Heat dissipation and information flow for delayed bistable Langevin systems near coherence resonance

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In this paper, stochastic thermodynamics of delayed bistable Langevin systems near coherence resonance is discussed. We calculate the heat dissipation rate and the information flow of a delayed bistable Langevin system under various noise intensities. Both the heat dissipation rate and the information flow are found to be bell-shaped functions of the noise intensity, which implies that coherence resonance manifests itself in the thermodynamic properties.

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I. INTRODUCTION

Stochastic thermodynamics (ST) of small systems has been one of the most significant developments in the field of nonequilibrium statistics in the past two decades [1–4]. Due to the non-negligible fluctuation effect, thermodynamic properties of small systems become stochastic variables, thus the first law and second law of thermodynamics could be interpreted in the stochastic trajectory level [5,6]. In those small systems with Markovian dynamics, a total dissipation function σ is defined as the summation of system entropy change Δs and medium entropy change $\Delta s_m = q/T$ such that $\sigma = \Delta s + \Delta s_m$. It is noted that events with negative dissipation $\sigma < 0$ could happen in small systems, while the averaged dissipation is positively defined ($\langle\sigma\rangle \geqslant 0$), which can be regarded as the second law of thermodynamics for small systems. In a steady state, $\langle\dot{\sigma}\rangle \geqslant 0$ reduces to $\langle\dot{s}_m\rangle \geqslant 0$. Such a physical picture has been revealed in various Markovian systems, such as colloid particles [7–9], biomolecules [10–12], chemical reaction networks [13–16], granular medium [17], driven Lorentz gas [18], electronic devices [19,20], and two-level optical systems [21,22].

In this study, we focus on the delayed bistable Langevin systems. Time delay is found in many real systems as results of the finite speed of signal transfer or feedback control processes [23–29]. Due to the interaction of nonlinearity, time delay, and random force (or equivalently noise), a bistable system under time delayed feedback could show coherence resonance, i.e., the peak hight in the power spectrum reaches a maximum at certain noise intensity, which is verified both theoretically [30] and experimentally [31]. As the Langevin systems are widely used to model the nanomachines which consume energies, it would be of great interest to see what we can deduce from the thermodynamic perspective. It is known that the mean heat dissipation rate $\langle\dot{s}_m\rangle$ of a stationary delayed system is no more positive definite [32–35]. Further studies demonstrate that the concept of Shannon entropy or the mutual information can be used to study the ST of small systems under control, where the information flow $\dot{I}$ is a low bound of the medium entropy change rate $\dot{S}_m$ [36–42]. ST of delayed systems has been discussed in several literatures [32,34,35,43], where detailed results are mainly derived for linear systems.

Herein we study the mean thermodynamic properties of delayed bistable Langevin systems, which are found to show nontrivial dependence on the noise intensity. Specifically, the medium entropy change rate $\langle\dot{s}_m\rangle$ and the conditional information flow $\dot{\bar{I}}_c$ can reach a maximum or a minimum at certain noise intensity. Then one may conclude that coherence resonance can manifest itself in the thermodynamic properties of delayed bistable systems.

This paper is organized as follows. In Sec. II, a model of delayed bistable Langevin systems is introduced. The stochastic thermodynamics of delayed Langevin systems is presented in Sec. III, where the heat dissipation rate and the information flow are introduced. The dependence of heat dissipation and information flow on the noise intensity is shown in Sec. IV, where coherence-resonance-like behavior is found. Finally some concluding remarks are given in Sec. V.

II. MODEL AND DESCRIPTION

A schematic diagram of a delayed Langevin system is shown in Fig. 1. In this study, we take an one-dimensional forced Brownian particle A as an illustrative example. The particle is subjected to a force $F(x,x_\tau)$ generated by a control device such as an optical laser as done in real experiments [8,9], where $x(t)$ is the coordinate or the state variable of the particle at time t, $x_\tau(t) = x(t-\tau)$ is a delayed state variable, τ is the delay time. The particle contacts to a heat bath with a temperature T, and the dynamic effect from the heat bath is spilt into a friction force $\gamma\dot{x}$ and a random force $\xi(t)$. In the over-damped limit, the motion of the particle is described by the Langevin equation [6]

$$\gamma\dot{x} = F(x,x_\tau) + \sqrt{2D}\xi(t), \tag{1}$$

where $\dot{x} = \frac{dx}{dt}$, $\xi(t)$ is a Gaussian white noise with $\langle\xi(t)\rangle = 0$ and $\langle\xi(t)\xi(t')\rangle = \delta(t-t')$. The time derivative of a function $h(t)$ will be denoted as $\dot{h} = \frac{dh}{dt}$ throughout this study. The deterministic force is taken as $F(x,y) = ax + by - x^3$. For $|b| < a$, the deterministic system has two stable solutions $x_s = \pm\sqrt{a+b}$. The noise intensity D is related to the temperature T of the heat bath via the Einstein relation $D = \gamma k_B T$, where γ is the friction coefficient and k_B is the Boltzmann constant.

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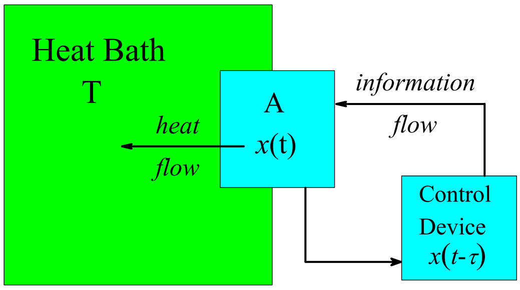

FIG. 1. Schematic diagram for a delayed Langevin system.

In the following, we set $\gamma = 1$ and $k_B = 1$ without loss of generality.

It could be convenient to discuss the ST using the probability of the variables. One can introduce a new state variable $y(t) = x(t-\tau)$, and the two point probability distribution function $p(x,y;t)$ for x and y satisfies a Fokker-Planck equation as [44]

$$\partial_t p(x,y;t) = -\partial_x J_x(x,y;t) - \partial_y J_y(x,y;t) \tag{2}$$

with probability current defined as $J_x(x,y;t) = F(x,y)p(x,y;t) - D\partial_x p(x,y;t)$, $J_y(x,y;t) = \int F(y,z)p(x,y,z;t)dz - D\partial_y p(x,y;t)$, where $p(x,y,z;t)$ is a three point probability distribution function with $z(t) = x(t-2\tau)$. One should note that such a Fokker-Planck equation is not closed in general as the evaluation of the probability current involves the three point probability distribution function.

With the joint probability distribution, one can introduce two marginal distributions as $p(x;t) \equiv \int p(x,y;t)dy$ and $p(y;t) \equiv \int p(x,y;t)dx$. A delayed system could usually reach a steady state, where the probability distributions reach stationary values $p_s(x,y) \equiv p(x,y;t \to \infty)$, $p_s(x) \equiv p(x;t \to \infty)$, and $p_s(y) \equiv p(y;t \to \infty)$ such that

$$\partial_t p_s(x,y) = 0, \partial_t p_s(x) = 0, \partial_t p_s(y) = 0. \tag{3}$$

III. ENERGETICS OF A DELAYED LANGEVIN SYSTEM

Concepts of medium entropy change rate and the information flow have been applied successfully to delayed linear systems [32,34,35,43]. In the following we briefly summarize these two concepts and will use them to study the stationary delayed bistable systems.

Denote a stochastic trajectory as $\chi(t) = \{x(t')\}|_{t'=-\tau}^{t}$, then one can introduce trajectory-dependent thermodynamic properties following Sekimoto's prescription [5]. The heat exchange rate $\dot{q}$ of the delayed system A is defined as the work done by the friction force and the random force as $\dot{q} = [\dot{x} - \sqrt{2D}\xi(t)]\dot{x} = F(x,y)\dot{x}$. We adopt the notion that $\dot{q} > 0$ or $\dot{q} < 0$ implies that heat is dissipated into or absorbed from the environment. The medium entropy change rate is then evaluated as [5]

$$\dot{s}_m = \dot{q}/T = \frac{1}{D}F(x,y)\dot{x}. \tag{4}$$

One can use a mutual information term $I = \ln[\frac{p(x,y;t)}{p(x;t)p(y;t)}]$ [4,36] to characterize the correlation between the system A and the control device. The information change rate (or information flow) $\dot{I}$ is $\dot{I} = \partial_t I + (\partial_x I)\dot{x} + (\partial_y I)\dot{y}$. For a system that stays in a stationary state, it is noted that $\partial_t I = 0$ and then the conditional information change rate due to the evolution of x can be evaluated as [45]

$$\dot{I}_c = (\partial_x I)\dot{x} = \left(\partial_x\left[\frac{p(x,y;t)}{p(x;t)}\right]\right)\dot{x}. \tag{5}$$

To derive the second equality in Eq. (5), $\partial_x ln[p(y;t)] = 0$ is used. Herein, we take the notion that $\dot{I}_c > 0$ or $\dot{I}_c < 0$ implies that information flow is ejected from or injected into the delayed system A. When doing the average over x and y, one can recover the results by Rosinberg, Munakata, and Tarjus [35]. Let us define $\dot{S}_m \equiv \langle \dot{s}_m \rangle$ and $\dot{\bar{I}}_c \equiv \langle \dot{I}_c \rangle$. When the system reaches a stationary state, one can proved that [35]

$$\dot{S}_m \geqslant \dot{\bar{I}}_c. \tag{6}$$

According to Eq. (6), the information flow is a lower bound for the released heat rate, so the system A may be able to absorb heat ($\dot{S}_m < 0$) when the information flow ($\dot{\bar{I}}_c < 0$) is injected in.

One should note that the second-law–like inequalities for a delayed system is not unique. Based on the Fokker-Planck equation of the delayed system, one can introduce the log ratio of the probability of a forward trajectory and a backward trajectory as the total dissipation function σ, which also satisfies $\langle \sigma \rangle \geqslant 0$ [32]. It is also possible to introduce another dissipation function with the path integral of delayed Langevin equation [33]. However, the term $\sigma - \Delta s - \Delta s_m$ in those cases does not have a clear physic significance.

IV. NUMERICAL RESULTS FOR A DELAYED LANGEVIN SYSTEM IN A STEADY STATE

In this section, we apply the prescription in Sec. III to study the bistable system with $F(x,x_\tau) = ax + bx_\tau - x^3$. For a bistable system with fixed dimensionless parameters $a = 1$, $|b| = 0.1, \tau = 250$, and with tunable noise intensity D, the main peak in the power spectrum of the system can reach a maximum. This nontrivial result is first reported by Tsimring and Pikovsky and is recognized as coherence resonance [30]. In order to study the thermodynamic properties near coherence resonance, we use the same parameters for the bistable system. We did stochastic simulations of the Langevin equations with a time step $dt = 0.01$. By introducing a variable $y = x_\tau$, the stationary probability distribution $p_s(x,y)$ evaluated from numerical simulation is fitted to an empirical function $p_s(x,y) = Ne^{-\sum_{i+j=2,4} a_{ij}x^i y^j}$. Denote $\alpha(x,y) \equiv -\partial_x \ln[p_s(x,y)]$, the medium entropy change rate $\dot{S}_m$ and the conditional information flow $\dot{\bar{I}}_c$ in a steady state are evaluated via $\dot{S}_m = \frac{1}{D}\langle F(x,y)\dot{x}\rangle$ and $\dot{\bar{I}}_c = \langle \alpha(x,y)\dot{x}\rangle$.

For control parameters $a = 1, b = 0.1, \tau = 250$ and $a = 1, b = -0.1, \tau = 250$, the stationary thermodynamic properties of the delayed systems are plotted in Figs. 2 and 3. In the range of $0.04 \leqslant D \leqslant 0.3$, one can find that $\dot{\bar{I}}_c < 0$ and $\dot{S}_m > 0$, so the inequality $\dot{S}_m \geqslant \dot{\bar{I}}_c$ is valid as expected. Furthermore, it is noted that $\dot{S}_m$ reaches a maximum while $\dot{\bar{I}}_c$ reaches a minimum at the noise intensity $D \approx 0.12$. Note that the main peak in the spectrum of the delayed bistable

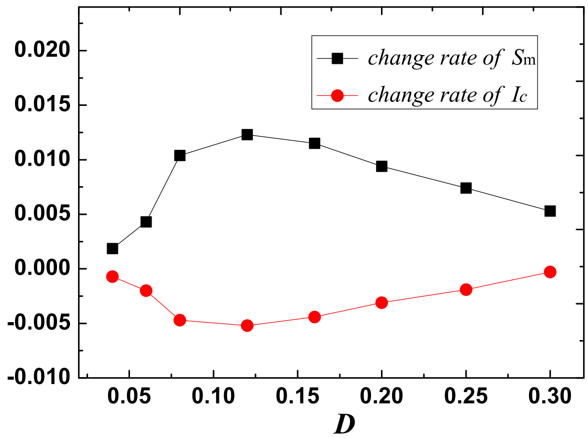

FIG. 2. Thermodynamic properties as a function of the noise intensity D for a delayed bistable system with $F(x,x_\tau) = x + 0.1x_\tau - x^3$. The lines are guides to the eye.

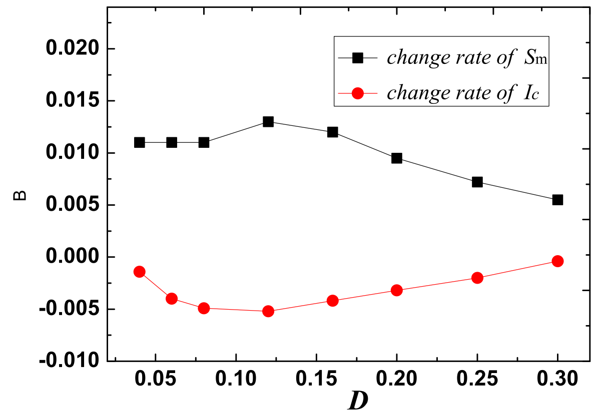

FIG. 3. Thermodynamic properties as a function of the noise intensity D for a delayed bistable system with $F(x,x_\tau) = x - 0.1x_\tau - x^3$. The lines are guides to the eye.

systems could reach a maximum in the range of $0.04 < D < 0.32$ [30], one may conclude that coherence resonance manifests itself in terms of thermodynamic properties. Though not shown, we also perform numerical simulations for the bistable systems with different control parameters, where both the heat dissipation rate and the information flow show bell-shaped dependence of the noise intensity.

It is found that similar behavior appears in bistable systems under periodic force, i.e., the signal to noise ratio could reach a maximum at certain noise intensity and is known as stochastic resonance [46], the heat dissipation rate in such systems also shows bell-shaped dependence on the noise intensity [47], and hence stochastic resonance can manifest itself in thermodynamic properties. Note that there are other dynamic systems which could show coherence resonance, one may wonder whether coherence resonance in general systems can always be manifested in the thermodynamic properties. Our answer to this question is no. A counter example is the stochastic bifurcation systems [48], e.g., coherence resonance could happen in stochastic systems that near Hopf bifurcation, however, the medium change rate $\dot{S}_m$ increases as the noise intensity increases and hence no coherence resonance behavior is observed for the thermodynamic properties [16].

As a brief summary, we studied the ST of delayed bistable systems, where both the heat dissipation rate and the information flow show bell-shaped dependence on the noise intensity. Such an observation implies that coherence resonance can manifest itself in thermodynamic properties of delayed bistable systems. Note that the thermodynamics properties can be tuned by the dynamic parameters such as a,b,τ, and D, our finding may find potential applications in controlling the performance of nonlinear molecular motors or artificial nanomachines.

V. CONCLUDING REMARKS

In conclusion, stochastic thermodynamics of delayed bistable Langevin systems near coherence resonance is discussed. The medium entropy change rate and the information flow are introduced to characterize the transport of the delayed systems. We find that both the medium entropy change rate and the information flow are bell-shaped functions of the noise intensity, which implies that coherence resonance can manifest itself in the thermodynamic properties.

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