

Charged particle dynamics in turbulent current sheets

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We study dynamics of charged particle in current sheets with magnetic fluctuations. We use the adiabatic theory to describe the nonperturbed charged particle motion and show that magnetic field fluctuations destroy the adiabatic invariant. We demonstrate that the evolution of particle adiabatic invariant's distribution is described by a diffusion equation and derive analytical estimates of the rate of adiabatic invariant's diffusion. This rate is proportional to power density of magnetic field fluctuations. We compare analytical estimates with numerical simulations. We show that adiabatic invariant diffusion results in transient particles trapping in the current sheet. For magnetic field fluctuation amplitude a few times larger than a normal magnetic field component, more than 50% of transient particles become trapped. We discuss the possible consequences of destruction of adiabaticity of the charged particle motion on the state of the current sheets.

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I. INTRODUCTION

In many space-plasma systems, electric currents of hot charged particles produce the coherent plasma structures called current sheets (CSs). Examples of CSs were observed in planetary magnetospheres [1], solar corona [2], laboratory devices [3], and distant astrophysical objects [4]. The modern theory suggests that the principal role in the CS formation is often played by a relatively small population of charged particles moving along specific orbits and carrying a strong electric current [5,6]. The standard approach to describe the motion of current-carrying particles in CS includes the applicability of the theory of adiabatic invariants [7–9]. Within this approach, the particle trajectories can be integrated analytically. Results obtained with this approach are well tested and verified both numerically and by comparison with *in situ* spacecraft observations [10,11]. However, up to now, the adiabatic theory was used to describe charged particles motion in CSs only in *laminar* magnetic field configurations, without any fluctuations of magnetic field. Recent spacecraft observations demonstrate that CSs are often filled by electromagnetic turbulence [12,13]. Thus, investigation of the influence of magnetic field fluctuations on charged particle motion in CSs is important.

The most intense and dynamical CSs are formed in the vicinity of regions where the magnetic energy is released during reconnections of magnetic field lines [14]. This process plays a key role in transformation of the magnetic field energy into the energy of plasma particles in many space-plasma systems [2–4]. The corresponding CS configurations are usually characterized by stretched magnetic field lines; see a schematic view in Fig. 1. Significant difference in magnitudes of spatial scales (across and along the CS) results in the separation of timescales of the particle rotation around a strong

B_x magnetic field and the motion along the field lines. This separation allows the introduction of the adiabatic invariant [7] and analytical integration of particle trajectories. However, additional magnetic field fluctuations generated in the vicinity of the $z = 0$ (where $B_x = 0$) plane by plasma flows from the reconnection region [15] can significantly change the particle trajectories. Moreover, CSs can be formed within turbulent plasma flows, where magnetic field fluctuations are intrinsic property of CSs [16,17]. In this paper, we describe particle motion in a turbulent CS.

II. MAIN EQUATIONS AND CHARGED PARTICLE TRAJECTORIES

We consider nonrelativistic motion of a particle with the mass m and charge q in 2D CS magnetic field $\mathbf{B} = B_z \mathbf{e}_z + B_0(z/L) \mathbf{e}_x$ with stationary fluctuations $\delta B_z(x, y)$ (note that dynamics of 3D turbulent current sheets can be different [18,19]). A CS thickness L is a characteristic system scale. The corresponding vector potential has two components $A_y = B_z x - B_0(z^2/2L) + \delta A_y(x, y)$ and $A_x = \delta A_x(x, y)$. We do not consider a magnetic field component B_y , because charged particle motion in CSs with $B_y \neq 0$ is much more complicated for analysis even without magnetic field fluctuations [20,21]. Magnetic field fluctuations are set as an ensemble of plane waves with a power-law spectrum [22]:

$$\begin{aligned} \delta A_y &= \delta \bar{B}_z L \sum_{k, \theta} \frac{\cos \theta}{1 + (kL)^2} \sin [k(x \cos \theta + y \sin \theta) + \phi_0] \\ \delta A_x &= -\delta \bar{B}_z L \sum_{k, \theta} \frac{\sin \theta}{1 + (kL)^2} \sin [k(x \cos \theta + y \sin \theta) + \phi_0], \end{aligned} \quad (1)$$

where $\delta \bar{B}_z$ is the amplitude of magnetic field fluctuations. We set 20 values of θ to be uniformly distributed over $\theta \in [0, 2\pi]$ and took 100 values of $kL \in [0.1, 10]$ with a step 0.1. For each harmonic, a value of the phase ϕ_0 was chosen randomly. Magnetic fluctuation Eqs. (1) satisfy the Coulomb gauge: $\partial \delta A_x / \partial x + \partial \delta A_y / \partial y = 0$. The corresponding single

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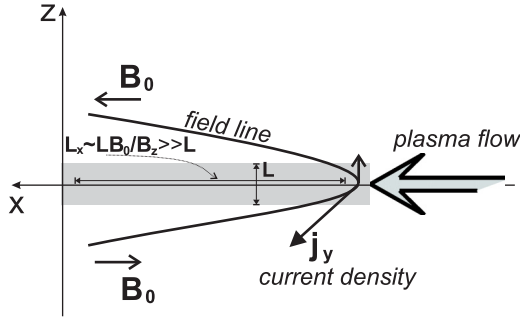


FIG. 1. Schematic view of the system. Gray region shows the current sheet (i.e., localized electric current density). Magnetic field B_x varies across this current sheet with the spatial scale L . Gray arrow shows plasma flow coming from the deep tail region and bringing electromagnetic fluctuations.

component of magnetic field fluctuations $\delta B_z = \partial \delta A_y / \partial x - \partial \delta A_x / \partial y$ has the form

$$\delta B_z = \delta \bar{B}_z \sum_{k, \theta} \frac{kL}{1 + (kL)^2} \cos [k(x \cos \theta + y \sin \theta) + \phi_0]. \quad (2)$$

The Hamiltonian of charged particles in this system is

$$H = \frac{1}{2m} p_z^2 + \frac{1}{2m} \left(p_x - \frac{q}{c} \delta A_x \right)^2 + \frac{1}{2m} \left(p_y - \frac{q}{c} A_y \right)^2, \quad (3)$$

where (p_x, p_y, p_z) are components of particle momentum. We introduce dimensionless variables $(x, y, z) \rightarrow (x, y, z) / \sqrt{L\rho_0}$, $(p_x, p_y, p_z) \rightarrow (p_x, p_y, p_z) / (\rho_0 m \Omega_0)$ and dimensionless time $t \rightarrow t \sqrt{\rho_0 / L \Omega_0}$, where $\Omega_0 = q B_0 / mc$, $\rho_0 = \sqrt{2H_0 / m} / \Omega_0$, and H_0 is a particle energy value (as $\partial H / \partial t = 0$ the energy H is conserved). We also use two dimensionless parameters $\kappa = (B_z / B_0) \sqrt{L / \rho_0}$ and $\beta = (\delta \bar{B}_z / B_0) \sqrt{L / \rho_0}$. In the new variables, Hamiltonian Eq. (3) takes the form (H is normalized by $2H_0$)

$$H = \frac{1}{2} p_z^2 + \frac{1}{2} (p_x - \beta g_x)^2 + \frac{1}{2} (p_y - \kappa x + \frac{1}{2} z^2 + \beta g_y)^2, \quad (4)$$

where $(g_x, g_y) = (\delta A_x, \delta A_y) / (\delta \bar{B}_z \sqrt{L\rho_0})$. In the absence of fluctuations ($\beta = 0$), Hamiltonian Eq. (4) does not depend on y and, thus, p_y is conserved. For small values of $\kappa \ll 1$ (observed, e.g., in thin CSs with small B_z , see Ref. [11]), dynamics of charged particles was described in details in Refs. [7–9]. In the present paper, we consider the same regime, $\kappa \ll 1$, taking into account magnetic field fluctuations with $\beta \sim \kappa$.

A. System without fluctuations

Conservation of p_y in Hamiltonian Eq. (4) with $\beta = 0$ allows us to apply change of variables $\kappa x \rightarrow \kappa x - p_y$ and consider two pairs of conjugated variables (z, p_z) , (x, p_x) . As $\kappa \ll 1$, variables (z, p_z) change much faster than variables $(\kappa x, p_x)$: the value of κ defines the ratio of characteristic periods in z and x motion. For frozen $(\kappa x, p_x)$, the oscillations

in the (z, p_z) plane are described by the following Hamiltonian of the fast motion:

$$h_z = H - \frac{1}{2} p_x^2 = \frac{1}{2} p_z^2 + \frac{1}{2} (\kappa x - \frac{1}{2} z^2)^2. \quad (5)$$

The corresponding action $I_z = (1/2\pi) \oint p_z dz$ is an adiabatic invariant of the exact system, i.e., for slowly changing $(\kappa x, p_x)$ (see, e.g., Ref. [9]). The equation $I_z(\kappa x, p_x) = \text{const}$ defines trajectories in the $(\kappa x, p_x)$ plane for a given value of energy H_0 :

$$I_z = \frac{(2h_z)^{3/4}}{\pi} \int_{\zeta_-}^{\zeta_+} \sqrt{1 - \left(s - \frac{\zeta^2}{2} \right)^2} d\zeta = (2h_z)^{3/4} f(s), \quad (6)$$

$$s = \kappa x / \sqrt{2h_z} = \kappa x / \sqrt{1 - p_x^2},$$

where $2h_z = 1 - p_x^2$, $\zeta = z / (2h_z)^{1/4}$, ζ_{\pm} are solutions of equation $1 - (s - \zeta^2/2)^2 = 0$ (if there are only two roots $\zeta_- = -\zeta_+$, the integral from Eq. (6) should be divided to two; see renormalization details in Ref. [7]). Note that $H = 1/2$. It follows from Eq. (6), that $2h_z = [I_z / f(s)]^{4/3}$. Variables s , I_z determine the position of particle in the $(\kappa x, p_x)$ plane.

In the course of slow evolution of $(\kappa x, p_x)$, the particle trajectory in the (z, p_z) plane changes. There are two types of these trajectories and the separatrix in the (z, p_z) plane demarcates regions filled by trajectories of different types [7]. When particles cross the separatrix ($s = 1$), the adiabatic invariant, I_z , experiences a small jump. An example of particle trajectory in the $(\kappa x, p_x)$ plane and the corresponding evolution of I_z are shown in Fig. 2(a). The particle starts at large κx (and large positive s), moves toward small κx and crosses the separatrix (at this moment $s = 1$), then makes a turnaround in the $(\kappa x, p_x)$ plane (at $p_x = 0$, where s reaches a minimal on a given curve negative value and then starts growing again). One can see weak variations of I_z along the trajectory, and the enhanced oscillations at about time = 200 and 400 occur near the separatrix crossings (there are two crossings for each trajectory shown in Fig. 2), where the motion of a particle slows down. Over a long time (many periods of particle motion in the $(\kappa x, p_x)$ plane), the slow destruction of adiabatic invariant can modify the particle trajectory and substantially change the value of I_z [9]. However, for a single passage of a particle along the trajectory shown in Fig. 2(a) we can neglect the variations of I_z for small enough κ , as well as the jump of I_z at the separatrix.

B. System with fluctuations

Small-scale fluctuations of magnetic field ($\beta \neq 0$) can scatter particles and result in variations of I_z . We showed two trajectories with $\beta \neq 0$ [Figs. 2(b) and 2(c)]. One can see that I_z strongly varies, while the corresponding trajectories are deformed. Comparison of trajectories with $\beta = 0$ and $\beta \neq 0$ shows that main variations of I_z occur in the same parts of trajectories for both systems, near the separatrix crossings, where the (z, p_z) motion drastically slows down.

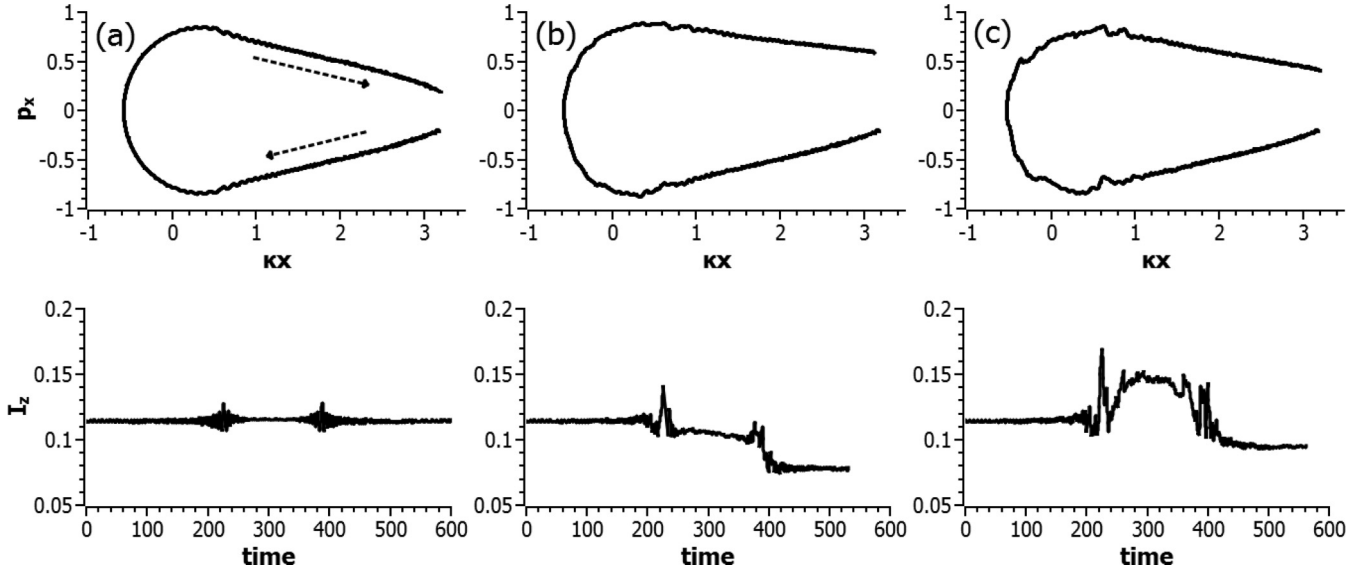


FIG. 2. Three particle trajectories in the (x, p_x) plane and the corresponding time profiles of I_z : (a) $\beta = 0$, (b) $\beta/\kappa = 1$, (c) $\beta/\kappa = 2.5$.

For analytical estimates we expand Hamiltonian Eq. (4) for small values of β :

$$H \approx H_0 + \beta H_1 = \frac{1}{2}p_z^2 + \frac{1}{2}p_x^2 + \frac{1}{2}(\kappa x - \frac{1}{2}z^2)^2 + \beta[(\kappa x - \frac{1}{2}z^2)g_y - p_x g_x], \quad (7)$$

where we excluded term p_y because $\dot{p}_y \sim g_x, g_y \ll 1$.

In what follows, we consider g_x, g_y to be random statistically independent functions that change their values at each time-step τ . We assumed that the amplitude of magnetic field fluctuations was defined by parameter β , with g_x, g_y normalized in such a way that $\text{var}(g_x) = \text{var}(g_y) = 1$. For analytical study we substituted the spatial dependence of g_x, g_y with the time dependence. To choose a value of the time-step τ for a fixed β , we compared the power density of magnetic field fluctuations along particle trajectories for model Eq. (1) and for our approximation. We assembled time series of magnetic field fluctuations along trajectories of Hamiltonian system Eq. (4) and calculated the power density of these fluctuations using Fourier transformation. Then we chose the frequency ω corresponding to maximum in spectrum and defined τ as $2\pi/\omega$. This approach gave us the same power density $\sim \delta B_z^2/\omega$ in model Eq. (1) and in approximation of functions g_x, g_y by time series along trajectories. To reproduce a nonuniform distribution of magnetic field fluctuations along trajectories, we use a multiplication factor for $\beta \rightarrow \beta \exp[-(s-1)^2/0.25]$, which defines that the maximum of fluctuations are observed by particles near the separatrix $s = 1$.

III. DIFFUSION OF ADIABATIC INVARIANT

To compute the change of adiabatic invariant ΔI_z due to magnetic field fluctuations for one time step τ , we use the

definition $2\pi/T = \partial H/\partial I_z$, where $T = \oint dz/p_z$ is a period of particle oscillations in the (z, p_z) plane:

$$\Delta I_z = \frac{T}{2\pi} \Delta H = -\frac{T\beta}{2\pi} z \Delta z g_y = \frac{T\beta}{2\pi} z p_z \tau g_y, \quad (8)$$

where ΔH corresponds to variation of Hamiltonian Eq. (7) due to magnetic field fluctuations. The right-hand side of Eq. (8) should be calculated for a particular moment of time $t_i \in [\tau i, \tau(i+1)]$, where i is an integer number. As T is much larger than τ and much smaller than $1/\kappa$, variables $\kappa x, p_x, p_y$ can be assumed to be constant during a time interval $\sim \tau$. For any time interval, the average value of ΔI_z is zero (as g_y has a zero mean), while the corresponding variance is

$$\text{var}(\Delta I_z) = \left(\frac{T\tau\beta}{2\pi}\right)^2 \text{var}(z p_z g_y) = \left(\frac{T\tau\beta}{2\pi}\right)^2 \text{var}(z p_z), \quad (9)$$

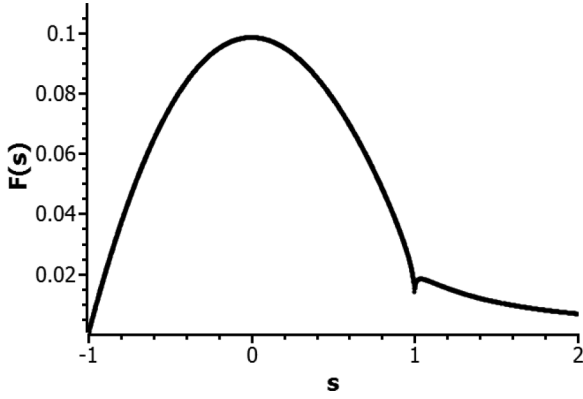
where we assumed that g_y and $z p_z$ were statistically independent and used $\text{var}(g_y) = 1$. The term $\text{var}(z p_z)$ in Eq. (9) should be considered as a sum of many $(z p_z)_i^2$ terms calculated at i th moments of time. As z and p_z oscillate regularly, we can express $\text{var}(z p_z)$ as

$$\begin{aligned} \text{var}(z p_z) &= \frac{1}{T} \oint z^2 p_z^2 \frac{dz}{p_z} = \frac{1}{T} \oint z^2 p_z dz \\ &= \frac{2}{T} (2h_z)^{5/4} \int_{\zeta_-}^{\zeta_+} \zeta^2 \sqrt{1 - \left(s - \frac{1}{2}\zeta^2\right)^2} d\zeta \\ &= \frac{2}{T} (2h_z)^{5/4} f_V(s), \end{aligned} \quad (10)$$

where h_z, ζ , and s were defined in Eq. (6).

For the period T , we have

$$T = \frac{2}{(2h_z)^{1/4}} \int_{\zeta_-}^{\zeta_+} \frac{d\zeta}{\sqrt{1 - (s - \zeta^2/2)^2}} = \frac{2f_T(s)}{(2h_z)^{1/4}}. \quad (11)$$

FIG. 3. Profile of function $F(s)$.

Approximate conservation of I_z determines trajectories in the (x, p_x) plane. Thus, for each given value of I_z , the function s varies along the trajectory. To derive the expression for \dot{s} , we use the definitions from Eq. (6):

$$p_x \dot{p}_x = \frac{4}{3} \frac{f'}{f} (I_z/f)^{4/3} \dot{s},$$

where $f' = df/ds$ and

$$p_x = \pm \sqrt{1 - 2h_z} = \pm \sqrt{1 - (I_z/f)^{4/3}},$$

$$\dot{p}_x = \frac{\partial I_z}{\partial \kappa x} = -\frac{(2h_z)^{1/4}}{\pi} \int_{\zeta^-}^{\zeta^+} \frac{(s - \zeta^2/2)d\zeta}{\sqrt{1 - (s - \zeta^2/2)^2}}$$

$$= -(2h_z)^{1/4} f'. \quad (12)$$

Thus, for \dot{s} we have

$$\dot{s} = \alpha \frac{3}{4} \frac{\sqrt{1 - (I_z/f)^{4/3}} f}{(I_z/f)}, \quad (13)$$

where $\alpha = \text{sign}(p_x)$.

Statistical behavior of I_z can be quantitatively described in terms of the probability distribution function $\Psi = \Psi(I_z, t)$: ΨdI_z is equal to the number of particles with values of adiabatic invariant in $(I_z - dI_z/2, I_z + dI_z/2)$ after time t . Jump Eqs. (8) result in a random walk of I_z , which can be described by the diffusion equation

$$\frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial I_z} \left(D \frac{\partial \Psi}{\partial I_z} \right), \quad (14)$$

where the diffusion coefficient $D(I_z, s) = \text{var}(\Delta I_z)/\tau$ is

$$D = \frac{\beta^2 \tau}{\pi^2} \left(\frac{I_z}{f} \right)^{4/3} f_V(s) f_T(s). \quad (15)$$

Considering evolution in terms of s instead of t and using Eq. (13), we get instead of Eq. (14),

$$\frac{\partial \Psi}{\partial s} = \alpha \frac{4}{3} \frac{(I_z/f)}{\sqrt{1 - (I_z/f)^{4/3}} f} \frac{\partial}{\partial I_z} \left(D \frac{\partial \Psi}{\partial I_z} \right). \quad (16)$$

Introducing $J = I_z/f(s) = (2h_z)^{3/4}$ as a new variable, we can write the diffusion equations as

$$\alpha F(s) \frac{\partial \Psi}{\partial s} = \frac{4}{3} \frac{\beta^2 \tau}{\pi^2} \frac{J}{\sqrt{1 - J^{4/3}}} \frac{\partial}{\partial J} \left(J^{4/3} \frac{\partial \Psi}{\partial J} \right), \quad (17)$$

where $F(s) = f^3(s)/f_V(s) f_T(s)$ (see Fig. 3), and $\alpha = -1$ and $+1$, for s decreasing and increasing, respectively.

Equation (17) was integrated from $s = 2$ (boundary of the region filled by magnetic field fluctuations) to s_{\min} (defined by the equation $I_z/f(s) = 1$) with $\alpha = 1$ and then back from s_{\min} to $s = 2$ with $\alpha = -1$. For any given s , the distribution $\Psi(I_z)$ can be converted into $\Psi(J)$ by a simple scaling.

As the diffusion coefficient D depends on I_z , there are both diffusive spreading of the distribution Ψ around the initial maximum ($\sim D \partial^2 \Psi / \partial I_z^2$) and a drift ($\sim \partial D / \partial I_z \partial \Psi / \partial I_z$). The direction of the drift is defined by the sign of $\partial D / \partial I_z$. As that quantity in the current setup is always positive, the drift is always directed toward the smaller values of I_z . To check the solutions of the diffusion equation, we integrated numerically original Hamiltonian system Eq. (4) for a large ensemble of

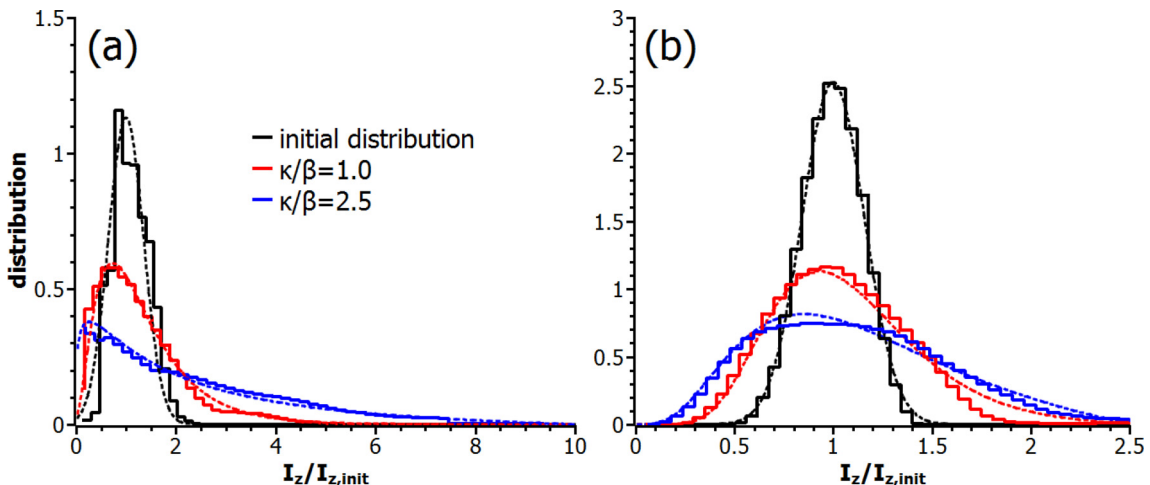


FIG. 4. Initial distribution $\Psi(I_z)$, results of solution of Eq. (17) (dotted curves), and results of numerical integration of 10^6 trajectories (solid curves) for two β values: (a) $I_{z,\text{init}} = 0.1$, (b) $I_{z,\text{init}} = 0.5$.

trajectories for different values of β . Each trajectory was integrated over the time interval corresponding to passage through the central region of the system $z \sim 0$ (see examples of trajectories in Fig. 2). Examples of initial and final distributions Ψ are shown in Fig. 4. Distributions were obtained for initial Ψ peaked around $I_{z,\text{init}} = 0.1$ and $I_{z,\text{init}} = 0.5$. Numerically obtained distributions are very close to analytical results. The final distributions of both types have similar maximum values and are shifted toward smaller I_z values. The main discrepancies are at the wings of the distributions and can be explained as follows. The minimum of s , $s_{\min} = s_{\min}(I_z)$, is defined based on the unperturbed value of I_z . However, if, in the process of evolution, the value of I_z becomes smaller than the original value, that particle penetrates into the values of s smaller than s_{\min} . Those particles move for a longer time than assumed in model Eq. (17). This creates a shorter, more abrupt tail, compared with the one predicted by Eq. (17). Similarly, the particles with larger values of I_z spend less time than assumed in model Eq. (17). This creates a shallower, longer tail, compared with the one predicted by Eq. (17). This effect can be most clearly seen in Fig. 4 with $I_{z,\text{init}} = 0.5$.

IV. EVOLUTION OF TRANSIENT TRAJECTORIES

The CS structure (and stability) strongly depends on properties of the so-called transient trajectories [9]. Particles moving along such trajectories come from large $|z|$ (which corresponds to large values of x ; see Fig. 1) with relatively small I_z (maximum I_z value of transient particles depends on magnetic field configuration outside the CS, $|z| \gg L$; see Ref. [7]), make a turnaround in the (x, p_x) plane, and move back to large $|z|$ [see an example of such trajectory in Fig. 2(a)]. If I_z is conserved, particles stay on the transient trajectories, whereas a destruction of I_z can lead to scattering of initially transient particles (scattered particles escape from transient trajectories and move along quasiclosed trajectories within CS). Transient particles significantly participate in generation of the current density [23,24] and, thus, play an important role in CS formation [5,6]. Therefore, it is important to describe the evolution of the amount of transient trajectories in CSs with magnetic field fluctuations. We numerically integrated 10^4 trajectories with I_z distributed on the $[0.1, 0.5]$ interval with different values of β and plotted the number of particles returning to the initial boundary $|z|$ after passing through the turbulent CS. Figure 5 shows that the number of

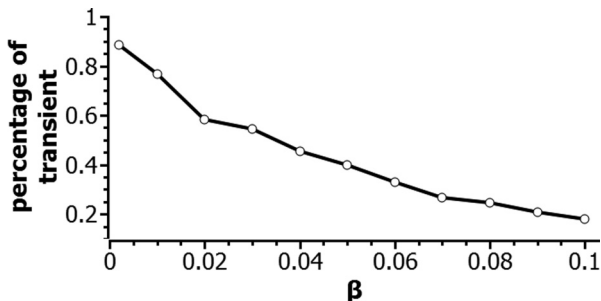


FIG. 5. Percentage of particles on transient trajectories after one interaction with CS for $\kappa = 0.01$.

transient particles decreases significantly only for magnetic field fluctuations stronger than background magnetic field in the $z = 0$ plane ($\beta/\kappa \geq 1$ means $\delta \bar{B}_z \geq B_z$). For $\beta/\kappa \sim 1$ the final number of transient particles is about 80% of initial population, and only for $\beta/\kappa \sim 10$ almost all transient particles become scattered.

V. DISCUSSION AND CONCLUSIONS

High levels of magnetic field fluctuations are often observed by spacecrafts in CSs in the distant Earth magnetotail [25,26]. We showed that these fluctuations may significantly influence the particle dynamics and destroy the adiabatic invariant I_z . Thus, CS configurations in the presence of fluctuations should differ from the *laminar* CS structures. Indeed, in more turbulent CSs spacecrafts detected weaker current density amplitudes and such CSs had larger spatial scales [27].

We considered the role of magnetic field fluctuations in scattering of transient particles and described this process as a diffusion of invariant I_z . Figure 5 shows that fluctuations reduce the percentage of transient trajectories in the system. However, we should mention that this result was obtained for a system where unperturbed (without fluctuations) state was dominated by the transient trajectories and almost absence of scattered trajectories. This is a typical condition for thin intense CSs observed in the distant magnetotail [28]. However, if a CS was initially filled by scattered particles, magnetic field fluctuations can potentially scatter them to transient trajectories. We did not consider this scenario because it is less probable to observe intense magnetic field fluctuations in weak large-scale CS filled by scattered particles [13].

Main mechanisms for generation of magnetic field fluctuations in CS are various current-driven instabilities [29] and gradient instabilities (e.g., ballooning [30] and double gradient [31] instabilities inducing CS flapping oscillations). In contrast to externally driven (e.g., by solar wind) CS motion, amount of the free energy in these instabilities directly depends on the intensity of the current density [32,33] and, thus, depends on the population of transient particles [5,6]. The larger is the amplitude of magnetic field fluctuations, the more transient particles leave the transient regime, thus reducing the current density. But the reduction of the current density decreases the intensity of the generation of magnetic field fluctuations. Therefore, the system is characterized by a negative feedback and, as a result, should have a stationary solution in the presence of a source of transient particles. In this case, incoming and scattered transient particles should provide the necessary intensity of current density generating magnetic field fluctuations with the level needed to scatter exactly the population of transient particles equal to the incoming population. This nonlinear system with external energy source (external source of transient particles) can be described with the same approach as one applied to the self-consistent CS evolution induced by particle scattering due to the separatrix crossings [9].

In the present paper, we studied the influence of magnetic field fluctuations on charged particle dynamics in CS. We demonstrated that fluctuations destroy the adiabatic invariant I_z and result in particle scattering. This process can be described in terms of diffusion of I_z . Such a diffusion decreases

the number of transient particles in CS and, as a result, can significantly change the CS configuration. However, if amplitude of fluctuations is smaller than the magnetic field amplitude in the neutral plane ($\beta/\kappa < 1$), the scattering does not result in significant decrease of a population of transient particles.

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