone. Finally, in potential $U_3(x)$ both the width and the depth of the left well depend on α. It is to investigate the influence of both well-depth and well-width asymmetry on SR. The above-mentioned three different potentials are easily achieved in experiments. For example, they can be obtained by adjusting the position and strength of left-side magnet in the mechanical model of the harmonic oscillators in Ref. [35]. In submicron biwires, Zimmerman *et al.* [36] varied the depth of one well in a bistable potential by controlling an external magnetic field. Therefore, it is important to explore the influence of the three asymmetries on SR.

III. THE INFLUENCE OF POTENTIAL ASYMMETRIES

Unlike most studies on asymmetric SR just considering one noise source, multiplicative and additive noise widely used to simulate internal and external noise of nonlinear devices is considered as a random force in this section to explore the influence of three asymmetries on output SNR under the simultaneous action of the two noise sources and a harmonic excitation. First, assuming that there is no correlation between the two noises, the influence of potential asymmetries on SR is evaluated by calculating output SNR. However, since there may exist a certain correlation between them in practice, the influence of potential asymmetries on SR subject to two correlated noises is also further investigated.

A. The influence of potential asymmetries on SR subject to two independent noises

In the presence of both harmonic excitation and multiplicative and additive noise, the overdamped motion of a Brownian particle in three types of asymmetric potentials is considered, which can be described as a Langevin equation,

$$dx(t)/dt = -U_i'(x) + A\cos(\Omega t) + x(t)\xi(t) + \eta(t), \quad (2)$$

where $U_i'(x) = \partial U_i(x)/\partial x$ and $\xi(t)$ and $\eta(t)$ denote multiplicative and additive noise and satisfy the following statistical properties, respectively:

$$\begin{aligned}&\langle\xi(t)\rangle = 0, \langle\xi(t)\xi(t+\tau)\rangle = 2D\delta(\tau),\\ &\langle\eta(t)\rangle = 0, \langle\eta(t)\eta(t+\tau)\rangle = 2\epsilon\delta(\tau),\end{aligned} \quad (3)$$

where D and ϵ are the intensity of both multiplicative and additive noise, respectively, and A and Ω are the amplitude and angular frequency of the periodic force, i.e., harmonic excitation, respectively. In addition, let $R = D/\epsilon$ represent the noise intensity ratio between the two noises. In this section, we suppose that additive noise is independent of the multiplicative noise, which reads as

$$\langle\xi(t)\eta(t+\tau)\rangle = \langle\eta(t)\xi(t+\tau)\rangle = 0. \quad (4)$$

According to the statistically equivalent description for the probability density function (PDF) $\rho(x,t)$, the Fokker-Planck (FPK) equation corresponding to the Langevin equation in Eq. (2) with Eqs. (3) and (4) can be written as

$$\begin{aligned}\frac{\partial\rho(x,t)}{\partial t} &= \frac{\partial}{\partial x}[U_i'(x) - Dx - A\cos(\Omega t)]\rho(x,t)\\ &\quad + \epsilon\frac{\partial^2}{\partial x^2}(Rx^2+1)\rho(x,t).\end{aligned} \quad (5)$$

On the one hand, in the presence of the periodic force $A\cos(\Omega t)$, the potential $U_i(x)$ is modulated by the periodic force. As a result, the potential is time periodic. Assuming that the amplitude of the periodic force is small enough, i.e., $A \ll 1$, in the absence of noise it is insufficient to induce a Brownian particle to hop from one well to the other. Therefore, the stable and unstable states of this system are considered to be invariable. On the other hand, assuming that the variation of the periodic force is slow enough, i.e., $\Omega \ll 1$ or the adiabatic limit, there is enough time to make the system reach local equilibrium in the period of $1/\Omega$. Then, quasi-steady-state PDF $\rho_s(x,t)$ corresponding to Eq. (5) is written as

$$\rho_s(x,t) = N|Dx^2+\epsilon|^{-1/2}\exp\left[-\frac{\phi(x,t)}{D}\right], \quad (6)$$

where N is a normalization constant and $\phi(x,t)$ is a generalized potential given by

$$\phi(x,t) = \begin{cases}\frac{b}{2}x^2 - \left(\frac{a}{2}+\frac{b}{2R}\right)\ln|Rx^2+1| - A\sqrt{R}\tan^{-1}(\sqrt{R}x)\cos(\Omega t) & x \geqslant 0,\\ \frac{bB_i}{2}x^2 - \left(\frac{aA_i}{2}+\frac{bB_i}{2R}\right)\ln|Rx^2+1| - A\sqrt{R}\tan^{-1}(\sqrt{R}x)\cos(\Omega t) & x < 0.\end{cases} \quad (7)$$

According to two-state theory in Ref. [28], the above bistable case can be simplified to a two-state system with the occupation probabilities $n_\pm(t)$ which satisfy the condition $n_+(t) + n_-(t) = 1$. The master equation governing the evolution of $n_+(t)$, i.e., $1 - n_-(t)$, is

$$\begin{aligned}\frac{dn_+(t)}{dt} = -\frac{dn_-(t)}{dt} &= W_-(t)n_-(t) - W_+(t)n_+(t)\\ &= W_-(t) - [W_-(t) + W_+(t)]n_+(t),\end{aligned} \quad (8)$$

where $W_\pm(t)$ is the transition rates out of stable states $x_\pm$ and it is time periodic since the external force is time periodic. The transition rates can be expanded by using Taylor series under small parameter conditions as

$$\begin{aligned}W_+(t) &= \mu_1 - \beta_1 A\cos(\Omega t) + o(A),\\ W_-(t) &= \mu_2 + \beta_2 A\cos(\Omega t) + o(A),\end{aligned} \quad (9)$$

where parameters $\mu_{1,2}$ and $\beta_{1,2}$ depend on the detailed potential structures of the asymmetric bistable system, such as well depth, well width, barrier height, and wall steepness, i.e., the steepness of potential wall. According to Ref. [15,28], this analytic expression of output SNR is able to be generalized to include the asymmetric cases where $\mu_1 \neq \mu_2$ and $\beta_1 \neq \beta_2$. The output SNR can be obtained by both integrating Eq. (8) and then calculating the corresponding autocorrelation function, which is expressed as

$$\mathrm{SNR} = \frac{\pi A^2(\mu_1\beta_2 + \mu_2\beta_1)^2}{4\mu_1\mu_2(\mu_1+\mu_2)}, \quad (10)$$

where parameters $\mu_{1,2}$ and $\beta_{1,2}$ are able to be analytically calculated as

$$\begin{aligned}\mu_1 &= W_+(t)|_{s(t)=0}, \quad \beta_1 = -\frac{dW_+(t)}{ds(t)}|_{s(t)=0},\\ \mu_2 &= W_-(t)|_{s(t)=0}, \quad \beta_2 = \frac{dW_-(t)}{ds(t)}|_{s(t)=0},\end{aligned} \quad (11)$$

with $s(t)=A\cos(\Omega t)$.

In order to calculate the transition rates $W_\pm(t)$ out of the states $x_\pm$, the MFPT $\tau_\pm$ of the resonance output $x(t)$ to reach the states $x_\mp$ with initial conditions $x(0) = x_\pm$ is able to be first calculated, which is given by the Kramers time,

$$\tau_\pm = 2\pi|U_i''(x_\pm)U_i''(x_u)|^{-1/2}\exp\left[\frac{\phi(x_u,t) - \phi(x_\pm,t)}{D}\right], \quad (12)$$

where the notation $U_i''(x)$ denotes the second derivative of $U_i(x)$ with respect to x. Then the transition rates $W_\pm(t)$ in this bistable system with three types of asymmetric potentials are obtained:

$$W_+(t) = \frac{a}{\sqrt{2\pi}} \exp\left\{-\frac{1}{D}\left[-\frac{a}{2} + \left(\frac{a}{2} + \frac{b}{2R}\right)\ln\left|\frac{aR}{b} + 1\right| + A\sqrt{R}\tan^{-1}\left(\sqrt{\frac{aR}{b}}\right)\cos(\Omega t)\right]\right\},$$
$$W_-(t) = \frac{a\sqrt{A_i}}{\sqrt{2\pi}} \exp\left\{-\frac{1}{D}\left[-\frac{aA_i}{2} + \left(\frac{aA_i}{2} + \frac{bB_i}{2R}\right)\ln\left|\frac{aA_iR}{bB_i} + 1\right| - A\sqrt{R}\tan^{-1}\left(\sqrt{\frac{aA_iR}{bB_i}}\right)\cos(\Omega t)\right]\right\}. \quad (13)$$

Therefore, according to Eq. (10) the analytic expression of output SNR in an asymmetric bistable system subject to two independent noises can be derived as

$$\mathrm{SNR} = \frac{\pi\mu_1\mu_2A^2\left[\tan^{-1}\sqrt{aR/b} + \tan^{-1}\sqrt{aA_iR/(bB_i)}\right]^2}{4D\epsilon(\mu_1 + \mu_2)}, \quad (14)$$

where parameters $\mu_{1,2}$ are calculated by virtue of Eq. (11) as follows:

$$\mu_1 = W_+|_{s(t)=0} = \frac{a}{\sqrt{2\pi}} \exp\left\{-\frac{1}{D}\left[-\frac{a}{2} + \left(\frac{a}{2} + \frac{b}{2R}\right)\ln\left|\frac{aR}{b} + 1\right|\right]\right\},$$
$$\mu_2 = W_-|_{s(t)=0} = \frac{a\sqrt{A_i}}{\sqrt{2\pi}} \exp\left\{-\frac{1}{D}\left[-\frac{aA_i}{2} + \left(\frac{aA_i}{2} + \frac{bB_i}{2R}\right)\ln\left|\frac{aA_iR}{bB_i} + 1\right|\right]\right\}. \quad (15)$$

In terms of Eq. (14), the influence of potential asymmetries on SR subject to two independent noises can be discussed in detail. For simplicity, the potential parameters are fixed to $a = 1$ and $b = 0.5$ in all numerical analysis, respectively. In potential $U_1(x)$, the influence of only well-depth asymmetry on output SNR is able to be analyzed, which is shown in Fig. 1. Figure 1 (a) depicts the output SNR as a function of both multiplicative noise intensity D and asymmetric ratio α. It is noticed that for a fixed α the output SNR initially raises and then drops as D increases, which is the typical characteristic of SR. Moreover, for a given D output SNR attains a peak and then declines rapidly as α increases, whereas for a small enough D it is difficult to induce the occurrence of SR. It is the reason that the cooperation between small noise and periodic force cannot also induce the particle to jump across the potential barrier. Note that all potentials are symmetric when $\alpha = 1$. Obviously, it is noticed from Figs. 1(a)–1(c) that SNR peaks are nearly obtained at $\alpha < 1$ instead of $\alpha = 1$ for different D, ϵ, and A. Such a behavior suggests that there exists an optimally asymmetric well depth for the fixed D, ϵ, or A to make the output SNR of asymmetric SR larger than symmetric one. In addition, it can also be seen from Figs. 1(a)–1(c) that the enhancement capability of SR declines rapidly when $\alpha > 1$. It is the reason that both the left wall of the potential barrier and the left well gradually grow steeper and deeper, respectively, as α increases, thereby making the interwell transition of the particle more difficult. Figure 1(d) further illustrates that SNR peaks initially raise and then decline with the increase of α, whereas the corresponding optimal noise intensity $D_{\max}$ at which the SNR peak is obtained continuously increases. Afterwards, the influence of both well-depth asymmetry and additive noise intensity ϵ on output SNR is also shown in Figs. 1(b) and 1(e). One can observe that a small ϵ more easily excites the SR, whereas a large one will weaken SR. Similar to multiplicative noise intensity D, the SNR peaks in Fig. 1(e) also exhibit a nonmonotonic behavior with respect to α and optimal additive noise intensity $\epsilon_{\max}$ enlarges monotonously with the continuous increase in α. Since the barrier height of the left well and left-wall steepness of the potential barrier increase as α varies from small to large, sufficiently large noise intensity ($D_{\max}$ or $\epsilon_{\max}$) is required for the particle to move from the left well to the right one. Finally, Fig. 1(c) exhibits the variation of SNR versus both A and α. Without a doubt, a larger A can excite higher SNR for different α. Similarly, the output SNR still behaves as a nonmonotonic function of asymmetric ratio α in Fig. 1(c). For different degrees of well-depth asymmetry, the amplitude A of the periodic force always plays a positive role in the SNR increase. Figure 1(f) depicts the influence of noise intensity ratio R on output SNR. One can see that for a small additive noise intensity $\epsilon = 0.01$ SNR peaks begin to increase and then decline as the increase in R, whereas for $\epsilon = 0.5$ SNR peaks decay monotonously. It indicates that output SNR ont only depends on the noise intensity ratio, but also strongly on additive noise intensity. Furthermore, $\alpha_{\max}$ at which SNR peaks are attained also rises monotonously as R increases, which is consistent with the results in Figs. 1(d) and 1(e). Comparing Fig. 1(a) with 1(b), it is found that small additive noise easily induces the occurrence of SR, but this SR system with well-depth asymmetry is more likely to suppress large multiplicative noise when $\alpha < 1$. Such a behavior is important for SR to achieve weak signal detection.

Figure 2 depicts the influence of well-width asymmetry on output SNR. Similar to the well-depth asymmetry, it is apparent that there still exists an optimal asymmetric ratio α for a given D, ϵ, or A to make the output SNR of asymmetric SR larger than symmetric one. Unlike the influence of well-depth asymmetry on output SNR, however, Figs. 2(a)–2(c) show that the output SNR of SR with well-width asymmetry has a slower decay as α increases. Moreover, it is noted that the SNR peaks in Figs. 2(a)–2(c) are nearly obtained at $\alpha > 1$ rather than $\alpha < 1$ in Figs. 1(a)–1(c). The above results suggest that the increment in well-width asymmetry has a weak effect on output SNR and more easily induces the occurrence of SR than well-depth one under the same situation. To some extent, well-width asymmetry is more likely to increase the output SNR. For example, the maxima of SNR in Figs. 2(b)

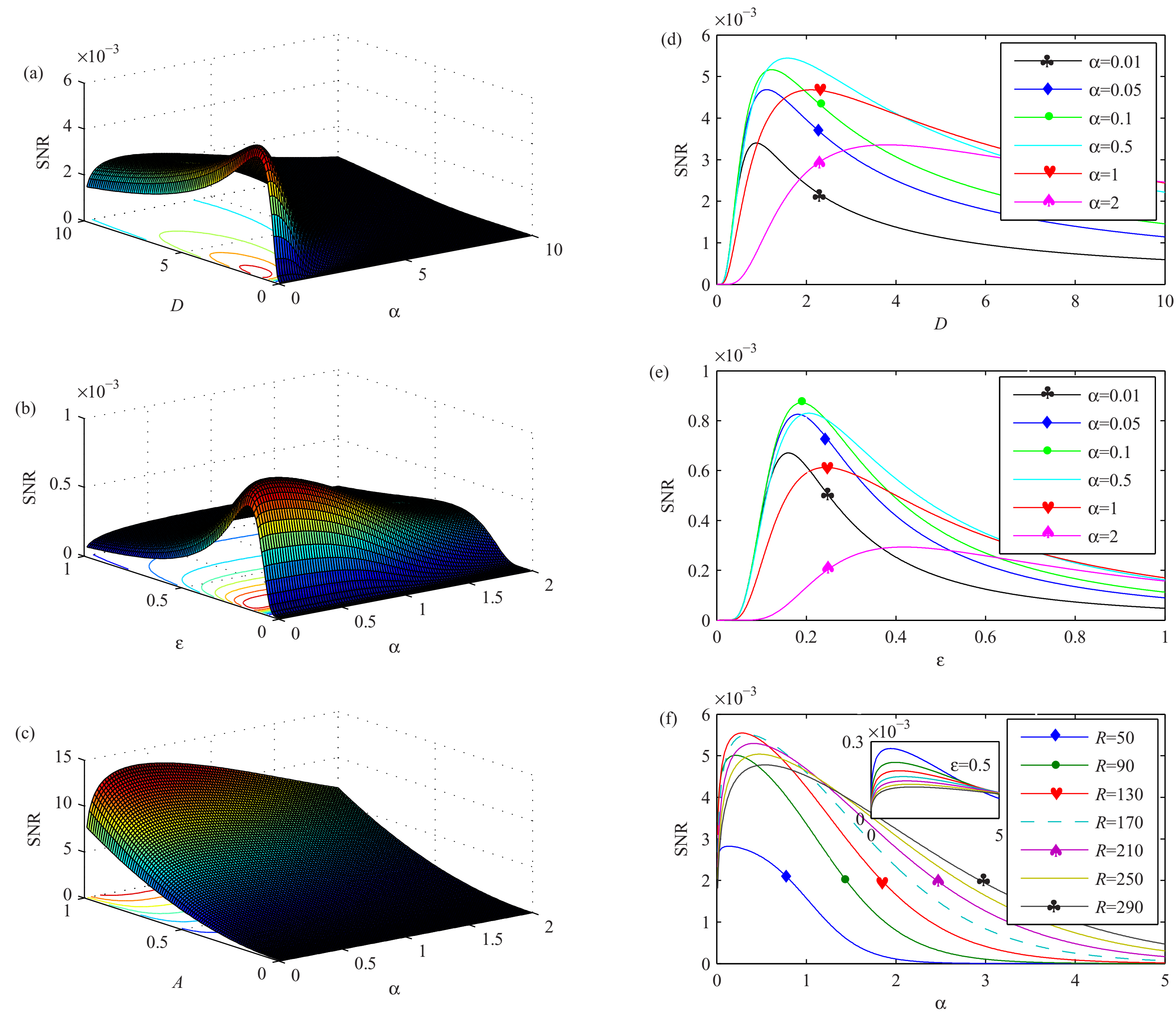

FIG. 1. SNR for well-depth asymmetry subject to two independent noises as a function of (a) both multiplicative noise intensity D and asymmetric ratio α, where $A = 0.02$ and $\epsilon = 0.01$; (b) both additive noise intensity ϵ and α, where $D = 0.01$ and $A = 0.02$; (c) both periodic force amplitude A and α, where $D = 2$ and $\epsilon = 0.01$; (d) D, where $A = 0.02$ and $\epsilon = 0.01$; (e) ϵ, where $D = 0.01$ and $A = 0.02$; and (f) α, where $A = 0.02$ and $\epsilon = 0.01$. Note that the color of the curves in the inset is consistent with that in figure (f).

and 2(e) are relatively larger than those in Figs. 1(b) and 1(e). In addition, it is interesting in Figs. 2(d) and 2(e) that the SNR peaks present a nonmonotonic characteristic with respect to α, whereas the corresponding $D_{\max}$ and $\epsilon_{\max}$ decrease continually as α increases. The same phenomenon also occurs in Fig. 2(f), where $\alpha_{\max}$ gradually decreases as R increases. Though the left well becomes wider as asymmetric ratio α increases, the left wall of potential barrier grows smoother, thereby making the particle hop the potential barrier more easily. Therefore, lower noise intensity is required for pushing the particle from the left well to the right one. In Fig. 2(f), since $\alpha_{\max}$ becomes smaller and smaller as the noise intensity ratio R increases, thereby producing a steeper wall of the potential barrier, a larger noise intensity ratio R is required for particle hopping from one well to the other. However, the steepness of potential-barrier wall is always ignored in most literature. This observation is helpful to design and construct a better potential for developing the potential of SR in weak signal detection. Finally, there is no doubt that the increase of the amplitude A still has a positive effect on output SNR under well-width asymmetry as shown in Fig. 2(c).

In this case of Fig. 3, width and depth of the left well are all controlled by asymmetric ratio α. For a fixed multiplicative or additive noise intensity in Fig. 3(a) or 3(b), the output SNR presents a nonmonotonic function versus α and its peak is nearly attained at $\alpha < 1$, which is similar to the influence of only well-depth asymmetry on output SNR. Unlike the influence of well-depth or well-width asymmetry on output

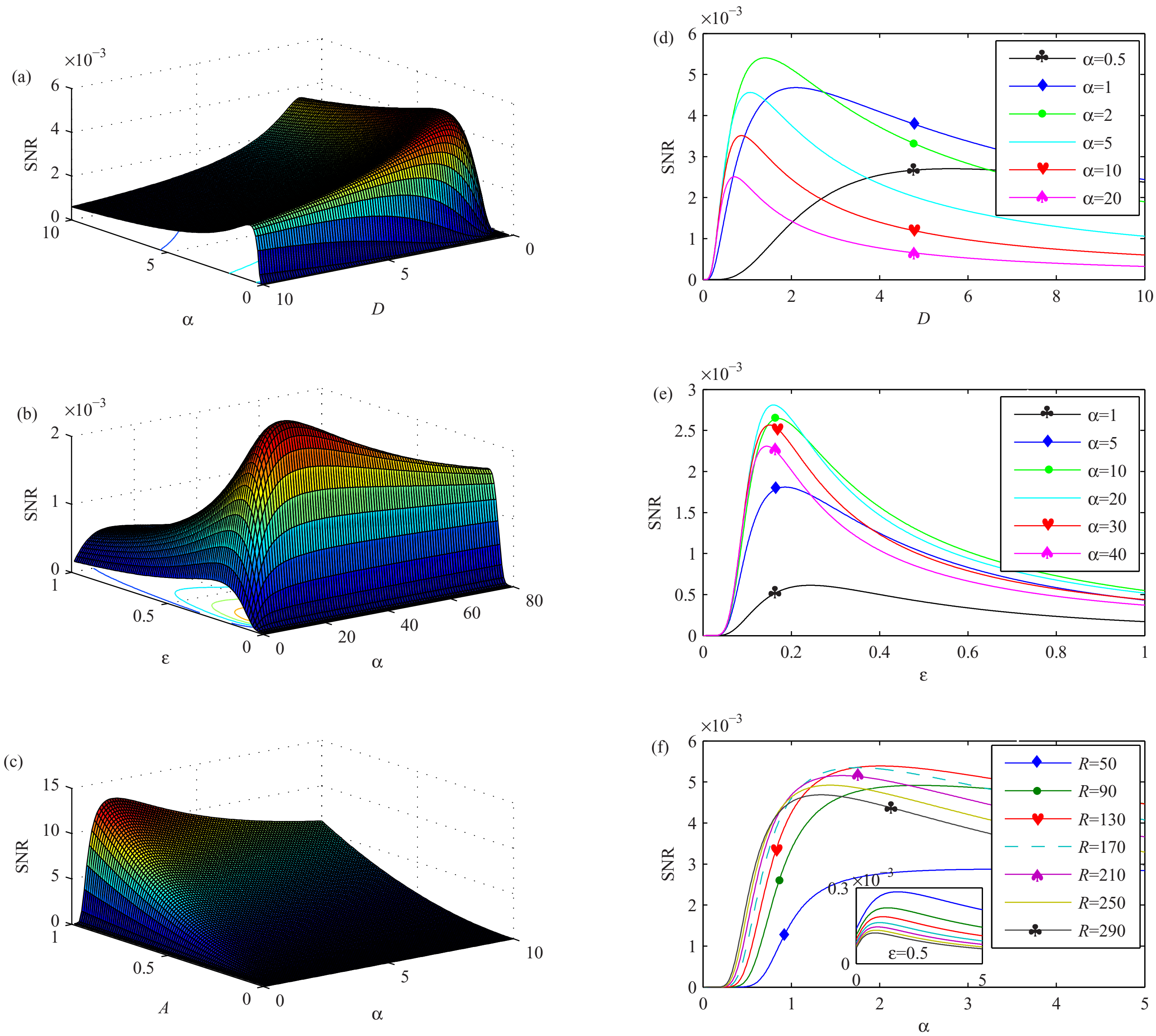

FIG. 2. SNR for well-width asymmetry subject to two independent noises as a function of (a) both multiplicative noise intensity D and asymmetric ratio α, (b) both additive noise intensity ϵ and α, (c) both periodic force amplitude A and α, (d) D, (e) ϵ, and (f) α. Note that other parameters are the same as in Fig. 1 and the color of the curves in the inset is consistent with that in figure (f).

SNR alone, however, $D_{\max}$ and $\epsilon_{\max}$ in Figs. 3(c) and 3(d) start to be nearly invariable as α increases, but $D_{\max}$ and $\epsilon_{\max}$ enlarge as α further increases. It is the reason that for a small α the influence of well-depth asymmetry on both $D_{\max}$ and $\epsilon_{\max}$ is merely counteracted by that of well-width asymmetry and therefore they are nearly invariable, whereas for a slightly large α the influence of well-depth asymmetry is dominant and thereby results in the increase of $D_{\max}$ and $\epsilon_{\max}$ similar to that in Fig. 1. By the comparison among the influences of three asymmetries on output SNR, it is discovered that output SNR is sensitive to the variation of well-depth asymmetry, while it is more robust to the variation of well-width asymmetry. Therefore, a precise SR control can be achieved by adjusting well-width asymmetry. In the three cases, the amplitude A of periodic force always plays a positive role in improving output SNR. Moreover, there exist optimal α and R to make output SNR largest. Particularly, output SNR not only depends on R but also is associated closely with multiplicative and additive noise intensity.

B. The influence of potential asymmetries on SR subject to two correlated noises

In practice, it is possible that there exists a certain correlation between multiplicative and additive noise. In this section, therefore, the influence of three types of asymmetries on SR driven by two correlated noises is further investigated by virtue of two-state theory. Hence, Eq. (4) should be rewritten as

$$\langle \xi(t)\eta(t+\tau)\rangle = \langle \eta(t)\xi(t+\tau)\rangle = 2\lambda\sqrt{D\epsilon}\,\delta(\tau), \qquad (16)$$

where λ is the cross-correlation intensity between two noises and obeys $|\lambda| \leqslant 1$. The FPK equation corresponding to Eq. (2)

with Eqs. (3) and (16) can be described as

$$\frac{\partial\rho(x,t)}{\partial t}=\frac{\partial}{\partial x}[U'(x)-Dx-\lambda\sqrt{D\epsilon}-A\cos(\Omega t)]\rho(x,t)+\epsilon\frac{\partial^2}{\partial x^2}(Rx^2+2\lambda\sqrt{R}x+1)\rho(x,t). \tag{17}$$

Similarly, we suppose that the amplitude and frequency of the periodic force are small enough, i.e., $A,\Omega \ll 1$, which can make the bistable system reach local equilibrium in the periodic of $1/\Omega$. Hence, the quasi-steady-state PDF $\rho_s(x,t)$ of the system subject to two correlated noises can be written as

$$\rho_s(x,t)=N|Dx^2+2\lambda\sqrt{D\epsilon}x+\epsilon|^{-1/2}\exp\left[-\frac{\phi(x,t)}{D}\right], \tag{18}$$

where the generalized potential function $\phi(x,t)$ can be expressed as

$$\phi(x,|\lambda|<1,t)=\begin{cases}\frac{b}{2}x^2-\frac{2b\lambda}{\sqrt{R}}x-\kappa_1\left[\frac{b(1-4\lambda^2)}{2R}+\frac{a}{2}\right]+\kappa_2\left[a+\frac{b(3-4\lambda^2)}{R}-\frac{A\sqrt{R}}{\lambda}\cos(\Omega t)\right] & x\geqslant 0,\\ \frac{bB_i}{2}x^2-\frac{2bB_i\lambda}{\sqrt{R}}x-\kappa_1\left[\frac{bB_i(1-4\lambda^2)}{2R}+\frac{aA_i}{2}\right]+\kappa_2\left[aA_i+\frac{bB_i(3-4\lambda^2)}{R}-\frac{A\sqrt{R}}{\lambda}\cos(\Omega t)\right] & x<0,\end{cases} \tag{19}$$

and

$$\phi(x,\lambda=\pm1,t)=\begin{cases}\frac{b}{2}x^2\mp\frac{2b}{\sqrt{R}}x-\frac{5b}{2R}+\frac{1}{\sqrt{R}x\pm1}\left[\pm\left(-a+\frac{b}{R}\right)+A\sqrt{R}\cos(\Omega t)\right]-\kappa_3\left(a-\frac{3b}{R}\right) & x\geqslant 0,\\ \frac{bB_i}{2}x^2\mp\frac{2bB_i}{\sqrt{R}}x-\frac{5bB_i}{2R}+\frac{1}{\sqrt{R}x\pm1}\left[\pm\left(-aA_i+\frac{bB_i}{R}\right)+A\sqrt{R}\cos(\Omega t)\right]-\kappa_3\left(aA_i-\frac{3bB_i}{R}\right) & x<0,\end{cases} \tag{20}$$

with

$$\kappa_1=\ln|Rx^2+2\lambda\sqrt{R}x+1|,\quad \kappa_2=\frac{\lambda}{\sqrt{1-\lambda^2}}\tan^{-1}\frac{\sqrt{R}x+\lambda}{\sqrt{1-\lambda^2}},\quad \kappa_3=\ln|\sqrt{R}x\pm1|. \tag{21}$$

Note that the generalized potential should satisfy the continuity of function, but the constant terms in Eqs. (19) and (20) are not given out. According two-state theory, the MFPT $\tau_\pm$ of the resonance output $x(t)$ to reach the states $x_\mp$ with initial conditions $x(0)=x_\pm$ is able to be first calculated by the Kramers time and the transition rates $W_\pm(t)$ can be further obtained,

$$\begin{aligned}W_+&=\frac{a}{\sqrt{2}\pi}\exp\left(-\frac{1}{D}\left\{-\frac{a}{2}+2\lambda\sqrt{\frac{ab}{R}}+\ell_1\left[\frac{b(1-4\lambda^2)}{2R}+\frac{a}{2}\right]-\hbar_1\left[a+\frac{b(3-4\lambda^2)}{R}-\frac{A\sqrt{R}}{\lambda}\cos(\Omega t)\right]\right\}\right),\\ W_-&=\frac{a\sqrt{A_i}}{\sqrt{2}\pi}\exp\left(-\frac{1}{D}\left\{-\frac{aA_i}{2}+2\lambda\sqrt{\frac{abA_iB_i}{R}}+\ell_2\left[\frac{bB_i(1-4\lambda^2)}{2R}+\frac{aA_i}{2}\right]\right.\right.\\ &\quad\left.\left.-\hbar_2\left[aA_i+\frac{bB_i(3-4\lambda^2)}{R}-\frac{A\sqrt{R}}{\lambda}\cos(\Omega t)\right]\right\}\right),\end{aligned} \tag{22}$$

for $|\lambda|<1$ and

$$\begin{aligned}W_+&=\frac{a}{\sqrt{2}\pi}\exp\left(-\frac{1}{D}\left\{-\frac{a}{2}\pm2\sqrt{\frac{ab}{R}}+\left(a-\frac{3b}{R}\right)\ln\left|\sqrt{\frac{aR}{b}}\pm1\right|+\ell_3\left[-a+\frac{b}{R}\pm A\sqrt{R}\cos(\Omega t)\right]\right\}\right)\\ W_-&=\frac{a\sqrt{A_i}}{\sqrt{2}\pi}\exp\left(-\frac{1}{D}\left\{-\frac{aA_i}{2}\mp2\sqrt{\frac{abA_iB_i}{R}}+\left(aA_i-\frac{3bB_i}{R}\right)\ln\left|\sqrt{\frac{aA_iR}{bB_i}}\mp1\right|\right.\right.\\ &\quad\left.\left.+\hbar_3\left[-aA_i+\frac{bB_i}{R}\pm A\sqrt{R}\cos(\Omega t)\right]\right\}\right)\end{aligned} \tag{23}$$

for $\lambda=\pm1$, where parameters $\ell_{1,2,3}$ and $\hbar_{1,2,3}$ are given as follows:

$$\begin{aligned}\ell_1&=\ln\left|\frac{aR}{b}+2\lambda\sqrt{\frac{aR}{b}}+1\right|,\quad \hbar_1=\frac{\lambda}{\sqrt{1-\lambda^2}}\left(\tan^{-1}\frac{\lambda+\sqrt{aR/b}}{\sqrt{1-\lambda^2}}-\tan^{-1}\frac{\lambda}{\sqrt{1-\lambda^2}}\right),\\ \ell_2&=\ln\left|\frac{aA_iR}{bB_i}-2\lambda\sqrt{\frac{aA_iR}{bB_i}}+1\right|,\quad \hbar_2=\frac{\lambda}{\sqrt{1-\lambda^2}}\left(\tan^{-1}\frac{\lambda-\sqrt{aA_iR/(bB_i)}}{\sqrt{1-\lambda^2}}-\tan^{-1}\frac{\lambda}{\sqrt{1-\lambda^2}}\right),\\ \ell_3&=\frac{1}{1\pm\sqrt{b/(aR)}},\quad \hbar_3=\frac{1}{1\mp\sqrt{bB_i/(aA_iR)}}.\end{aligned} \tag{24}$$

Therefore, in terms of Eq. (10) the output SNR in bistable system with all three types of asymmetric potentials and two correlated noises can be calculated as

$$\mathrm{SNR}=\frac{\pi A^2(\mu_1\beta_2+\mu_2\beta_1)^2}{4\mu_1\mu_2(\mu_1+\mu_2)}=\frac{\pi\mu_1\mu_2A^2\left[\tan^{-1}\frac{\lambda+\sqrt{aR/b}}{\sqrt{1-\lambda^2}}-\tan^{-1}\frac{\lambda-\sqrt{aA_iR/(bB_i)}}{\sqrt{1-\lambda^2}}\right]^2}{4D\epsilon(\mu_1+\mu_2)(1-\lambda^2)} \tag{25}$$

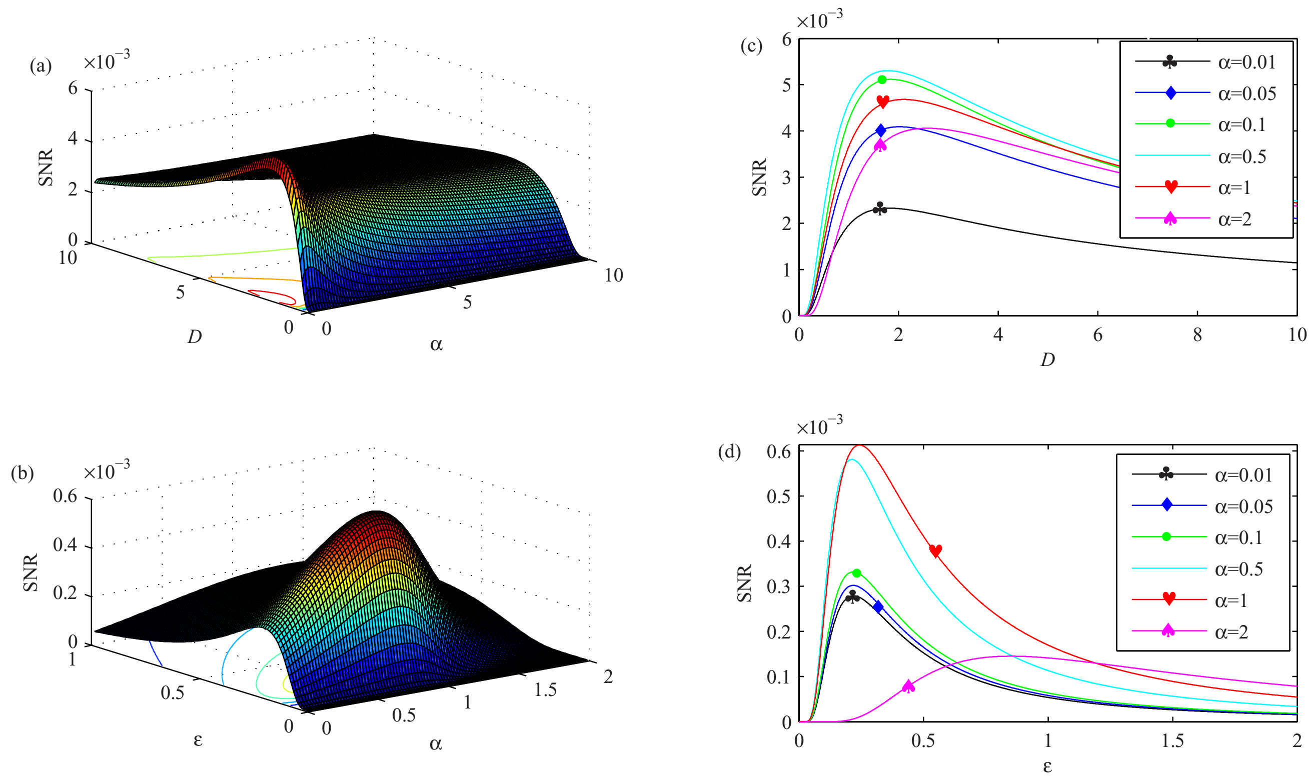

FIG. 3. SNR for both well-depth and well-width asymmetry subject to two independent noises as a function of (a) both multiplicative noise intensity D and asymmetric ratio α, (b) both additive noise intensity ϵ and α, (c) D, and (d) ϵ. Note that other parameters are the same as in Fig. 1.

for $|\lambda| < 1$, where parameters $\mu_{1,2}$ are given as

$$\mu_1 = \frac{a}{\sqrt{2}\pi}\exp\left(-\frac{1}{D}\left\{-\frac{a}{2}+2\lambda\sqrt{\frac{ab}{R}}+\left[\frac{a}{2}+\frac{b(1-4\lambda^2)}{2R}\right]\ln\left|\frac{aR}{b}+2\lambda\sqrt{\frac{aR}{b}}+1\right| -\frac{\lambda}{\sqrt{1-\lambda^2}}\left[a+\frac{b(3-4\lambda^2)}{R}\right]\left(\tan^{-1}\frac{\lambda+\sqrt{aR/b}}{\sqrt{1-\lambda^2}}-\tan^{-1}\frac{\lambda}{\sqrt{1-\lambda^2}}\right)\right\}\right)$$

$$\mu_2 = \frac{a\sqrt{A_i}}{\sqrt{2}\pi}\exp\left(-\frac{1}{D}\left\{-\frac{aA_i}{2}-2\lambda\sqrt{\frac{abA_iB_i}{R}}+\left[\frac{aA_i}{2}+\frac{bB_i(1-4\lambda^2)}{2R}\right]\ln\left|\frac{aA_iR}{bB_i}-2\lambda\sqrt{\frac{aA_iR}{bB_i}}+1\right| -\frac{\lambda}{\sqrt{1-\lambda^2}}\left[aA_i+\frac{bB_i(3-4\lambda^2)}{R}\right]\left[\tan^{-1}\frac{\lambda-\sqrt{aA_iR/(bB_i)}}{\sqrt{1-\lambda^2}}-\tan^{-1}\frac{\lambda}{\sqrt{1-\lambda^2}}\right]\right\}\right) \tag{26}$$

and for $\lambda = \pm 1$

$$\mathrm{SNR} = \frac{\pi\mu_1\mu_2A^2\{1/[1\pm\sqrt{b/(aR)}]-1/[1\mp\sqrt{bB_i/(aA_iR)}]\}^2}{4D\epsilon(\mu_1+\mu_2)}, \tag{27}$$

with

$$\mu_1 = \frac{a}{\sqrt{2}\pi}\exp\left\{-\frac{1}{D}\left[-\frac{a}{2}\pm 2\sqrt{\frac{ab}{R}}+\left(a-\frac{3b}{R}\right)\ln\left|\sqrt{\frac{aR}{b}}\pm 1\right|+\frac{-a+b/R}{1\pm\sqrt{b/(aR)}}\right]\right\},$$
$$\mu_2 = \frac{a\sqrt{A_i}}{\sqrt{2}\pi}\exp\left\{-\frac{1}{D}\left[-\frac{aA_i}{2}\mp 2\sqrt{\frac{abA_iB_i}{R}}+\left(aA_i-\frac{3bB_i}{R}\right)\ln\left|\sqrt{\frac{aA_iR}{bB_i}}\mp 1\right|+\frac{-aA_i+bB_i/R}{1\mp\sqrt{bB_i/(aA_iR)}}\right]\right\}. \tag{28}$$

Therefore, the influence of potential asymmetries on SR subject to two correlated noises is able to be explored by virtue of Eqs. (25) and (27). Our goals in this section mainly focus on the influence of both asymmetries and cross-correlation

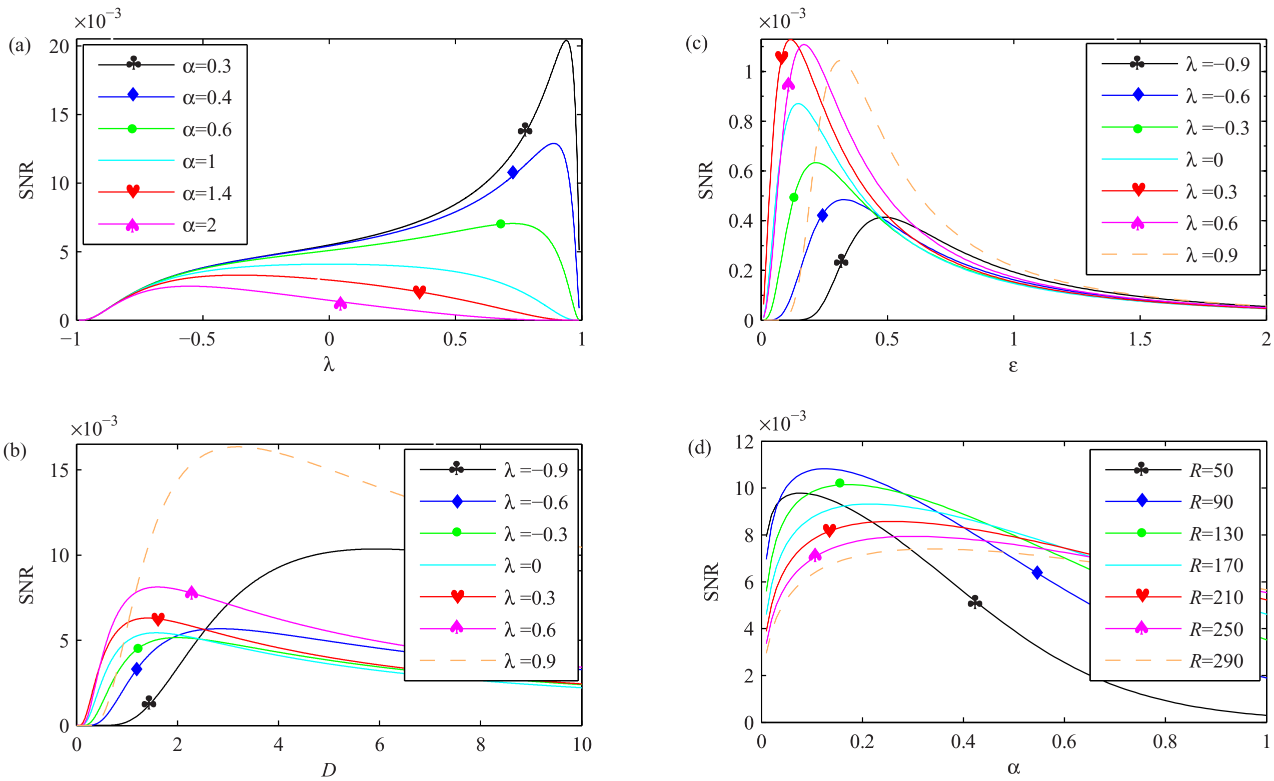

FIG. 4. SNR for well-depth asymmetry subject to two correlated noises as a function of (a) cross-correlation intensity λ, where $\epsilon = 0.01$ and $D = 1.2$; (b) multiplicative noise intensity D, where $\epsilon = 0.01$ and $\alpha = 0.5$; (c) additive noise intensity ϵ, where $D = 0.1$ and $\alpha = 0.5$; and (d) asymmetric ratio α, where $\lambda = 0.6$ and $\epsilon = 0.01$. Note that the amplitude of periodic force always remains as $A = 0.02$.

intensity between two noises on output SNR. Figures 4(a)–4(d) show output SNR as a function of cross-correlation intensity λ, multiplicative noise intensity D, additive noise intensity ϵ, and asymmetric ratio α, respectively. In Fig. 4(a), for a given α output SNR presents a nonmonotonic function with respect to λ. Furthermore, $\lambda_{\max}$ corresponding to SNR peaks shifts from the positive correlation to negative one as the increment in α. It is vital for weak signal detection that SNR peaks at positive correlation are always larger than ones at negative correlation. It is noticed that for a fixed λ output SNR is not only a nonmonotonic function of either D or ϵ, but also SNR peaks for D initially start to decline and then increase with λ varying from negative to positive, as shown in Fig. 4(b), whereas SNR peaks for ϵ first raise and then decline in Fig. 4(c). The above results demonstrate that the existence of an optimal cross-correlation intensity λ can make the output SNR largest. In well-depth asymmetry, it is interesting that the corresponding $D_{\max}$ and $\epsilon_{\max}$ initially decrease and then increase as the increment in λ in Figs. 4(b) and 4(c), which is completely distinct from that under the action of two independent noises. In addition, it can be seen from Figs. 4(a)–4(c) that the highest SNR peaks are always obtained at $\lambda \neq 0$. It indicates that the certain cross-correlation intensity between multiplicative and additive noise is able to improve the output SNR and is further of benefit for weak signal detection. It is possible that the correlation between the two noises causes the system to remember its initial position and therefore the output SNR depends on the initial conditions $x(0) = x_{\pm}$. Finally, for different noise intensity ratio R the output SNR versus α also has a nonmonotonic trend in Fig. 4(d), which is similar to the SR behavior induced by two independent noises in Fig. 1(f). However, $\alpha_{\max}$ at which SNR is maximum gradually increases as R enlarges. Since the left well becomes deeper and deeper as α increases, a larger noise intensity ratio is required for inducing particle hopping from the left well to the right well.

Figure 5 exhibits the influence of well-width asymmetry on SR under the action of two correlated noises. In Fig. 5(a), though SNR versus λ still has a nonmonotonic behavior, $\lambda_{\max}$ corresponding to SNR peaks moves from negative correlation to positive one as the increment in α, which is completely different from that in Fig. 4(a). As opposed to Fig. 4(b), for a given $\alpha = 0.5$ the largest SNR peak under different λ in Fig. 5(b) is obtained when the cross-correlation intensity λ is the negative maximum instead of the positive one. Likewise, $\epsilon_{\max}$ and $D_{\max}$ for different λ initially decrease and then increase in Figs. 5(b) and 5(c). Moreover, Fig. 5(d) illustrates that there exists an optimal R to amplify the output SNR. Particularly, the double SR phenomenon is also observed from the inset of Fig. 5(d) for a certain range of noise intensity ratio R and cross-correlation intensity, where $\epsilon = 0.5$, $\lambda = 0.9$, and D varying from 0.2 to 0.8 with an increment of 0.2. Obviously, double SR phenomenon depends not only on noise intensity ratio R but also strongly on additive noise intensity ϵ and cross-correlation intensity λ. In addition, it is exciting that $\alpha_{\max}$ in Fig. 5(d) continually decreases as R increases, which is different from the influence of well-depth asymmetry in Fig. 4(d). It is the reason that a smaller $\alpha_{\max}$ results in a steeper

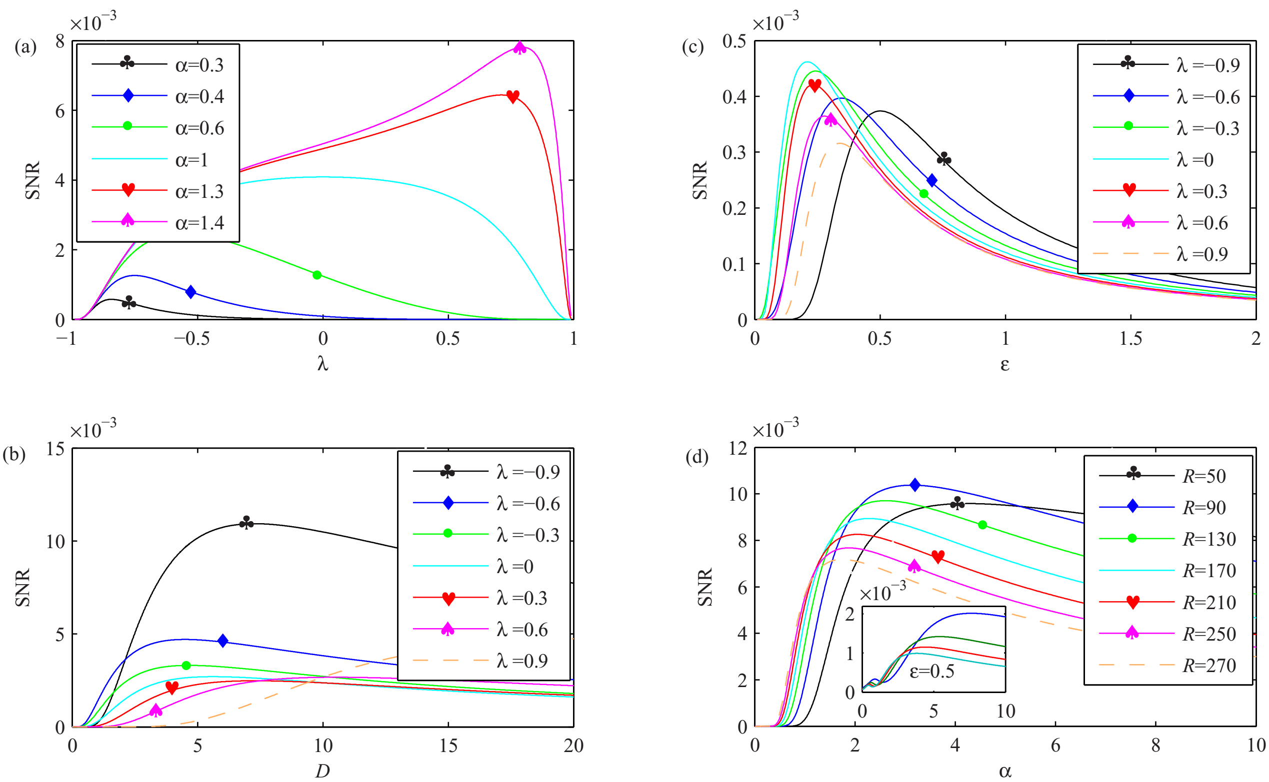

FIG. 5. SNR for well-width asymmetry subject to two correlated noises as a function of (a) cross-correlation intensity λ, (b) multiplicative noise intensity D, (c) additive noise intensity ϵ, and (d) asymmetric ratio α. Note that other parameters are the same as in Fig. 4.

left wall of the potential barrier and therefore a larger noise intensity ratio is needed to excite the particle motion from the left well to the right one.

Finally, Fig. 6 exhibits the influence of both well-depth and well-width asymmetry on output SNR. One can easily observe from Fig. 6(a) that output SNR versus cross-correlation intensity has a nonmonotonic behavior for a given asymmetric ratio. Moreover, $\lambda_{\max}$ corresponding to SNR peaks moves from positive correlation to negative one as the increment in α and SNR peaks also decline gradually. Such a behavior is similar to that of well-depth asymmetry, while it is opposite to that of well-width asymmetry. Meanwhile, output SNR has wider variation range for λ than that of well-depth asymmetry, which is similar to that of well-width asymmetry. In Fig. 6(b), it is found that the SNR peaks at $\lambda = \pm 0.9$ are nearly equal, which is different from that under the action of either well-depth or well-width asymmetry alone. In addition, SNR peaks in Fig. 6(c) become larger and larger as λ varies from negative to positive. Though SNR peaks in Fig. 6(d) have a nonmonotonic phenomenon as R amplifies, the corresponding $\alpha_{\max}$ is nearly invariable. The above results may be produced by the simultaneous action of both well-depth and well-width asymmetries. Therefore, there are both similarities and differences between them.

In terms of Eq. (27), Figs. 7 and 8 show the influences of well-depth and well-width asymmetries on output SNR when cross-correlation intensity $\lambda = \pm 1$, respectively. It is observed from Fig. 7 that output SNR versus either D or ϵ has a double-peak characteristic when $\lambda = 1$ for well-depth or well-width asymmetry, where double resonance peaks are named as first and second resonance peaks from left to right, respectively. Moreover, optimal multiplicative noise intensity $D^2_{\max}$ at which the second resonance peak is obtained keeps invariable in Figs. 7(a) and 7(c) as the increment in α, even in Figs. 8(a) and 8(c) when $\lambda = -1$. However, optimal noise intensity $D^1_{\max}$ at which the first resonance peak is attained increases for well-depth asymmetry as the increment in α in Fig. 7(a) and decreases for well-width asymmetry in Fig. 7(c). More interestingly, SNR versus D transfers from a double-peak structure to a single-peak one in the inset of Fig. 8(a) when $\epsilon = 0.2$ and $\lambda = -1$. The first resonance peak disappears in Fig. 8(c) for well-width asymmetry when $\lambda = -1$. Different from multiplicative noise intensity D, it is noticed for additive noise intensity ϵ that optimal additive noise intensity $\epsilon^2_{\max}$ increases gradually as the increment in α, as shown in Figs. 7(b) and 7(d), whereas $\epsilon^1_{\max}$ declines for well-depth asymmetry and increases for well-width asymmetry as α increases. However, the double SR phenomenon disappears for additive noise intensity ϵ when $\lambda = -1$, as shown in Figs. 8(b) and 8(d). In addition, one can easily discover that the increment in α can weaken the first resonance peaks, such as in Figs. 7(a) and 7(c) and Fig. 8(a). Without a doubt, double SR not only depends on cross-correlation intensity λ and asymmetric ratio α, but also is associated closely with noise intensity D and ϵ; for example, double SR disappears in Fig. 8(a) when $\epsilon = 0.2$.

IV. DISCUSSION

In Sec. III A, we examine the influence of three types of asymmetries on output SNR to quantify the SR under

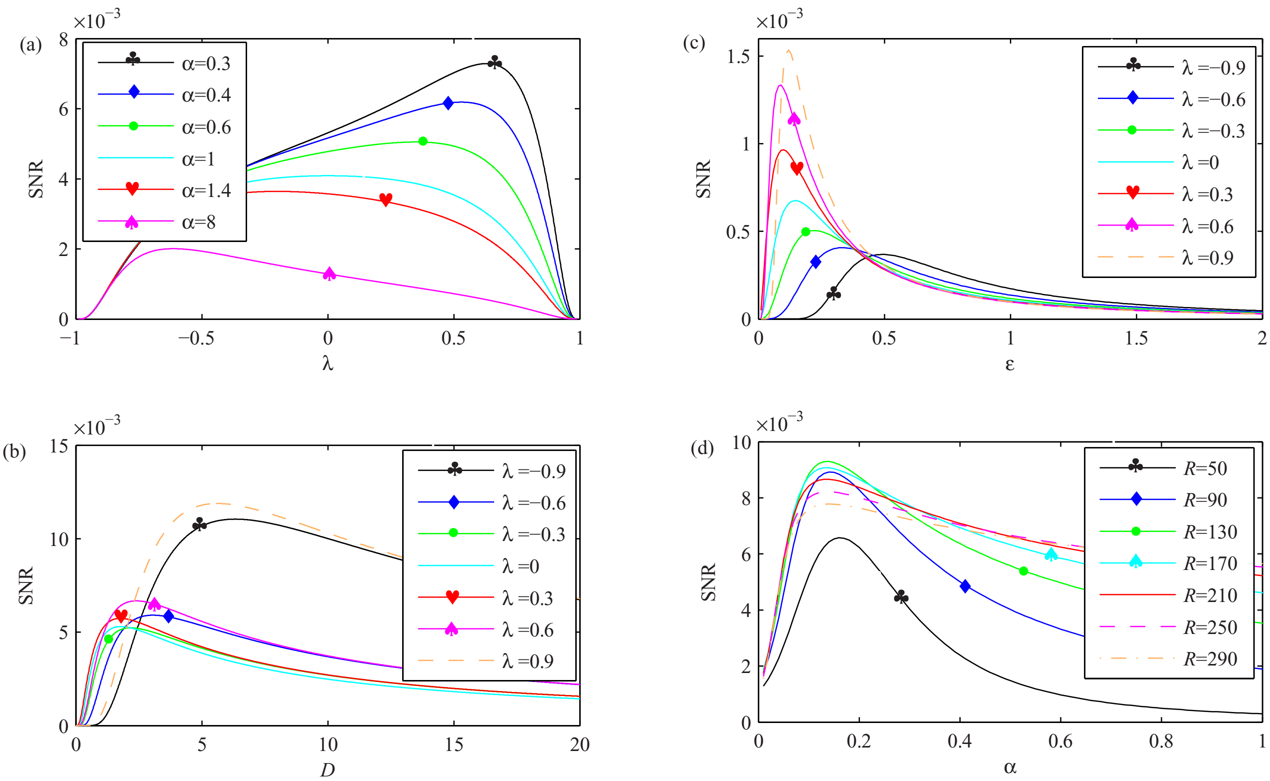

FIG. 6. SNR for both well-depth and well-width asymmetry subject to two correlated noises as a function of (a) cross-correlation intensity λ, (b) multiplicative noise intensity D, (c) additive noise intensity ϵ, and (d) asymmetric ratio α. Note that other parameters are the same as in Fig. 4.

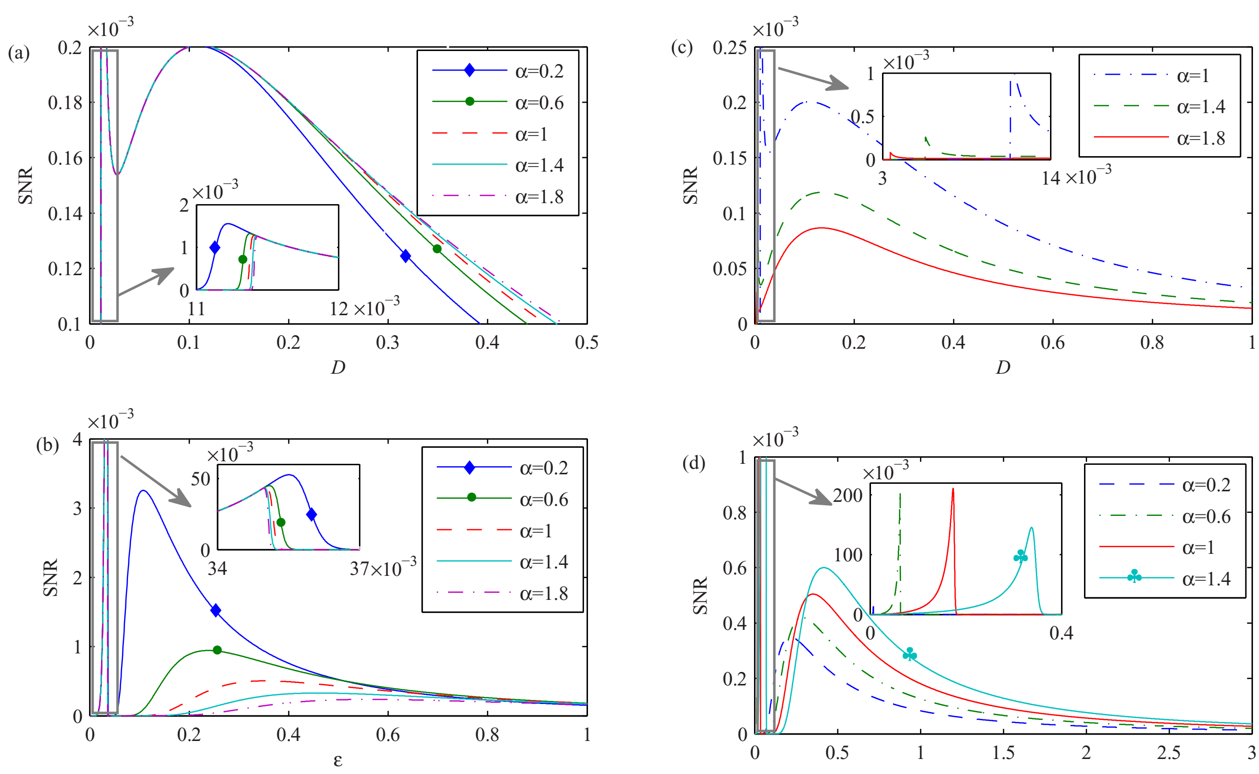

FIG. 7. SNR when cross-correlation intensity $\lambda = 1$: for well-depth asymmetry as a function of (a) multiplicative noise intensity D, where $\epsilon = 0.02$, and (b) additive noise intensity ϵ, where $D = 0.02$; for well-width asymmetry as a function of (c) multiplicative noise intensity D, where $\epsilon = 0.02$, and (d) additive noise intensity ϵ, where $D = 0.1$. Note that the amplitude of periodic force always remains as $A = 0.02$.

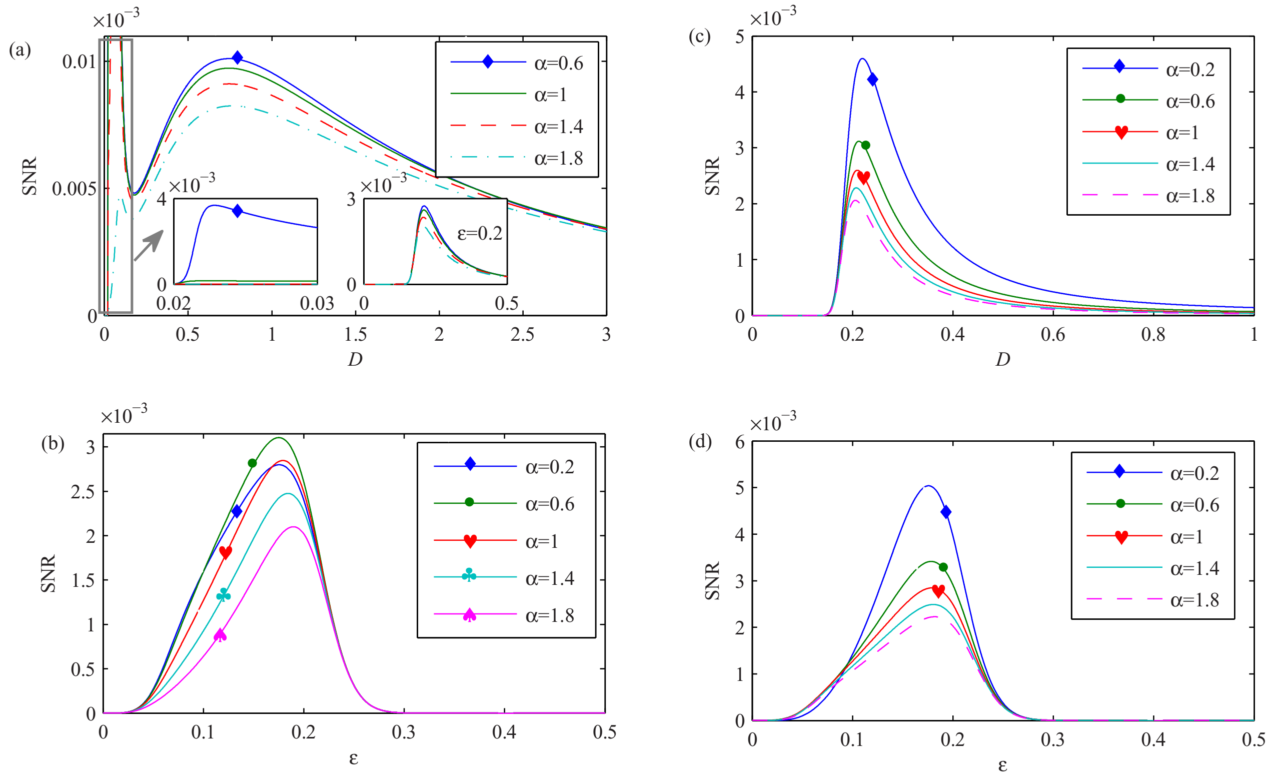

FIG. 8. SNR when cross-correlation intensity $\lambda = -1$: for well-depth asymmetry as a function of (a) multiplicative noise intensity D, where $\epsilon = 0.02$, and (b) additive noise intensity ϵ, where $D = 0.2$; for well-width asymmetry as a function of (c) multiplicative noise intensity D, where $\epsilon = 0.2$, and (d) additive noise intensity ϵ, where $D = 0.2$. Note that the amplitude of periodic force always remains as $A = 0.02$ and the color of the curves in the inset is consistent with that in figure (a).

the action of multiplicative and additive noise without correlation between them. Interestingly, the SNR peaks nearly are obtained at $\alpha < 1$ for well-depth asymmetry, whereas they are found at $\alpha > 1$ for well-width asymmetry. Such a behavior indicates that there exist optimally asymmetric well depth and well width to make the output SNR larger than the symmetric one. This behavior is completely different from that in Refs. [28–31], where the largest SNR of asymmetric SR under the action of additive noise is always obtained when the tilting bistable potential is symmetric. It is the reason that Refs. [28–31] not only consider one additive noise source, but also their asymmetries are obtained by adding a constant bias to an otherwise symmetric potential, thereby resulting in asymmetric well width and well depth. Moreover, the tilt does affect the slope of two wells, where the higher-lying one becomes shallower and shallower and the lower-lying one becomes steeper and steeper as the tilt increases, implying longer dwell times in the lower-lying well. However, the shape of the right well in our study is unaffected by asymmetric ratio α in all three potentials. The slope of the left wells in all three potentials varies with α: (1) for well-depth asymmetry, the slope of the left well becomes steeper and steeper as α increases; (2) for well-width asymmetry, the slope of left well becomes shallower and shallower; (3) for both well-depth and well-width asymmetry, the slope is mainly controlled by well-depth asymmetry. Therefore, the slope changes of the left well in (1) and (3) are similar to that of the tilting potential. In Ref. [34], Borromeo and Marchesoni explored the characteristics of SR in a deformable bistable system with an asymmetric barrier under the action of additive white noise, where the depths of the two wells are equal or symmetric and a double SR phenomenon is discovered. It is similar to the well-width asymmetry in our study, but the slope of its two wells are all variable while the slope of the right well in our study is invariable. However, the double SR in Sec. III B is discovered in certain cases for two correlated noises, whereas it disappears for two independent noises. The above comparison between the present and previous work shows that the characteristics of SR are closely associated with external excitation and structures of potential. Tiny differences on external excitation and the structures of potential may result in different SR behavior. Double SR may be attributed to the cross-correlation intensity, which causes the resonance output to depend on the initial condition of the system. In addition, one can observe that the magnitude of harmonic excitation always plays a positive role in improving SNR, which is consistent with those earlier results.

V. CONCLUSIONS

In order to explore the influence of asymmetries on SR under simultaneous action of multiplicative and additive noise, conventional bistable potential is modified as a double-well potential with three types of asymmetries. On the one hand, for two independent noises, SNR peaks present a nonmonotonic behavior as the increment in α and the largest SNR peak is always obtained at $\alpha \neq 1$. Such a behavior demonstrates that asymmetric SR is superior to the symmetric one for a certain range of asymmetric ratio α. However, it is interesting that optimal noise intensity $D_{\max}$ and $\epsilon_{\max}$ increase as α increases in well-depth asymmetry, but the reverse effect is observed in well-width asymmetry. Even in both well-depth and well-width asymmetry $D_{\max}$ and $\epsilon_{\max}$ remain invariable as the increment in α and then rise with the further increase of α. Similarly, $\alpha_{\max}$ also increases with the increase of noise intensity ratio R in well-depth asymmetry, whereas $\alpha_{\max}$ declines in well-width asymmetry. On the other hand, for two correlated noises, output SNR versus cross-correlation intensity λ has a nonmonotonic characteristic and moreover SNR peaks move from positive correlation to the negative one in well-depth asymmetry as the increment in α, but reverse motion direction is found in well-width asymmetry. In addition, $D_{\max}$ and $\epsilon_{\max}$ also exhibit a nonmonotonic behavior as λ varies from negative to positive, but $\alpha_{\max}$ increases gradually as R amplifies in well-depth asymmetry, and, inversely, $\alpha_{\max}$ declines in well-width asymmetry. More interestingly, double SR is discovered in certain cases, which not only depends on λ and α but also is closely related with noise intensity D and ϵ. The above clues are helpful in achieving weak signal detection in the applications ranging from paleoclimatology to electronic circuits, lasers, and chemical systems by using asymmetric SR which can be obtained by experimental design or some existing natural systems such as Schmitt triggers, Duffing oscillators, fluxgate magnetometers, and SQUIDs. The asymmetric SR is expected to achieve deep-space exploration, e.g., gravity waves. In this paper, we examine the influence of potential asymmetry on SR under the action of both multiplicative and additive noise. In future work, we will examine the relationship between asymmetric potentials and asymmetric excitations such as symmetric saw-tooth wave, rectified sine wave, etc., thereby achieving effective control of asymmetric SR.

ACKNOWLEDGMENTS

This research is supported by National Natural Science Foundation of China (Grants No. 51475355 and No. 51421004), Young Talent Support Plan of Central Organization Department, and Fundamental Research Funds for the Central Universities (Grants No. 2012jdgz01 and No. CXTD2014001).

[1] R. Benzi, A. Sutera, and A. Vulpiani, J. Phys. A **14**, L453 (1981).
[2] C. Nicolis and G. Nicolis, Tellus **33**, 225 (1981).
[3] R. L. Badzey and P. Mohanty, Nature (London) **437**, 995 (2005).
[4] K. Wiesenfeld and F. Moss, Nature (London) **373**, 33 (1995).
[5] F. B. Duan, F. Chapeau-Blondeau, and D. Abbott, Phys. Rev. E **84**, 051107 (2011).
[6] F. Monifi, J. Zhang, S. K. Ozdemir, B. Peng, Y. X. Liu, F. Bo, F. Nori, and L. Yang, Nat. Photonics **10**, 399 (2016).
[7] Y. G. Lei, D. Han, J. Lin, and Z. J. He, Mech. Syst. Signal Process. **38**, 113 (2013).
[8] Y. G. Lei, J. Lin, D. Han, and Z. J. He, Proc. Inst. Mech. Eng., Part C **228**, 815 (2014).
[9] Z. J. Qiao, Y. G. Lei, J. Lin, and F. Jia, Mech. Syst. Signal Process. **84**, 731 (2017).
[10] L. Gammaitoni, P. Hanggi, P. Jung, and F. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998).
[11] J. Casado-Pascual, D. Cubero, and J. P. Baltanas, Europhys. Lett. **77**, 50004 (2007).
[12] M. Ridley, A. MacKinnon, and L. Kantorovich, Phys. Rev. B **93**, 205408 (2016).
[13] P. Huang, J. W. Zhou, L. Zhang, D. Hou, S. C. Lin, W. Deng, C. Meng, C. K. Duan, C. Y. Ju, X. Zheng, F. Xue, and J. F. Du, Nat. Commun. **7**, 11517 (2016).
[14] B. McNamara, K. Wiesenfeld, and R. Roy, Phys. Rev. Lett. **60**, 2626 (1988).
[15] B. McNamara and K. Wiesenfeld, Phys. Rev. A **39**, 4854 (1989).
[16] F. B. Duan, F. Chapeau-Blondeau, and D. Abbott, Phys. Rev. E **90**, 022134 (2014).
[17] Y. H. Ren and F. B. Duan, Phys. A (Amsterdam, Neth.) **456**, 319 (2016).
[18] V. Galdi, V. Pierro, and I. M. Pinto, Phys. Rev. E **57**, 6470 (1998).
[19] A. V. Barzykin and K. Seki, Europhys. Lett. **40**, 117 (1997).
[20] J. H. Li and Y. X. Han, Phys. Rev. E **74**, 051115 (2006).
[21] H. Calisto, F. Mora, and E. Tirapegui, Phys. Rev. E **74**, 022102 (2006).
[22] V. Berdichevsky and M. Gitterman, Phys. Rev. E **60**, 1494 (1999).
[23] Y. Jia, S. N. Yu, and J. R. Li, Phys. Rev. E **62**, 1869 (2000).
[24] Y. Jia, X. P. Zheng, X. M. Hu, and J. R. Li, Phys. Rev. E **63**, 031107 (2001).
[25] Y. F. Guo, Y. J. Shen, and J. G. Tian, Commun. Nonlinear Sci. Numer. Simulat. **38**, 257 (2016).
[26] M. E. Inchiosa, A. R. Bulsara, and L. Gammaitoni, Phys. Rev. E **55**, 4049 (1997).
[27] L. Gammaitoni and A. R. Bulsara, Phys. Rev. Lett. **88**, 230601 (2002).
[28] H. S. Wio and S. Bouzat, Braz. J. Phys. **29**, 136 (1999).
[29] Y. F. Jin, W. Xu, and M. Xu, Chaos, Solitons Fractals **26**, 1183 (2005).
[30] J. H. Li, Phys. Rev. E **66**, 031104 (2002).
[31] S. Bouzat and H. S. Wio, Phys. Rev. E **59**, 5142 (1999).
[32] S. Arathi and S. Rajasekar, Int. J. Bifurcat. Chaos **21**, 2729 (2011).
[33] C. Li, Z.-L. Jia, and D.-C. Mei, Front. Phys. **10**, 95 (2015).
[34] M. Borromeo and F. Marchesoni, Phys. Rev. E **81**, 012102 (2010).
[35] C. A. Kitio Kwuimy, G. Litak and C. Nataraj, Nonlinear Dyn. **80**, 491 (2015).
[36] N. M. Zimmerman, B. Golding, and W. H. Haemmerle, Phys. Rev. Lett. **67**, 1322 (1991).