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First-order phase transition in a model of self-propelled particles with variable angular range of interaction

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We have carried out a Monte Carlo simulation of a modified version of Vicsek model for the motion of self-propelled particles in two dimensions. In this model the neighborhood of interaction of a particle is a sector of the circle with the particle at the center (rather than the whole circle as in the original Vicsek model). The sector is centered along the direction of the velocity of the particle, and the half-opening angle of this sector is called the "view angle." We vary the view angle over its entire range and study the change in the nature of the collective motion of the particles. We find that ordered collective motion persists down to remarkably small view angles. And at a certain transition view angle the collective motion of the system undergoes a first-order phase transition to a disordered state. We also find that the reduction in the view angle can in fact increase the order in the system significantly. We show that the directionality of the interaction, and not only the radial range of the interaction, plays an important role in the determination of the nature of the above phase transition.

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I. INTRODUCTION

There are many systems, natural as well as artificial, that consist of moving and interacting "self-propelled" agents. Biological systems such as schools of fish [1], flocks of birds [2,3], and bacterial colonies [4–6] and artificial systems such as Kobots (robots specially developed for the study of flocking) [7], platinum-silica particles in hydrogen peroxide solution [8], carbon-coated Janus particles in water-lutidine mixtures [9], and vibrating rods [10–12] are some examples. One remarkable characteristic of these systems is that under certain conditions they are capable of displaying extraordinary collective dynamics, such as highly cooperative collective motion and complex moving patterns [13–19].

One of the simplest models proposed to describe the motion of a collection of agents which have a tendency "to move as their neighbors do" (birds in a flock is an obvious example) is the one by Vicsek *et al.* [13], now commonly known as the Vicsek model. In this model a collection of point particles move with the same constant speed, and each particle at discrete time intervals "adjusts" its direction of motion so as to move along the mean direction of motion of the particles in its local (short-range) neighborhood. This direction adjustment is imperfect due to the presence of noise in the system. Vicsek *et al.* found that the nature of the collective motion of the particles depends on the level of this noise and the particle density of the system. For high densities and low noise levels the collective motion attains an ordered state in which the particles move largely in a common direction. For low densities and high noise levels there is no such collective motion and particles essentially perform uncorrelated random walks. And as the noise level is varied (for a fixed particle density) or as the particle density is varied (for a fixed noise level), the system displays a nonequilibrium order-disorder phase transition. This subject has been recently reviewed by Vicsek [20] and Menzel [21].

A number of variants of the Vicsek models have been studied, and among them of particular interest to the present work are the ones where certain constraints are imposed on the angular range of the local neighborhood of a particle, or on the extent of the reorientation of the particle's direction of motion. These studies have found that these restrictions, which might naively be expected to reduce the degree of order in the system, can in fact enhance it. Tian *et al.* [22] as well as Li *et al.* [23] found that restricting the angular range of interaction can reduce the "consensus time" (the time taken by the system to attain the stationary value of the order parameter). Similarly Gao *et al.* [24] found that restricting the angle of velocity reorientation can increase the order parameter. In the model that they studied, in a single update particle directions are allowed to change within a limited range. And Yang *et al.* [25] found that discarding short-range interactions can increase the order parameter.

In this work we too study the effects of variation in the angular range of interaction on the collective motion of the particles in the Vicsek model. One of the motivations for the Vicsek model was to understand the collective motions of the large moving groups of living beings, such as flocks of bird or schools of fish, where each agent is expected to move, as far as possible, in the same direction as its close neighbors. In the Vicsek model the neighborhood observed by the agent is a circle (for a two-dimensional system) centered on the current position of that agent, regardless of its direction of motion. This is quite unrealistic, because an agent such as a bird or a fish does not have a 360° view of its surroundings. For example, the cyclopean field of view (i.e., combined field of view of both eyes [26]) of the grey-headed albatross is about 270° in the horizontal plane [27], and that of *Dasyatis sabina* fish is about 327° in the horizontal plane [26]. This kind of restricted view of the neighborhood also plays a role in the phototactic motion of certain marine organisms such as *Platynereis* larvae [28]. Thus exploration of the effects of limitation of the angular range of interaction neighborhood should be of interest in the study of all those processes where the Vicsek model is relevant.

In this report we present the results of the numerical simulations of the Vicsek model in which the angular range of

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the interaction neighborhood is restricted. We have measured the order parameter of the system as a function of the angular range of the interaction neighborhood (radius of the neighborhood held fixed). We find that the order parameter of the system varies nonmonotonically as the angular range of the interaction neighborhood is decreased, and at a certain point the system undergoes a first-order (i.e., discontinuous) phase transition to a disordered state. We have also measured the variation of the order parameter as the radius of the interaction is reduced (without restricting the angular range of the interaction neighborhood) and find that the resulting change in the nature of the collective motion is qualitatively different.

II. MODEL

The model of self-propelled particles we have studied is a modification of the Vicsek model. In this model the interaction neighborhood of a particle is not a circle centered on that particle, but a sector of this circle, as illustrated in Fig. 1. The neighborhood sector S_i has an opening angle of 2ϕ and is centered about the direction of velocity of the ith particle. We shall call the half opening angle ϕ the "view angle," which can vary from 0 to π. For $\phi = \pi$ this model reduces to the original Vicsek model.

Simulations are carried out in a box of size $L \times L$ with N particles, with the usual periodic boundary conditions in both directions. The mean particle density is given by $\rho = N/L^2$. The initial positions $\mathbf{r}_i$ $(i = 1,2,3,\ldots,N)$ are assigned randomly with uniform probability within the box. The initial directions of the particles θ_i are also assigned randomly in the range $[-\pi,\pi]$ with uniform probability. All the particles have the same constant speed v_0. The velocities and positions of the particles at time $t+1$ are obtained from the velocities and positions at time t using the following update rules. First we update velocities of all the particles simultaneously with

$$\mathbf{v_i}(t+1) = v_0 \mathcal{R}(\theta)\hat{\mathbf{v}}(t), \tag{1}$$

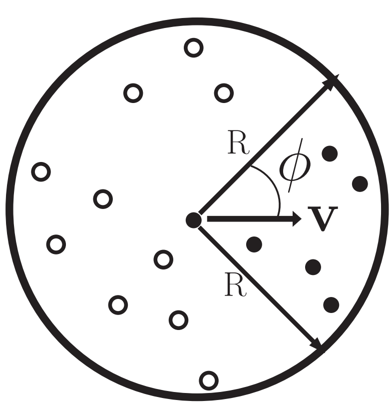

FIG. 1. The neighborhood S_i of the ith particle. The ith particle is shown at the center of the circle, and S_i is the sector bound by the two radii marked as R. The solid black circles (including the particle at the center of the circle) indicate the particles lying within the neighborhood, and the open black circles indicate particles outside it. The vector $\mathbf{v}$ indicates the direction of the velocity of the ith particle. The view angle ϕ is the half opening angle of the neighborhood at the center.

where $\hat{\mathbf{v}}(t)$ is the unit vector in the direction of the mean velocity of the particles in the neighborhood S_i of the ith particle, *including* the ith particle itself (see Fig. 1), and is given by

$$\hat{\mathbf{v}}(t) = \frac{\sum_{j\in S_i} \mathbf{v}_j(t)}{\left|\sum_{j\in S_i} \mathbf{v}_j(t)\right|}. \tag{2}$$

Here $|\ldots|$ denotes the norm of the vector, $\mathcal{R}(\theta)$ is the rotation operator which rotates the vector it acts upon [i.e., $\hat{\mathbf{v}}(t)$] by an angle θ. The angle θ is a random variable uniformly distributed over the interval $[-\eta\pi,\eta\pi]$, where η is the level (i.e., amplitude) of the noise that can be varied from 0 to 1.

Following the velocity updates, the positions are updated with

$$\mathbf{r}_i(t+1) = \mathbf{r}_i(t) + \mathbf{v_i}(t+1)\Delta t, \tag{3}$$

where $\Delta t = 1$. This update scheme is known as the "forward update" in the literature [29].

To quantify the degree of order in the collective motion of the particles a scalar order parameter $\psi(t)$ is defined as

$$\psi(t) = \frac{1}{N v_0}\left|\sum_{i=1}^{N} \mathbf{v}_i(t)\right|. \tag{4}$$

It can be readily seen that in the perfectly ordered state when all the particles are moving in the same direction $\psi(t) = 1$, and in the completely disordered state when the directions of motion are completely random $\psi(t) = 0$ (in the limit of $N \to \infty$). In this report we use the phrase "ordered state" to mean the stationary state of the system for which $\psi(t) > 0$ in the limit of $N \to \infty$.

III. SIMULATION DETAILS

In this work the data presented in Figs. 2–6 are produced with the following parameters fixed: the number density $\rho = N/L^2 = 1$, the particle speed $v_0 = 0.5$, the noise level $\eta = 0.3$, and the interaction neighborhood radius $R = 1$. For the

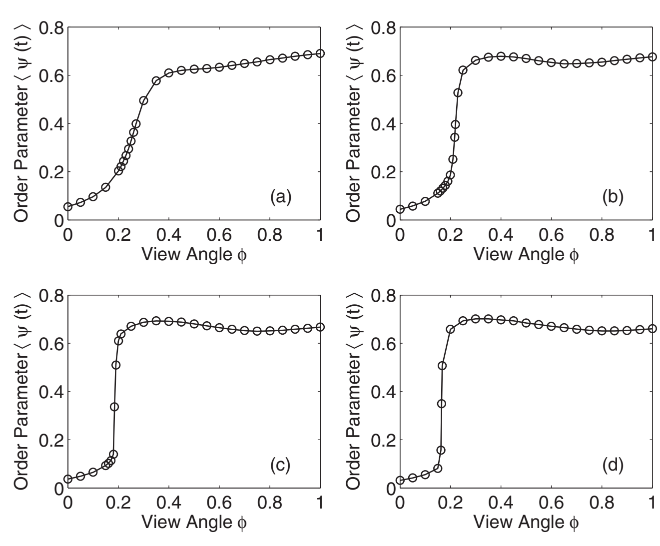

FIG. 2. Order parameter $\langle\psi(t)\rangle$ vs view angle ϕ (in units of π) for system sizes (a) $L = 16$, (b) $L = 20$, (c) $L = 24$, and (d) $L = 28$.

data presented in Figs. 7 and 8 the parameter R is varied from 0 to 1, with all other parameters the same as above (the parameter values for the data in Fig. 9 are described later). Box sizes L in the range of 16 to 36 (i.e., particle numbers N in the range 256 to 1296) were used. We have measured the order parameter $\psi(t)$ as a function of the view angle ϕ, which was varied over the entire range 0 to π. Measurements were averaged over 20 independent realizations, with each realization consisting of 10^5 to 10^7 time steps. In the results we present in the following section we have used the time-averaged order parameter $\langle\psi(t)\rangle$, measured as a function of the view angle ϕ. For satisfactory averages we have to have a time series $\psi(t)$ of a length T much larger than the correlation time for that series. The length of the time series T is effectively the length of a single realization multiplied by the number of independent realizations. Near the transition value of ϕ, the T values are 5.2×10^5, 4.9×10^6, 1.3×10^7, and 1.9×10^8 for system sizes $L = 16$, 20, 24, and 28, respectively. The correlation time τ can be estimated from the autocorrelation function $C(\Delta t)$ for $\psi(t)$, which is defined as

$$C(\Delta t) = \frac{\langle[(\psi(t+\Delta t) - \overline{\psi}][\psi(t) - \overline{\psi}]\rangle}{\sigma_\psi^2}, \tag{5}$$

where $\overline{\psi}$ and σ_ψ^2 are the mean and variance of $\psi(t)$ for a single time series. The angular brackets indicate averaging over all the initial time instants t. The correlation time τ can be estimated by fitting the autocorrelation function $C(\Delta t)$ to the exponential decay function $e^{-\Delta t/\tau}$ (when that is possible), or by using the "integrated correlation time" definition

$$\tau = \sum_{\Delta t=1} C(\Delta t), \tag{6}$$

where the sum is cut off at the first negative value of $C(\Delta t)$. We have used this definition to estimate τ [30]. Near a phase transition the correlation time τ increases rapidly with the system size L, and for a given system size it increases rapidly as the phase transition is approached. We estimate that close to the transition (i.e., for $|\phi - \phi_c| \lesssim 0.01$) $\tau \approx 4 \times 10^2$, 3×10^3, 2.7×10^4, and 5.5×10^5 for $L = 16$, 20, 24, and 28, respectively. Thus we have the total length of the time series T much larger than the correlation time. For the results presented in the following section, $T = 13000\tau$, 1650τ, 510τ, 360τ are for the system sizes $L = 16$, 20, 24, 28, respectively, close to the transition. Farther away from the transition, the T values come out to be much larger multiples of τ.

IV. RESULTS AND DISCUSSION

Tian *et al.* [22] and Li *et al.* [23] have studied one aspect of the question that interests us in the present work. They varied the view angle ϕ and measured the "consensus time," that is, the time taken for the system to achieve the stationary value of the order parameter ψ in the absence and presence of the noise. They made a counterintuitive observation that the consensus time can be shorter for $\phi < \pi$, i.e., restricting the angular range of the particle interactions can speed up the establishment of the ordered collective motion. They found that there is an optimum value for the view angle for which the consensus time is the shortest. A similar observation was made by Wang *et al.* [31], who considered not only variable view angle but also the interaction strengths weighted by the separations between the particle and its neighbors.

With the above-mentioned earlier studies in mind, we were interested in the effect of varying the view angle ϕ on the nature of collective motion of the particles in the modified Vicsek model considered in this study. We have made three important observations: (i) It is not only that the collective motion attains consensus faster with a restricted view angle (as found in the reports [22,23]), but also the stationary value of order parameter can increase with the decreasing view angle. (ii) The ordered state can persist down to quite low view angles for nonzero noise. (iii) As the view angle is reduced, the system undergoes a first-order phase transition from the ordered state to the disordered state.

In Fig. 2 we have shown the time-averaged order parameter $\langle\psi(t)\rangle$ as a function of the view angle ϕ for four different system sizes. (In this and all other plots the symbols are the data and the connecting lines are guides to the eye.) The phase transition is clearly seen for all four system sizes, but there are considerable finite size effects. For the system sizes $L = 24$ and $L = 28$ the transition is quite sharp and recognizably discontinuous to the eye, but further analysis discussed below makes it clear that it is indeed of the first order. It can be noted that as the view angle is decreased, the order parameter varies slowly but nonmonotonically. It at first decreases slightly, then increases to a maximum near the transition, and then drops to low values (not exactly zero, due to the finite size of the system) that characterize a disordered state.

In numerical simulations it is not a simple matter to estimate precisely the transition value of the control parameter and to determine the order of the phase transition, because the finite size effects "round and shift" the transition. A complete treatment of the numerical simulation of a phase transition would require a full-fledged finite size scaling (FSS) analysis [32,33]. This theory of FSS was originally developed for the equilibrium phase transition, but now it is known that much of this analysis is applicable to far-from-equilibrium phase transitions also, such as the ones observed in the Vicsek model [14,34–36]. Here we do not wish to carry out a full finite size scaling analysis for the phase transitions we have observed, because that would require very large system sizes and times, and is quite prohibitive for us at present. Here we only wish to estimate the transition value of the control parameter ϕ with moderate accuracy and establish that the phase transition is of the first order. This can be done using a function of the moments of the order parameter, known as the Binder cumulant, defined by [32]

$$G(\eta, L) = 1 - \frac{\langle\psi^4(t)\rangle}{3\langle\psi^2(t)\rangle^2}, \tag{7}$$

where $\langle\psi^2(t)\rangle$ and $\langle\psi^4(t)\rangle$ are time-averaged second and fourth moments of the order parameter, for given values of η and L (for averaging details please see the previous section). For a second-order phase transition, the Binder cumulant is known to take a value independent of the system size L, and so if the Binder cumulant is plotted as a function of the control parameter for different system sizes all the curves are expected to cross at the critical value of the order parameter. On the other

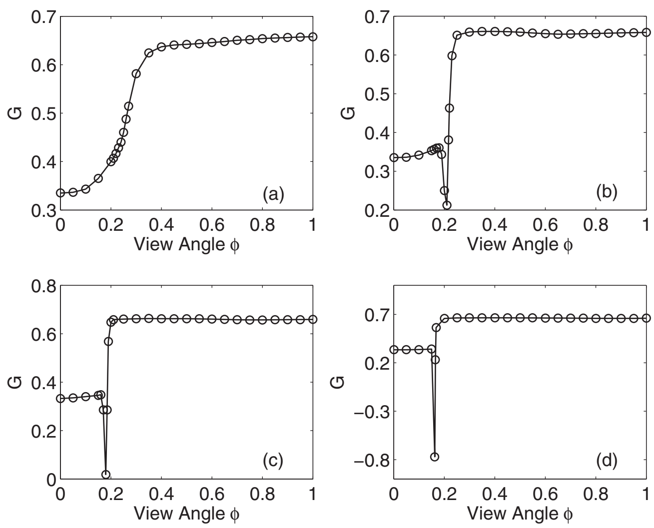

FIG. 3. Binder cumulant G vs view angle ϕ (in units of π) for the system sizes (a) $L = 16$, (b) $L = 20$, (c) $L = 24$, and (d) $L = 28$.

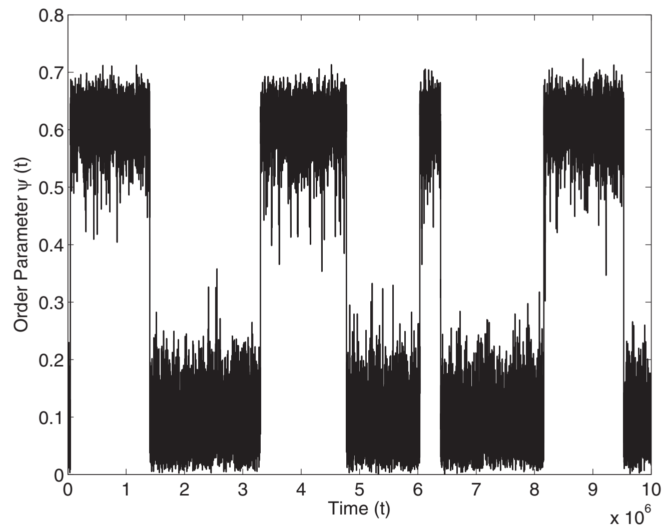

FIG. 5. A section of instantaneous order parameter $\psi(t)$ time series close to the phase transition. The jumps between ordered state [with $\langle\psi(t)\rangle \approx 0.6$] and disordered state [with $\langle\psi(t)\rangle \approx 0.1$] are clearly seen. Here the system size $L = 28$ and the view angle $\phi = 0.165\pi$.

hand, for first-order phase transitions the Binder cumulant dips toward the negative values at the transition point, the dip becoming sharper and deeper as the system size increases. In Fig. 3 we have shown the plots for the Binder cumulant G as a function of the view angle ϕ. For the system size $L = 16$ we do not see any dip, as the finite size effects are too large. But for system sizes $L = 20$, 24, and 28 we have very clear dips. Taking the value ϕ for the minimum of G as the estimated transition value ϕ_c, we get $\phi_c = 0.21\pi$, 0.18π, and 0.1625π for the system sizes $L = 20$, 24, and 28, respectively. Also, we get $G \approx 2/3$ in the ordered phase and $G \approx 1/3$ in the disordered state [14], as expected.

We have also calculated the distribution of the instantaneous order parameter $\psi(t)$ in Fig. 4 close to the estimated transition points. For a first-order phase transition at the transition point both phases coexist, and over a period of time the system fluctuates between the ordered and disordered states. The distribution for the system size $L = 16$ [Fig. 4(a)] is a broad unimodal curve, which means that due to the finite size effects, the distinction between two phases is blurred, consistent with the Binder cumulant plot for the same system size [Fig. 3(a)]. But for the system sizes $L = 20$, 24, and 28 [Figs. 4(b), 4(c), and 4(d), respectively] we have clear bimodal distributions, showing the presence of both the ordered and disordered phases. This is also seen in Fig. 5, which gives a sample of the time series $\psi(t)$ for the system size $L = 28$. Here we clearly see the system abruptly and stochastically switching between the two phases, one with average order parameter $\langle\psi(t)\rangle \approx 0.6$ (the ordered state) and the one with average order parameter $\langle\psi(t)\rangle \approx 0.1$ (the disordered state), which agrees with the distribution peak positions in Fig. 4.

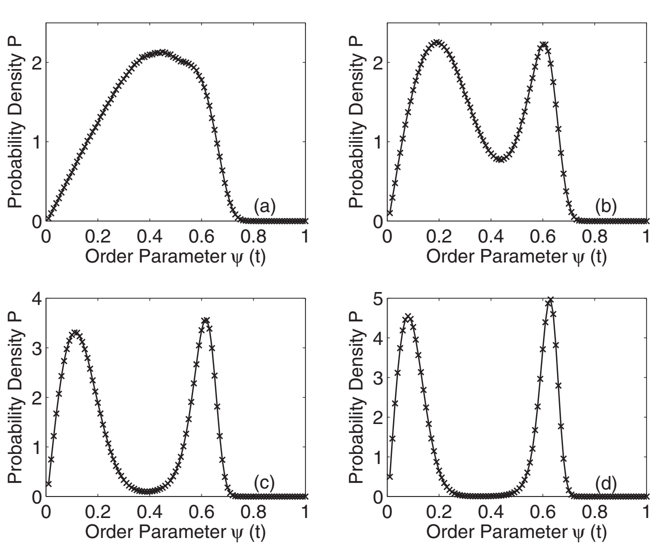

FIG. 4. Probability distribution of the instantaneous order parameter $\psi(t)$ for (a) $L = 16, \phi = 0.270\pi$, (b) $L = 20, \phi = 0.217\pi$, (c) $L = 24, \phi = 0.185\pi$, and (d) $L = 28, \phi = 0.165\pi$.

Another signature of a first-order phase transition is the presence of hysteresis phenomenon near the transition point. If the control parameter is ramped up and down across the transition point at a small, constant ramp rate, the instantaneous order parameter shows hysteresis. Figure 6 shows the hysteresis in the instantaneous order parameter $\psi(t)$ as the view angle ϕ is ramped up and down at small ramp rates in the range of 1.5×10^{-5} to 6.66×10^{-6} radians/unit time. We obtain the clear hysteresis loops for all four system sizes. The loops are centered about the view-angle ϕ values which match the transition ϕ values as estimated from Figs. 2 and 3.

The above results show the effects of the variation of the view angle on the collective motion of the particles. As we reduce the view angle we are in effect doing two things: we are reducing the size of the neighborhood, and we are also introducing an increasing degree of anisotropy or directionality in the interaction of the particle with its neighbors. Therefore it would be of interest to know how the effects would differ if the size of the neighborhood is varied while the interaction remains isotropic. In Fig. 7 we have shown a comparison of

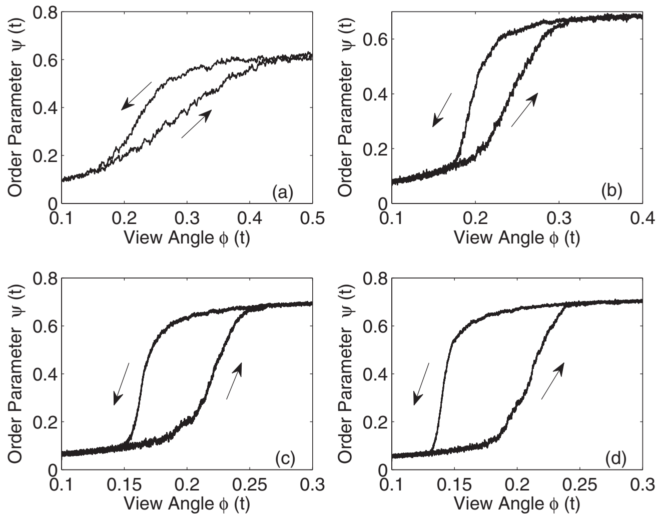

FIG. 6. Hysteresis in the variation of the instantaneous order parameter $\psi(t)$ with the instantaneous view angle $\phi(t)$ (in units of π) for (a) $L = 16$, ramp rate 1.15×10^{-4}, (b) $L = 20$, ramp rate 1.50×10^{-5}, (c) $L = 24$, ramp rate 6.66×10^{-6}, and (d) $L = 28$, ramp rate 6.66×10^{-6}. The ramp rate is in radians/unit time. The arrows indicate ramp-up and ramp-down sections of the hysteresis curves. Each hysteresis loop is obtained by averaging over 100 independent realizations.

the variation of the order parameter $\langle\psi(t)\rangle$ as a function of the size of the neighborhood. The size, as measured by the area of the neighborhood A, is varied in two ways: by varying the view angle ϕ from 0 to π (as radius $R = 1$ is held constant), and by varying the radius R from 0 to 1 (as the view angle $\phi = \pi$ is held constant). The results differ qualitatively. In the first case, as we have already discussed above, we have a first-order phase transition at the transition area $A_c \approx 0.5$ (which

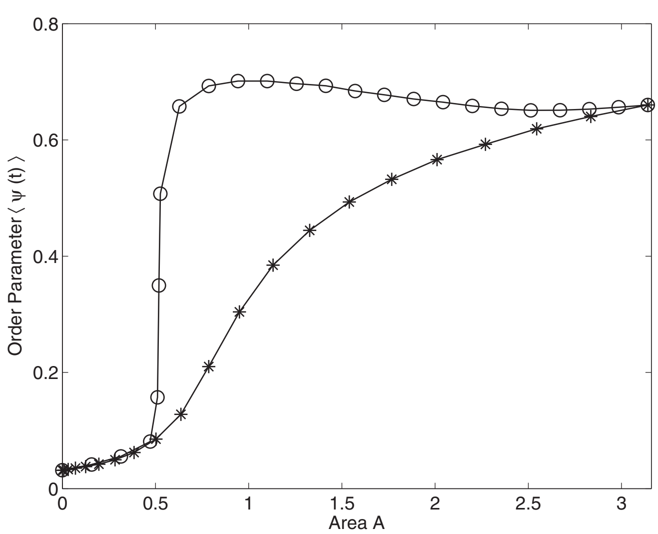

FIG. 7. Order parameter $\langle\psi(t)\rangle$ as a function of the area of the interaction neighborhood A. The symbol circle is for the fixed radius $R = 1$ neighborhood (for which $A = \phi$ numerically) and the symbol asterisk is for the fixed view angle $\phi = \pi$ (for which $A = \pi R^2$). Both curves are for the system size $L = 28$.

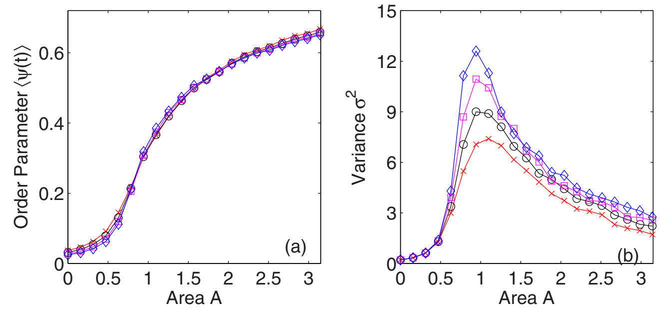

FIG. 8. (a) Order parameter $\langle\psi(t)\rangle$ vs interaction neighborhood area A. (b) Variance $\sigma^2(A,L)$ vs neighborhood area A. The interaction neighborhood area A is varied by varying the radius R, with view angle $\phi = \pi$ fixed, so that $A = \pi R^2$. The symbols cross (red), circle (black), square (magenta), and diamond (blue) respectively correspond to $L = 24$, 28, 32, and 36.

corresponds to $\phi \approx 0.16\pi$) and a nonmonotonic variation of the order parameter with a maximum around $A \approx 1$. In the second case we have a monotonic fall in the order parameter as the area of the neighborhood decreases, with no obvious indication of a phase transition. But in fact there does appear to be a second-order phase transition which is obscured by the finite size effects. This can be seen from the behavior of the variance of the order parameter $\sigma^2(A,L)$ as a function of the neighborhood area A, defined by [14,36]

$$\sigma^2(A,L) = L^2[\langle\psi^2(t)\rangle - \langle\psi(t)\rangle^2]. \quad (8)$$

This variance is expected to diverge at a second-order phase transition. In Fig. 8 we have shown both the order parameter and the variance of the order parameter as functions of the neighborhood area, which is varied by varying the neighborhood radius holding the view angle fixed at the maximum value $\phi = \pi$. In Fig. 8(a), which shows the order parameter as a function of the neighborhood area A for four different system sizes $L = 24$, 28, 32, and 36 the phase transition is not clearly discernible. But in Fig. 8(b), which shows the variance of the order parameter as a function of the neighborhood area, we have a clear peak around $A \approx 1$ which becomes more pronounced as the system size increases (it is expected to diverge as $L \to \infty$). To establish the presence of this phase transition conclusively and to characterize its nature (i.e., second- or first-order transition) one would need to do detailed finite size scaling analysis. Here we only wish to underline the point that the first-order phase transition with the view angle as the control parameter discussed in this work is not only the effect of the variation in the size of the interaction neighborhood, but also the effect of the change in the degree of directionality of the interaction.

It would be interesting to see the effects of the choice of the velocity parameter v_0 on the phase transition induced by the view angle ϕ. To this end we have computed $\langle\psi(t)\rangle$ as a function of the view angle ϕ for a range of velocity parameter v_0 values. These data are presented in Fig. 9. These calculations are done for a moderate system size of $L = 24$, with a relatively short realization of $T = 2 \times 10^5$ time steps to obtain each data point, and therefore are not meant to make any quantitative determinations. Nevertheless, qualitatively

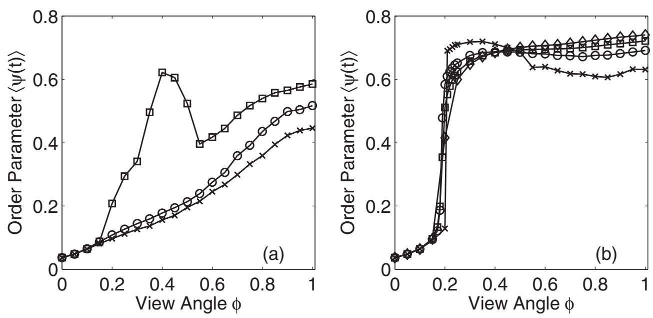

FIG. 9. Order parameter $\langle\psi(t)\rangle$ vs view angle ϕ for various values of velocity parameter v_0 for the system size $L = 24$. Number of realizations = 1, $R = 1.0$. (a) The symbols cross, circle, and square respectively correspond to $v_0 = 0.1$, 0.2, and 0.3. (b) The same symbols correspond to $v_0 = 0.4$, 0.6, 0.8 and diamond corresponds to $v_0 = 1.0$.

we make the following observations: The first-order phase transition induced by the variation of the view angle ϕ is robustly present over the range $0.4 \leqslant v_0 \leqslant 1.0$ (we have not performed calculations for $v_0 > 1.0$). For $v_0 \leqslant 0.3$, there is no obvious sign of the first-order phase transition. More accurate determination of the bounds of the velocity parameter v_0 for which this first-order phase transition exists would require large-scale calculations.

Before concluding, we shall discuss one recent report by Nguyen *et al.* [37], who have studied the same model as we have in this study. In their work they estimate the critical noise η_c as a function of the view angle ϕ. Based on their observations they claim that for $\phi < 0.5\pi$ the critical noise is zero, or in other words the ordered state does not exist for the view angles $\phi < 0.5\pi$ in the presence of the noise (i.e, $\eta > 0$). They discuss the implication of this observation to the presence of the phenomenon of collective motion (which they call flocking) in certain animal species and its absence in others. They draw the conclusion that the prey species, which presumably have view angle $\phi > 0.5\pi$, do display flocking behavior; whereas the predator species with view angle $\phi < 0.5\pi$ do not. This claim would be more substantial if this critical view angle value of 0.5π were robust to some variation in other parameters (velocity v, density ρ, radius of interaction R) because the parameter values they have chosen $\rho = 1.0$, $R = 1$, and $v = 0.1$ have no special physical significance. But as we have seen in the present study, where ρ and R have the same values but $v = 0.5$, the ordered state does persist all the way down to $\phi = 0.2\pi$ (for nonzero noise, i.e, $\eta = 0.3$). In fact, we obtain a higher degree of order at $\phi = 0.35\pi$ than at $\phi = 1.0\pi$.

V. CONCLUSIONS

We have performed a Monte Carlo simulation study of a modified Vicsek model in two dimensions. We have studied the effect of the variation of the view angle on the collective motion of the particles. We have found that as the view angle is reduced, the order in the system varies nonmonotonically; the order parameter at first decreases slowly and then increases, attaining a maximum at a remarkably low view angle, just before the system undergoes a first-order phase transition to a disordered state. The results are qualitatively different when we reduce the radius of the (circular) neighborhood; in this case the order parameter decreases monotonically and goes to zero continuously. And the variance of the order parameter shows a peak for a certain value of the neighborhood radius, suggesting there could be a second-order order-disorder phase transition. Considering the importance of the limited view angle in modeling the motion of real world systems, such as flocks of birds, it would be interesting to study if similar phase transitions arise in other models of self-propelled particles, when the angular range of interparticle interaction in such models is varied.

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