# Breathers and rogue waves excited by all-magnonic spin-transfer torque

Zai-Dong Li,[1,2,*] Qiu-Yan Li,[1] Tian-Fu Xu,[3] and Peng-Bin He[4]

[1]*Department of Applied Physics, Hebei University of Technology, Tianjin 300401, China*
[2]*International Center for Quantum Materials, School of Physics, Peking University, Beijing 100871, China*
[3]*Department of Physics, Yanshan University, Qinhuangdao 066004, China*
[4]*School of Physics and Electronics, Hunan University, Changsha 410082, China*

(Received 5 June 2016; published 25 October 2016; corrected 4 November 2016)

In terms of Darboux transformation we investigate the dynamic process of spin wave passing through a magnetic soliton. It causes nonlinear excitations, such as Akhmediev breathers solution and Kuznetsov-Ma soliton. The former case demonstrates a spatial periodic process of a magnetic soliton forming the petal with four pieces. The spatial separation of adjacent magnetic petals increases rapidly, while one valley splits into two and the amplitude of valley increases gradually with the increasing amplitude of spin wave. The other case shows a localized process of the spin-wave background. In the limit case, we get rogue waves and clarify its formation mechanism.

DOI: 10.1103/PhysRevE.94.042220

## I. INTRODUCTION

During the past several decades there has been significant progress in describing dynamics of magnetization in magnetic nanostructures. In these studies, self-organization [1] is one of the most interesting phenomena in nature. In magnetism, this phenomenon has been intensively studied in terms of the spontaneous formation of magnetic domains such as stripe domains, bubble domains, soliton, and magnetic vortex. In addition, the study on two-dimensional magnetic systems of thin films have revealed further interesting magnetic self-organization patterns such as spin waves [2] and skyrmion lattices [3–5], which can be nucleated as a metastable state in thin films. It opens a path to concepts of magnetic memories and contributes to designing memories based on skyrmion motion in nanotracks.

The dynamics of domain wall is of great significance in ferromagnetic nanowires for its potentially technological applications [6–11]. For example, a magnetic domain wall forms a spatially localized configuration of magnetization in ferromagnet, and it can be seen as a potential hill, which separates two generated magnetic states [12,13]. The propagation of domain wall with the influence of spin-Hall effect [14], Rashba effect [15], and Dzyaloshinskii-Moriya interaction [16–19] has drawn considerable interest in low-dimensional magnetism. These studies have been inspired not only by the fundamental physics questions but also by the potential application for the spintronic memory and logic nanodevices. Recently, considerable attention has been paid to the dynamics of magnetization associated with spin-polarized current in layered materials [20,21]. The spin-polarized current can cause many unique phenomena [22,23], such as spin-wave excitation [24,25], magnetization switching [20] and reversal [26–28], and enhanced Gilbert damping [29,30] in magnetic multilayers. Nowadays, spin-polarized currents are commonly used to create, manipulate, and control nanoscale magnetic excitations such as domain walls [31–34] and vortices [35–37].

Nonlinear excitations [12,13] are general phenomena in magnetic-ordered materials. In ferromagnet a cluster of magnons tends to self-localization because of attractive interaction. In a certain sense, the attraction of magnons is critical for a one-dimensional ferromagnet because it produces a bound state of quasiparticles (magnons), i.e., self-localization. A spin wave may be regarded as a cluster of a macroscopic number of coherent magnons. Because of the attractive interaction, the magnon cluster tends to be localized, and thus the spin wave becomes unstable. The developing instability causes magnetization localization and brings about a domain wall and a magnetic soliton.

However, the nonlinear excitations have not been well explored. When a spin wave passes through a magnetic soliton, a spin angular momentum can be transferred from the propagating magnons to the soliton, which is called by all-magnonic spin-transfer torque [43]. This all-magnonic spin-transfer torque can affect the dynamics of magnetization and magnetic states can occur. In this paper, we will study the exact breather solutions and magnetic states. As an example, we give the exact solutions of bright (dark) rogue waves caused by this magnonic spin-transfer torque.

## II. EXACT BREATHER SOLUTIONS AND ROGUE WAVES

As a simple model, we consider the Landau-Liftshitz equation,

$$\frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times \frac{\partial^2 \mathbf{m}}{\partial x^2}, \tag{1}$$

which admits spin-wave and soliton solutions. The exact breather solutions and rogue waves of Eq. (1) can be structured by the Darboux transformation. The main idea of the Darboux transformation is that it first transforms the nonlinear equation into the Lax representation, and then by a series of transformations the soliton solution can be constructed algebraically with the obvious seed solution of the nonlinear equation. It is an effective technique to generate a solution for Eq. (1) once a *seed solution* $\mathbf{m}_0$ is known. In the following, we take the initial "seed" as spin wave, i.e., $\mathbf{m}_0 \equiv (m_{01}, m_{02}, m_{03}) = (A_s \cos\delta, A_s \sin\delta, \sqrt{1 - A_s^2})$ with $\delta = k_s x - \omega_s t$ and the dispersion relation $\omega = -k_s^2 m_{03}$.

In terms of the developed procedure of Darboux transformation [38–42], we obtain the exact solutions of Eq. (1) as the

*lizd@hebut.edu.cn

 

form

$$\mathbf{m}\cdot\sigma = K(\mathbf{m}_0\cdot\sigma)K^{-1}, \tag{2}$$

where $\sigma$ is pauli matrix and the matrix $K$ is given by

$$K = \frac{1}{|\xi|^2(P+Q)}\begin{pmatrix} \xi^* P + \xi Q & -\mu R^* e^{-i\delta} \\ \mu R e^{i\delta} & \xi P + \xi^* Q \end{pmatrix},$$

with $\xi = i\mu/2 + \nu/2$, $N = \sqrt{(k_s - 2\xi m_{03})^2 + 4\xi^2 A_s^2}$, $\beta = -i2\xi - i m_{0,3} k_s$, $P = h_{11}h_{11}^*$, $Q = h_{12}h_{12}^*$, $R = -ie^{-i\delta} h_{11}^* h_{12}$, $h_{11} = i(C_1 e^B - C_2 e^{-B})e^{-i\delta/2}$, $h_{12} = (C_1 e^{-B} - C_2 e^B)e^{i\delta/2}$, $C_1 = \sqrt{(\mu m_{03} + i(A_s^2 k_s - N))/2}$, $C_2 = \sqrt{(\mu m_{03} + i(A_s^2 k_s + N))/2}$, and $B = -iN(x + i\beta t)/2$. The solution in Eq. (2) denotes a soliton solution embedded in a spin wave background. With the increasing of $\mu$, the spin wave background is gradually localized and forms breathers due to the interaction between soliton and spin wave background. With the analytical solutions in Eq. (2) we can obtain the Akhmediev breathers, Kuznetsov-Ma soliton, and magnetic rogue waves of magnetization. From the solution in Eq. (2) we find that the critical point $|\mu| = A_s k_s$ forms a dividing line between the modulation instability process ($|\mu| < A_s k_s$), the periodization process ($|\mu| > A_s k_s$), and magnetic states ($|\mu| \to A_s k_s$).

### A. Modulation instability and Akhmediev breathers

Modulation instability has been extensively studied in nonlinear physics [44], which is characterized by the periodic energy exchange between a perturbation and a continuous wave background. It can be used to generate the high-repetition-rate pulse trains in optical fibers [45] and can be described by near exactly the Akhmediev breathers [46] solution of the nonlinear Schrödinger equation. In optical fibers, Akhmediev breathers are temporal periodic and show the properties of single growth-return cycle in the propagation direction, namely a visual illustration of the famous Fermi-Pasta-Ulam recurrence [47]. Recently, modulation instability has been found to play a central role in the emergence of highly localized rogue-wave structures in various contexts of nonlinear physics.

In ferromagnet this ubiquitous process of magnetization dynamics can be realized by the condition $|\mu| < A_s k_s$ and $\nu = k_s m_{03}$ in Eq. (2). The parameters are given by

$$\begin{aligned} P &= A_s(k_s\cosh\theta - N\sinh\theta) - \mu\cos\phi - N m_{03}\sin\phi, \\ Q &= A_s(k_s\cosh\theta + N\sinh\theta) - \mu\cos\phi + N m_{03}\sin\phi, \\ R &= \mu\cosh\theta + iN m_{03}\sinh\theta - A_s(k_s\cos\phi + iN\sin\phi), \end{aligned} \tag{3}$$

where $\theta = \mu N T, \phi = -N(X + 2k_s m_{03} T)$ with $N = \sqrt{A_s^2 k_s^2 - \mu^2}$. The above result reveals that the solution to Eq. (2) is spatial periodic denoted by $2\pi/N$, and aperiodic in the temporal variable, as shown in Fig. 1. This process can also be seen as the spatial manifestation of Fermi-Pasta-Ulam recurrence realized by the magnetization dynamics. The spatial periodic distribution of magnetization shows that the component $m_3$ has two peaks and one valley in each unit distribution. As the spin wave amplitude $A_s$ increases the connection line of two peaks in the component $m_3$ rotates clockwise and the two peaks move with the opposite direction, as shown in Fig. 2. Also, the one valley splits into two and the distance of two valleys increases with the increasing $A_s$.

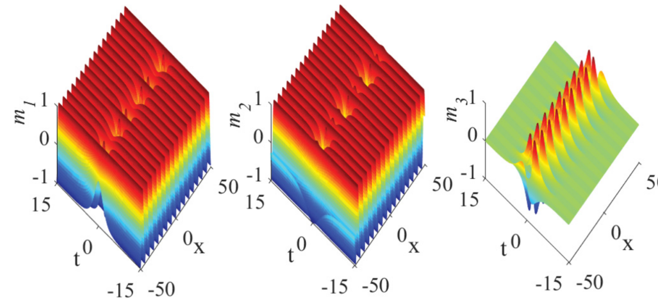


FIG. 1. Evolution of Akhmediev breathers for magnetization $\mathbf{m} = (m_1, m_2, m_3)$ in Eqs. (2) and (3). The component $m_3$ takes the spatial periodic distribution, which is characterized by two peaks and two valleys in each unit distribution. Parameters are given as follows: $A_s = 0.8$, $k_s = 1$, $\nu = k_s\sqrt{1 - A_s^2}$, and $\mu = 0.64$.

In order to study the asymptotic form of modulation instability of magnetization we consider the case of $t \to \pm\infty$. The background of $m_3$ approaches to $m_{03}(1 - 4\mu^2/k_s^2)$ as $t \to \pm\infty$. When $A_s = 1$ or $|\mu_1| = k_s/2$ with $1/2 \leqslant A_s < 1$, the magnetization lies in the $m_1$-$m_2$ plane and the component $m_3$ takes zero background. Under the condition $A_s = 1$ or $|\mu_1| = k_s/2$ with $1/2 \leqslant A_s < 1$ the magnon density distribution $|m_+(x,t)|^2$ takes a maximum 1 at $t \to \pm\infty$, where $m_+ \equiv m_1 + i m_2$. The solution in Eq. (2) with the parameters of Eq. (3) can be considered as the modulation instability process [44]. This instability process can also be expressed by

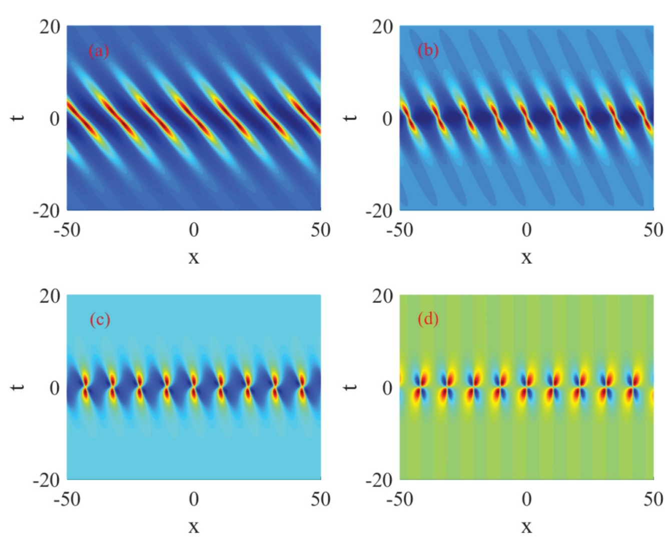


FIG. 2. The formation of magnetic petal in the component $m_3$, shown in (a)–(d). As the spin wave amplitude $A_s$ increases, the connection line of two peaks in the component $m_3$ rotates clockwise and the two peaks move with the opposite direction. The one valley splits into two and the distance of two valleys increases with the increasing $A_s$. Parameters are given as follows: $k_s = 1$, $\nu = k_s\sqrt{1 - A_s^2}$, and $\mu = 0.8 A_s k_s$. The parameters $A_s$ is given by (a) $A_s = 0.7$, (b) $A_s = 0.9$, (c) $A_s = 0.98$, and (d) $A_s = 1$, respectively.

linearizing the initial value of corresponding solution as

$$m_+(0,t) \approx \left(-1 \pm i\epsilon \frac{4\mu N}{k_s^2} \sin\phi\right) e^{ik_s x},$$
$$m_3 \approx \pm\epsilon \frac{4N^2}{k_s^2} \sin\phi, \tag{4}$$

where we use the condition $A_s = 1$, $\epsilon = \exp(-x_0)$ is a small quantity for $x_0 > 0$.

The magnetic Akhmediev breathers in Eq. (2) with the parameters of Eq. (3) in fact denotes the instability process of spin wave background. Small perturbations that disturb the spin wave can be amplified exponentially. The spin wave background is unstable against small perturbations. At this instability process there occurs the spatial periodic distribution of high magnon density, as shown in Fig. 1. A periodic magnon exchange occurs between the magnetic soliton and the spin-wave background. It should be noted that the magnetic soliton will lose this character on the ground-state background. It is worth mentioning that the interaction between spin wave and magnetic soliton causes this very interesting phenomenon.

### B. Kuznetsov-Ma soliton solution

Under the conditions $|\mu| > A_s k_s$ and $\nu = k_s m_{03}$ we obtain the magnetic Kuznetsov-Ma soliton solution of Eq. (2), which can be proposed as prototypes of hydrodynamic of rogue waves. This solution is characterized by the following parameters:

$$P = \mu \cosh\theta + \zeta m_{03} \sinh\theta - A_s(k_s \cos\phi + \zeta \sin\phi),$$
$$Q = \mu \cosh\theta - \zeta m_{03} \sinh\theta - A_s(k_s \cos\phi - \zeta \sin\phi),$$
$$R = A_s(k_s \cosh\theta + i\zeta \sinh\theta) - \mu \cos\phi + i\zeta m_{03} \sin\phi, \tag{5}$$

where $\zeta = \sqrt{\mu^2 - A_s^2 k_s^2}$, $\theta = \zeta(x + 2m_{03}k_s t)$, and $\phi = \mu\zeta t$. With the above parameters we see that the main characteristic properties of magnetic Kuznetsov-Ma soliton solution is spatially aperiodic and temporally periodic, while the soliton propagates with the velocity $-2k_s m_{03}$ and width $1/\zeta$. Similar to the discussion in the section of Akhmediev breathers the component $m_3$ shows two peaks and one valley in each periodic distribution, and the connection line of two peaks also rotates clockwise and the two peaks move with the opposite direction as spin-wave amplitude $A_s$ increases.

The illustration of magnetic Kuznetsov-Ma soliton is depicted in Fig. 3. When $A_s = 1$, the parameter $\theta$ depends only on $x$ which implies the envelope velocity becomes zero, i.e., the soliton is trapped in space by spin wave background. In order to study the asymptotic form of Kuznetsov-Ma soliton we consider the limitation case $x \to \pm\infty$. From Eqs. (2) and (5) we see that the component $m_3$ approaches to $(1 - 4A_s^2)m_{03}$, while the transverse components denoted by $m_+$ approach to $m_{0+}(4A_s^2 - 3)(N_1 \mp ik_s)/(N_1 \pm ik_s)$ with $m_{0+} \equiv m_{01} + im_{02}$ as $x \to \pm\infty$. This result shows that a spin wave undergoes a phase change $2\arctan[2Nk_s/(N^2 - k_s^2)]$ when it pass across a magnetic soliton. This phase change of spin wave can affect the propagation velocity of magnetic soliton, which denotes the transfer of spin angular momentum from spin wave background to a dynamic soliton called magnonic spin-transfer torque [43]. We also obtain that the zero background of $m_3$ can be realized by two cases, i.e., $A_s = 1$ or $|\mu_1| = k_s/2$ with $1/2 \leqslant A_s < 1$, while the magnon density distribution attains the maximum value 1 at $x \to \pm\infty$. One also finds that the maximum and minimum evolution of the component $m_3$ is the same as the propagation direction of soliton. This feature illustrates the characteristic breather behavior of the soliton as it propagates on the background of a periodic solution of magnetization in ferromagnet.

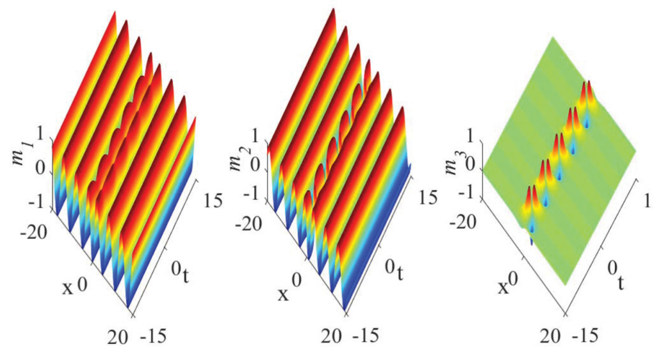


FIG. 3. Evolution of Kuznetsov-Ma soliton for magnetization $\mathbf{m} = (m_1, m_2, m_3)$ in Eqs. (2) and (5). This soliton is spatially aperiodic and temporally periodic, while the component $m_3$ shows two peaks and two valleys in each periodic distribution. Parameters are given as follows: $A_s = 1$, $k_s = 1$, $\nu = k_s\sqrt{1 - A_s^2}$, and $\mu = 1.3$.

Different from the magnetic Akhmediev breathers, the magnetic Kuznetsov-Ma soliton in Eq. (2) with the parameters of Eq. (5) expresses the localized periodic magnon exchange, which takes the temporal periodic evolution. Also, the high magnon density shows the temporal periodicity along the propagation direction of soliton.

### C. Bright and dark rogue waves

The above discussion shows that the condition $|\mu| = A_s k_s$ forms a critical point that divides the modulation instability process ($|\mu| < A_s k_s$) and the periodization process ($|\mu| > A_s k_s$). It leads to the different physical behavior of how the breather character depends strongly on the modulation parameter $\mu$, as shown in Fig. 4. Two different asymptotic behaviors are plotted in Fig. 4 in the limit processes $|\mu| \to (A_s k_s)^-$ and $(A_s k_s)^+$ under the condition $\nu = k_s m_{03}$, respectively. The former case demonstrates a spatial periodic process of a magnetic soliton forming the petal with four pieces. The spatial separation of adjacent magnetic petals increases rapidly, while the one valley splits in two and the amplitude of valley increases gradually as the modulation parameter $|\mu|$ approaches $A_s k_s$. The other case shows a localized process of the spin-wave background. In this case, the temporal separation of adjacent magnetic petals also increases rapidly as the modulation parameter $\mu$ approaches $(A_s k_s)^+$.

In the limit case of $|\mu| \to A_s k_s$, we get the magnetic rogue wave of Eq. (1), where the main parameters are given by

$$P = \left(2tA_s k_s^2 + A_s k_s m_{03} x \pm 1\right)^2 + A_s^3 k_s^2 \left(A_s x^2 - 3A_s k_s^2 t^2 \mp 6t\right),$$

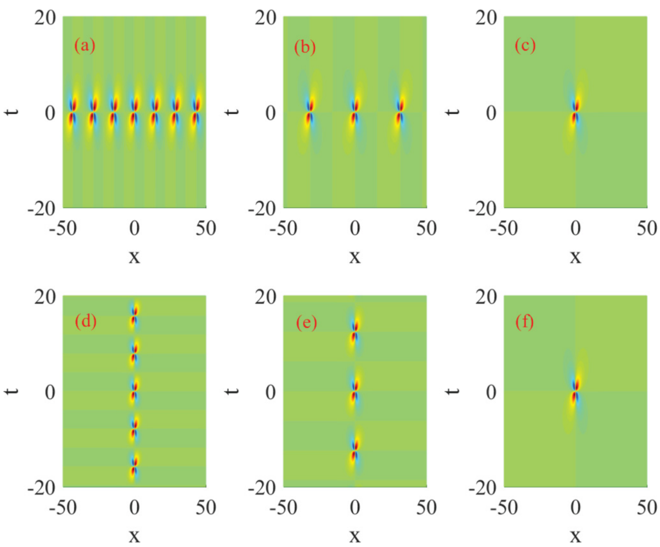


FIG. 4. The asymptotic processes of the magnetic component $m_3$ in the limit processes $\mu \rightarrow A_s k_s$ and $\nu = k_s\sqrt{1 - A_s^2}$ in Eq. (2). As $\mu \rightarrow A_s k_s$ the spatiotemporal separation between adjacent magnetic petal increases gradually, as shown in (a)–(f), where the parameters are $A_s = 0.8$, $k_s = 1$, (a) $\mu = 0.9$, (b) $\mu = 0.98$, (c) $\mu = 0.9999$, (d) $\mu = 1.2$, (e) $\mu = 1.1$, and (f) $\mu = 1.0001$, respectively.

$$
\begin{aligned}
Q &= \left(2tA_s k_s^2 + A_s k_s m_{03} x \mp 1\right)^2 \\
&\quad + A_s^3 k_s^2 \left(A_s x^2 - 3t^2 A_s k_s^2 \pm 6t\right), \\
R &= i2A_s^2 k_s (x + 3tk_s m_{03}) + (P + Q)/2 - 2,
\end{aligned} \tag{6}
$$

where the sign $\pm$ denotes the limit case $\mu \rightarrow \pm A_s k_s$, respectively. In order to study the asymptotic form of the rogue waves in Eqs. (2) and (6) we consider the case of $x \rightarrow \pm\infty$ $(t \rightarrow \pm\infty)$ and $x \rightarrow 0$ $(t \rightarrow 0)$. The component $m_3$ approaches to $(1 - 4A_s^2)m_{03}$ as $x \rightarrow \pm\infty$ $(t \rightarrow \pm\infty)$ and $m_{03}$ as $x \rightarrow 0$ $(t \rightarrow 0)$ for the case $+$, while approaches to $m_{03}$ as $x \rightarrow \pm\infty$ $(t \rightarrow \pm\infty)$ and $(1 - 4A_s^2)m_{03}$ as $x \rightarrow 0$ $(t \rightarrow 0)$ for the case $-$. The transverse components $m_+$ approaches to $m_{0+}(3 - 4A_s^2)$ as $x \rightarrow \pm\infty$ $(t \rightarrow \pm\infty)$ and $-m_{0+}$ as $x \rightarrow 0$ $(t \rightarrow 0)$ for the case $+$, while approaches to $-m_{0+}$ as $x \rightarrow \pm\infty$ $(t \rightarrow \pm\infty)$ and $m_{0+}(3 - 4A_s^2)$ as $x \rightarrow 0$ $(t \rightarrow 0)$ for the case $-$. The above analysis shows that the case $+$ expresses the bright rogue wave, while the case $-$ corresponds to dark rogue wave. The graphical representation of bright and dark rogue waves are shown in Fig. 5.

Especially, when $A_s = 1$ we can get the compact magnetic rogue waves as follows:

$$
\begin{aligned}
m_+ &= -e^{ik_s x}\left[1 - \left(8x^2 k_s^2 - i4xk_s(F_1 - 2)\right]/F_1^2\right), \\
m_3 &= \pm 8txk_s^3/F_1^2,
\end{aligned} \tag{7}
$$

where $F_1 = 1 + t^2 k_s^4 + x^2 k_s^2$. The component $m_3$ is characterized by the antisymmetric distribution of two peaks and two valleys, as shown in Fig. 4.

The above results show that there exist two processes of the formation of the magnetic rogue wave: one is the localized process of the spin-wave background, and the other is the reduction process of the periodization of the magnetic bright soliton. The magnetic rogue wave is exhibited by the strong temporal and spatial localization of the magnon exchange and high magnon density. Also, the magnetic rogue waves can be excited by a small localized perturbation of spin-wave background, as shown in Fig. 4.

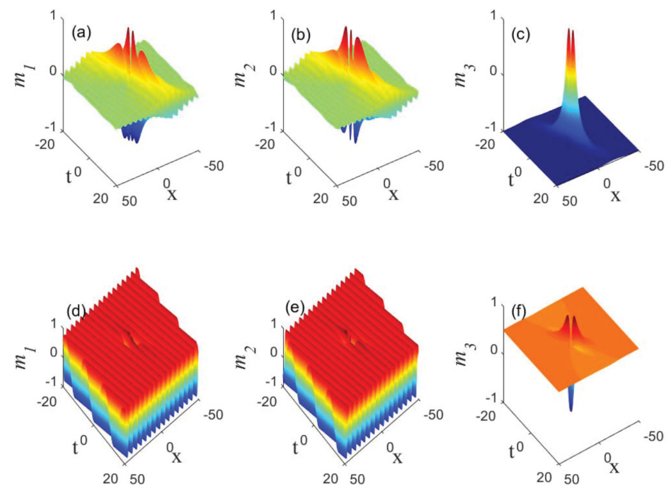


FIG. 5. The graphical evolution of rogue waves for the magnetization $\mathbf{m} = (m_1, m_2, m_3)$ in Eqs. (2) and (6), i.e., bright rogue waves (a)–(c) and dark rogue waves (d)–(f). The parameters are $A_s = \sqrt{3}/2$, $k_s = 1$, $\nu = k_s\sqrt{1 - A_s^2}$, and $\mu = \pm\sqrt{3}/2$ with the sign $\pm$ corresponding to the bright and dark rogue waves, respectively.

It should be interesting to discuss how to detect such breathers and rogue waves in experiment. In spinor Bose-Einstein condensates trapped in optical potentials [48–50] the average of $m_3$ component is measured directly by the difference numbers of the population between the spin $+1$ and $-1$ Zeeman sublevel. It implies that there exists the temporal or spatial periodic population of atoms for magnetic breather solutions, while the atoms take the nonuniform population for rogue waves. For the fermionic ferromagnet the current flow is strongly affected by the orientation of the magnetic moments. Therefore, a periodic change of electrical resistance in magnetic layer may occur for magnetic breathers solutions, while a higher electrical resistance for rogue waves.

## III. CONCLUSIONS

In summary, we investigate the dynamics of magnetization in a ferromagnet excited by the all-magnonic spin-transfer torque with the developed Darboux transformation. As an example, we obtain the exact expressions of Akhmediev breathers solution, Kuznetsov-Ma soliton, and rogue waves. We also obtain the critical condition between the modulation instability process, the periodization process, and magnetic states. These results can be useful for the exploration of nonlinear excitation in Bosonic and fermionic ferromagnet.

## ACKNOWLEDGMENTS

We are grateful to Professor Lu Li and Biao Wu for his helpful discussions. This work was supported by the Key Project of Scientific and Technological Research in Hebei Province, China (Grant No. ZD2015133), and the National Natural Science Foundation of China (Grant No. 11304270).

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