Duality and reciprocity of fluctuation-dissipation relations in conductors

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By analogy with linear response, we formulate the duality and reciprocity properties of current and voltage fluctuations expressed by Nyquist relations, including the intrinsic bandwidths of the respective fluctuations. For this purpose, we individuate total-number and drift-velocity fluctuations of carriers inside a conductor as the microscopic sources of noise. The spectral densities at low frequency of the current and voltage fluctuations and the respective conductance and resistance are related in a mutually exclusive way to the corresponding noise source. The macroscopic variances of current and voltage fluctuations are found to display a dual property via a plasma conductance that admits a reciprocal plasma resistance. Analogously, the microscopic noise sources are found to obey a dual property and a reciprocity relation. The formulation is carried out in the frame of the grand canonical (for current noise) and canonical (for voltage noise) ensembles, and results are derived that are valid for classical as well as degenerate statistics, including fractional exclusion statistics. The unifying theory so developed sheds new light on the microscopic interpretation of dissipation and fluctuation phenomena in conductors. In particular, it is proven that for fermions, as a consequence of the Pauli principle, nonvanishing single-carrier velocity fluctuations at zero temperature are responsible for diffusion but not for current noise, which vanishes in this limit.

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I. INTRODUCTION

The dual property in electrical transport in the linear-response regime asserts that perturbation (applied voltage V or imposed current I) and response (measured current or voltage drop) can be interchanged with the associated kinetic coefficients (conductance G or resistance R, respectively), being reciprocally interrelated. According to Ohm's law, for a homogeneous conductor the dual property gives

$$I = GV \quad \text{and} \quad V = RI, \tag{1}$$

with the reciprocity relation given by

$$GR = 1. \tag{2}$$

The most used applications of the above properties are Thévenin's and Norton's theorems, which in electrotechnics are two equally valid methods of reducing a complex linear network down to something simpler to analyze [1].

The aim of this paper is to formulate an analogous dual property and reciprocity relation for electrical fluctuations at thermal equilibrium. Here the perturbation is the microscopic source of spontaneous fluctuations inside a conductor (taken as the physical system), and the response is the variance of the macroscopic response (i.e., the variance of current or voltage fluctuations) measured in the outside circuit. The individuation of the noise sources at a kinetic level, and thus beyond the simple temperature model, is a major issue in statistical physics that has received only partial, and sometimes controversial, answers even in the basic literature [2–6]. Ultimately, a unifying theory that applies generally to classical and degenerate statistics, including explicitly Fermi-Dirac and Bose-Einstein distribution functions, is not available to our knowledge. Here, all these issues will be addressed and formally solved in the framework of the basic laws of statistical mechanics.

For the analysis of current or voltage fluctuations on a kinetic level, a correct system definition becomes of prime importance. On the one hand, the microscopic model for carrier transport implies a well-defined equivalent circuit. On the other hand, the measurement of current or voltage fluctuations in the outside circuit is reflected in the boundary conditions for the microscopic modeling, which determine the choice of the appropriate statistical ensemble. Current noise is measured in the outside short circuit, which implies an open system where carriers may enter or leave the sample, thus referring to the grand-canonical ensemble (GCE). Voltage noise is measured in the outside open circuit when the carrier number in the sample is fixed, thus referring to the canonical ensemble (CE). While it is well known that in the thermodynamic limit different statistical ensembles become equivalent [7], this does not hold anymore in the case of fluctuations, when a finite system size has to be considered. Nevertheless, we will show that the dual property provides a direct link between the noise sources in the GCE and the CE.

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II. THEORY

Electrical fluctuations of a conductor in the limit of low frequency (i.e., $\omega \to 0$) are described by the Nyquist relations [3]

$$S_I(\omega = 0) = 4\frac{\overline{I^2}}{\Delta f_I} = 4K_B T G, \tag{3}$$

$$S_V(\omega = 0) = 4\frac{\overline{V^2}}{\Delta f_V} = 4K_B T R, \tag{4}$$

where S_I and S_V are the spectral densities of instantaneous current and voltage fluctuations, respectively, $\overline{I^2}$ and $\overline{V^2}$ are the variances of the corresponding current and voltage fluctuations (we recall that being at thermal equilibrium, their average values are identically zero), Δf_I and Δf_V are the corresponding intrinsic bandwidths determined by the decay of the corresponding correlation functions, K_B is Boltzmann's constant, and T is the absolute temperature. Here and henceforth, the bar over physical quantities denotes the ensemble average. The dual property we are interested in refers to the above Nyquist relations, also called fluctuation-dissipation theorems (FDTs) [5].

To describe the real conductor on a macroscopic level, the ideal resistance R has to be complemented by a kinetic inductance $\mathcal{L}$ associated with the inertia of the carriers and a capacitance $\mathcal{C}$ associated with the contacts resulting in the equivalent circuit shown in Fig. 1. Note that current and voltage fluctuations are measured under different operation conditions: Voltage noise is measured at the contacts in the open circuit [i.e., for $I(t) \to 0$] while current noise is measured in the outside short circuit [i.e., for $V(t) \to 0$].

While the open circuit, being characterized by a closed system with no particle exchange, is a well-defined concept, the short circuit deserves some more comments. To be an ideal short circuit, the resistance of the external circuit has to be negligible compared to the resistance of the conductor under consideration. The resistance of a material is inversely proportional to the momentum relaxation time and the number of carriers [see, e.g., Eq. (7)]. Since the momentum relaxation time typically cannot be varied to a large extent (unless extremely low temperatures are considered), the ideal short circuit requires a very large number of carriers, such that it indeed can be treated as a reservoir in the sense of the GCE.

On a microscopic level, current and voltage fluctuations are generated by the stochastic motion of the carriers in the conductor. For the sake of clarity of the model, we assume a homogeneous conductor with length L the in x direction, cross section A, and with the length much smaller than the transverse dimension. The link between the microscopic and the macroscopic picture is provided by the Ramo-Shockley theorem [8,9], which for our geometry reads

$$\frac{d}{dt}V(t) = \frac{L}{\varepsilon_0 \varepsilon_r A}\left[\frac{N(t)q}{L}v_d(t) - I(t)\right], \tag{5}$$

where ϵ_0 and ϵ_r are the vacuum and the relative dielectric constant of the host lattice material, respectively, and

$$v_d(t) = \frac{1}{N(t)}\sum_i v_{i,x}(t) = \frac{1}{N(t)}\sum_{\mathbf{k}} v_{\mathbf{k},x} n_{\mathbf{k}}(t) \tag{6}$$

is the instantaneous drift velocity in the x direction of the $N(t)$ carriers with charge q and effective mass m moving with velocities $\mathbf{v}_i(t)$ in the conductor, the second form being written in terms of the fluctuating occupation number of carriers $n_{\mathbf{k}}(t)$ in the state $\mathbf{k}$ with the velocity component $v_{\mathbf{k},x}$ in this state, which explicitly accounts for the indistinguishability of the carriers. Extensions of the theorem to more general boundary conditions and quantum-mechanical currents can be found in Refs. [10,11]. We notice that under voltage noise operation (CE), $N(t) = N$, with N the fixed number of carriers inside the conductor, and for average quantities the usual assumption $N = \overline{N}$ is well justified [7]. This microscopic model indeed leads to the equivalent circuit shown in Fig. 1, and, using kinetic theory and a Drude model for the carriers, its lumped elements are related to the microscopic properties according to

$$R = \frac{L^2 m}{q^2 N \tau}, \quad \mathcal{L} = \frac{L^2 m}{q^2 N}, \quad \mathcal{C} = \frac{A}{L}\epsilon_0 \epsilon_r, \tag{7}$$

with τ being the momentum relaxation time. Note that on the microscopic level, the different operation conditions are reflected in different boundary conditions for the carriers. Voltage noise operation refers to a closed system with a fixed carrier number, while current noise operation refers to an open system in which carriers enter or leave the system through the contacts.

Within a correlation function scheme, the intrinsic bandwidths are directly related to the decay of the correlation functions associated with the current and voltage fluctuations. In the case of current noise, $\Delta f_I = 1/\tau = R/\mathcal{L}$ is determined by the momentum relaxation time τ or, equivalently, by the $R\mathcal{L}$ time constant of the equivalent circuit. Analogously, the intrinsic bandwidth of voltage fluctuations $\Delta f_V = 1/\tau_d = 1/(R\mathcal{C})$ is determined by the dielectric relaxation time $\tau_d = \tau_p^2/\tau$ or, equivalently, by the $R\mathcal{C}$ time constant of the circuit. Here, τ_p denotes the plasma time (i.e., the inverse of the plasma frequency ω_p),

$$\tau_p = \omega_p^{-1} = \sqrt{\frac{\epsilon_0 \epsilon_r m A L}{N q^2}} = \sqrt{\mathcal{L}\mathcal{C}}, \tag{8}$$

which is related to the $\mathcal{L}\mathcal{C}$ time constant of the circuit. We remark that also outside the static limit (i.e., for $\omega \neq 0$) the equivalent circuit in Fig. 1 gives the impedance (or the admittance) whose real parts reproduce the frequency dependence of the current and voltage spectra in the classical limit $K_B T \gg \hbar\omega$, with $\hbar$ being the reduced Planck constant.

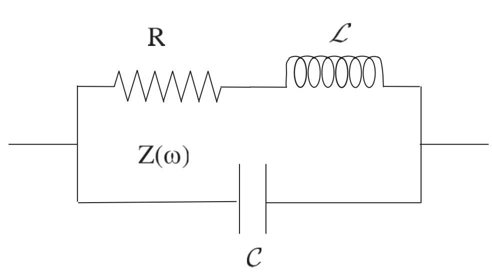

FIG. 1. Equivalent circuit with impedance $Z(\omega)$ of a real homogeneous conductor of resistance R. The capacitance $\mathcal{C}$ and inductance $\mathcal{L}$ account for the presence of the contacts and for the inertia of carriers, respectively.

These spectra are associated with the microscopic time scales of the corresponding correlation functions, as was validated by Monte Carlo simulations [12]. Furthermore, the equivalent circuit consistently recovers the standard relations

$$\overline{V^2} = \frac{K_B T}{\mathcal{C}} \quad \text{and} \quad \overline{I^2} = \frac{K_B T}{\mathcal{L}}, \tag{9}$$

which in this form are valid for any type of inductance and capacitance in a circuit, as given in Fig. 1. As such, this circuit is of the highest physical importance, and it should replace alternative equivalent circuits (such as, e.g., simple $R\mathcal{C}$ parallel and $R\mathcal{L}\mathcal{C}$ serial circuits), which are also sometimes used in the literature.

Making use of statistics, the temperature can be associated with the microscopic quantities defining two noise sources that are present in the conductor, one for each operation condition. These sources should be taken as mutually exclusive for each of the two boundary conditions. Accordingly, a constant-voltage (i.e., current noise) operation is associated with a GCE reflecting the open system in which [13]

$$\overline{\delta N^2} = K_B T \frac{\partial \overline{N}}{\partial \mu_0}. \tag{10}$$

Here, $\overline{\delta N^2}$ is the variance of the instantaneous number of carriers inside the sample, and μ_0 is the chemical potential. A constant-current (i.e., voltage noise) operation, on the other hand, is associated with a CE with a fixed carrier number ($N = \overline{N}$) and

$$\overline{\delta v_d^2} = \frac{1}{N^2} \sum_{\mathbf{k}} v_{\mathbf{k},x}^2 \, \overline{\delta f^2(\varepsilon_{\mathbf{k}})}, \tag{11}$$

where $\overline{\delta v_d^2}$ is the variance of the fluctuations of the instantaneous carrier drift velocity averaged over the sample, $v_{\mathbf{k},x} = \hbar k_x/m$ is the x component of its velocity, $\varepsilon_{\mathbf{k}}$ is the corresponding energy, $\overline{\delta f^2(\varepsilon_{\mathbf{k}})} = \overline{n_{\mathbf{k}}^2} - \overline{n}_{\mathbf{k}}^2$ is the variance of the occupation number, and $f(\varepsilon_{\mathbf{k}}) = \overline{n}_{\mathbf{k}}$ is the equilibrium distribution function normalized to the carrier number, which, according to statistics, satisfies the property

$$\overline{\delta f^2(\varepsilon_{\mathbf{k}})} = -K_B T \frac{\partial f(\varepsilon_{\mathbf{k}})}{\partial \varepsilon_{\mathbf{k}}}. \tag{12}$$

Using the symmetry of the problem, $v_{\mathbf{k},x}^2$ can be replaced by $(2\varepsilon_{\mathbf{k}}/md)$, where d denotes the dimension of the system. With the density of states of a d-dimensional carrier gas satisfying $\mathcal{D}(\varepsilon) \propto \varepsilon^{(d/2)-1}$, Eq. (11) can be evaluated to

$$\overline{\delta v_d^2} = \frac{K_B T}{m \, N}, \tag{13}$$

independent of the dimension d. We remark that since these noise sources are directly obtained from the properties of the statistical ensembles, they hold for carriers obeying any type of statistics, in particular for Fermi-Dirac (FD) and Bose-Einstein (BE) statistics, but also for carriers obeying a fractional exclusion statistics (FES) [14,15]. In all cases, Maxwell-Boltzmann (MB) statistics is implicitly recovered in the limit $f \ll 1$.

By using Eq. (10) to replace the temperature in Eq. (3), the current fluctuations can be expressed as

$$\overline{I^2} = \frac{G}{\tau} \frac{\overline{\delta N^2}}{} \frac{\partial \mu_0}{\partial \overline{N}}. \tag{14}$$

By taking G from the generalized Einstein relation [16],

$$G = \left(\frac{q}{L}\right)^2 D \frac{\partial \overline{N}}{\partial \mu_0}, \tag{15}$$

where

$$D = \overline{v_x^{2'}} \tau \tag{16}$$

is the longitudinal-diffusion coefficient [16] with the differential (with respect to carrier number) quadratic velocity component along the x direction given by

$$\begin{aligned} \overline{v_x^{2'}} &= \sum_{\mathbf{k}} v_{\mathbf{k},x}^2 \frac{\partial f(\varepsilon_{\mathbf{k}})}{\partial \overline{N}} \\ &= \sum_{\mathbf{k}} v_{\mathbf{k},x}^2 \frac{\partial f(\varepsilon_{\mathbf{k}})}{\partial \mu_0} \frac{\partial \mu_0}{\partial \overline{N}} = \frac{K_B T}{m} \frac{\overline{N}}{\overline{\delta N^2}}, \end{aligned} \tag{17}$$

where the last equality, obtained in the same way as Eq. (13), follows from Eq. (11) and additionally using Eq. (10). Notice the explicit appearance of the Fano factor, $\overline{\delta N^2}/\overline{N}$, to account for the effective interaction among carriers due to the symmetry properties of their wave functions and thus for the correct statistics.

Then, Eq. (14) takes the equivalent forms

$$\overline{I^2} = \frac{K_B T}{\mathcal{L}} = \frac{q^2 \overline{v_x^{2'}} \, \overline{\delta N^2}}{L^2} = \frac{q^2 \overline{\delta N^2}}{\tau_N^2}, \tag{18}$$

with $\tau_N = \sqrt{L^2/\overline{v_x^{2'}}}$ being an effective transport time through the sample [17] determining the conversion of total-number fluctuations of carriers inside the sample into total-current fluctuations measured in the external short circuit.

For the variance of voltage fluctuations, substitution of Eq. (13) into Eq. (4) gives the equivalent expressions

$$\overline{V^2} = \frac{K_B T}{\mathcal{C}} = \frac{m N L \overline{\delta v_d^2}}{A \epsilon_0 \epsilon_r} = \frac{m^2 L^2 \overline{\delta v_d^2}}{q^2 \tau_p^2} \tag{19}$$

with the plasma time τ_p given in Eq. (8). Introducing the variance of electric-field fluctuations averaged over the sample length $\overline{\delta E^2} = \overline{\delta V^2}/L^2$ and the plasma carrier mobility $\mu_p = q\tau_p/m$, Eq. (19) can be written in terms of a generalized Ohm's law,

$$\overline{\delta v_d^2} = \mu_p^2 \, \overline{\delta E^2}, \tag{20}$$

describing the conversion of carrier drift-velocity fluctuations inside the sample into electric field (or voltage) fluctuations at the terminals of the open circuit.

By using Eqs. (3) and (10), the macroscopic conductance is associated with carrier total-number fluctuations by

$$G = \frac{q^2 \overline{v_x^{2'}} \tau}{L^2 K_B T} \, \overline{\delta N^2}. \tag{21}$$

Analogously, using Eqs. (4) and (13), the macroscopic resistance is associated with drift-velocity fluctuations by

$$R = \frac{L^2 m^2}{q^2 \tau K_B T}\overline{\delta v_d^2}. \tag{22}$$

From a microscopic point of view, Eqs. (2), (21), and (22) imply that the noise sources satisfy the relations

$$GR = \frac{\overline{v_x^{2'}}\,\overline{\delta N^2} m^2 \overline{\delta v_d^2}}{(K_B T)^2} = \frac{\overline{v_x^{2'}}}{\overline{\delta v_d^2}}\,\frac{\overline{\delta N^2}}{N^2} = 1 \tag{23}$$

and thus

$$\overline{\delta N^2}\;\overline{v_x^{2'}} = N^2\;\overline{\delta v_d^2}\; = \frac{N K_B T}{m}, \tag{24}$$

where again Eq. (13) has been used to eliminate the temperature in Eq. (23).

Equations (23) and (24) express the reciprocity and duality properties of the microscopic noise sources associated with the fluctuation-dissipation relations. In other words, at thermodynamic equilibrium, carrier total-number fluctuations inside a conductor under constant-voltage conditions are interrelated with carrier drift-velocity fluctuations under constant-current conditions.

The dual property of the macroscopic FDTs is obtained from Eqs. (18), (19), and (24) as

$$\overline{I^2} = G_p^2\,\overline{V^2} \quad \text{and} \quad \overline{V^2} = R_p^2\,\overline{I^2} \tag{25}$$

with a plasma conductance $G_p = (qN\mu_p)/L^2$ and a plasma resistance R_p satisfying the reciprocity relation

$$G_p R_p = 1. \tag{26}$$

By satisfying the relations (3) and (4), the expressions (25) and (26) justify the identification of the intrinsic bandwidths assumed here.

Equations (25) and (26) express the duality and reciprocity properties of fluctuation-dissipation relations and parallels Eqs. (1) and (2) of linear-response relations. Notice that all the above expressions hold for any type of statistics (in the case of bosons for temperatures above the critical temperature for Bose-Einstein condensation [18]), thus complementing the standard FDT in the limit of low frequencies. From statistics, the two boundary conditions are associated with a GCE and a CE, respectively, and Eq. (24) shows the interesting results that both statistics provide the same result even outside the thermodynamic limit conditions [7].

III. CONCLUSIONS

We have formulated the dual property and the reciprocity relation of the FDTs in the limit of low frequency by expressing conductance and resistance in terms of the proper microscopic noise source, which we call the dissipation-fluctuation relations. Analogously, by accounting for the intrinsic bandwidth, the variances of current and voltage fluctuations are related to the same noise sources, which are usually called the fluctuation dissipation relations. All these relations are given in a form that is independent of the type of distribution functions, thus including MB, FD, and BE statistics (the latter at temperatures above Bose-Einstein condensation) as well as FES. From a physical point of view, the temperature entering the Nyquist relations (3) and (4) is expressed here in kinetic terms and associated with the variance of instantaneous fluctuations (i) of the total number of carriers inside the sample, or (ii) of the carrier drift velocity inside the sample. Calculations are carried out in the framework of the GCE and the CE, and the result summarized in Eq. (24) provides an interesting example in which both ensembles give the same result even outside the thermodynamic limit.

While thermal noise and shot noise are typically described as different noise phenomena, one originating from the thermal agitation of the carriers [2,3] and the other from the discreteness of the charge [19–21], our results clearly indicate the close relationship between both of them. Indeed, by expressing current fluctuations in terms of the ratio between the variance of carrier number fluctuations and the effective transport time τ_N, we show that the source of the shot noise is already present at thermal equilibrium [22].

The present formulation shows that for fermions, the vanishing of the low-frequency current spectral density at $T \to 0$ is associated with the vanishing of the variance of carrier total-number fluctuations, i.e., with the instantaneous correlation (coherence) between the appearance and disappearance of an elemental carrier number fluctuation inside the sample as dictated by the Pauli principle. This is essential because for fermions the value of the diffusion coefficient [Eq. (16)] does not vanish at $T \to 0$, and thus the notion of diffusion being synonymous with noise fails completely. In contrast, for the classical case the absence of motion at $T = 0$, i.e., $D = 0$ and $\tau_N \to \infty$, is definitely responsible for the vanishing of current noise.

For the case of voltage fluctuations, the vanishing of thermal noise at $T \to 0$ is associated with the tendency of the drift-velocity fluctuations to approach zero, which is a property independent of the kind of statistics.

As a final remark, we notice that the equivalent circuit introduced herein reproduces the correct frequency dependence of both current and voltage spectral densities, the latter including the plasmonic contribution in the case $\tau_d \ll \tau$ [12]. We want to stress that the time scales of the fluctuating macroscopic variables are generally not related to those of the respective noise sources. Instead, they should be treated in the framework of the time or frequency dependence of the corresponding correlation functions or spectral densities.

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