PHYSICAL REVIEW E **93**, 043205 (2016)

Pseudorelativistic laser-semiconductor quantum plasma interactions

Yunliang Wang[1,*] and Bengt Eliasson[2,†]
[1]*Department of Physics, School of Mathematics and Physics, University of Science and Technology Beijing, Beijing 100083, China*
[2]*SUPA, Department of Physics, University of Strathclyde, Glasgow G4 0NG, Scotland, United Kingdom*
(Received 5 September 2015; revised manuscript received 17 March 2016; published 13 April 2016)

A model is presented for the nonlinear interaction between a large-amplitude laser and semiconductor plasma in the semirelativistic quantum regime. The collective behavior of the electrons in the conduction band of a narrow-gap semiconductor is modeled by a Klein-Gordon equation, which is nonlinearly coupled with the electromagnetic (EM) wave through the Maxwell equations. The parametric instabilities involving the stimulated Raman scattering and modulational instabilities are analyzed theoretically and the resulting dispersion relation relation is solved numerically to assess the quantum effects on the instability. The study of the quasi-steady-state solution of the system and direct numerical simulations demonstrate the possibility of the formation of localized EM solitary structures trapped in electrons density holes.

DOI: 10.1103/PhysRevE.93.043205

The magnitude and response time of different types of optical nonlinearities such as solitons in semiconductors is a subject of considerable interest, as the optical solitary waves have important applications in all-optical signal processing [1]. Parametric amplifications of oscillations have been observed in n-doped narrow-gap semiconductors, where the nonlinear optic effects come from the nonparabolicity of the conduction band [2]. The conduction electrons can be reasonably taken as a high-density semiconductor plasma with a fixed neutralizing ionic background. The band structure of InSb leads to a nonparabolic conduction band [3], which gives rise to a momentum-dependent effective mass of the electrons near the bottom of the conduction band, formally resembling that of relativistic electrons [4–6], but with a smaller effective rest mass and with an effective speed of light that is several orders of magnitude smaller than the speed of light in vacuum. Similar properties apply to the two-dimensional gas of massless and massive Dirac fermions in graphene [7,8], where the electrons are modeled by a Dirac equation. Hence, the collective dynamics of conduction electrons with a velocity-dependent mass may simulate some aspects a relativistic quantum plasma. In the past, relativistic effects in gaseous quantum plasmas have been invoked by using collective Klein-Gordon, Dirac, quantum electrodynamic, and quantum fluid equations [9–16] to model the interaction with large-amplitude electrostatic (ES) and electromagnetic (EM) waves. For semiconductor plasmas, the interaction with a large-amplitude laser can give rise to a variety of linear and nonlinear excitations, such as the beat wave generation of plasmons [17] and electron wake field acceleration [18]. The pseudorelativistic velocity-dependent mass increase due to a large-amplitude EM wave can lead to nonlinear effects such as self-focusing of EM radiation [5], parametric instabilities [6], a modulational instability and saturation in the form of a chain of soliton structures [19], multidimensional self-trapping of EM pulses [20], and the self-organization of vortex solitons [21] in narrow-gap semiconductors. The pseudorelativistic and quantum effects on ES nonlinear structures have recently been investigated [22].

*ylwang@ustb.edu.cn
†bengt.eliasson@strath.ac.uk

The quantum effects have attracted much attention due to important applications in modern semiconductor quantum devices, such as spintronics, nanotubes, quantum dots, and quantum wells [23,24]. In semiconductor devices, the quantum effects are significant when the de Broglie wavelength of the charged carriers is comparable to the characteristic spatial scales of the system and the effect of quantum tunneling then has to be taken into account for semiconductor quantum plasmas [25,26]. The formation of solitons has been investigated in semiconductor quantum plasma [27,28], taking into account the wave dispersion due to charge separation and quantum recoil and the nonlinearities coming from the large-amplitude ES potential and from the quantum statistical and exchange-correlation effects.

In this paper we present a model that includes both the pseudorelativistic and quantum tunneling effects for the interaction of intense laser light with semiconductor plasmas. Parametric amplification due to the Raman and modulational instabilities is studied with our model, as well as the possibility of localized nonlinear EM structures in the form of solitons accompanied by a local depletion of the conduction electron densities.

In narrow-gap semiconductors, the conduction electrons have a small effective mass and exhibit a significant degree of nonparabolicity. The dynamics of the conduction band electrons is governed by a pseudorelativistic Hamiltonian in the form [4–6]

$$\mathcal{E} = (\mathbf{p}^2 c_*^2 + m_*^2 c_*^4)^{1/2}, \tag{1}$$

where $c_* = (E_g/2m_*)^{1/2}$ plays the role of the speed of light, m_* is the effective rest mass of an electron at the bottom of the conduction band, and E_g is the width of the energy gap separating the valence and conduction bands. For the semiconductor InSb, the effective speed of light is $c_* \approx 3 \times 10^{-3} c$, where c is the speed of light in vacuum. The Hamiltonian (1) can be used to model electrons on length scales large enough that quantum tunneling effects can be neglected and can then be used to construct a Vlasov equation for the wave dynamics of the Fermi-Dirac distributed electrons [5,6]. To model relativistic and quantum spin and tunneling effects on equal footing would involve using the

Dirac equation for a single electron as a basis to construct a corresponding relativistic quantum kinetic equation via a Wigner transform or some other approach. Such a model would be too complicated to use in practice. We here use two simplifying assumptions: (i) that spin effects are relatively small due to the absence of a strong magnetic field and (ii) kinetic effects are small due to the low temperature of the plasma. The dispersive effects due to the electron degeneracy pressure is also neglected with the assumption of a not too dense plasma; a moderately doped InSb plasma has a density of $\sim 10^{23}$ m^{-3}, while the density of a weakly doped InSb plasma is $\sim 10^{20}$ m^{-3} [19], which are about 5–8 orders of magnitude smaller than typical metallic densities. That spin effects are neglected allows us to use a Klein-Gordon equation (instead of the Dirac equation) for the electrons, which can then be used for a single-particle wave function representing an ensemble of electrons. By the substitutions $\mathcal{E} \to i\hbar\partial/\partial t$ and $\mathbf{p} \to -i\hbar\nabla$ in Eq. (1), where $\hbar$ is the Planck constant divided by 2π, we obtain the Klein-Gordon equation (KGE) for a free conduction electron as

$$\hbar^2 \frac{\partial^2 \psi}{\partial t^2} - \hbar^2 c_*^2 \nabla^2 \psi + m_*^2 c_*^4 \psi = 0, \qquad (2)$$

where ψ is the conduction electron wave function. In order to study the laser beam interaction with a semiconductor quantum plasma, we next let ψ represent an ensemble of conduction electrons and use the charge and current densities as sources for the self-consistent EM scalar and vector potentials ϕ and $\mathbf{A}$, respectively. The EM potentials are introduced into the KGE (2) with the substitutions $i\hbar\partial/\partial t \to i\hbar\partial/\partial t + e\phi$ and $-i\hbar\nabla \to -i\hbar\nabla + e\mathbf{A}$, resulting in

$$\mathcal{W}^2\psi - c_*^2\mathcal{P}^2\psi - m_*^2c_*^4\psi = 0, \qquad (3)$$

where the energy and momentum operators are $\mathcal{W} = i\hbar\partial/\partial t + e\phi$ and $\mathcal{P} = -i\hbar\nabla + e\mathbf{A}$, respectively. Using the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$, the self-consistent vector and scalar potentials are governed by the EM wave equation and Poisson's equation, respectively, as

$$\nabla^2\mathbf{A} - \frac{1}{u^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} - \frac{1}{u^2}\nabla\frac{\partial \phi}{\partial t} = -\mu_0 \mathbf{j}_e \qquad (4)$$

and

$$\nabla^2\phi = -\frac{1}{\varepsilon}(\rho_e + e n_0), \qquad (5)$$

where the electric charge and current densities are obtained as

$$\rho_e = -\frac{e}{2m_* c_*^2}[\psi^*\mathcal{W}\psi + \psi(\mathcal{W}\psi)^*] \qquad (6)$$

and

$$\mathbf{j}_e = -\frac{e}{2m_*}[\psi^*\mathcal{P}\psi + \psi(\mathcal{P}\psi)^*], \qquad (7)$$

respectively. The charge and current densities obey the continuity equation $\partial\rho_e/\partial t + \nabla \cdot \mathbf{j}_e = 0$. Here μ_0 is the magnetic vacuum permeability, ε is the effective dielectric permittivity of the lattice, $-e$ is the electron charge, and n_0 is the unperturbed density of the electrons and of the positive neutralizing ionic background. For the semiconductor InSb, the effective dielectric permittivity is $\varepsilon = 16\varepsilon_0$, where ε_0 is the vacuum electric permittivity and $u = 1/\sqrt{\varepsilon\mu_0}$ is the effective light speed in the semiconductor lattice. Systems similar to (3)–(7) have been used in the past to describe relativistic interactions with electromagnetic waves in gaseous quantum plasmas [9,11,13,14], the difference being that the present model for a semiconductor plasma contains the effective speed of light c_* for the dynamics of the electrons and the effective speed of light u for the propagation of the EM wave in the semiconductor lattice.

We now consider the instability of a pseudorelativistically intense circularly polarized electromagnetic (CPEM) wave propagating in a semiconductor quantum plasma. To investigate the growth rates of the stimulated Raman and modulational instabilities, we linearize our system by introducing $\psi = [\tilde{\psi}_0 + \tilde{\psi}_1(\mathbf{r},t)]\exp(-i\gamma_A m_* c_*^2 t/\hbar)$, $\mathbf{A} = \mathbf{A}_0(\mathbf{r},t) + \mathbf{A}_1(\mathbf{r},t)$, and $\phi(\mathbf{r},t) = \phi_1(\mathbf{r},t)$, where $\gamma_A = \sqrt{1 + e^2A_0^2/m_*^2c_*^2}$ is the pseudorelativistic gamma factor due to the quivering motion of an electron in the laser field, A_0 is the amplitude of the CPEM carrier wave $\mathbf{A}_0$, and $\tilde{\psi}_0$ is assumed to be constant. The equilibrium quasineutrality requires that $\tilde{\psi}_0$ is normalized such that $|\tilde{\psi}_0|^2 = n_0/\gamma_A$ [14]. We now introduce the Fourier representations $\tilde{\psi}_1 = \hat{\psi}_+ \exp(-i\Omega t + i\mathbf{K}\cdot\mathbf{r}) + \hat{\psi}_- \exp(i\Omega t - i\mathbf{K}\cdot\mathbf{r})$, $\phi_1 = \hat{\phi}\exp(-i\Omega t + i\mathbf{K}\cdot\mathbf{r}) + \text{c.c.}$, $\mathbf{A}_0 = (1/2)\hat{\mathbf{A}}_0\exp(-i\omega_0 t + i\mathbf{k}_0\cdot\mathbf{r}) + \text{c.c.}$, and $\mathbf{A}_1 = [\hat{\mathbf{A}}_+\exp(-i\omega_+ t + i\mathbf{k}_+\cdot\mathbf{r}) + \hat{\mathbf{A}}_-\exp(-i\omega_- t + i\mathbf{k}_-\cdot\mathbf{r})] + \text{c.c.}$, where $\omega_\pm = \omega_0 \pm \Omega$, $\mathbf{k}_\pm = \mathbf{k}_0 \pm \mathbf{K}$, and c.c. stands for complex conjugate. Separating different Fourier modes and eliminating the Fourier coefficients, we find the nonlinear dispersion relation

$$1 + \frac{1}{\tilde{\chi}_e} = \frac{c_*^2K^2 - \Omega^2 + \omega_{pe}^2/\gamma_A}{4\gamma_A^2 m_*^2 c_*^4 - \hbar^2(\Omega^2 - c_*^2K^2)} \times \left[\frac{c_*^2e^2|\mathbf{k}_+ \times \hat{\mathbf{A}}_0|^2}{k_+^2 D_A(\omega_+,\mathbf{k}_+)} + \frac{c_*^2e^2|\mathbf{k}_- \times \hat{\mathbf{A}}_0|^2}{k_-^2 D_A(\omega_-,\mathbf{k}_-)}\right], \qquad (8)$$

where $\omega_{pe} = \sqrt{n_0e^2/m_*\varepsilon}$ is the effective quantum semiconductor plasma frequency and the EM sidebands are governed by $D_A(\omega_\pm,\mathbf{k}_\pm) = u^2k_\pm^2 - \omega_\pm^2 + \omega_{pe}^2/\gamma_A$. The carrier wave $\mathbf{A}_0$ obeys the nonlinear dispersion relation (see Ref. [14]) $\omega_0 = \sqrt{u^2k_0^2 + \omega_{pe}^2/\gamma_A}$. The electric susceptibility in the presence of the laser field is given by

$$\tilde{\chi}_e(\Omega,K) = \frac{\omega_{pe}^2\left[4\gamma_A^2 m_*^2 c_*^4 - \hbar^2(\Omega^2 - c_*^2K^2)\right]}{\gamma_A\left[\hbar^2(\Omega^2 - c_*^2K^2)^2 - 4\gamma_A^2 m_*^2 c_*^4 \Omega^2\right]}. \qquad (9)$$

In the absence of EM waves ($\mathbf{A}_0 = 0$), the dispersion relation $1 + \tilde{\chi}_e = 0$ supports ES electron plasma waves and pair branches in quantum plasmas [11,14] and semiconductor quantum plasmas [22]. In the limits $c_* \to c$ and $u \to c$, the dispersion relation (8) governs the relativistic parametric interaction between an EM wave and a gaseous quantum plasma [14].

It should be noted that for frequencies $\omega_0/\omega_{pe} > 2/\sqrt{\gamma_A}$ and the corresponding wave numbers $k_0u/\omega_{pe} > \sqrt{3/\gamma_A}$, the dispersion relation (8) governs the three-wave resonant Raman scattering instability, in which the laser decays into a frequency-downshifted EM sideband and a plasma oscillation. For this instability, $|D_A(\omega_-,\mathbf{k}_-)| \ll |D_A(\omega_+,\mathbf{k}_+)|$, so that the term proportional to $1/D_A(\omega_+,\mathbf{k}_+)$ can usually be neglected. The resonance condition for the maximum growth rate is then

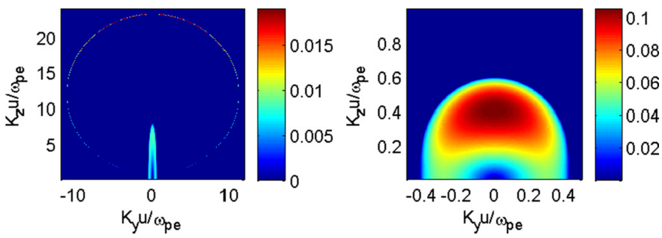

FIG. 1. Normalized growth rate Ω_I/ω_{pe} of the stimulated Raman scattering (left column) and modulational instability (right column) in the nonlinear interaction of a pseudorelativistic intense CPEM wave with an InSb semiconductor quantum plasma.

approximately given by $1+\chi_e(\Omega,K)\approx 0$ and $D_A(\omega_-,\mathbf{k}_-)\approx 0$. For very short wavelengths of the laser, the quantum tunneling (quantum recoil) effects become important for the plasma oscillations. On the other hand, when $\omega_0/\omega_{pe}\lesssim 2/\sqrt{\gamma_A}$ (corresponding to $k_0u/\omega_{pe}\lesssim\sqrt{3/\gamma_A}$), the dispersion relation (8) supports the modulational instability [19], which is a four-wave coupling where the laser decays into two neighboring EM sidebands and a nonresonant plasma oscillation. In this case, $|D_A(\omega_-,\mathbf{k}_-)|$ and $|D_A(\omega_+,\mathbf{k}_+)|$ are of the same order and have to be kept in the dispersion relation. The modulational instability typically saturates by the formation of solitary wave structures.

To proceed with the numerical evaluation of the nonlinear dispersion relation, we choose a coordinate system such that the right-hand CPEM wave takes the form $\hat{\mathbf{A}}_0=(\hat{\mathbf{x}}+i\hat{\mathbf{y}})\hat{A}_0$ and the CPEM wave vector is $\mathbf{k}_0=k_0\hat{\mathbf{z}}$, where $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ are units vectors in the x, y, and z directions. Without loss of generality, we choose a coordinate system such that the perturbation wave vector for the plasma density oscillations can be written $\mathbf{K}=K_z\hat{\mathbf{z}}+K_y\hat{\mathbf{y}}$. We now assume that the wave frequency is complex valued $\Omega=\Omega_R+i\Omega_I$, where Ω_R is the real frequency and Ω_I the growth rate, and solve numerically the dispersion relation (8) for Ω. We consider typical parameters for semiconductor InSb plasma as [19] $m_*=m_e/74$, $\varepsilon=16\varepsilon_0$, $c_*=c/253$, and $n_0=10^{20}$ m^{-3}. The amplitude of the CPEM wave is $e|\hat{A}_0|/m_*c_*=1$. The dimensionless quantum parameter $H=\hbar\omega_{pe}/m_*c_*^2=0.0074$ obtained from the InSb semiconductor parameters is a measure of the importance of quantum effects in the system. On the left-hand side of Fig. 1, we have plotted the growth rate as a function of the wave numbers K_z and K_y for $k_0u/\omega_{pe}=12$, which corresponds to a wavelength of 3.23×10^{-5} m for $H=0.0074$, which is in the THz regime. In Fig. 1, the stimulated Raman instability takes place in a narrow circular region for large wave numbers where the resonance condition of the instability is fulfilled. The modulational instability is visible for small values of $|K_yu/\omega_{pe}|\lesssim 1$ and $|K_yu/\omega_{pe}|\lesssim 7$. On the right-hand side of Fig. 1, we investigate the instability of a CPEM dipole wave field with $k_0=0$, where the modulational instability dominates. The modulational instability has a positive growth-rate for relatively small wave numbers with $|K_yu/\omega_{pe}|\lesssim 0.4$ and $|K_yu/\omega_{pe}|\lesssim 0.6$. For moderately doped samples, the quantum parameter H becomes relatively large. This leads to a reduction of the growth rate of the Raman instability, while the quantum diffraction effects have little influence on the modulational instability at small wave numbers.

We have used a collisionless model for the electrons, even though the collision time for electrons in semiconductors is comparatively short [19]. The collisionality depends on the level of doping, the quality of the sample, and the temperature of the semiconductor plasma. The mobility of the electrons in InSb plasma is high, but due to their small effective masses, the measured mean collision time due to collisions with charged particles is in the range $\tau_0=10^{-11}$–10^{-13} s in a InSb plasma at a temperature of $T_e=77$ K. However, the high laser field significantly increases the collision time and drives the plasma towards a collisionless state. The relaxation time scales as $\tau_r=\tau_0(E/E_0)^{3/2}$ [29,30], where $E_0\approx 3k_BT_e/2\approx 1.6\times10^{-21}$ is the average kinetic energy of the unperturbed state and $E\approx m_*c_*^2(\gamma_A-1)\approx 7\times10^{-21}$ is the average kinetic energy in the presence of the field. Hence, we get $\tau_r\approx 10\tau_0=10^{-10}$–$10^{-12}$ s. The Raman instability (see the left panel of Fig. 1) develops on a time scale of $\tau_i=\omega_I^{-1}\approx 50\omega_{pe}^{-1}\approx 4\times10^{-11}$ s, while the modulational instability develops on $\tau_i\approx 10\omega_{pe}^{-1}\approx 10^{-11}$ s. Hence, in an InSb plasma in the lower range of collisionality $\tau_0\sim 10^{-11}$, the collisionless assumption could remain justified, while at higher collisionality such as $\tau_0\sim10^{-13}$, the growth rate of the instability could be decreased or the instability could be quenched by collisions.

We next investigate the possibility of localized CPEM wave excitations in semiconductor quantum plasma. It is convenient to first introduce a new wave function $\Psi(z,t)$ via the transformation $\psi(\mathbf{r},t)=\Psi(z,t)\exp(-im_*c_*^2t/\hbar)$. To study quasi-steady-state structures propagating with a constant speed v_0, we assume that $\phi=\phi(\xi)$ and $A^2=A^2(\xi)$, where $\xi=z-v_0t$ and $A^2=|\mathbf{A}|^2$. The CPEM wave vector potential is of the form $\mathbf{A}=A(\xi)[\hat{\mathbf{x}}\cos(k_0z-\omega_0t)-\hat{\mathbf{y}}\sin(k_0z-\omega_0t)]$. It is convenient to introduce the eikonal representation $\Psi=P(\xi)\exp[i\theta(\xi)]$, where P and θ are real valued. The boundary conditions for the localized structures are $A=0$, $P=\sqrt{n_0}$, $\phi=0$, and $d/d\xi=d^2/d\xi^2=0$ at $|\xi|=\infty$. Then, using a formalism similar to that in Ref. [14], with $k_0=\omega_0v_0/u^2$, we obtain the phase from $d\theta/d\xi=(v_0m_*\gamma_2^2/\hbar)[(e\phi/m_*c_*^2+1)-n_0/P^2]$ and the coupled system of equations

$$\frac{d^2A}{d\xi^2}+\frac{\omega_{pe}^2}{u^2}\left[\lambda+\gamma_1^2\left(1-\frac{P^2}{n_0}\right)\right]A=0, \tag{10}$$

$$\frac{d^2P}{d\xi^2}+\frac{m_*^2c_*^2\gamma_2^4}{\hbar^2}\left[\left(\frac{e\phi}{m_*c_*^2}+1\right)^2-\frac{v_0^2}{c_*^2}\frac{n_0^2}{P^4}-\frac{\gamma_A^2}{\gamma_2^2}\right]P=0, \tag{11}$$

$$\frac{d^2\phi}{d\xi^2}=\frac{en_0\gamma_2^2}{\varepsilon}\left[\left(\frac{e\phi}{m_*c_*^2}+1\right)\frac{P^2}{n_0}-1\right], \tag{12}$$

where $\lambda=\omega_0^2/\omega_{pe}^2-\gamma_1^2$ is a nonlinear eigenvalue and $\gamma_1=1/\sqrt{1-v_0^2/u^2}$, $\gamma_2=1/\sqrt{1-v_0^2/c_*^2}$, and $\gamma_A=\sqrt{1+e^2A^2/m_*^2c_*^2}$ are pseudorelativistic gamma factors. The coupled system (10)–(12) describes the profile of EM solitary waves in a quantum semiconductor plasma. We solved the system (10)–(12) as a nonlinear boundary value problem with the boundary conditions $eA/m_*c_*=e\phi/m_*c_*^2=0$ and $P/\sqrt{n_0}=1$ at the boundaries $\xi\omega_{pe}/u=\pm20$. The spatial domain is numerically resolved with 4000 intervals and the

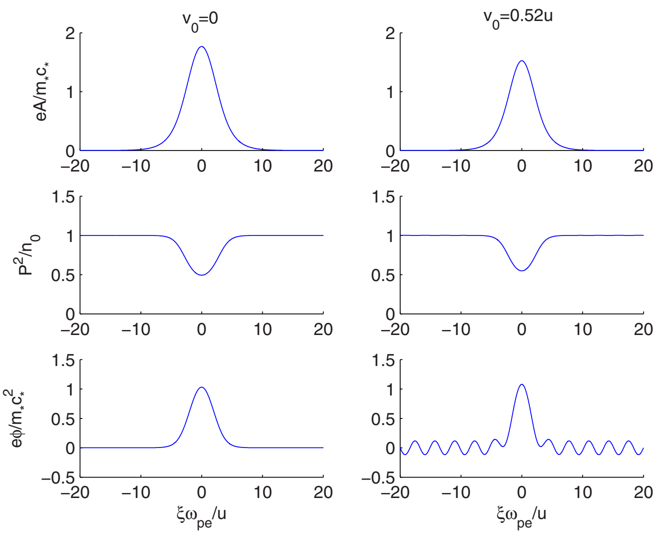

FIG. 2. Spatial profiles of the vector potential, the conduction electron density, and the scalar potential (top to bottom panels) for $v_0 = 0$ (left column) and $v_0 = 0.52u$ (right column) with the frequency shift $\lambda = -4$.

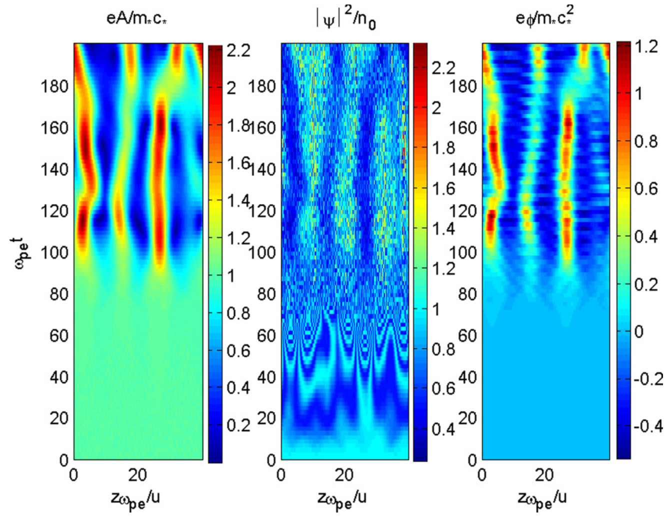

FIG. 3. Dynamics of the normalized CPEM vector potential eA/m_*c_* (left), conduction electron density $|\psi|^2/n_0$ (middle), and scalar potential $e\phi/m_*c_*^2$ (right) in an InSb semiconductor quantum plasma.

second derivatives in the system (10)–(12) are approximated by centered second-order approximations. The resulting nonlinear system of equations is then solved numerically by Newton's method. We choose the same parameters as in Fig. 1. The numerical solutions are displayed in Fig. 2. As shown in Fig. 2, the local depletions of the conduction electron density P are associated with a single maximum of the localized vector potential A and a localized positive potential ϕ for $v_0 = 0$. For $v_0 = 0.52u$, we instead have the plasma wake oscillations as illustrated by scale potential $e\phi/m_*c_*^2$ in right-hand column of Fig. 2. Hence, an EM pulse creates an oscillatory wake that extends far away from the propagating soliton. In classical InSb semiconductor plasmas, the wake field also forms when the laser pulse length is comparable to the inverse of the plasma frequency [18].

In order to study the dynamics of the localized CPEM packets, we have carried out numerical simulations of the KGE-Maxwell system of equations (3)–(5) in the slow varying envelope approximation limit. We have here restricted our study to one space dimension, along the z direction, and have written our governing equations in the form

$$2i\omega_0\left(\frac{\partial A}{\partial t} + v_g\frac{\partial A}{\partial z}\right) + u^2\frac{\partial^2 A}{\partial z^2} + \omega_{pe}^2\left(1 - \frac{P^2}{n_0}\right)A = 0,$$

$$\left(i\hbar\frac{\partial}{\partial t} + e\phi + m_*c_*^2\right)\tilde{\psi} = W,$$

$$\left(i\hbar\frac{\partial}{\partial t} + e\phi + m_*c_*^2\right)W + \hbar^2c_*^2\frac{\partial^2\tilde{\psi}}{\partial z^2} - \gamma_A^2 m_*^2c_*^4\tilde{\psi} = 0,$$

$$\frac{\partial^2\phi}{\partial z^2} = \frac{e}{2m_*c_*^2\varepsilon}(\psi^*W + \psi W^*) - \frac{en_0}{\varepsilon}, \qquad (13)$$

where we have used the transformation $\psi = \tilde{\psi}\exp(-im_*c_*^2t/\hbar)$. The group velocity is $v_g = k_0u^2/\omega_0$. We use a pseudospectral method for calculating the spatial derivatives and the standard fourth-order Runge-Kutta method to advance the solution in time. The spatial domain is from $z\omega_{pe}/u = 0$ to $z\omega_{pe}/u = 40$ with 2048 intervals in space, with periodic boundary conditions. We carry out the simulation from time $\omega_{pe}t = 0$ to $\omega_{pe}t = 200$ using the time step $\omega_{pe}\Delta t = 2\times 10^{-6}$. The initial conditions are $A = m_*c_*/e$, $\tilde{\psi} = \sqrt{n_0}$, $W = 0$, and $\phi = 0$. Small-amplitude noise (random numbers) of order $10^{-2}m_*c_*/e$ is added to A to give a seed for any instability. We use the same semiconductor parameters as in Figs. 1 and 2, with $k_0 = 0$ so that A initially represents an oscillating dipole field.

The numerical results are shown in Fig. 3. We found that the instability grows and saturates at time $\omega_0 t \approx 100$. The CPEM waves collapse and form localized solitary wave structures, where the maxima of the vector A is accompanied by depletions of the conduction electron density and maxima of the scalar potential. Small-scale spatial oscillations are excited in the electron density due to quantum diffraction effects. The scalar potential also has high-frequency oscillations in time near the plasma frequency as illustrated in the right-hand column of Fig. 3.

In summary, we have developed a model for the interaction between intense EM waves and a pseudorelativistic semiconductor quantum plasma. Our nonlinear model is based on the coupled Klein-Gordon and Maxwell equations for the relativisticlike electron dynamics and the EM fields. A nonlinear dispersion relation is derived for the growth rates of the Raman scattering and modulational instabilities in the presence of pseudorelativistically intense CPEM waves. In the nonlinear regime, we have demonstrated theoretically and by simulations the localization and collapse of large-amplitude CPEM waves into solitary EM wave packets in semiconductor quantum plasmas.

This research was partially supported by NSFC (Grant No. 11104012) and the Fundamental Research Funds for the Central Universities (Grants No. FRF-TP-09-019A and No. FRF-BR-11-031B).

[1] D. Mihalache, D. Mazilu, F. Lederer, Y. V. Kartashov, L. C. Crasovan, L. Torner, and B. A. Malomed, Phys. Rev. Lett. **97**, 073904 (2006).

[2] C. K. N. Patel, R. E. Slusher, and P. A. Fleury, Phys. Rev. Lett. **17**, 1011 (1966).

[3] E. O. Kane, J. Phys. Chem. Solids **1**, 249 (1957).

[4] P. A. Wolff and G. A. Pearson, Phys. Rev. Lett. **17**, 1015 (1966).

[5] N. Tzoar and J. I. Gersten, Phys. Rev. Lett. **26**, 1634 (1971).

[6] J. I. Gersten and N. Tzoar, Phys. Rev. Lett. **27**, 1650 (1971).

[7] K. S. Novoselov *et al.*, Nature (London) **438**, 197 (2005).

[8] S. Ulstrup *et al.*, Phys. Rev. Lett. **112**, 257401 (2014).

[9] T. Takabayasi, Prog. Theor. Phys. **9**, 187 (1953).

[10] T. Takabayasi, Prog. Theor. Phys. Suppl. **4**, 1 (1957).

[11] V. Kowalenko, N. E. Frankel, and K. C. Hines, Phys. Rep. **126**, 109 (1985).

[12] P. K. Shukla and L. Stenflo, Phys. Plasmas **13**, 044505 (2006).

[13] M. V. Kuzelev, JETP **112**, 333 (2011).

[14] B. Eliasson and P. K. Shukla, Phys. Rev. E **83**, 046407 (2011).

[15] B. Eliasson and P. K. Shukla, Phys. Rev. E **84**, 036401 (2011).

[16] F. A. Asenjo, V. Muñoz, J. A. Valdivia, and S. M. Mahajan, Phys. Plasmas **18**, 012107 (2011).

[17] V. I. Berezhiani and S. M. Mahajan, Phys. Rev. B **55**, 9247 (1997).

[18] V. I. Berezhiani and S. M. Mahajan, Phys. Rev. Lett. **73**, 1837 (1994).

[19] V. I. Berezhiani, V. Skarka, and R. Miklaszewski, Phys. Rev. B **57**, 6251 (1998).

[20] V. Skarka, V. I. Berezhiani, and V. Boucher, Phys. Rev. B **61**, 10201 (2000).

[21] V. Skarka, N. B. Aleksic, M. Derbazi, and V. I. Berezhiani, Phys. Rev. B **81**, 035202 (2010).

[22] Y. Wang, X. Wang, and X. Jiang, Phys. Rev. E **91**, 043108 (2015).

[23] G. Manfredi and P.-A. Hervieux, Appl. Phys. Lett. **91**, 061108 (2007).

[24] F. Haas, G. Manfredi, P. K. Shukla, and P.-A. Hervieux, Phys. Rev. B **80**, 073301 (2009).

[25] P. K. Shukla and B. Eliasson, Rev. Mod. Phys. **83**, 885 (2011).

[26] M. E. Yahia, I. M. Azzouz, and W. M. Moslem, Appl. Phys. Lett. **103**, 082105 (2013).

[27] W. M. Moslem, I. Zeba, and P. K. Shukla, Appl. Phys. Lett. **101**, 032106 (2012).

[28] Y. Wang and B. Eliasson, Phys. Rev. B **89**, 205316 (2014).

[29] A. Miller, in *Nonlinear Optics in Signal Processing* (Chapman and Hall, London, 1993).

[30] A. A. Borshch, M. Brodin, and V. Volkov, *International Handbook: Laser Science and Technology* (Harwood Academic, Chur, 1990).