Fostering cooperation of selfish agents through public goods in relation to the loners

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Altruistic behaviors in multiplayer groups have obtained great attention in the context of the public goods game, which poses a riddle from the evolutionary viewpoint. Here we focus on a particular type of public goods game model in which the benefits of cooperation are either discounted or synergistically enhanced at the appearance of multiple cooperators in a group. Moreover, we focus on the three-strategies profile by adding the role of loners, besides the often-used cooperation and defection. Using the replicator dynamic equations, we investigate a range of dynamical portraits that characterizes the properties of the steady state. Analysis results indicate that loners and cooperators both have chances to be the stable equilibrium points in the presence of perturbations, while defectors fail to do so in this three-strategy competition. Moreover, the coexistence state, in which all three strategies exist in equilibrium, can be led by suitable parameters and stabilized for perturbations. These results elucidate the interplay between the characteristics of the public goods game and evolutionary dynamics in well-mixed systems.

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I. INTRODUCTION

Public goods and common-pool resources are fundamental features of biological and social systems, and pose important challenges in achieving sustainability. Such societal problems, where the private interest may be at odds with the collective interest constitute an important class of social dilemma. To investigate this, game theory provides a quantitative framework for analyzing the decision of rational agents [1–3]. Furthermore, evolutionary game theory popularly provides an interdisciplinary mathematical framework embodying several relevant features of the problem and, as such, is used in much cooperation-oriented research [4–6].

The often-used game models for elucidating cooperative dilemmas are the two-player prisoner's dilemma game for pairwise interactions, as well as the multiplayer public goods games (PGGs) in larger groups of interacting individuals, which are both typical paradigms for investigating the emergence of cooperation in spite of the fact that self-interest seems to dictate defective behavior [7,8]. Generally, these social dilemmas can be treated as binary situations in which two strategies are optional: either choose cooperation (C) in order to serve the public interest, or choose defection (D), which serves the immediate private interest. In a typical PGG, players can voluntarily make a contribution to a common pool, knowing beforehand that the collected goods will be multiplied and then allocated evenly among all the players regardless of their actual contributions. Herein, cooperators are defined as altruistic individuals that offer contributions or benefits to the group, while at some cost to themselves concurrently. Instead, defectors act as freeloaders to exploit the collective goods produced by altruistic behaviors, without any contribution. This model hypothesis implies that the optimal outcome for a player is to contribute nothing (i.e., defect) while others contribute to the common pool (i.e., cooperate). Hence a benefit conflict between the individual and the group occurs in such kind of situations.

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It has been known that the general public goods game can be viewed as one of the most frequently used models in describing "the puzzle of common goods." However, there are still some collective dilemmas in real life for which other models would be more appropriate. For example, in natural systems, the actual value of the benefits provided by cooperators may depend on the number of cooperators in the group. As referred to in a previous study [9], in the case of foraging yeast cells, the benefit provided by the first cooperator may be of the greatest importance for the survival, whereas the value of additional food decreases until, eventually, more food turns out to be useless for the saturated cells. This leads to discounted or synergistically enhanced benefits based on the number of cooperators in groups of interacting individuals [10]. This kind of public goods dilemma is fascinating and significant, and is a key supplement to the modeling of the collective dilemma in real social society, though currently getting less attention than the general public goods game model. In this study we will try to understand how steady-state strategies emerge in the framework of this game, and to identify the characteristic features of steady-state strategies.

In Recent years scores of mechanisms have been proposed, which aim to enhance the evolution of cooperation among selfish and unrelated individuals. Kin selection operates when the donor and the recipient of an altruistic behavior are genetic relatives, and is deemed as the key to altruism [11–14]. Direct reciprocity needs repeated encounters between the same two individuals [15,16]. Indirect reciprocity is based on reputation; a helpful individual gets a better reputation and an individual with a good reputation is more likely to receive help [17–22]. Network reciprocity is a reciprocity based on some type of spatial or other clustering [23–25]. In addition, a number of more recent studies have focused on the coevolution of strategy and population structure, which also plays a crucial role in the resolution of cooperative dilemma [26–30].

Literature on competitive altruism has also revealed that strategy choices provide an important means to explain the sustained levels of cooperation within the context of social dilemmas. Examples include loners [31], punishers [32], insured players [33], or destructive agents [34,35]. Recent work has uncovered a remarkable class of extortion strategies that afford one player a disproportionate payoff when facing an unwitting opponent [36,37]. Moreover, a strong body of empirical evidence points to the widespread behaviors of voluntary participation [38–42]. Among these previous studies, some have supported a noticeable effect of an attractive exit option on contribution levels, and voluntary participation can induce a recovery of cooperation levels when the payoff yielded by the exit option is high enough. Briefly, the mentioned strategies providing rich types of options and describing the individual heterogeneity, have been verified to play important roles in the dynamics of evolutionary games [43].

Here we establish our analysis about strategy competition in the framework of the PGGs played by cooperators, defectors, and loners, and in which the benefits of cooperation are either discounted or synergistically enhanced. To our knowledge, previous investigations paid little attention to the resolution of the collective dilemma illustrated by this type of public goods game. In this study we adopt the replicator dynamics and hope to find some situations in which dilemma can be resolved and cooperation can be enhanced. Moreover, a voluntary participation case can better represent a conservative attitude and the attempts to keep fixed values in complicated real-life situations. In order to test for the direct effect of voluntary participation on the contribution levels in this particular model, we thus adopt these available strategies. Our simple model may be applied to many social systems, where strategic interactions occur all around us in a multitude of forms among agents.

The paper is structured in the following way: In the next section we give a description about the model. In Sec. III we study analytically the case when the game is a PGG played by cooperators, defectors, and loners, and in which the benefits of cooperation are either discounted or synergistically enhanced, along with a detailed discussion of the results. Finally, in Sec. VI we summarize our findings and conclude.

II. MODEL DESCRIPTION

The present model consists of a population of N agents; all the members of this sample can make a decision to play the game or not. Each player has three possible actions available to her: to cooperate, defect, or to be a loner. Loners do not take part in the game, but obtain an autarkic benefit independent of the other player's strategy and the game result. Notably a special case will probably arise: if only one player intends to play, but all others refuse, then this single player is forced to play the role of a loner by only obtaining the autarkic payoff σ. Donation therefore represents the altruistic behavior because the cooperator incurs an individual cost c to benefit the group. As outlined in detail below, each agent receives a payoff corresponding to the choices made by them.

As argued earlier, the first cooperator in a group makes her contribution c which will produce a benefit rc to the group. And then this benefit is shared by all the participants in the group (including herself), and each participant gets rc/N_p. Here r denotes the amplification effect on the collected common goods, and N_p is the number of participants in this group. Then, the second one enhances each participator's benefit by providing an income of $rc\omega/N_p$. By that analogy, the last of k cooperators in the group will provide a benefit of $rc\omega^{k-1}/N_p$ to all the participants. The special case of $\omega = 1$, where all cooperators furnish the group member with the same incremental benefit rc/N_p, naturally transforms the model to the general formulation of the PGG.

Then, it is accessible that in the case of $\omega < 1$, the benefits are discounted and the value of the benefits provided by each additional cooperator is lower than her previous one. While in the setting of $\omega > 1$, the benefits are synergistically enlarged, and each additional cooperator brings incremental benefits of increasing value.

For the analysis we have assumed a group of size N, which is composed of N_c cooperators, N_d defectors, and N_l loners. With P_c, P_d, and P_l, we denote the average payoff for cooperators, defectors, and loners, respectively, as follows:

$$P_c = \frac{r(1+\omega+\omega^2+\cdots+\omega^{N_c-1})c}{N_c+N_d} - c,$$
$$P_d = \frac{r(1+\omega+\omega^2+\cdots+\omega^{N_c-1})c}{N_c+N_d},$$
$$P_l = \sigma. \tag{1}$$

The parameter c is the cooperative cost paid by a cooperator. For simplicity and without loss of generality, we set the cooperative cost $c = 1$ in the following parts. As usual, defectors avoid contributions and exploit other players' contributions. And, the loners do not make any contribution and only sparingly keep the fixed payoff σ, which will not be affected by the game results.

III. GENERAL ANALYSIS

As mentioned we consider a sample of N players randomly selected from the population. Thus, the probability that two players in a sufficiently large population ever encounter each other again can be neglected. And, it is clear that such random sampling leads to groups whose composition follows a binomial distribution. We now focus on the strategy evolution dynamics of the groups composed by these three types of players. Based on the replicator dynamics, which is a useful approach, here a strategy's payoff determines the growth rate of its fraction in the population [44]. In the continuous time model, the evolution of the frequency f_i of the strategy i acts up to the reduced differential equation

$$\dot{f_i} = f_i(P_i - \bar{P}), \tag{2}$$

where $i \in (C,D,L)$, and $\bar{P} = f_c P_c + f_d P_d + f_l P_l$ is the average payoff of a player in the population. Here we employ x, y, and z to denote the fractions of cooperators, defectors, and loners, respectively, in this large population.

Next, similar to [31], the probability that there are $S-1$ coplayers (not including the loners) in the sample group in which a given player finds herself, is determined by the probability of

$$\binom{N-1}{S-1}(1-z)^{S-1}z^{N-S}, \tag{3}$$

which is independent of whether the agent is a cooperator, defector, or loner.

Then, the probability that there are m cooperators and $S-1-m$ defectors in the current group where a given player finds herself, is determined by

$$\binom{S-1}{m}\left(\frac{x}{x+y}\right)^m\left(\frac{y}{x+y}\right)^{S-1-m}. \tag{4}$$

According to Eq. (1), the expected payoff for a defector in such a group of size S ($S=2,\ldots,N$) is

$$\begin{aligned}P_{ds} &= \sum_{m=0}^{S-1}\frac{r(1-\omega^m)}{S(1-\omega)}\binom{S-1}{m}\left(\frac{x}{x+y}\right)^m\left(\frac{y}{x+y}\right)^{S-1-m}\\ &= \frac{r}{S(1-\omega)}\left[1-\left(\frac{\omega x}{x+y}+\frac{y}{x+y}\right)^{S-1}\right],\end{aligned} \tag{5}$$

where $\omega \neq 1$, and the summation runs over all the possible numbers of cooperators belonging to the group of size S, which includes the focal defector. As argued earlier, the term $\frac{r(1-\omega^m)}{S(1-\omega)}$ represents the payoff related to the value of S, and the cooperation cost is set as $c=1$ as mentioned above. In this case, the probability that there are m cooperators and $S-1-m$ defectors in the group where a given defector finds herself is given by Eq. (1).

Thus, the average payoff of a defector in the whole population is

$$\begin{aligned}P_{dw} = \sigma z^{N-1} + \sum_{S=2}^{N}\binom{N-1}{S-1}(1-z)^{S-1}z^{N-S}\\ \times\frac{r}{S(1-\omega)}\left[1-\left(\frac{\omega x+y}{1-z}\right)^{S-1}\right].\end{aligned} \tag{6}$$

As already mentioned, loners make no contribution and get a fixed payoff σ irrespective of the game results. In this case, a defector facing $N-1$ loners in the population is forced to be a loner. This consideration is described by the first term σz^{N-1} of Eq. (6).

From this analysis we get the formula which represents the payoffs of a focal defector facing $N-1$ coplayers. For the convenience of analysis, it can be rewritten in another form of

$$\begin{aligned}P_d = \sigma z^{N-1} + \frac{r}{1-\omega}\left[\frac{1-z^N}{N(1-z)} - \frac{(\omega x+y+z)^N}{N(\omega x+y)}\right.\\ \left. + \frac{z^N}{N(\omega x+y)}\right].\end{aligned} \tag{7}$$

The above analytical approach allows us to study the payoff of a cooperator in such population, which is given by

$$\begin{aligned}P_{cw} = \sigma z^{N-1} + \sum_{S=2}^{N}\binom{N-1}{S-1}(1-z)^{S-1}z^{N-S}\\ \times\left\{\frac{r\left[1-\omega\left(\frac{\omega x+y}{1-z}\right)^{S-1}\right]}{S(1-\omega)} - 1\right\}.\end{aligned} \tag{8}$$

For the sake of the convenient calculation, it can be rewritten as another form of

$$\begin{aligned}P_c = \sigma z^{N-1} + \frac{r}{1-\omega}\left[\frac{1-z^N}{N(1-z)} - z^{N-1}\right]\\ - \frac{\omega r}{1-\omega}\frac{1}{N(\omega x+y)}[(\omega x+y+z)^N - z^N\\ - N(\omega x+y)z^{N-1}] - 1 + z^{N-1}.\end{aligned} \tag{9}$$

Under the replicator condition, the strategy evolution will depend on the respective payoffs depending on different strategies. Here we first focus on the competition between strategies C and D. The sign of $P_d - P_c$ determines whether it pays to switch from defection to cooperation or vice versa, with $P_d - P_c = 0$ being the equilibrium condition.

$$P_d - P_c = 1 - z^{N-1} - \frac{r[(\omega x+y+z)^N - z^N]}{N(\omega x+y)} + rz^{N-1}. \tag{10}$$

For any state (x,y,z) where $x \neq 0$, the probability that there are N_c cooperators in the current group where a given cooperator finds herself, is denoted by $\xi(N_c)$. Then it is easy to get $\sum_{N_c=1}^{N}\xi(N_c) = 1$.

We also notice that

$$\xi(N_c) = \sum_{N_c=1}^{N-N_c}\tau(N_c,N_d), \tag{11}$$

where $\tau(N_c,N_d)$ is the joint probability that the group is composed of N_c cooperators and N_d defectors. And, it is worth noting that $\tau(N_c,N_d)$ is only related to (x,y,z), and is not affected by the parameter ω. It suggests that the ratio of cooperators, defectors, and loners in the whole population decides the numbers of three strategies in the chosen players of size N.

For a chosen group which contains N_c cooperators and N_d defectors, the payoff of cooperators is given by P_c and P_d by recalling Eq. (1). If $\omega > 0$, it is easy to get the result $\frac{\partial P_c(N_c,N_d)}{\partial\omega} > 0$, implying that P_c is monotonically increasing with ω. This result is consistent with our intuition about the roles of subsequent cooperators, driven by synergistically enhanced benefits based on the number of cooperators.

In the following we focus our attention on the dynamic results of the system. The first consideration is the evolution dynamics on the boundary of the strategy simplex, where only two strategies (i.e., pairs of CD, pairs of CL, and pairs of DL in this model) are provided for the population. The results are listed as follows.

(1) The available strategies are C and D (i.e., $z=0$), and in this case $P_d - P_c = 1 - \frac{r(\omega x+y)^{N-1}}{N}$. Here, we can get the following results. Particularly, $\omega \leqslant 1$ results in $P_d - P_c > 0$ due to $r < N$. Second, $\omega > 1$ helps to get $\lim_{y\to 0}(P_d - P_c) = 1 - \frac{r\omega^{N-1}}{N}$, and $\lim_{x\to 0}(P_d - P_c) = 1 - \frac{r}{N} > 0$. Thus, it is easy to conclude that a nonstable equilibrium point exists on the edge of CD when $1 - \frac{r\omega^{N-1}}{N} < 0$.

(2) The optional strategies are only D and L (i.e., $x=0$), where $P_d = \sigma z^{N-1} < \sigma$, therefore the population will evolve into a pure state of L.

(3) Players are provided with two strategies of C and L (i.e., $y = 0$), where the results will depend on the value of ω. If $\omega = 1$, cooperators get dominance over loners with the aid of $P_c - P_l = r - 1 - \sigma$ and $r > 1 + \sigma$. If $\omega \neq 1$, we can obtain $\lim_{x\to 1} P_c = \frac{r(1+\omega+\cdots+\omega^{N-1})}{N} - 1$. Specifically, the two cases given by $\lim_{x\to 1} P_c = \frac{r(1-\omega^N)}{N(1-\omega)} - 1$ and $\lim_{x\to 0} P_c = \sigma$, respectively, need to be distinguished by comparing the values of $\lim_{x\to 0} P_c$ and σ.

As analyzed, for any cooperator, the probability of finding $S-1$ coplayers among the $N-1$ players in her group is given by

$$\binom{N-1}{S-1}(1-z)^{S-1}z^{N-S} = \binom{N-1}{S-1}x^{S-1}(1-x)^{N-S}. \quad (12)$$

Here these $S-1$ coplayers are cooperators since we have assumed that only cooperators and loners are provided for the gaming population.

By considering the condition of

$$\xi(S) = \binom{N-1}{S-1}x^{S-1}(1-x)^{N-S}, \quad (13)$$

and when $i > 2$,

$$\lim_{x\to 0^+}\frac{\xi(i)}{\xi(2)} = \lim_{x\to 0^+}\frac{\binom{N-1}{i-1}x^{i-1}(1-x)^{N-i}}{(N-1)x(1-x)^{N-2}} = 0. \quad (14)$$

The approximative results can be gained, where $\lim_{x\to 0^+}\frac{\partial P_c}{\partial x} > 0$ supported by $P_c(N_c = 2) = \frac{r(1+\omega)}{2} - 1$ and thus $r > \frac{2\sigma+2}{\omega+1}$. Then the final results depend on the following: If $\lim_{x\to 1} P_c = \frac{r(1-\omega^N)}{N(1-\omega)} - 1 > \sigma$, the system will evolve into the pure state dominated by cooperators; if $\lim_{x\to 1} P_c < \sigma$, there exists a steady equilibrium point $x^* \in (0,1)$ which leads to $P_c(x = x^*) = \sigma$.

The dependence of game dynamics on the characteristics of this public goods game model deserves investigation, where the amplification coefficient can be diminishing, increasing, or fixed. In the following we focus our attention on two different situations for a thorough study.

Situation 1. where no border equilibrium consisting of cooperation and defection exists, i.e., $1 - \frac{r\omega^{N-1}}{N} > 0$. As illustrated in Fig. 1, we obtain four scenarios in relation to the values of ω and σ, where these two parameters play key roles in deterring defection.

(1.1) If $r(\omega+1) > 2\sigma + 2$ and $\frac{r(1-\omega^{N-1})}{N(1-\omega)} - 1 > \sigma$, the outcome of the game dynamics depends on the initial conditions. For any initial state situating on the boundary of the strategy simplex, the corners C, D, and L are unstable equilibria of the game dynamics for possible perturbations. However, the initial state satisfying the condition of $x > 0$, $y > 0$, $z > 0$ will lead to a state where the three strategies coexist in the final state.

(1.2) If $r(\omega+1) < 2\sigma + 2$ and $\frac{r(1-\omega^{N-1})}{N(1-\omega)} - 1 < \sigma$, full loner equilibrium (L) is the only stable point in the system when considering the presence of perturbations. As mentioned, the advantage of one strategy over another depends on the payoff difference between them. Different from (1.1) by increasing σ from $\sigma = 0.8$ to $\sigma = 0.9$, larger σ facilitates the survival of loners.

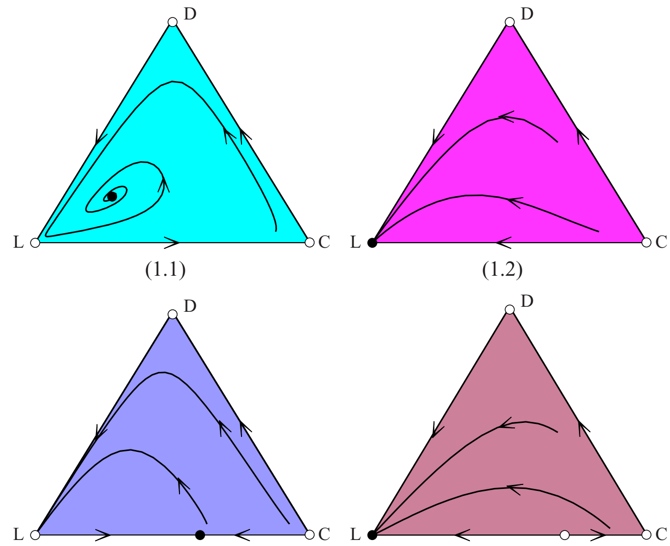

FIG. 1. Three strategy evolution dynamics in this special public goods game characterized by the parameter ω, where in the presence of smaller ω, cooperators are hard to stabilize. However, the presence of loners is beneficial for inhibiting the dominance of free riders in this system. In the presence of loners, there are four scenarios for the game dynamics that are characterized by the magnitude of multiplication factor ω and the payoffs σ of acting as loners. Resulting game dynamics depending on the initial state are shown in the four scenarios. The corners C, D, and L are equilibrium points. Open dots are unstable equilibrium points and closed dots are stable equilibrium points when considering the mutation. In (1.1) the parameters satisfy $r(\omega+1) > 2\sigma+2$ and $\frac{r(1-\omega^{N-1})}{N(1-\omega)} - 1 > \sigma$; here $N = 5$, $r = 3$, $\sigma = 0.8$, $\omega = 0.8$. Besides the unstable equilibria of C, D, and L, the coexistence (i.e., the interior equilibrium point) of the three competing strategies is also possible here. In (1.2) the parameters satisfy $r(\omega+1) < 2\sigma+2$ and $\frac{r(1-\omega^{N-1})}{N(1-\omega)} - 1 < \sigma$; here $N = 5$, $r = 2$, $\sigma = 0.9$, $\omega = 0.8$. Results show that full loner equilibrium (L) is the only stable point in the system when facing perturbations. In (1.3) the parameters satisfy $r(\omega+1) > 2\sigma+2$ and $\frac{r(1-\omega^{N-1})}{N(1-\omega)} - 1 < \sigma$, and we employ here $N = 5$, $r = 2$, $\sigma = 0.5$, and $\omega = 0.8$ in our analysis. From the results we can see a border equilibrium consisting of cooperators and loners. (1.4) the parameters satisfy $r(\omega+1) < 2\sigma+2$ and $\frac{r(1-\omega^{N-1})}{N(1-\omega)} - 1 > \sigma$. The employed parameter takes the value of $N = 5$, $r = 2$, $\sigma = 1.3$, $\omega = 1.05$. Besides the stable point of pure loners, an unstable border equilibrium consisting of loners and cooperation is also observed.

(1.3) If $r(\omega+1) > 2\sigma + 2$ and $\frac{r(1-\omega^{N-1})}{N(1-\omega)} - 1 < \sigma$, there is an additional border equilibrium consisting of cooperators and loners. Compared with case (1.2) above, with lower σ drive the population evolves to this steady state.

(1.4) If $r(\omega+1) < 2\sigma + 2$ and $\frac{r(1-\omega^{N-1})}{N(1-\omega)} - 1 > \sigma$, this creates a border equilibrium consisting of loners and cooperators. This equilibrium is unstable for perturbations. And, there is now an additional stable equilibrium purely consisting of loners. It is understandable that large enough payoffs gained by loners will promote its survival and spread in the populations always seeking the maximum profit. Further, here we employ $\omega = 1.05$ in our analysis, signifying the synergistically enhanced benefits based on the number of cooperators in groups of interacting players. It is clear that the public goods benefits are provided only by cooperators here, not defectors or loners. Thus, more cooperators will improve the payoffs of the whole public group. Along this line, decreasing ω impairs the magnifying effect of contributions impartially provided by cooperators. Here, the payoff competition between the cooperators and loners depends on the number of cooperators, resulting in the unstable border equilibrium consisting of loners and cooperation as shown in Fig. 1 (1.4).

Summarizing the four scenarios corresponding to Figs. 1(1.1)–1(1.4), we can conclude that the introduction of loners plays a significant role in deterring the spread of defectors who free ride other's contributions. Naturally, larger σ [e.g., scenario (1.4)] can effectively guarantee the existence of a stable equilibrium point of pure loners. Further, the nature of the employed public goods game also notably influences the strategy evolution. Smaller ω could not efficiently ensure the payoff advantages of cooperators which will propel the spread and domination of the corresponding strategy.

Situation 2. where there is a border equilibrium consisting of cooperation and defection (on the edge of CD), i.e., $1 - \frac{r\omega^{N-1}}{N} < 0$. Similar to the above analysis, we need to clarify the possible scenarios of the game dynamics.

(2.1) If $r(\omega+1) > 2\sigma + 2$ and $\frac{r(1-\omega^{N-1})}{N(1-\omega)} - 1 > \sigma$, the outcome of the game dynamics depends on the initial conditions. For any initial state situating on the boundary of the strategy simplex, full cooperation equilibrium (C) is the only stable point in the system in the presence of perturbations. It is clear that $\omega > 1$ means synergistically enhanced benefits based on the number of cooperators in groups of interacting agents. More cooperators will magnify the gains of the group nonlinearly. Different from Figs. 1(1.1)–1(1.3) by increasing ω from $\omega = 0.8$ to $\omega = 1.2$, this updated situation provides a dominating advantage in payoffs for cooperators over others.

(2.2) If $r(\omega+1) < 2\sigma + 2$ and $\frac{r(1-\omega^{N-1})}{N(1-\omega)} - 1 < \sigma$, full loner equilibrium (L) is the only stable point in the system. Under this situation, the increasing σ from $\sigma = 0.5$ [as used in case (2.1)] to $\sigma = 3$ facilitates the loners to finally dominate the whole population.

(2.3) If $r(\omega+1) < 2\sigma + 2$ and $\frac{r(1-\omega^{N-1})}{N(1-\omega)} - 1 > \sigma$, this creates a border equilibrium consisting of loners and cooperators. And this equilibrium is unstable in the presence of perturbations. Moreover, cooperation (C) and loners (L) are both stable equilibria of the game dynamics. By comparison we can see the dependence of the dynamic results on the increasing σ and ω.

(2.4) If $r(\omega+1) > 2\sigma + 2$ and $\frac{r(1-\omega^{N-1})}{N(1-\omega)} - 1 < \sigma$, this will create a stable border equilibrium consisting of loners and cooperators. In our system, the evolution dynamics thus depends on the following set of inequalities:

$$1 - \frac{r\omega^{N-1}}{N} < 0, \quad r(\omega+1) > 2\sigma + 2,$$
$$\frac{r(1-\omega^N)}{N(1-\omega)} - \sigma - 1 < 0, \quad \omega > 1, \quad r < N. \qquad (15)$$

Considering the precondition of $r > \frac{2(\sigma+1)}{\omega+1}$, we introduce $r^* = \frac{2(\sigma+1)}{\omega+1}$, thus $r > r^*$ denotes $r(\omega+1) > 2\sigma + 2$. With a view to $r < r^{**} = \frac{N(\sigma+1)}{\omega^{N-1}+\omega^{N-2}+\cdots+\omega+1}$, $\frac{r(1-\omega^N)}{N(1-\omega)} - \sigma - 1 < 0$ is equal to $r < r^{**}$. It is clear that the above Eq. (15) has no solution if $r^* > r^{**}$. Accordingly, $r^* - r^{**} = (\sigma+1)(\frac{2}{\omega+1} - \frac{N}{\omega^{N-1}+\omega^{N-2}+\cdots+\omega+1})$. If N is even, we can obtain the following results: $r^* - r^{**} > (\sigma+1)(\frac{2}{\omega+1} - \frac{N}{\omega+1}\frac{2}{N}) = 0$ (due to $\omega > 1$). If N is odd, $r^* - r^{**} > (\sigma+1)[\frac{2}{\omega+1} - \frac{N}{(\omega+1)(\frac{N-1}{2})+\omega^{N-2}}] > (\sigma+1)[\frac{2}{\omega+1} - \frac{N}{(\omega+1)(\frac{N-1}{2}+\frac{1}{2})}] = 0$. The precondition of $\omega > 1$ results in $\omega^{N-2} > \omega$. Omitting other calculation details, we finally obtain $r^* > r^{**}$ and arrive at a conclusion that the set of inequalities Eq. (15) has no solution, thus there is no stable border equilibrium consisting of loners and cooperators.

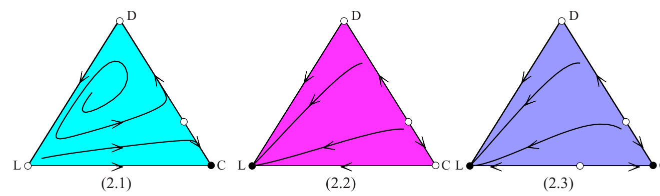

FIG. 2. Three-strategies evolution dynamics in this special public goods game in the presence of larger ω. Similarly, the corners C, D, and L are equilibrium points. Open dots are unstable equilibrium points and closed dots are stable equilibrium points when considering perturbations. In (2.1) the parameters satisfy $r(\omega+1) > 2\sigma + 2$ and $\frac{r(1-\omega^{N-1})}{N(1-\omega)} - 1 > \sigma$; here $N = 5$, $r = 3$, $\sigma = 0.5$, $\omega = 1.2$. In this case, the outcome of the game dynamics depends on the initial conditions. In (2.2) the parameters satisfy $r(\omega+1) < 2\sigma + 2$ and $\frac{r(1-\omega^{N-1})}{N(1-\omega)} - 1 < \sigma$; here $N = 5$, $r = 2.5$, $\sigma = 3$, $\omega = 1.2$. Resulted by larger σ, full loner equilibrium (L) is the only stable point in the system for perturbations. In (2.3) the parameters satisfy $r(\omega+1) < 2\sigma + 2$ and $\frac{r(1-\omega^{N-1})}{N(1-\omega)} - 1 > \sigma$; here $N = 5$, $r = 2.5$, $\sigma = 2$, $\omega = 1.2$. Depending on the initial state, the stable equilibria of C and D, and a border equilibrium consisting of L and C are possible outcomes under this situation. (2.4) $r(\omega+1) > 2\sigma + 2$ and $\frac{r(1-\omega^{N-1})}{N(1-\omega)} - 1 < \sigma$; since there is no stable border equilibrium consisting of loners and cooperation under this situation, no figure is presented here.

Summarizing above, the results in Figs. 1 and 2 have proved to be sensitive to the key parameter ω of the underlying public goods game, and the given payoffs σ of loners. Particularly, in this setting larger ω promotes the emergence of stable equilibrium of pure cooperators, or loners under suitable conditions. Synergy ($\omega > 1$) generally favors cooperation as compared to discounting ($\omega < 1$). For the discounted benefits, $\omega < 1$, defection gains the chance to reign (depending on the payoffs σ of loners), whereas cooperators lose this chance. As described, depending on the initial condition, the state converges either to an equilibrium consisting of cooperators and loners, or to an coexistence of the three strategies, or the fixation state of cooperators or loners.

IV. CONCLUSION

Cooperation in social interactions involving more than two agents is a widely studied problem. The mostly discussed public goods game can be seen as a standard model for studying cooperation and cheating in evolving populations. Relaxing the uniform influence of each contribution provided by the cooperators, we discuss a version of the public goods game which includes the discounting or synergy in the benefits provided by cooperators in the gaming group. Among them, how the tunable parameter helps the traditional public goods game is a special case of this studied game. The conflict between the pursuit of self-interests and collective interests in the context of social dilemmas still exists here. And our investigations are focused on the three-strategies games, which are constructed by adding an option of being loners. We aim to provide some conditions by which cooperation may originate and be maintained in this type of public goods game. A rich variety of competition scenarios emerge from the description of evolutionary games.

In our model, $\omega < 1$ is a crucial ingredient where the discounted benefits (driven by parameter $\omega < 1$) indicate a stricter cooperation dilemma than the synergistically enhanced one. It has indeed been observed that the discounted benefits based on the number of cooperators lead to a decline in cooperation. In opposition to this, larger ω together with the appropriate payoffs σ of loners, may keep cooperators alive. The resulting dynamics can also allow the coexistence of the three strategies, and even the full cooperator equilibrium which is a stable point of the dynamics under perturbations. Last, the variation of the payoffs σ helps us to verify that being loners is an effective mechanism for sustaining contribution levels when players can opt out. In addition, it is understandable that larger payoffs of loners can better deter the spread of free riders.

To conclude, we focused on the synergy and discounting in a three-strategies public goods game. Our results point out that the nature of the public goods game (decided by the discounted or synergistically enhanced benefits in the presence of multiple cooperators) plays a fundamental and positive role in the evolution of cooperation. While our well-mixed model does not address here the question of which strategy succeeds in the structured populations, it would be interesting to see if our conclusions would still hold (or not) in a networked system.

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