Work extraction and energy storage in the Dicke model

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We study work extraction from the Dicke model achieved using simple unitary cyclic transformations keeping into account both a nonoptimal unitary protocol and the energetic cost of creating the initial state. By analyzing the role of entanglement, we find that highly entangled states can be inefficient for energy storage when considering the energetic cost of creating the state. Such a surprising result holds notwithstanding the fact that the criticality of the model at hand can sensibly improve the extraction of work. While showing the advantages of using a many-body system for work extraction, our results demonstrate that entanglement is not necessarily advantageous for energy storage purposes, when nonoptimal processes are considered. Our work shows the importance of better understanding the complex interconnections between nonequilibrium thermodynamics of quantum systems and correlations among their subparts.

I. INTRODUCTION

In the last decades there has been a tremendous interest in the thermodynamical analysis of devices, the conversion of heat into work and the extraction of work from a substance, with a substantial effort in the study of quantum heat engines, i.e., machines operating on a quantum system [1–14]. One of the aims is the identification of strategies for the efficient storage of energy. One of the first steps towards the realization of a quantum heat engine was made recently, with the proposal and demonstration of a single-ion heat engine in the classical regime [15–17]. Experiments studying nonequilibrium thermodynamics in the quantum regime have been realized recently verifying the Jarzynski relation [18,19] and measuring entropy production resulting from processes implemented in quantum systems [20].

Whether or not quantum fluctuations and quantum correlations are effective resources, when it comes to the efficiency of a heat engine, is still an open point. An enhancement of work extraction when using two- or three-qubit entangled working media has been shown experimentally [21]. However, the enhanced extraction from entangled states is effective only for small quantum systems [22].

For a machine using a classical working medium on the verge of a phase transition, a boost in the efficiency has been predicted [23]. On the other hand, the study of many-body quantum heat engines is still in its infancy [24–27], and we need to understand whether a many-body quantum system can give an improvement in this respect as compared to a sequence of many heat engines each operating with a single particle. Thus it is timely to proceed towards a systematic study of such devices. The level of control over cold-atomic systems suggests that they could be extremely valuable as a test bed for such devices. An emblematic example is the experimental realization of the Dicke model in an intracavity atomic system [28]. The technology available at hand is mature enough to assess the thermodynamics of such system in the fully quantum regime. Also, the presence of a superradiant phase transition has been shown to play a role in the work output of a such an engine [29].

In this paper we take a significantly different approach with respect to previous studies. We quantify the relation between the energy extracted and the energy initially stored, for *practical* cyclic processes putting constrains on the optimality of the protocol, motivated by the experimental control available over the system. We then compare such practical protocols with the optimal ones. We find that the quantum phase transition can improve the extraction of work. However, by considering the energetic cost of creating the initial state, and through an analysis of the role of entanglement, we show that highly entangled states can be inefficient for energy storage. Our results show the existence of a nontrivial link between nonequilibrium thermodynamics and entanglement for nonoptimal unitarily operating devices.

II. WORK EXTRACTION FORMALISM

We assume to drive cyclically a quantum system with a time-dependent periodic Hamiltonian $\hat{H}(t)$, with t_i and t_f the initial and final time of the evolution, respectively, without contact to external reservoirs. Since work extraction from equilibrium state is forbidden by Thomson's formulation of the second law [30], we consider initial out-of-equilibrium states.

Suppose the initial state and the initial Hamiltonian are $\hat{\rho}(t_i) = \sum_j r_j |r_j\rangle\langle r_j|$ and $\hat{H}(t_i) = \sum_j \epsilon_j |\epsilon_j\rangle\langle\epsilon_j|$, where the ordering $r_1 \geqslant r_2 \geqslant \cdots$, and $\epsilon_1 \leqslant \epsilon_2 \leqslant \cdots$ is assumed. Due to unitarity all of the eigenvalues of the initial state are preserved at any time. The least energetic final state is $\hat{\rho}(t_f)_{\text{pass}} = \sum_j r_j |\epsilon_j\rangle\langle\epsilon_j|$. This final state commutes with the Hamiltonian $\hat{H}(t_i)$, and so it is stationary, and the ordering of the eigenvalues is such that no work can be further extracted from it, making it passive. Correspondingly we have the maximum extraction of work by the amount $\mathcal{E} = \sum_{ij} r_j \epsilon_i (|\langle r_j|\epsilon_i\rangle|^2 - \delta_{ij})$, called ergotropy [31].

III. DICKE MODEL

We consider the Dicke model: an emblematic model in quantum optics [32], also widely used as a benchmark for studying the behavior of quantum many-body systems with a quantum phase transition [33–36]. The Dicke Hamiltonian describes the coupling between an ensemble of N two-level

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atoms and a single cavity mode and reads ($\hbar = 1$)

$$\hat{H} = \omega_0 \hat{J}_z + \omega \hat{a}^\dagger \hat{a} + \frac{\lambda}{\sqrt{2j}}(\hat{a} + \hat{a}^\dagger)(\hat{J}_+ + \hat{J}_-), \qquad (1)$$

where ω_0 is the single atom two-level energy splitting, ω is the cavity frequency, and λ is the atom-cavity interaction strength [37]. However, the Dicke model is implemented experimentally with a hybrid cold-atomic system in an optical cavity [28], in which case the parameters must be interpreted differently as explained later in this article. The operators $\hat{J}_i$ ($i = x, y, z$) are collective angular momentum operators, which allow us to describe the atomic ensemble as a pseudospin of length $j = N/2$. We can define the mean fields as $\langle \hat{a} \rangle = \alpha$, $\langle \hat{J}_- \rangle = \beta$, $\langle \hat{J}_z \rangle = w$, and we write semiclassical equations of motion for them derived from the Heisenberg equations, replacing operators with expectation values. The critical coupling $\lambda_{\rm cr} = \sqrt{\omega \omega_0}/2$ defines the separation point between the two fixed-point solutions of the semiclassical equations: for $\lambda < \lambda_{\rm cr}$, the so-called normal phase, the mean fields are null; while for $\lambda > \lambda_{\rm cr}$, the so-called superradiant phase, both the atoms and field acquire macroscopic mean fields of both signs. With a standard Holstein-Primakoff transformation $\hat{J}_+ = \hat{b}^\dagger \sqrt{2j - \hat{b}^\dagger \hat{b}}$, $\hat{J}_- = \sqrt{2j - \hat{b}^\dagger \hat{b}}\, \hat{b}$, $\hat{J}_z = \hat{b}^\dagger \hat{b} - j$ [38], we can introduce the fluctuations operators $\delta \hat{a} = \hat{a} - \alpha$, $\delta \hat{b} = \hat{b} - \widetilde{\beta}/\sqrt{N}$, where α and $\widetilde{\beta} = \langle \hat{b} \rangle$ are chosen as the steady-state mean fields. Explicitly, we get

$$\hat{H} = \frac{\widetilde{\omega}_0}{2}\left(\hat{A}_x^2 + \hat{A}_y^2\right) + \frac{\omega}{2}\left(\hat{P}_x^2 + \hat{P}_y^2\right) + 2\widetilde{\lambda}\hat{P}_x \hat{A}_x - 2\mu \hat{A}_x^2$$

$$= \epsilon^- \hat{d}^\dagger \hat{d} + \epsilon^+ \hat{c}^\dagger \hat{c} + \frac{1}{2}(\epsilon^- + \epsilon^+ - \omega - \widetilde{\omega}_0), \qquad (2)$$

where the eigenvalues ϵ^+ and ϵ^- and the coefficients $\widetilde{\omega}_0, \widetilde{\lambda}, \mu$ in Eq. (2) are reported in Appendix A, and the quadrature operators are defined by $\hat{P}_x = (\delta \hat{a}^\dagger + \delta \hat{a})/\sqrt{2}$, $\hat{P}_y = i(\delta \hat{a}^\dagger - \delta \hat{a})/\sqrt{2}$, $\hat{A}_x = (\delta \hat{b}^\dagger + \delta \hat{b})/\sqrt{2}$, $\hat{A}_y = i(\delta \hat{b}^\dagger - \delta \hat{b})/\sqrt{2}$. In the last line of Eq. (2) we have introduced the polariton operators $\hat{d}$ and $\hat{c}$, which are connected to the local modes operators $\delta \hat{a}$ and $\delta \hat{b}$ via the matrix equation $\boldsymbol{\delta \hat{a}} = \boldsymbol{M} \cdot \boldsymbol{\hat{d}}$, where we have used the vector notation $\boldsymbol{\delta \hat{a}} = (\delta \hat{a}, \delta \hat{a}^\dagger, \delta \hat{b}, \delta \hat{b}^\dagger)^T$ and $\boldsymbol{\hat{d}} = (\hat{d}, \hat{d}^\dagger, \hat{c}, \hat{c}^\dagger)^T$. The approach to the phase transition, in terms of Eq. (2), is signaled by the change of the parameters (see Appendix A), or by the softening of the polariton frequency ϵ_+, corresponding to different Holstein-Primakoff approximations for each phase [39]. In what follows we assume a constant value of the atomic frequency ω_0. In Ref. [28] the Dicke model is found as an effective Hamiltonian model describing a system that consists of a Bose-Einstein condensate (BEC) loaded into a high-finesse optical cavity, transversally pumped with a standing-wave laser far-off resonant with respect to the atomic transition. The mapping to the Dicke model is realized with ω given by the detuning between the pump frequency ω_p and the cavity mode frequency ω_c, dispersively shifted by the atomic system $\omega = \omega_p - \omega_c + Ng_0^2/2\Delta_a$, where $\Delta_a = \omega_p - \omega_a$ is the pump-atom detuning, and g_0 is the atom-cavity coupling. Thus ω can be changed by varying the pump frequency ω_p.

The coupling parameter is $\lambda = N/2\sqrt{g_0 \Omega_p/\Delta_a}$, where Ω_p is the pump Rabi frequency. A variation of λ can be obtained by

quenching the intensity of the pump, controlled via Ω_p. A variation of the pump frequency ω_p instead determines a variation of two parameters of the effective Dicke model, ω and λ. To realize an independent variation of ω such that it does not affect λ, we can realize simultaneously two protocols $\omega_{p_1} \to \omega_{p_2}$ and $\Omega_{p_1} \to \Omega_{p_1}(\omega_{p_2} - \omega_a)/(\omega_{p_1} - \omega_a)$. We thus assume the independent maneuverability of such parameters.

The practical realizability of the closed version of the Dicke model calls for a clarification. In fact, the $\hat{A}^2$ term is typically overlooked in the minimal-coupling Hamiltonian, where $\hat{A}$ is the potential vector operator of the electromagnetic field. If instead this term is included, the phase transition was shown to be prohibited [40], while this is instead reachable for a system of genuine spins interacting with radiation [41], or in the open system scenario [42]. However, the theoretical analysis of the closed model from one hand is necessary, as explained above, to apply the formalism of work extraction introduced here, and on the other hand it serves as a groundwork before making a proper comparison with the open version, necessary to analyze the experimental results coming from the setup in Ref. [28].

IV. MEAN FIELD CONTRIBUTION TO THE WORK

The fixed points (α_s, β_s) of the semiclassical equations are local minima of the mean energy $E := \langle \hat{H} \rangle$, as a function of α and β. This means that the system starting slightly off the fixed point is in a classical nonequilibrium state, and, according to Thomson's formulation of the second law, a cyclic variation of the parameters can determine a classical contribution to the work extraction, which would be macroscopic and would completely mask the quantum contribution. However, as we are interested in studying the contribution to the work extraction coming from the quantum fluctuations of the system, we start the protocol from the fixed point, so that the extracted energy is due only to the quantum fluctuations. This should not be confused with an initial ground state, as in our description we consider quantum fluctuations on top of the classical mean fields.

Consider a time-dependent protocol, starting from mean fields corresponding to a fixed point in the normal phase, i.e., for $\lambda < \lambda_{\rm cr}$. If we realize a general protocol so as to remain inside the normal phase, the mean fields will stay fixed in time. If instead we bring the system from the normal to the superradiant phase and then back, the mean fields will still remain fixed, but inside the superradiant phase this point is unstable. For a very small change of the initial values, the mean fields evolve with a corresponding positive work exchange. A positive work coming from the mean fields would also be present for a cycle starting and ending inside the superradiant phase. Therefore, for work extraction purposes that originate from quantum fluctuations, we must limit ourselves to protocols within the normal phase. This result agrees with what was done in Ref. [43], where the authors found that crossing the two phases is not interesting from the point of view of the statistics of the work, due to the macroscopic generation of excitations.

One of the main goals of this work is the study of the role of the quantum contribution to the extraction of work while approaching the phase transition. In order to do this, however, many questions can be asked, one of which is related to the choice of the best time protocol to use for this analysis. The
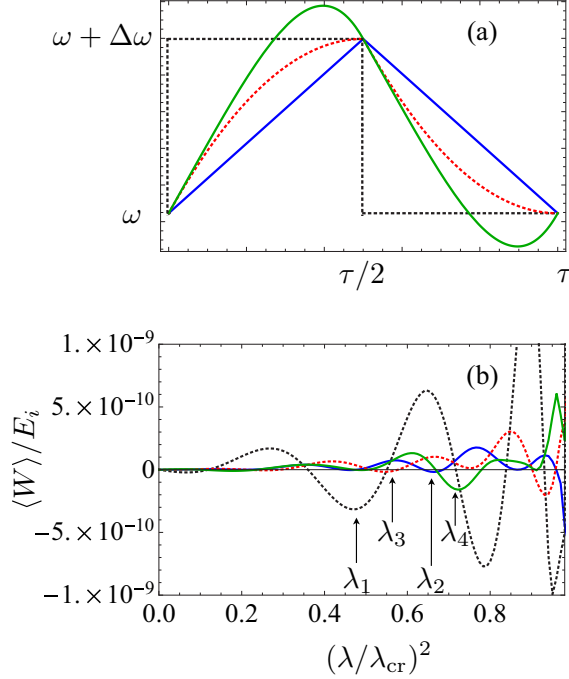
FIG. 1. (a) Four different cyclic time protocols, where the dotted black line is an instantaneous quench, the solid blue line is a linear quench, the dotted red line is a quadratic quench, and the solid green line is a quartic quench. (b) Work-energy ratio, for the protocols reported in (a), as a function of the renormalized coupling parameter. The initial state is an eigenenergy Fock state $|m = 10\rangle_d |n = 0\rangle_c$ in the polariton basis.

answer to this is, however, not trivial because the work-energy ratio depends nontrivially on many factors. In particular, the initial point, in the parameter space (λ, ω), plays a crucial role. This is in fact responsible for the best protocol, to be used to optimize the work extraction, being dependent on the particular initial point. We report in Fig. 1(a) four different time protocols (instantaneous quench, linear, quadratic, quartic), corresponding to which we evaluate the work-energy ratio.

Figure 1(b) shows clearly how for different initial points the best time protocol correspondingly changes. In particular for λ_1 the best protocol, in the sense of the one giving the most negative ratio, is the instantaneous quench. For λ_2 the best protocol is the linear, for λ_3 the quadratic and for λ_4 the quartic. For this reason we choose to fix the time protocol, to the instantaneous quench, to analyze coherently the approach to the phase transition for a given protocol and compare it with different initial states.

We have considered the following cycles: (i) preparation of an initial state, (ii) instantaneous quench $\hat{H}_i \rightarrow \hat{H}_f$ ($i = A$ and $f = B$, or $i = C$ and $f = D$ with reference to Fig. 2) and evolution under $\hat{H}_f$ for t_f, and (iii) instantaneous quench $\hat{H}_f \rightarrow \hat{H}_i$. The average value of the work done in quenching the Hamiltonian can be written as $\langle W \rangle = \langle \psi(t_i)|(\hat{H}_H(t_f) - \hat{H}(t_i))|\psi(t_i)\rangle$ [44], where $\hat{H}_H(t) = \hat{U}^\dagger(t)\hat{H}(t)\hat{U}(t)$ is the Hamiltonian in the Heisenberg representation, and $\hat{U}(t)$ is the evolution operator describing the process. We show in Appendix B how to get an analytical expression of the average work. The validity of the sudden quench is explained in
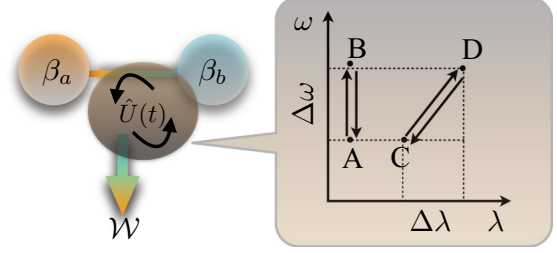

FIG. 2. Diagrammatic representation of the cycles. Initially the system is prepared in a locally thermal state, at inverse temperatures β_a and β_b. The cyclic unitary transformation $\hat{U}(t)$ in the parameter space (λ, ω) is highlighted on the right: quench $A(C) \rightarrow B(D)$, evolution in $B(D)$, quench $B(D) \rightarrow A(C)$, with a final extraction of work.

Ref. [45]: the time scale of the change of the parameters should be smaller than the time scale of the internal evolution, given roughly by $1/\epsilon_+$. The softening $\epsilon_+ \rightarrow 0$ at the phase transition, and the consequent freezing of the internal dynamics, indeed is compatible with the sudden quench.

V. RESULTS FOR LOCALLY THERMAL STATES

In order to emulate, in our unitary framework, the effects of two thermal reservoirs, we consider the scenario sketched in Fig. 2. An initial locally thermal state is prepared, where the two local modes have different inverse temperatures $\beta_a = 1/k_B T_a$ and $\beta_b = 1/k_B T_b$, where k_B is Boltzmann's constant, and the two oscillators are coupled, effectively realizing Hamiltonian (2). Afterwards, a cyclic unitary process is applied, which can result in an extraction of work. We want to check whether the natural flow of energy, due to the initial state chosen here, can help us improve the extraction of energy. We define locally thermal states $\hat{\rho}^{\beta_a \beta_b} = \hat{\rho}^{\beta_a} \otimes \hat{\rho}^{\beta_b}$, where the thermal states are $\hat{\rho}^{\beta_j} = e^{-\beta_j \hat{H}_j}/\mathcal{Z}_j$, with partition functions $\mathcal{Z}_j = \text{Tr}[\exp(-\beta_j \hat{H}_j)]$ ($j = a, b$). The local Hamiltonians are $\hat{H}_a = \omega \, \delta\hat{a}^\dagger \delta\hat{a}$, $\hat{H}_b = \omega_0 \, \delta\hat{b}^\dagger \delta\hat{b}$.

The ergotropy for a locally thermal state $\hat{\rho}^{\beta_d \beta_c}$ in the polariton partition is

$$\mathcal{E}(\hat{\rho}^{\beta_d \beta_c}) = E(\hat{\rho}^{\beta_d \beta_c}) - E(\hat{\rho}_{\text{pass}}^{\beta_d \beta_c}) = 0, \qquad (3)$$

where $E(\hat{\rho})$ denotes the average energy of the state $\hat{\rho}$ since, despite not being a thermal state because of the different local temperatures of the polariton modes, it is, however, a passive state. The ergotropy of the locally thermal state $\hat{\rho}^{\beta_a \beta_b}$ is instead

$$\mathcal{E}(\hat{\rho}^{\beta_a \beta_b}) = E(\hat{\rho}^{\beta_a \beta_b}) - E(\hat{\rho}_{\text{pass}}^{\beta_a \beta_b})$$
$$= \epsilon_c(\langle \hat{c}^\dagger \hat{c} \rangle_{\beta_a \beta_b} - \langle n_b^T \rangle) + \epsilon_d(\langle \hat{d}^\dagger \hat{d} \rangle_{\beta_a \beta_b} - \langle n_a^T \rangle),$$
$$(4)$$

where

$$\langle n_a^T \rangle = \frac{1}{e^{\beta_a \omega} - 1}, \quad \langle n_b^T \rangle = \frac{1}{e^{\beta_b \omega_0} - 1}, \qquad (5)$$

and

$$\langle \hat{c}^\dagger \hat{c} \rangle_{\beta_a \beta_b} = \text{Tr}[\hat{c}^\dagger \hat{c} \, \hat{\rho}^{\beta_a \beta_b}] \neq \langle n_b^T \rangle,$$
$$\langle \hat{d}^\dagger \hat{d} \rangle_{\beta_a \beta_b} = \text{Tr}[\hat{d}^\dagger \hat{d} \, \hat{\rho}^{\beta_a \beta_b}] \neq \langle n_a^T \rangle, \qquad (6)$$

making the ergotropy of state $\hat{\rho}^{\,\beta_a\beta_b}$ different from zero. In fact, defining the covariance matrix $\boldsymbol{\sigma}^{\beta_a\beta_b}_{ab}$ of state $\hat{\rho}^{\,\beta_a\beta_b}$ in the phase space basis $\boldsymbol{\delta\hat{a}}$ as

$$\left(\boldsymbol{\sigma}^{\beta_a\beta_b}_{ab}\right)_{ij} = \tfrac{1}{2}\langle\delta\hat{a}_i\delta\hat{a}_j + \delta\hat{a}_j\delta\hat{a}_i\rangle, \qquad (7)$$

where the averages are zero by definition of fluctuation operators, we have

$$\boldsymbol{\sigma}^{\beta_a\beta_b}_{ab} = \begin{pmatrix} 0 & \langle n_a^T\rangle + \tfrac{1}{2} & 0 & 0 \\ \langle n_a^T\rangle + \tfrac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \langle n_b^T\rangle + \tfrac{1}{2} \\ 0 & 0 & \langle n_b^T\rangle + \tfrac{1}{2} & 0 \end{pmatrix}. \qquad (8)$$

Thus, from the properties of symplectic transformations we have

$$\boldsymbol{\sigma}^{\beta_a\beta_b}_{dc} = \boldsymbol{M}^{-1} \cdot \boldsymbol{\sigma}^{\beta_a\beta_b}_{ab} \cdot (\boldsymbol{M}^{-1})^T. \qquad (9)$$

Finally, the following relations are obtained:

$$\langle\hat{d}^\dagger\hat{d}\rangle_{\beta_a\beta_b} = \left(\boldsymbol{\sigma}^{\beta_a\beta_b}_{dc}\right)_{21}, \qquad \langle\hat{c}^\dagger\hat{c}\rangle_{\beta_a\beta_b} = \left(\boldsymbol{\sigma}^{\beta_a\beta_b}_{dc}\right)_{43}. \qquad (10)$$

This results in the impossibility to extract energy from locally thermal states of noninteracting systems, while this is possible for interacting systems. This has motivated the study of the energetics of correlations in interacting systems [46].

In Fig. 3 we report the ratio between the total work and the average initial energy, for locally thermal states at different temperatures of the local mode a, as a function of the coupling parameter λ, which has been renormalized with respect to the critical value of the initial Hamiltonian. This renormalization causes a shift of the effective transition point, since the Hamiltonian after the quench is characterized by a different value of the critical coupling. The work-energy ratio can be thought of as an efficiency of energy storage. In Figs. 3(a) and 3(b) we report the case of two-strokes cycles with $\Delta\omega = \omega$ (A-B cycle in Fig. 2), and two-strokes cycles with $\Delta\omega = 0.1\,\omega$ and $\Delta\lambda = -0.1\,\lambda_{\rm cr}$ (C-D cycle in Fig. 2), respectively. The green curves in the insets are the work-ergotropy ratio.

The extraction regime is witnessed by a negative sign of the work. The oscillations of the work, as a function of the coupling, are a consequence of the free evolution part of the cycle and show the importance of choosing appropriate initial values of the coupling parameter to start the cycles from, in order to extract work. For increasing temperature of one of the local modes, the fraction of work extracted to the initial energy decreases. However, the closer we are to the phase transition, the better the extraction of work is, as shown by the height of the negative peaks of the work-energy ratio. Thus the phase transition helps retrieving the energy previously stored.

On the other hand the work-ergotropy ratio, reported in the insets of Fig. 3, does not heavily depend on the temperature, for the particular regime of parameters considered, with values of the negative peaks $-1 \leqslant \langle W\rangle/\mathcal{E} \leqslant -0.5$. As the ergotropy is peaked around the phase transition, a nonoptimal process will be more inefficient close to the transition, and this is witnessed by the decrease in absolute value by approaching the phase transition. This shows some of the consequences of the second law, inasmuch as despite being close to optimality (work-ergotropy close to one in absolute value) there is a
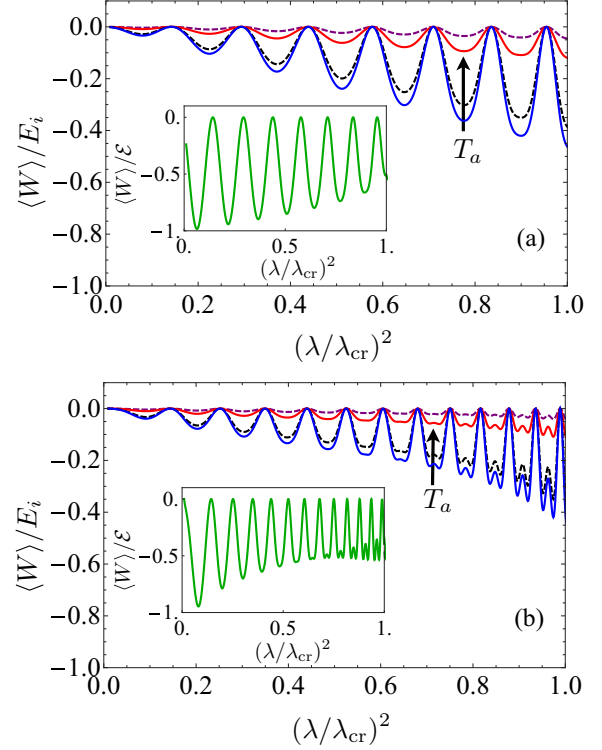

FIG. 3. Panel (a): Work-energy ratio for the A-B cycle in Fig. 2, with $\Delta\omega/2\pi = \omega/2\pi = 15\,\mathrm{MHz}$ and $\omega_0/2\pi = 8.3\,\mathrm{kHz}$. (b) Work-energy ratio for the C-D cycle in Fig. 2, with $\Delta\omega = 0.1\,\omega$, $\Delta\lambda = -0.1\,\lambda_{\rm cr}$. The free evolution time at point B is $\tau_B = 0.003\,\mathrm{s}$. Dashed purple: $\hat{\rho}^{\beta_a\beta_b}_1$, solid red: $\hat{\rho}^{\beta_a\beta_b}_2$, dashed black: $\hat{\rho}^{\beta_a\beta_b}_3$, solid blue: $\hat{\rho}^{\beta_a\beta_b}_4$, where $\beta_J = 1/k_B T_J$, with $T_b = 0.01\,K$, $T_a^1 = 10^{-1}\,\mathrm{K}$, $T_a^2 = 10^{-1.5}\,\mathrm{K}$, $T_a^3 = 10^{-2.5}\,\mathrm{K}$, $T_a^4 = 10^{-3}\,\mathrm{K}$. The green curve is the work-ergotropy ratio.

fraction of the initial energy, spent to create the state, that we are not able to extract.

VI. RESULTS FOR LOCALLY PASSIVE ENTANGLED STATES

Previous studies have shown the importance of quantum correlations for work extraction purposes, analyzing, for example, the role of discord in work extraction from a d-level system [47]. We now want to take into consideration the role of entanglement, studying its role in the work extraction for nonoptimal processes. We evaluate the entanglement between the two modes via the logarithmic negativity of a two-mode Gaussian state [48–50]. At this aim we recall the position and momentum quadratures of the fluctuation operators of the two modes

$$\hat{P}_x = \frac{1}{\sqrt{2}}(\delta\hat{a}^\dagger + \delta\hat{a}), \qquad \hat{P}_y = \frac{i}{\sqrt{2}}(\delta\hat{a}^\dagger - \delta\hat{a}),$$

$$\hat{A}_x = \frac{1}{\sqrt{2}}(\delta\hat{b}^\dagger + \delta\hat{b}), \qquad \hat{A}_y = \frac{i}{\sqrt{2}}(\delta\hat{b}^\dagger - \delta\hat{b}), \qquad (11)$$

where $\hat{P}_i$ refers to the photons, $\hat{A}_i$ to the atoms ($i = x, y$). In the case in which the first moments are null, as is it our case,
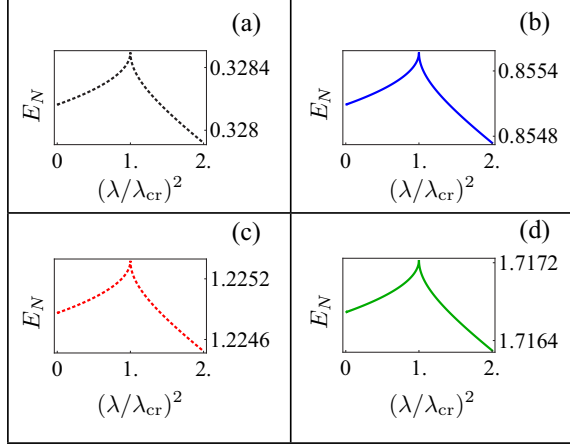
FIG. 4. Entanglement in the polariton partition $d - c$ for four locally passive entangled states, with different values of $\beta = 1/K_B T$. Dotted black: $T = 10^{-4}$, solid blue: $T = 2 \times 10^{-4}$, dotted red: $T = 3 \times 10^{-4}$, solid green: $T = 4 \times 10^{-4}$.

the covariance matrix for the quadratures is simply given by

$$S_{ij} = \tfrac{1}{2} \langle \hat{u}_i \hat{u}_j + \hat{u}_j \hat{u}_i \rangle, \tag{12}$$

with $\hat{u}$ the vector $\hat{u} = (\hat{P}_x, \hat{P}_y, \hat{A}_x, \hat{A}_y)^T$. It is useful to write the matrix explicitly as

$$\mathcal{S} = \begin{pmatrix} P & X \\ X^T & A \end{pmatrix}, \tag{13}$$

where X refers to the correlations between the two modes. If we now introduce the quantity

$$\Sigma(S) = \det P + \det A - 2 \det X, \tag{14}$$

we can define

$$\nu_- = \frac{1}{\sqrt{2}} \sqrt{\Sigma(\mathcal{S}) - \sqrt{\Sigma(\mathcal{S})^2 - 4 \det \mathcal{S}}}. \tag{15}$$

The logarithmic negativity is then obtained as

$$E_N = \max(0, -\log 2\nu_-), \tag{16}$$

which is a measure of the quantum entanglement, for a Gaussian state defined by matrix $\mathcal{S}$, in the partition of modes a and b. Analogously we can evaluate the entanglement in the partition of the polariton modes d and c, via appropriate replacements of the relative operators, and this is indeed what we consider in the following. In particular, we consider entangled states of the polaritonic modes that result in passive single-mode states. With this premise any work extraction can only be ascribed to entanglement. Consider the state

$$\left| \psi_{dc}^{\text{ent}} \right\rangle = \frac{1}{\sqrt{\mathcal{N}_{dc}}} \sum_{n=0}^{\infty} \exp\left[-\beta(\epsilon_+ + \epsilon_-)n/4 \right] |n\rangle_d |n\rangle_c, \tag{17}$$

with $\mathcal{N}_{dc} = \{1 - \exp[-\beta(\epsilon_+ + \epsilon_-)n/2]\}^{-1}$, whose marginals operators are passive states.

In Fig. 4 we show the entanglement for four locally passive entangled states defined in Eq. (17) for different values of the parameter β. These plots show that for the particular definition of this state, for a fixed value of β, the entanglement does not
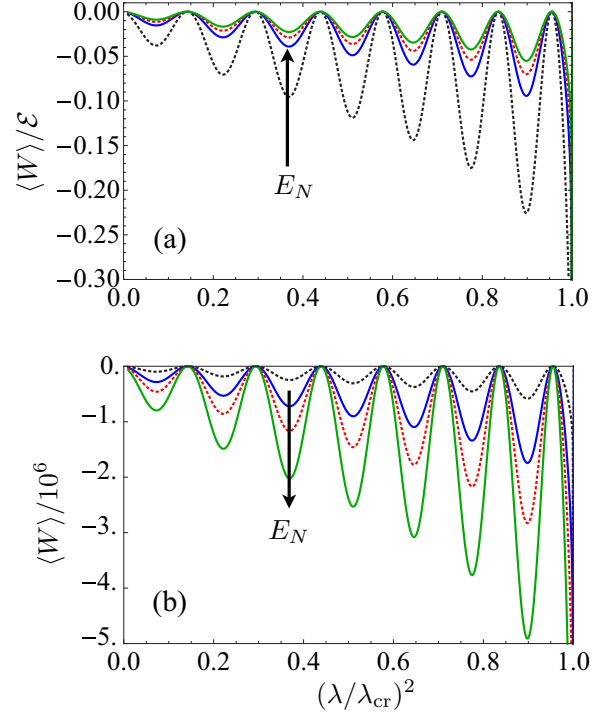

FIG. 5. (a) Work-ergotropy ratio for the locally passive entangled state in Eq. (17) for the A-B cycle in Fig. 2, with $\Delta\omega/2\pi = \omega/2\pi = 15$ MHz, and $\omega_0/2\pi = 8.3$ kHz. (b): Total work for the same cycles as in panel (a).

vary significantly with the coupling λ. In fact, for our regime of parameter $\epsilon_- \gg \epsilon_+$, and so the state is dependent almost only on ϵ_-, which is almost independent on the coupling λ. This is particularly useful as it allows us to use the entanglement as a parameter, which increases as we go from panel (a) to (d), to analyze its role in the extraction of work.

In Fig. 5(a) we report the work-ergotropy ratio, and in Fig. 5(b) the total work, for two-strokes cycles. If we were able to perform optimal work extraction, we would extract more work for more entangled states. This is true also in the case of the nonoptimal protocols considered here [cf. Fig. 5(b)]. However, Fig. 5(a) shows that if we consider the fraction of extracted work to the maximum extractable, the behavior is reversed: the ratio is smaller for more entangled states. This behavior becomes more interesting if we consider that this figure of merit is also an efficiency of energy storage, as for the initial pure state here chosen the ergotropy is equal to the average energy of the initial state. This is true even for the best case in which the state is a maximally entangled state (i.e., $\beta \to 0$).

VII. CONCLUSIONS

We have shown a nontrivial role played by entanglement and quantum phase transitions, for extraction and storing of energy, when considering both a nonoptimal process and the energetic cost of creating the initial state. We have studied, for the emblematic example of the Dicke model, the advantages (and lack thereof) arising from the use of a many-body quantum system as a working medium. If entanglement is

the only resource for work extraction, the phase transition improves work extraction due to entanglement achieving a maximum at the phase transition. This intuition could lead us to prepare initially entangled states with high degrees of entanglement, to extract an increasing amount of work. However, we have shown that even for the best case in which the state approaches a maximally entangled state, the energy spent to create this state overcomes the gain in the possible extraction of work. Our results provide guidelines for the development of the new technology based on quantum machines.

APPENDIX A: DIAGONALIZATION OF THE DICKE HAMILTONIAN

Here we show the details of the diagonalization of the Dicke Hamiltonian. The parameters of the Hamiltonian are given by

$$
\begin{aligned}
\widetilde{\omega}_0 &= \omega_0 - \frac{2\lambda \alpha_s \beta_s}{N^{3/2}\sqrt{1 - \frac{\beta_s^2}{N^2}}}, \\
\mu &= \frac{\lambda \alpha_s \beta_s}{N^{3/2}\sqrt{1 - \frac{\beta_s^2}{N^2}}}\left[1 + \frac{\beta_s^2}{2(N^2 - \beta_s^2)}\right], \\
\widetilde{\lambda} &= \lambda \frac{1 - 2\frac{\beta_s^2}{N^2}}{\sqrt{1 - \frac{\beta_s^2}{N^2}}}, \\
E_0 &= \omega \alpha_s^2 + \omega_0\left(\frac{\beta_s^2}{N} - \frac{N}{2}\right) + 4\lambda \frac{\alpha_s \beta_s}{\sqrt{N}}\sqrt{1 - \frac{\beta_s^2}{N^2}},
\end{aligned}
\tag{A1}
$$

where the steady-state mean fields are

$$
\alpha_s = \begin{cases} 0 & \text{for} \quad \lambda < \lambda_{\mathrm{cr}}, \\ \mp \frac{\lambda\sqrt{N}}{\omega}\sqrt{1 - \left(\frac{\lambda_{\mathrm{cr}}}{\lambda}\right)^4} & \text{for} \quad \lambda > \lambda_{\mathrm{cr}}, \end{cases}
\tag{A2}
$$

and

$$
\beta_s = \begin{cases} 0 & \text{for} \quad \lambda < \lambda_{\mathrm{cr}}, \\ \pm \frac{N}{2}\sqrt{1 - \left(\frac{\lambda_{\mathrm{cr}}}{\lambda}\right)^4} & \text{for} \quad \lambda > \lambda_{\mathrm{cr}}. \end{cases}
\tag{A3}
$$

Then we apply a transformation that renormalizes the effective masses of the oscillators by going into the phase space [39]

$$
\begin{aligned}
\hat{x} &= \frac{1}{\sqrt{2\omega}}(\delta\hat{a}^\dagger + \delta\hat{a}), \qquad \hat{p}_x = i\sqrt{\frac{\omega}{2}}(\delta\hat{a}^\dagger - \delta\hat{a}), \\
\hat{y} &= \frac{1}{\sqrt{2\widetilde{\omega}_0}}(\delta\hat{b}^\dagger + \delta\hat{b}), \qquad \hat{p}_y = i\sqrt{\frac{\widetilde{\omega}_0}{2}}(\delta\hat{b}^\dagger - \delta\hat{b}).
\end{aligned}
\tag{A4}
$$

After this transformation we get

$$
\begin{aligned}
\hat{H} = \frac{1}{2}\Big\{&\omega^2 \hat{x}^2 + \hat{p}_x^2 + \left(\widetilde{\omega}_0^2 - 4\mu\widetilde{\omega}_0\right)\hat{y}^2 + \hat{p}_y^2 \\
&+ 4\widetilde{\lambda}\sqrt{\omega\widetilde{\omega}_0}\,\hat{x}\hat{y} - \widetilde{\omega}_0 - \omega\Big\} + E_0.
\end{aligned}
\tag{A5}
$$

Then we rotate the system coordinate with the transformation (we will indicate the Bogoliubov angle as $\gamma^{(B)}$)

$$
\begin{aligned}
\hat{x} &= \hat{q}_1 \cos\gamma^{(B)} + \hat{q}_2 \sin\gamma^{(B)}, \\
\hat{y} &= -\hat{q}_1 \sin\gamma^{(B)} + \hat{q}_2 \cos\gamma^{(B)}
\end{aligned}
\tag{A6}
$$

and similar transformations apply to the momentum operators. In the new representation the interaction is removed if we choose the angle $\gamma^{(B)}$ such that

$$
\tan(2\gamma^{(B)}) = \frac{4\widetilde{\lambda}\sqrt{\omega\widetilde{\omega}_0}}{\widetilde{\omega}_0^2 - 4\mu\widetilde{\omega}_0 - \omega^2}.
\tag{A7}
$$

The Hamiltonian for the two decoupled oscillators is

$$
\hat{H} = \frac{1}{2}\left\{\epsilon^- \hat{q}_1^2 + \hat{p}_1^2 + \epsilon^+ \hat{q}_2^2 + \hat{p}_2^2 - \omega - \widetilde{\omega}_0\right\} + E_0,
\tag{A8}
$$

where the energies are

$$
\epsilon^\pm = \sqrt{\frac{1}{2}\left(z + 2\omega^2 \pm \mathrm{sign}(z)\sqrt{z^2 + 16\widetilde{\lambda}^2\omega\widetilde{\omega}_0}\right)},
\tag{A9}
$$

with $z = \widetilde{\omega}_0^2 - 4\mu\widetilde{\omega}_0 - \omega^2$. Then again we apply the transformation

$$
\begin{aligned}
\hat{q}_1 &= \frac{1}{\sqrt{2\epsilon^-}}(\hat{d}^\dagger + \hat{d}), \qquad \hat{p}_1 = i\sqrt{\frac{\epsilon^-}{2}}(\hat{d}^\dagger - \hat{d}), \\
\hat{q}_2 &= \frac{1}{\sqrt{2\epsilon^+}}(\hat{c}^\dagger + \hat{c}), \qquad \hat{p}_2 = i\sqrt{\frac{\epsilon^+}{2}}(\hat{c}^\dagger - \hat{c}),
\end{aligned}
\tag{A10}
$$

and we end up finally with the Hamiltonian

$$
\hat{H} = \epsilon^- \hat{d}^\dagger \hat{d} + \epsilon^+ \hat{c}^\dagger \hat{c} + \frac{1}{2}(\epsilon^- + \epsilon^+ - \omega - \widetilde{\omega}_0) + E_0
\tag{A11}
$$

In the phase space the diagonalization is obtained with the transformation $\boldsymbol{\delta\hat{a}} = \boldsymbol{M} \cdot \hat{\boldsymbol{d}}$, with $\boldsymbol{\delta\hat{a}} = (\delta\hat{a}, \delta\hat{a}^\dagger, \delta\hat{b}, \delta\hat{b}^\dagger)^T$ and $\hat{\boldsymbol{d}} = (\hat{d}, \hat{d}^\dagger, \hat{c}, \hat{c}^\dagger)^T$. The symplectic matrix $\boldsymbol{M}$ is

$$
\boldsymbol{M} = \begin{pmatrix} A_+ & A_- & B_+ & B_- \\ A_- & A_+ & B_- & B_+ \\ C_+ & C_- & D_+ & D_- \\ C_- & C_+ & D_- & D_+ \end{pmatrix},
\tag{A12}
$$

where the coefficients are

$$A_{\pm} = \frac{1}{2}\cos(\gamma^{(B)})\left(\sqrt{\frac{\omega}{\epsilon_-}} \pm \sqrt{\frac{\epsilon_-}{\omega}}\right),$$

$$B_{\pm} = \frac{1}{2}\sin(\gamma^{(B)})\left(\sqrt{\frac{\omega}{\epsilon_+}} \pm \sqrt{\frac{\epsilon_+}{\omega}}\right),$$

$$C_{\pm} = -\frac{1}{2}\sin(\gamma^{(B)})\left(\sqrt{\frac{\widetilde{\omega_0}}{\epsilon_-}} \pm \sqrt{\frac{\epsilon_-}{\widetilde{\omega_0}}}\right), \quad \text{(A13)}$$

$$D_{\pm} = \frac{1}{2}\cos(\gamma^{(B)})\left(\sqrt{\frac{\widetilde{\omega_0}}{\epsilon_+}} \pm \sqrt{\frac{\epsilon_+}{\widetilde{\omega_0}}}\right).$$

APPENDIX B: SUDDEN QUENCH CYCLE

In the case of a cycle in which the strokes are realized with successive sudden quenches, it is possible to obtain analytical expressions for the average work. In what follows we will consider explicitly the case of a pure initial state, just for convenience of calculation, but everything can be easily transposed to the case of a general mixed initial state with the proper averages taken.

Suppose that we want to realize a four strokes cycle (A-B-C-D), and we initially prepare the state of the system in the state $|\psi_A\rangle$ where A labels the starting point of the cycle in the parameter space. For a sudden quench the unitary evolution operator is the identity $\hat{U}(t) = \hat{\mathbb{1}}$, so that for the average work we have

$$\langle W \rangle_{AB} = \langle \psi_A | (\hat{H}_B - \hat{H}_A) | \psi_A \rangle = E_{0B} - E_{0A}$$
$$+ \langle \epsilon_B^- \hat{d}_B^\dagger \hat{d}_B + \epsilon_B^+ \hat{c}_B^\dagger \hat{c}_B \rangle - \langle \epsilon_A^- \hat{d}_A^\dagger \hat{d}_A + \epsilon_A^+ \hat{c}_A^\dagger \hat{c}_A \rangle$$
$$+ \frac{1}{2}(\epsilon_B^- - \epsilon_A^- + \epsilon_B^+ - \epsilon_A^+ - \omega_B$$
$$+ \omega_A - \widetilde{\omega}_{0B} + \widetilde{\omega}_{0A}). \quad \text{(B1)}$$

In order to calculate this expression we use the relation between mode operators at different points in the parameter space

$$\hat{d}_B = M_B^{-1} M_A \hat{d}_A + M_B^{-1}(\alpha_A - \alpha_B), \quad \text{(B2)}$$

which allows us to express the terms $\hat{d}_B^\dagger \hat{d}_B$ and $\hat{c}_B^\dagger \hat{c}_B$ in terms of operators $\hat{d}_A$. It is supposed that we know the covariance matrix

$$(\sigma_A^d)_{ij} = \frac{1}{2}\langle (\hat{d}_A)_i (\hat{d}_A)_j + (\hat{d}_A)_j (\hat{d}_A)_i \rangle, \quad \text{(B3)}$$

where the indices i and j denote the components of the respective vectors or matrix. In what follows we use the convention that number as indices denote elements of vectors or matrices, while letters as indices denote different points in the parameter space. If we indicate with $\hat{d}_{Bi}$ the ith element of vector $\hat{d}_B$, and similarly for others, we have

$$\hat{d}_B^\dagger \hat{d}_B = \left(M_B^{-1} M_A \hat{d}_A + M_B^{-1}(\alpha_A - \alpha_B)\right)_2$$
$$\times \left(M_B^{-1} M_A \hat{d}_A + M_B^{-1}(\alpha_A - \alpha_B)\right)_1,$$
$$\hat{c}_B^\dagger \hat{c}_B = \left(M_B^{-1} M_A \hat{d}_A + M_B^{-1}(\alpha_A - \alpha_B)\right)_4$$
$$\times \left(M_B^{-1} M_A \hat{d}_A + M_B^{-1}(\alpha_A - \alpha_B)\right)_3. \quad \text{(B4)}$$

Given the covariance matrix σ_A^d of the initial state $|\psi_A\rangle$, we can conveniently express everything in terms of elements of the matrix $Q^{AB} = M_B^{-1} M_A$ and vector $V^{AB} = M_B^{-1}(\alpha_A - \alpha_B)$ as follows:

$$\hat{d}_B = Q^{AB} \hat{d}_A + V^{AB}, \quad \text{(B5)}$$

$$\langle \psi_A | \hat{d}_B^\dagger \hat{d}_B | \psi_A \rangle = \sum_{ij} Q_{2i}^{AB} Q_{1j}^{AB} \left[(\sigma_A^d)_{ij} + \Lambda_{ij}\right] + V_2^{AB} V_1^{AB}, \quad \text{(B6)}$$

$$\langle \psi_A | \hat{c}_B^\dagger \hat{c}_B | \psi_A \rangle = \sum_{ij} Q_{4i}^{AB} Q_{3j}^{AB} \left[(\sigma_A^d)_{ij} + \Lambda_{ij}\right] + V_4^{AB} V_3^{AB}, \quad \text{(B7)}$$

so that the work is given by

$$\langle W \rangle_{AB} = \epsilon_B^- \langle \hat{d}_B^\dagger \hat{d}_B \rangle + \epsilon_B^+ \langle \hat{c}_B^\dagger \hat{c}_B \rangle - \epsilon_A^+ \left[(\sigma_A^d)_{43} + \Lambda_{43}\right]$$
$$- \epsilon_A^- \left[(\sigma_A^d)_{21} + \Lambda_{21}\right] + \Delta C_{AB}, \quad \text{(B8)}$$

where the first two terms are given in Eqs. (B6) and (B7), and ΔC_{AB} account for the total constant part in Eq. (B1).

For a second stroke ($B \to C$) we need to evaluate the following expression:

$$\langle W \rangle_{BC} = \langle \psi_A | e^{i\hat{H}_B \tau_B}(\hat{H}_C - \hat{H}_B) e^{-i\hat{H}_B \tau_B} | \psi_A \rangle$$
$$= \langle \psi_A | e^{i\hat{H}_B \tau_B}(\epsilon_C^- \hat{d}_C^\dagger \hat{d}_C + \epsilon_C^+ \hat{c}_C^\dagger \hat{c}_C) e^{-i\hat{H}_B \tau_B} | \psi_A \rangle$$
$$- \langle \psi_A | \epsilon_B^- \hat{d}_B^\dagger \hat{d}_B + \epsilon_B^+ \hat{c}_B^\dagger \hat{c}_B | \psi_A \rangle + \Delta C_{BC}. \quad \text{(B9)}$$

It is convenient to define the diagonal matrix

$$D^B = \text{diag}(e^{-i\epsilon_B^- \tau_B}, e^{i\epsilon_B^- \tau_B}, e^{-i\epsilon_B^+ \tau_B}, e^{i\epsilon_B^+ \tau_B}), \quad \text{(B10)}$$

so that we can write the evolution of the vector $\hat{d}_K$ ($K = A, B, C, D$) in matrix notation as

$$e^{i\hat{H}_K \tau_K} \hat{d}_K e^{-i\hat{H}_K \tau_K} = D^K \hat{d}_K. \quad \text{(B11)}$$

With this definition we can compute the first two terms of Eq. (B9) as

$$\langle \psi_A | e^{i\hat{H}_B \tau_B} \hat{d}_C^\dagger \hat{d}_C e^{-i\hat{H}_B \tau_B} | \psi_A \rangle$$
$$= \langle \psi_A | (R^{AC} \hat{d}_A + S^{AC})_2 (R^{AC} \hat{d}_A + S^{AC})_1 | \psi_A \rangle \quad \text{(B12)}$$

and

$$\langle \psi_A | e^{i\hat{H}_B \tau_B} \hat{c}_C^\dagger \hat{c}_C e^{-i\hat{H}_B \tau_B} | \psi_A \rangle$$
$$= \langle \psi_A | (R^{AC} \hat{d}_A + S^{AC})_4 (R^{AC} \hat{d}_A + S^{AC})_3 | \psi_A \rangle, \quad \text{(B13)}$$

with matrix $R^{AC} = Q^{BC} D^B Q^{AB}$, and vector $S^{AC} = Q^{BC} D^B V^{AB} + V^{BC}$. The meaning of expression for the matrix R^{AC} is straightforward. The matrix Q^{AB} is responsible for the connection between operators of points A and B in the parameter space due to the quench $A \to B$. Then matrix D^B expresses the time evolution of the system at point B, and finally again matrix Q^{BC} realizes the quench $B \to C$. Vector S^{AC} instead expresses the contribution coming from the mean fields in the evolution from A to C. There can be a contribution from the difference of mean fields between A and B (V^{AB}), then an evolution in B (D^B) and finally a quench $B \to C$ (Q^{BC}); in addition there is also a contribution coming from the difference between the mean fields of B and C.

The crucial point is that Eqs. (B12) and (B13) are totally equivalent to Eqs. (B6) and (B7), so that we can use the same results in the latter expressions to evaluate the former ones, with the substitutions $\boldsymbol{Q}^{AB} \to \boldsymbol{R}^{AC}$ and $\boldsymbol{V}^{AB} \to \boldsymbol{S}^{AC}$. The second term in Eq. (B9) for the work $\langle W \rangle_{BC}$ has already been evaluated for the work $\langle W \rangle_{AB}$. If we keep on calculating the averages of work for each stroke in the same way, eventually we need to sum all the contributions to get the total average work for the cycle, which, e.g., in the case of a four-strokes cycle gives $\langle W \rangle_{\text{tot}} = \langle W \rangle_{AB} + \langle W \rangle_{BC} + \langle W \rangle_{CD} + \langle W \rangle_{DA}$.

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