Scaling of thermal hysteretic behavior in a parametrically modulated cold atomic system

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We observe the hysteresis of a spontaneous symmetry breaking (SSB) transition in a parametrically modulated magneto-optical trap by sweeping the total number of atoms and study thermal hysteretic behavior in the system by measuring the scaling exponent of hysteresis. It is shown that the relaxation time of the order parameter in the SSB transition becomes larger near the critical number. The scaling exponent of the hysteresis area with number sweeping rate is found to be 0.64 ± 0.04, which is consistent with the value in the mean-field model.

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I. INTRODUCTION

It is important to understand nonequilibrium phenomena in a far-from-equilibrium system because most natural phenomena take place in nonequilibrium conditions. To understand nonequilibrium phenomena, various nonlinear oscillators, such as Duffing and parametric oscillators, have been studied in electric circuits, nano-mechanical beam oscillators, and trapped-ion systems [1–3]. In addition, modulated magneto-optical trap (MOT) systems are also a good platform for studying nonlinear oscillators [4,5]. In these systems, Duffing and parametric oscillators were implemented by modulating the intensity of lasers [6,7]. In this study, we consider a parametrically modulated MOT system, in which it is easier to control the parameter of the system than in other systems. Furthermore, it takes on a few seconds in our system to perform an experiment and observe the same phenomena that would require several minutes in other systems.

Hysteresis, a nonequilibrium phenomenon with varying the temperature of a system, is one of the most interesting topics that have been studied in various fields such as molecular switching using spin crossover [8–10], temperature-driven metal-insulator transition in solid-state devices [11,12], and antifreeze proteins in bionic systems [13,14]. The phenomenon of thermal hysteresis was reported in the mean-field model [15]. We note that the scanning rates of temperature are typically several K per minute in the systems reported in Refs. [8–11,13,16,17], and therefore, it would require several minutes to observe hysteresis in these systems. In contrast, the number sweeping rate of our system is about 10^5–10^6 s^{-1}. Therefore, it is possible to observe hysteresis within a few seconds.

In the mean-field model the closed hysteresis loop area (A) scales with the rate of temperature R as

$$A = A_0 + bR^n, \tag{1}$$

where n is the scaling exponent of the hysteresis and A_0 and b are constants. It is known that n approaches two thirds, which is universal for both the mean-field and field theoretical models [15,18].

In our system, the external field such as an oscillating magnetic field in the Ising model can be implemented by employing an additional modulation of the intensity of the trapping laser. We studied the dynamic scaling behavior of parametrically modulated MOT system under the oscillating bias field in a preceding work, which will be reported in a separate paper. For the experiments with sinusoidal magnetic field, the system has different scaling exponents of the hysteresis by an external field as the temperature of the system, i.e., the hysteresis loop area by sinusoidal external field can be scaled with different values below the critical temperature and above critical temperature [19]. Thus, the hysteresis by sweeping of the total number of atoms has different scaling behavior from the hysteresis by the bias field.

In this study, we observed the hysteretic behavior of a spontaneous symmetry breaking (SSB) transition in a modulated atomic system by sweeping the total number of atoms linearly in this study. We determined the universality class of the hysteresis by obtaining the scaling exponent of the hysteresis loops.

II. THEORY

A. Relaxation time

The SSB transition originates from the two collective behaviors of the trapped atoms in our system [6,20]. One is the noise-induced transition caused by the fluctuations of the atomic motion in the MOT. It originates from the reabsorption of the emitted photons and acts like a repulsive force between the trapped atoms [5,20]. A slight imbalance of the populations of two atomic clouds is removed by the transition. Therefore, the noise-induced transition conserves the symmetry of system. The transition rate of our system can be written as [21]

$$W \equiv C \exp(-S/D), \tag{2}$$

where C is a constant, S is the activation energy corresponding to the height of the potential barrier between the two atomic clouds, and D is the noise intensity of the system. The activation energy S is the important quantity to determine the dynamics of system. In the presence of external changes, the activation energy can be rewritten as $S = S^{(0)} + S^{(1)}$ where $S^{(0)}$ is the activation energy without number sweeping and $S^{(1)}$ is associated with external perturbations.

The other collective behavior is the light-induced long-range interaction called the shadow force, caused by the shielding effect on atoms from laser light by other atoms

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[20,22,23]. It acts like an attractive force, therefore an atom in one atomic cloud which has less number of atoms tends to move to the other side. In the one-dimensional approximation, the force exerted on one atom at the coordinate z^i by other atoms can be described by $F^i = -f_{\rm sh}\sum_j \mathrm{sgn}(z^i - z^j)$, where $f_{\rm sh}$ is a constant representing the shadow force due to the single atom [21]. The dynamics of the system is described by the combination of the two collective effects. When the total number of atoms exceeds the critical number, the shadow force is dominant and the symmetry of system, which has been preserved by the noise-induced transition, is now broken, and the SSB transition occurs.

To theoretically describe the symmetry breaking transition with sweeping of the total number of atoms, the dynamic evolution of the two clouds respectively with N_1 and N_2 atoms can be found from the following master equations:

$$\frac{dN_1(t)}{dt} = -W_{12}N_1(t) + W_{21}N_2(t) + \frac{R}{2}, \tag{3}$$

$$\frac{dN_2(t)}{dt} = -W_{21}N_2(t) + W_{12}N_1(t) + \frac{R}{2}, \tag{4}$$

where

$$W_{ij} \equiv W_0 \exp\left[-\lambda\frac{N_i - N_j}{N}\right] \tag{5}$$

are the transition rates between the two clouds with $i,j = 1,2$. $W_0(\sim \exp[-S^{(0)}/D])$ is the transition rate in the absence of number sweeping of the atomic clouds. In this paper, we define that N_1 is the atomic cloud which has the larger number of atoms. We note that $N = N_1 + N_2$ is the total number of atoms and $\lambda \equiv \theta + 1 = N/N_c$ represents the normalized total number of atoms where N_c is the critical number of the SSB transition and θ is the reduced total number of atoms. R is the sweeping rate of the total number of atoms, i.e., the speed of the changing number of atoms. In the experiment R has a uniform speed and is related with the angular speed of a neutral density (ND) filter which can control the intensity of the repuming laser.

Solving Eqs. (3) and (4) for the normalized number difference between the two clouds, $x = (N_1 - N_2)/N$, which corresponds to the order parameter of the SSB phase transition, we obtain:

$$\frac{1}{2W_0}\frac{dx}{dt} = -x\cosh[(\theta+1)x] + \sinh[(\theta+1)x]. \tag{6}$$

Since the total number of atoms was swept with a uniform rate, it becomes a function of time. Thus, the reduced number can be written as $\theta(t) = \theta_0 + R't$, where $R' = R/N_c$. When the total number of atoms is changed with time, the light-induced interatomic interaction caused by the shadow force is changed accordingly with the total number of atoms. Because of this variation in the interaction, the order parameter relaxes toward a new equilibrium state that is appropriate to the total number of atoms at that time. This relaxation time is expected to become large when the total number approaches the critical number.

To obtain the relaxation time for some reduced total number of atoms, we expand the right-hand side of Eq. (6) about the point which is the stationary solution of the Eq. (6), $x = x_s$ [24]. The stationary value, x_s, can be obtained from Eq. (6) by setting the left-hand side in Eq. (6) to zero. At $\theta < 0$, i.e., $N < N_c$, we have x_s=0 and Eq. (6) becomes

$$\eta\frac{dx}{dt} \cong -|\theta|x. \tag{7}$$

Hence, the order parameter is given by

$$x = x_0 \exp\left[-\frac{|\theta|t}{\eta}\right], \tag{8}$$

where $\eta = 1/(2W_0)$, and $x_0 = x(t=0)$. When $\theta > 0$, i.e., $N > N_c$, we have $x_s = \tanh[(\theta+1)x_s]$ and Eq. (6) can be expanded about x_s up to first order in $x - x_s$ as

$$\eta\frac{dx}{dt} \cong -\left[1 - (\theta+1)\left(1 - x_s^2\right)\right]\cosh[(\theta+1)x_s] \times (x - x_s). \tag{9}$$

The solution of Eq. (9) is simply given by

$$x = (x_0 - x_s)\exp\left[-\frac{\epsilon}{\eta}t\right] + x_s, \tag{10}$$

where $\epsilon = [1 - (\theta+1)(1 - x_s^2)]\cosh[(\theta+1)x_s]$.

Finally the relaxation time τ is obtained as

$$\tau = \begin{cases} \frac{\eta}{|\theta|}, & \text{for } \theta < 0, \\ \frac{\eta}{\epsilon}, & \text{for } \theta > 0. \end{cases} \tag{11}$$

It is expected to increase when the reduced total number of atoms approaches the critical point.

B. Model of hysteresis

To identify the universality class of the hysteresis in our system, we analyze the SSB transition. Denoting the numbers of trapped atoms in the two atom clouds of the parametrically modulated MOT as N_1 and N_2, respectively, $P_1(N_1)$, the probability that cloud 1 has N_1 atoms at certain time, can be obtained from the following master equation [21]:

$$\begin{aligned} \dot{P}_1(N_1) &= -[\mu(N_1) + \nu(N_1)]P_1(N_1) \\ &\quad + \nu(N_1 - 1)P_1(N_1 - 1) \\ &\quad + \mu(N_1 + 1)P_1(N_1 + 1), \\ \mu(N_1) &= N_1 W_{12}(N_1; N), \\ \nu(N_1) &= (N - N_1)W_{12}(N - N_1; N), \end{aligned} \tag{12}$$

where μ is the transition rate from cloud 2 to cloud 1 and ν is the transition rate from cloud 2 to cloud 1. The stationary solution of Eq. (12) is given by

$$P_1^{\rm st}(N_1) = Z^{-1}\binom{N}{N_1}\exp[-2\alpha N_1(N - N_1)], \tag{13}$$

where Z is a normalization constant, $\alpha = 1/N_c \propto f_{\rm sh}/T$, and T is temperature of the system.

The reduced difference of the cloud populations, i.e., the order parameter of the phase transition, is determined theoretically by the maximum of the stationary solution $P_1^{\rm st}(N_1)$ [21]. For $x = (N_1 - N_2)/N \ll 1$, the maximum probability is given by

$$P_1^{\rm st}(N_1) = Z^{-1}\exp\left[-N\left(\frac{1}{12}x^4 - \frac{1}{2}\theta x^2\right)\right]. \tag{14}$$

This has an analogous form with the free energy of mean-field model as [11]

$$F = \frac{1}{2}rM^2 + \frac{1}{4}uM^4 + \frac{1}{6}vM^6 - HM, \tag{15}$$

where r is the reduced temperature with respect to the critical temperature, u and v are coefficients, and H is the external field. We can suggest that $L \equiv x^4/12 - \theta x^2/2$ is free-energy-like function per single atom of the system. In our case the sixth power term in Eq. (15) is negligible and H is zero. Thus, the parameter θ plays the role of the reduced temperature, which is the deviation from the critical temperature and is the control parameter. In contrast, x plays the role of the order parameter of the phase transition [21]. Therefore, hysteresis of the SSB transition, which occurs under the total number sweeping, has an analogy to thermal hysteresis in other systems.

When the reduced total number of atoms (θ) is negative, the solution has one maximum at $x = 0$, and it has two maxima when θ is positive [21]. The solutions are described by the equation

$$x = \tanh[(\theta + 1)x], \tag{16}$$

which generates the red solid curves in Fig. 4.

III. EXPERIMENTS

Figure 1 shows our experimental setup for the parametrically modulated atomic trap of ^{85}Rb atoms. The conventional six-beam MOT had a pair of counter-propagating and intensity-modulated trapping lasers along the anti-Helmholtz coil axis (z axis) [25]. The magnetic field gradient b was 10 G/cm, and the trap laser intensity was about 0.294 mW/cm^2. The detunings were about -2.05Γ ($\Gamma = 2\pi \times 6.06$ MHz is the decay rate of the excited state of ^{85}Rb) in the z axis and -2.32Γ in the x and y axes [26]. The intensity of trapping lasers in the x and y axes were more than five times larger than that in the z axis so that we may consider the one-dimensional atomic motion in the z direction.

In our system, the SSB transition occurs as a result of the shadow effect caused by the light-induced interaction between atoms in the MOT when the total number of atoms exceeds the critical number [6]. Because the relaxation time of the phase transition increased as the system approached the critical point, a phenomenon known as the critical slowing down, a hysteresis was observed when we swept the control parameter, i.e., the total number of atoms, in our system. We adjusted the total number of atoms in the system by changing the intensity of the repumping laser controlled by a neutral density filter mounted on a step motor (Fig. 1). The sweeping rate of the total number of atoms could be adjusted by changing the time interval of each motor step. Then, we observed and obtained the dynamics of atoms inside the MOT by performing florescence imaging with a charge-coupled device (CCD) camera.

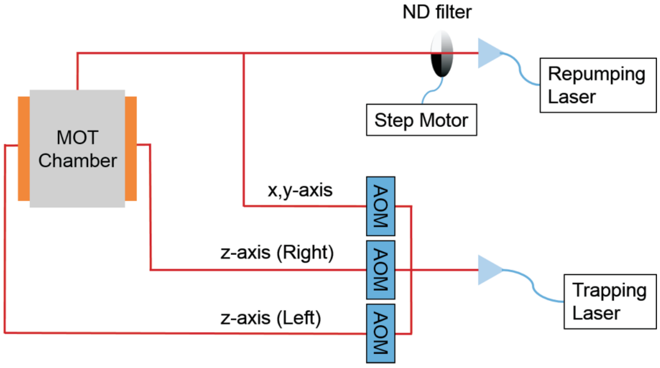

FIG. 1. A schematic diagram of the experimental setup.

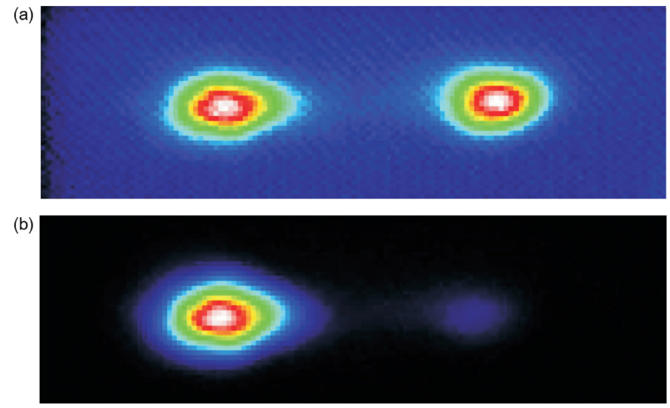

FIG. 2. Typical CCD images of the system. The total number of atoms is about (a) 2.46×10^7 (before SSB) and (b) 7.69×10^7 (after SSB).

IV. RESULTS AND DISCUSSIONS

Figure 2 shows the typical stroboscopic images of atomic clouds in the parametrically modulated MOT, which triggered at half of the modulation frequency. When the total number of atoms is below the critical value, the populations of the two atom clouds are equal as shown in Fig. 2(a). This is called a symmetric state. In contrast, the symmetry of the atomic population is broken above the critical value of the total atomic number as shown in Fig. 2(b), which is a SSB state. The symmetric state [Fig. 2(a)] is equivalent to the zero-magnetization state in the Ising model [6]. Thus, the phase transition occurs as the system moves from the SSB

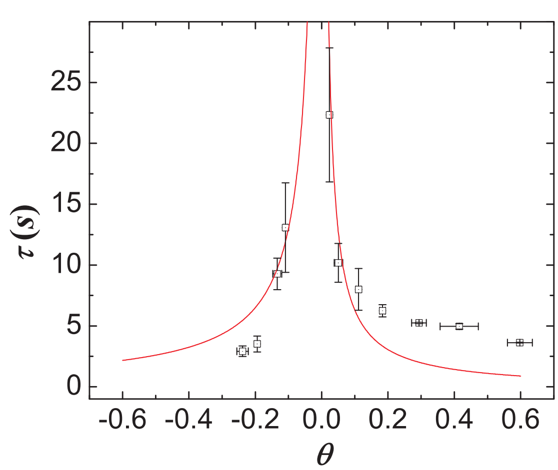

FIG. 3. Measured relaxation time to the equilibrium state for each reduced number superimposed with the numerical result obtained from Eq. (11) for $N_c = 4.81 \times 10^7$ and $R = 3.72 \times 10^6$ s^{-1}.

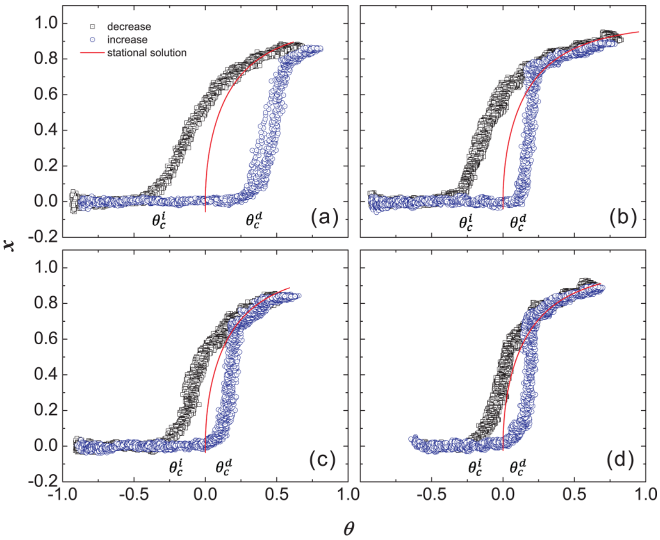

FIG. 4. Measured hysteresis loops for various sweeping rates (pulse intervals). (a) $1.87 \times 10^6\ \mathrm{s}^{-1}$ (10 ms) (b) $9.61 \times 10^5\ \mathrm{s}^{-1}$ (20 ms), (c) $5.84 \times 10^5\ \mathrm{s}^{-1}$ (30 ms), and (d) $5.7 \times 10^5\ \mathrm{s}^{-1}$ (40 ms). The red solid curves are the solutions of Eq. (16).

state to the symmetric state (from the symmetric state to the SSB state) with a decrease (an increase) in the total atomic number, which represents an analogy to the ferromagnetic–paramagnetic phase transition in the Ising model. In Fig. 2, the paramagnetic phase [Fig. 2(a)] and ferromagnetic phase [Fig. 2(b)] are presented.

We obtained relaxation times experimentally by measuring the time that the order parameter has been stabilized to a steady state after stopping changing the number of atoms at some points during the process of number sweeping. Figure 3 shows the measured relaxation time at various reduced numbers of atoms with the sweeping rate $R = 3.72 \times 10^6\ \mathrm{s}^{-1}$. As can be seen in Fig. 3, the relaxation time increases as the reduced total number of atoms approaches the critical number of the phase transition and diverges at the critical point $\theta = \theta_c\ (= 0)$ as expected. In Fig. 3 the red solid curves represent the solutions of Eq. (11).

Figure 4 shows the hysteresis curves as a function of the reduced atom number for sweeping rates of $1.87 \times 10^6\ \mathrm{s}^{-1}$, $9.61 \times 10^5\ \mathrm{s}^{-1}$, $5.84 \times 10^5\ \mathrm{s}^{-1}$, and $5.7 \times 10^5\ \mathrm{s}^{-1}$. The red curves in Fig. 4 are the results calculated numerically from Eq. (16). The parameters θ_c^i and θ_c^d are critical values of the reduced number for the symmetric state to SSB state transition (upon increasing) and the SSB state to symmetric state transition (upon decreasing), respectively. The shadow effect, i.e., the cause of the SSB transition, does not catch up with the variation in the number difference between the two clouds, because the time for the order parameter to relax toward the equilibrium state becomes larger and larger when the number of atoms approaches the critical number. Thus, when the total number of atoms is swept across the critical number with a uniform speed, the hysteresis loops can be obtained during the transition as shown in Fig. 4. It can be seen that the area of the hysteresis loop decreases as the sweeping rate of the total number of atoms is decreased.

Figure 5(a) presents the scaling behavior of the hysteresis loop area versus the sweeping rate of the total number of atoms on the log-log scale. Each point in Fig. 5 was obtained by averaging the experimental values more than three times and the constant term A_0 was derived by a linear fitting of the hysteresis area A versus sweeping rate R plot. The scaling exponent n for the system was 0.64 ± 0.04, which is quite close to the value given in the mean-field theory [15]. Therefore, it is clearly seen that the hysteresis induced by number sweeping in the modulated MOT system exhibited a thermal hysteretic behavior.

We define the hysteresis width by $\Delta\theta = |\theta_c^i - \theta_c^d|$. It is well known that the hysteresis width is described by the scaling law:

$$\Delta\theta \cong R^{\beta}, \tag{17}$$

where β is the scaling exponent [16]. The scaling exponent can be obtained by fitting the hysteresis width data to the double logarithmic form of Eq. (17). The log-log plot of the hysteresis width $\Delta\theta$ versus the sweeping rate R is shown in Fig. 5(b). The scaling exponent β of the hysteresis width was found to be 0.44 ± 0.025. This value is very close to the value of 0.465 predicted by the kinetic Ising model [27]. Following the scaling theory of thermal hysteresis developed by Zhong *et al.* in Ref. [15], the scaling exponent of the hysteresis width should have the physical meaning of the resistance characteristics in the glass transition. To describe resistance of the system, the scaling exponent β should be compared with

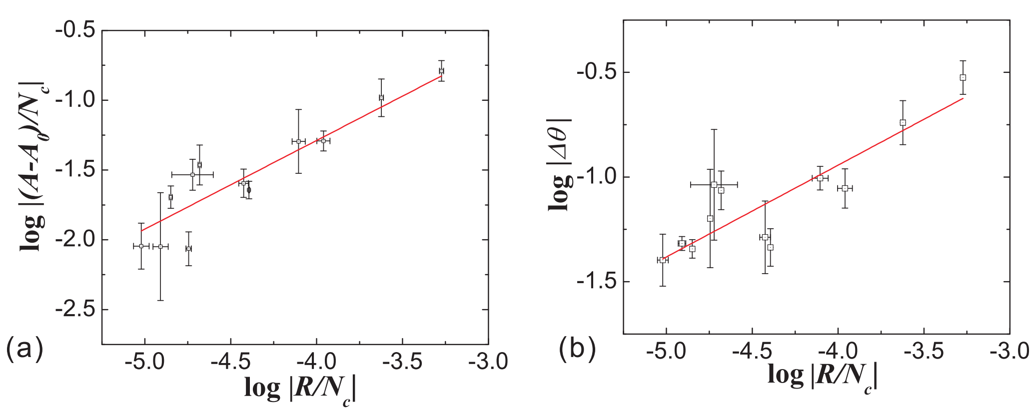

FIG. 5. Log-log (base: e) plot of (a) hysteresis area A and (b) hysteresis width $\Delta\theta$ versus number sweeping rate R. The solid red curves in (a) and (b) are the logarithmic fits of Eqs. (1) and (17), respectively.

the exponent n reported in the mean-field model [15,17,18]. The value obtained in our system was $\beta \sim 0.44 \pm 0.025$, which is slightly smaller than the scaling exponent n in the mean-field model. This implies that our system corresponds to a thermal model with a rather low resistance.

V. CONCLUSION

In summary, we observed the thermal hysteretic scaling behavior in a modulated MOT system. We obtained the hysteresis loop by sweeping the total number of atoms with a uniform speed. The scaling exponent of the hysteresis loop area with sweeping was 0.64 ± 0.04, which was in excellent agreement with the expected value in the mean-field model. The scaling exponent of the hysteresis width was obtained to be 0.44 ± 0.025. We provided a new method for studying thermal hysteresis. Moreover, if the number decreasing (increasing) process was reversed before the system went to an equilibrium state, the hysteresis loop was not closed at the end of the decreasing (increasing) process [16]. This suggests a glassy behavior, and thus a further study of the glass transition is in progress.

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