Resonance vector soliton of the Rayleigh wave

G. T. Adamashvili*

Technical University of Georgia, Kostava street 77, Tbilisi, 0179, Georgia

(Received 25 September 2015; revised manuscript received 5 January 2016; published 2 February 2016)

A theory of acoustic vector solitons of self-induced transparency of the Rayleigh wave is constructed. A thin resonance transition layer on an elastic surface is considered using a model of a two-dimensional gas of impurity paramagnetic atoms or quantum dots. Explicit analytical expressions for the profile and parameters of the Rayleigh vector soliton with two different oscillation frequencies is obtained, as well as simulations of this nonlinear surface acoustic wave with realistic parameters, which can be used in acoustic experiments. It is shown that the properties of a surface vector soliton of the Rayleigh wave depend on the parameters of the resonance layer, the elastic medium, and the transverse structure of the surface acoustic wave.

DOI: 10.1103/PhysRevE.93.023002

I. INTRODUCTION

The spread of acoustic nonlinear waves of invariable profile solitons and their different modifications (kinks, breathers, vector solitons, etc.) is one of the most bright manifestations of nonlinear acoustic effects in crystals and nanostructures. This problem has attracted significant interest in both theoretical and experimental studies (see, for instance, Ref. [1] and references therein). The determination of the mechanisms causing the excitation of the acoustic nonlinear waves and the investigation of their properties are among the basic problems of the physics of nonlinear acoustic waves. Depending on the character of the acoustic nonlinearity, the nonresonance or resonance mechanism of the generation of acoustic nonlinear waves can be studied. In the case of nonresonance nonlinearity, which is associated with anharmonic vibrations of the lattice, its competition with the dispersion leads to the formation of nonresonance acoustic nonlinear waves [2].

Nonlinear resonance acoustic waves can be excited with the help of the McCall and Hahn mechanism, when a nonlinear coherent interaction of an acoustic pulse with a small concentration of paramagnetic impurities or quantum dots takes place and the conditions of acoustic self-induced transparency (SIT) $\omega T \gg 1$ and $T \ll T_{1,2}$ are fulfilled [3]. Here, ω and T are the acoustic pulse frequency and width, respectively, while T_1 and T_2 are the longitudinal and transverse relaxation times of the resonant impurity atoms or quantum dots, respectively. When the area of the acoustic pulse $\Theta > \pi$, a soliton (2π pulse) is generated, and for $\Theta \ll \pi$, small-amplitude acoustic pulses (0π pulses), for instance, breather or vector solitons, can be excited. The acoustic soliton (or breather) is a one-component single acoustic pulse which propagates in such a way that it maintains its state. When these conditions are not fulfilled, we must consider the interaction between two acoustic wave components at different frequencies as a bound state. Under this condition, an acoustic vector soliton can be formed [4].

Resonance acoustic nonlinear waves have been considered for both bulk plane acoustic waves as well as surface acoustic waves (see, for instance, Refs. [1,2,4–9] and references therein). Characteristic peculiarities of surface acoustic waves are strong enhancement and spatial confinement of the elastic deformation energy of the acoustic wave near the surface for the Rayleigh wave [10] and near the interface in multilayered systems for the Stoneley wave or Love wave [11,12], while they decay evanescently in the directions perpendicular to the surface or interface. The properties of nonlinear surface acoustic waves have attracted much interest in the context of nanoacoustics and applications [5,13–15].

Nonlinear surface acoustic waves have been investigated in many works (see, for instance, Refs. [2,5–15]). In particular, resonance solitons and breathers of surface acoustic waves, which are scalar (one-component) nonlinear waves have, been considered in Ref. [16], but resonance surface acoustic vector solitons, which are two-component nonlinear waves of SIT, have not been considered up to now.

The purpose of the present work is to consider the conditions of realization of resonance acoustic vector solitons of SIT with the difference and sum of the oscillation frequencies of the Rayleigh wave and the determination of the explicit analytical and numerical expressions for the parameters of the surface acoustic vector pulse.

II. BASIC EQUATIONS

We consider the formation of vector solitons of SIT for a surface acoustic Rayleigh wave propagating along the surface (at $x = 0$) of the dielectric or semiconductor crystal semispace $x \leqslant 0$. We shall suppose that, on the surface of the crystal, there is a thin homogeneous transition layer of thickness $d \ll \lambda$ containing a small concentration n_0 of paramagnetic impurities or quantum dots with electron spin $S = \frac{1}{2}$ and nuclear spin $I = \frac{1}{2}$, where λ is the length of the Rayleigh wave. We assume that an external constant magnetic field H_0 is applied along the x axis. In this material, surface acoustic wave of vertical polarization can propagate. We shall consider a surface acoustic Rayleigh wave with width $T \ll T_{1,2}$, frequency $\omega \gg T^{-1}$, and wave vector $\vec{k}$, propagating along the positive z axis.

The Rayleigh wave is polarized elliptically in the sagittal plane and has longitudinal and transverse deformation vector components, which depend on the carrier wave frequency can cause excitations in the spin system of the paramagnetic impurities or quantum dots with the frequencies $\omega_S + \omega_I$ and ω_S, where ω_S and ω_I are the Zeeman frequencies of the electron and nuclear spins, respectively. To take into

*guram_adamashvili@ymail.com

account the potential extension of the obtained results for quantum dots, we consider the resonance interaction of the wave with impurities with a carrier wave frequency of the Rayleigh wave $\omega = \omega_S + \omega_I$ [17]. In this case the Rayleigh wave is capable of causing simultaneous excitations of the electron S and nuclear I spins of the paramagnetic impurities or quantum dots. But this Rayleigh wave, along with the resonance transitions, can at the same time also cause another nonresonance transition of frequency ω_S. Because solitary pulses (among them vector solitons) have bell-like shapes of the spectral line with a strong maximal peak at the resonance frequency $\omega_S + \omega_I$, therefore for others, the nonresonance frequencies (for instance, ω_S), the amplitude, and consequently the possibility of nonresonant transitions will be significantly smaller. Besides this, the quantum dots used in the different experiments are usually anisotropic in nature, and some have an anisotropic g-factor [18], and therefore different transitions will have different possibilities. The contribution from such nonresonant transitions could be taken into account by means of perturbation theory, but this is not the aim of the present work.

The boundary conditions for the Rayleigh wave on the free surface at $x = 0$ have the following form [19]:

$$\sigma_{xx} = \sigma_{xy} = \tilde{\sigma}_{xz} = 0,$$

where

$$\tilde{\sigma}_{xz} = \sigma_{xz} + \sigma'_{xz},$$

σ_{xx}, σ_{xy}, and σ_{xz} are the components of the stress tensor, and σ'_{xz} is the contribution to the quantity $\tilde{\sigma}_{xz}$ caused by the presence of the transition layer with electron and nuclear spins S and I, respectively.

From the boundary condition $\sigma_{xy} = 0$, it follows that the y component of the Rayleigh wave deformation vector $\vec{u}(u_x,u_y,u_z)$ is equal to zero and has vertical polarization.

We will consider a Fourier decomposition of the x and z components of the deformation vector u_x and u_z, which is given by

$$\begin{aligned} u_x(x,z,t) = \int &[\tilde{u}_{t,x}(\Omega,Q)e^{æ_t(\Omega,Q)x} \\ &+ \tilde{u}_{l,x}(\Omega,Q)e^{æ_l(\Omega,Q)x}]e^{i(Qz-\Omega t)}\,d\Omega\,dQ, \\ u_z(x,z,t) = \int &[\tilde{u}_{t,z}(\Omega,Q)e^{æ_t(\Omega,Q)x} \\ &+ \tilde{u}_{l,z}(\Omega,Q)e^{æ_l(\Omega,Q)x}]e^{i(Qz-\Omega t)}\,d\Omega\,dQ, \end{aligned} \quad (1)$$

where

$$æ_l^2 = Q^2 - \frac{\Omega^2}{c_l^2}, \qquad æ_t^2 = Q^2 - \frac{\Omega^2}{c_t^2},$$

the functions $\tilde{u}_{l,x}(\Omega,Q)$, $\tilde{u}_{l,z}(\Omega,Q)$, $\tilde{u}_{t,x}(\Omega,Q)$, and $\tilde{u}_{t,z}(\Omega,Q)$ have to be determined, c_l and c_t are the longitudinal and transverse polarized sound velocity in the medium, respectively.

We can present the deformation vector $\vec{u}$ as a sum of two vectors $\vec{u} = \vec{u}_l + \vec{u}_t$, where the vectors $\vec{u}_l(u_{l,x},0,u_{l,z})$ and $\vec{u}_t(u_{t,x},0,u_{t,z})$ satisfy well-known conditions [19]:

$$\mathrm{rot}\vec{u}_l = 0, \quad \mathrm{div}\vec{u}_t = 0. \quad (2)$$

Substituting Eqs. (1) into (2), the x and z components of the deformation vector can be transformed in the following form:

$$\begin{aligned} u_x(x,z,t) = -i\int &[Q\tilde{u}_t(\Omega,Q)e^{æ_t(\Omega,Q)x} \\ &+ æ_l(\Omega,Q)\tilde{u}_l(\Omega,Q)e^{æ_l(\Omega,Q)x}]e^{i(Qz-\Omega t)}\,d\Omega\,dQ, \\ u_z(x,z,t) = \int &[æ_t(\Omega,Q)\tilde{u}_t(\Omega,Q)e^{æ_t(\Omega,Q)x} \\ &+ Q\tilde{u}_l(\Omega,Q)e^{æ_l(\Omega,Q)x}]e^{i(Qz-\Omega t)}\,d\Omega\,dQ, \end{aligned} \quad (3)$$

Substituting Eqs. (3) into the boundary condition $\sigma_{xz} = 0$, we obtain the connection between the functions $\tilde{u}_l(\Omega,Q)$ and $\tilde{u}_t(\Omega,Q)$ in the form

$$\tilde{u}_t(\Omega,Q) = \alpha(\Omega,Q)\tilde{u}_l(\Omega,Q),$$

where

$$\alpha(\Omega,Q) = -\frac{2Q^2c_t^2 + c_l^2\left[æ_l^2(\Omega,Q) - Q^2\right]}{2Qc_t^2æ_t(\Omega,Q)}.$$

The component of the deformation tensor $\varepsilon_{xx} = \frac{\partial u_x}{\partial x}$ can be presented as

$$\varepsilon_{xx} = \int \tilde{\varepsilon}_{xx}(\Omega,Q)e^{i(Qz-\Omega t)}\,d\Omega\,dQ, \quad (4)$$

where

$$\tilde{\varepsilon}_{xx}(\Omega,Q) = -i\left[æ_l^2(\Omega,Q) + Qæ_t(\Omega,Q)\alpha(\Omega,Q)\right]\tilde{u}_l(\Omega,Q).$$

Substituting Eq. (3) into the boundary condition $\tilde{\sigma}_{xx} = 0$, and dividing the real and imaginary parts of this equation, we obtain dispersion law for the Rayleigh wave [19]:

$$4kæ_l(\omega,k)æ_t(\omega,k) = \left[k^2 + æ_t^2(\omega,k)\right]^2 \quad (5)$$

and the nonlinear wave equation for the ε_{xx} component of the deformation tensor:

$$\int \tilde{\varepsilon}_{xx}(\Omega,Q)G(\Omega,Q)e^{i(Qz-\Omega t)}\,d\Omega\,dQ = -\frac{\sigma'_{xx}}{\rho}, \quad (6)$$

where

$$G(\Omega,Q) = 2c_t^2 - \frac{\Omega^2\left(1 - \frac{2c_t^2}{c_l^2}\right)}{æ_l^2(\Omega,Q) + Qæ_t(\Omega,Q)\alpha(\Omega,Q)};$$

ρ is the density of the medium.

In order to determine the dependence of the stress tensor component σ'_{xx} on the ε_{xx} component of the deformation tensor at $x = 0$, we have to consider the magnetic Bloch equations [1,17,20].

If we neglect nonresonant excitations of the impurities and use the rotating-wave approximation, the Hamiltonian of the spin system can be transformed into the following form [10,16,17,21,22]:

$$\begin{aligned} \hat{H} = \hbar\omega_S \sum_i \hat{S}_i^z - \hbar\omega_I \sum_i \hat{I}_i^z + A \sum_i \hat{S}_i^z \hat{I}_i^z \\ g \sum_i (\hat{S}_i^+ \hat{I}_i^- \varepsilon^- + \hat{S}_i^- \hat{I}_+^z \varepsilon^+), \end{aligned} \quad (7)$$

where

$$g = \frac{A\beta_0 H_0 F_{xxxx}}{2\hbar\omega},$$

β_0 is the Bohr magneton, $\hbar$ is Planck's constant, $\varepsilon_{xx} = \varepsilon^+ + \varepsilon^-$, F_{xxxx} is a component of the spin-phonon coupling tensor, and A is the constant of the hyperfine interaction.

From the Hamiltonian (7), we obtain the system of equations

$$\frac{\partial}{\partial t}(S_i^+ I_i^-) = i(\omega_S + \omega_I)S_i^+ I_i^- - ig(S_i^z - I_i^z)\varepsilon^-,$$
$$\frac{\partial}{\partial t}(S_i^z - I_i^z) = 2ig(S_i^+ I_i^- \varepsilon^+ - S_i^- I_i^+ \varepsilon^-). \tag{8}$$

Equations (8) are exact only in case that we assume that the relaxation times T_1 and T_2 of the paramagnetic impurities are infinite.

The system of Eqs. (8) can be transformed to the slowly varying variables using the equations

$$\varepsilon_{xx} = \sum_{l=\pm 1} \hat{E}_l Z_l,$$
$$\langle S_i^\pm I_i^\mp \rangle = \pm i\rho^\pm Z_{\mp 1},$$
$$\left\langle S_i^z - I_i^z \right\rangle = 2N, \tag{9}$$

where

$$Z_l = e^{il(kz-\omega t)};$$

$<$ ... $>$ are the average values of the spin operators.

Substituting Eqs. (9) into (8), we obtain the system of equations for the slowly varying envelope functions $\hat{E}_{\pm 1}$, $\rho^\pm$, and N:

$$\frac{\partial \rho^+}{\partial t} = i(\omega_S + \omega_I - \omega)\rho^+ - 2gN\hat{E}_{-1},$$
$$\frac{\partial N}{\partial t} = g(\rho^+ \hat{E}_{+1} + \rho^- \hat{E}_{-1}). \tag{10}$$

The systems of Eqs. (6) and (10) are the equations for acoustic SIT for a Rayleigh wave with the frequency $\omega \approx \omega_S + \omega_I$, which is capable of causing "forbidden" transitions with simultaneous reorientation of electron S and nuclear spin I of paramagnetic impurities, which are the main objects of our investigation. This system of equations can describe a wide class of nonlinear resonance phenomena for surface acoustic waves.

In Ref. [16] the coherent state of the surface Reyleigh modes has been used for the consideration of the resonance soliton and breather, which are the scalar (one-component) solutions of the surface wave equations. In the present work we are using another approach for the derivation of the surface wave equation, and we consider the vector soliton solution of the systems of Eqs. (6) and (10) by using an approach which we have not used for the solution of the resonance surface acoustic wave equations in previous work [16]. In particular, for the solution of Eqs. (6) and (10), we will use reduced perturbation expansion to study the evolution of the surface acoustic vector solitons with two different oscillation frequencies for the Rayleigh wave. The surface vector soliton is an object qualitatively different from the scalar soliton or breather treated in Ref. [16].

III. SOLUTION OF THE WAVE EQUATION

We can simplify the wave equation (6) using a second order Taylor series expansion of a function $G(\Omega,Q)$ about the point (ω,k):

$$G(\Omega,Q) = G(\omega,k) + (\Omega-\omega)G'_\Omega + (Q-k)G'_Q + \tfrac{1}{2}[(\Omega-\omega)^2 G''_\Omega + 2(\Omega-\omega)(Q-k)G''_{\Omega,Q} + (Q-k)^2 G''_Q], \tag{11}$$

where

$$G'_\Omega = \left.\frac{\partial G}{\partial \Omega}\right|_{\Omega=\omega,\,Q=k}, \quad G'_Q = \left.\frac{\partial G}{\partial Q}\right|_{\Omega=\omega,\,Q=k},$$
$$G''_\Omega = \left.\frac{\partial^2 G}{\partial \Omega^2}\right|_{\Omega=\omega,\,Q=k}, \quad G''_{\Omega,Q} = \left.\frac{\partial^2 G}{\partial Q\partial \Omega}\right|_{\Omega=\omega,\,Q=k},$$
$$G''_Q = \left.\frac{\partial^2 G}{\partial Q^2}\right|_{\Omega=\omega,\,Q=k};$$

ω and k are the frequency and wave number of the carrier wave, respectively.

Substituting Eq. (11) into (6), we obtain the following nonlinear acoustic wave equation:

$$\int [G(\omega,k) + (\Omega-\omega)G'_\Omega + (Q-k)G'_Q + \left(\frac{\Omega^2}{2} - \omega\Omega + \frac{\omega^2}{2}\right)G''_\Omega + (\Omega Q - \omega Q - k\Omega + k\omega)G''_{\Omega,Q} + \left(\frac{Q^2}{2} - kQ + \frac{k^2}{2}\right)G''_Q]\tilde{\varepsilon}_{xx}(\Omega,Q)e^{i(Qz-\Omega t)}\,d\Omega\,dQ = -\frac{\sigma'_{xx}}{\rho}. \tag{12}$$

Using Eq. (4), Eq. (12) will be transformed into the following form:

$$\left(A + iB\frac{\partial}{\partial t} - iC\frac{\partial}{\partial z} - \frac{G''_\Omega}{2}\frac{\partial^2}{\partial t^2} + G''_{\Omega,Q}\frac{\partial^2}{\partial t\partial z} - \frac{G''_Q}{2}\frac{\partial^2}{\partial z^2}\right)\varepsilon_{xx} = -\frac{\sigma'_{xx}}{\rho}, \tag{13}$$

where

$$A = G(\omega,k) - \omega G'_\Omega - kG'_Q + \frac{\omega^2}{2}G''_\Omega + k\omega G''_{\Omega,Q} + \frac{k^2}{2}G''_Q,$$
$$B = G'_\Omega - \omega G''_\Omega - kG''_{\Omega,Q}, \qquad C = G'_Q - \omega G''_{\Omega,Q} - kG''_Q.$$

Substituting Eq. (9) into (13), we obtain

$$\sum_{l=\pm1} Z_l\left(a\frac{\partial}{\partial t}+ib\frac{\partial^2}{\partial t^2}-id\frac{\partial^2}{\partial t\partial z}+G''_{\Omega,Q}\frac{\partial^3}{\partial t^2\partial z}-\frac{G''_{\Omega}}{2}\frac{\partial^3}{\partial t^3}-\frac{G''_{Q}}{2}\frac{\partial^3}{\partial z^2\partial t}\right)\Theta_l=-\frac{\sigma'_{xx}}{\rho},\tag{14}$$

where

$$a=A+l(B\omega+Ck)+\frac{1}{2}G''_{\Omega}\omega^2+G''_{\Omega,Q}\omega k+\frac{1}{2}G''_{Q}k^2,$$

$$b=B+l(G''_{\Omega}\omega+G''_{\Omega,Q}k),$$

$$d=C+l(G''_{\Omega,Q}\omega+G''_{Q}k),$$

$$\hat{E}_l=\frac{\partial\Theta_l}{\partial t}.$$

As the next step for the solution of Eq. (14), we can use the reduced perturbation expansion for nonlinear equations considered in Ref. [23]. It is assumed that the area of the acoustic nonlinear pulse envelope $|\Theta_l|\ll 1$ and is of the order of a small parameter ϵ. This is the typical scaling for the coupled nonlinear Schrodinger equations and consequently can describe the acoustic vector soliton. In the considered case, the function $\Theta_l(z,t)$ has the following form:

$$\Theta_l=\sum_{\alpha=1}\varepsilon^{\alpha}\Theta_l{}^{(\alpha)}=\sum_{\alpha=1}^{\infty}\sum_{n=-\infty}^{+\infty}\varepsilon^{\alpha}Y_{l,n}f_{l,n}^{(\alpha)}(\zeta,\tau),\tag{15}$$

where

$$Y_{l,n}=e^{in(Q_{l,n}z-\Omega_{l,n}t)},$$

$$\zeta_{l,n}=\varepsilon Q_{l,n}(z-v_{g_{l,n}}t),$$

$$\tau=\varepsilon^2 t,\quad v_{g_{l,n}}=\frac{d\Omega_{l,n}}{dQ_{l,n}}.$$

We must assume that the parameters $\Omega_{l,n}$, $Q_{l,n}$, and $f_{l,n}^{(\alpha)}$ satisfy the following conditions for any l and n indices:

$$\omega\gg\Omega,\quad k\gg Q,$$

$$\left|\frac{\partial f_{l,n}^{(\alpha)}}{\partial t}\right|\ll\Omega\left|f_{l,n}^{(\alpha)}\right|,\quad\left|\frac{\partial f_{l,n}^{(\alpha)}}{\partial z}\right|\ll Q\left|f_{l,n}^{(\alpha)}\right|.$$

It is obvious that the parameters Q, Ω, ζ, and v_g depend on the indices l and n. But for simplicity, in the further considerations we omit these indices in the equations where this will not lead to confusion.

Substituting Eq. (15) into the system of Eqs. (10), we can determine the explicit form of the envelope of the σ'_{xx} component of the stress tensor:

$$\sigma'_{xx}=iR\rho\sum_{l=\pm1}lZ_l\left[\varepsilon^1\Theta_l{}^{(1)}+\varepsilon^2\Theta_l{}^{(2)}+\varepsilon^3\Theta_l{}^{(3)}-\varepsilon^3\frac{1}{2}\int\frac{\partial\Theta_l{}^{(1)}}{\partial t}\Theta_{-l}{}^{(1)}\Theta_l{}^{(1)}\,dt'\right]+O(\varepsilon^4),\tag{16}$$

where

$$R=\frac{(A\beta_0H_0F_{xxxx})^2n_0}{8\rho\hbar^2\omega^2}\int\frac{g(\Delta)\,d\Delta}{1+\Delta^2T^2};$$

$g(\Delta)$ is the inhomogeneous broadening function of the spectral line of the paramagnetic impurities, $\Delta=\omega_S+\omega_I-\omega$.

On substituting Eq. (15) into (14) and then taking into account Eq. (16), we obtain the acoustic nonlinear wave equation

$$\sum_{\alpha=1}^{\infty}\sum_{l=\pm1}\sum_{n=-\infty}^{+\infty}\varepsilon^{\alpha}Z_lY_{l,n}\left(W_{l,n}+\varepsilon J_{l,n}\frac{\partial}{\partial\zeta}+\varepsilon^2h_{l,n}\frac{\partial}{\partial\tau}+i\varepsilon^2H_{l,n}\frac{\partial^2}{\partial\zeta^2}\right)f_{l,n}^{(\alpha)}$$
$$=i\varepsilon^3\frac{R}{2}\sum_{l=\pm1}lZ_l\int\frac{\partial\Theta_l{}^{(1)}}{\partial t}\Theta_{-l}{}^{(1)}\Theta_l{}^{(1)}\,dt'+O(\varepsilon^4),\tag{17}$$

where

$$W_{l,n}=-in\left(a\Omega+nb\Omega^2+G''_{\Omega,Q}\Omega^2Q+\frac{G''_{\Omega}}{2}\Omega^3+\frac{G''_{Q}}{2}Q^2\Omega+dnQ\Omega+\frac{l}{n}R\right),$$

$$J_{l,n}=-Q[av_g+2bn\Omega v_g+dn(Qv_g+\Omega)+G''_{\Omega,Q}\Omega(\Omega+2Qv_g)+\frac{3}{2}G''_{\Omega}\Omega^2v_g+\frac{1}{2}G''_{Q}Q(Qv_g+2\Omega)],$$

$$h_{l,n}=a+2bn\Omega+dnQ+2G''_{\Omega,Q}Q\Omega+\frac{3}{2}G''_{\Omega}\Omega^2+\frac{1}{2}G''_{Q}Q^2,$$

$$H_{l,n}=Q^2[bv_g^2+dv_g+G''_{\Omega,Q}nv_g(2\Omega+Qv_g)+\frac{3}{2}G''_{\Omega}n\Omega v_g{}^2+\frac{1}{2}G''_{Q}n(2Qv_g+\Omega)].$$

According to the standard procedure (see, for instance, Ref. [4] and references therein), we equate to each other the terms corresponding to the same orders of ε. Consequently, we obtain that only the components $f^{(1)}_{\pm1,\pm1}$ and $f^{(1)}_{\pm1,\mp1}$ are nonzero. From this, we determine the equations $J_{\pm1,\pm1}=J_{\pm1,\mp1}=0$. The relation between the two characteristic parameters Ω and

Q has the following form:

$$a\Omega + nb\Omega^2 + G''_{\Omega,Q}\Omega^2 Q + \frac{G''_\Omega}{2}\Omega^3 + \frac{G''_Q}{2}Q^2\Omega + dnQ\Omega + \frac{l}{n}R = 0 \tag{18}$$

and the expression for the quantity

$$v_g = \frac{-dn\Omega - G''_Q Q\Omega - G''_{\Omega,Q}\Omega^2}{a + dnQ + 2nb\Omega + 2G''_{\Omega,Q}\Omega Q + \frac{3}{2}G''_\Omega\Omega^2 + \frac{1}{2}G''_Q Q^2}. \tag{19}$$

Substituting Eq. (18) into (17), we obtain the coupled nonlinear Schrödinger equations for the functions $q_1 = \varepsilon f^{(1)}_{+1,+1}$ and $q_2 = \varepsilon f^{(1)}_{+1,-1}$ that describe the connection between the two components of the acoustic nonlinear pulse:

$$i\left(\frac{\partial q_1}{\partial t} + v_1\frac{\partial q_1}{\partial z}\right) + p_1\frac{\partial^2 q_1}{\partial z^2} + g_1|q_1|^2 q_1 + r_1|q_2|^2 q_1 = 0,$$
$$i\left(\frac{\partial q_2}{\partial t} + v_2\frac{\partial q_2}{\partial z}\right) + p_2\frac{\partial^2 q_2}{\partial z^2} + g_2|q_2|^2 q_2 + r_2|q_1|^2 q_2 = 0, \tag{20}$$

where

$$v_1 = v_{g(+1,+1)}, \quad v_2 = v_{g(+1,-1)},$$
$$p_1 = \frac{H_{+1,+1}}{-h_{+1,+1}Q^2}, \quad p_2 = \frac{H_{+1,-1}}{-h_{+1,-1}Q^2},$$
$$g_1 = \frac{R}{-2h_{+1,+1}}, \quad g_2 = \frac{R}{-2h_{+1,-1}},$$
$$r_1 = \frac{R}{-2h_{+1,+1}}\left(1 - \frac{\Omega_{-1}}{\Omega_{+1}}\right),$$
$$r_2 = \frac{R}{-2h_{+1,-1}}\left(1 - \frac{\Omega_{+1}}{\Omega_{-1}}\right),$$
$$\Omega_{+1} = \Omega_{l=\pm1,n=\pm1},$$
$$\Omega_{-1} = \Omega_{l=\pm1,n=\mp1}. \tag{21}$$

The coupled nonlinear Schrödinger equations (20) describe the functions q_1 and q_2 oscillating with the frequencies $\omega + \Omega_{+1}$ and $\omega - \Omega_{-1}$, respectively. The nonlinear connection between these two waves q_1 and q_2 are determined by the terms $r_1|q_2|^2 q_1$ and $r_2|q_1|^2 q_2$.

An invariant profile solution of Eqs. (20) is an acoustic vector soliton. Let us consider the solution of Eqs. (20) in the following form:

$$q_i(z,t) = A_i\, S(\xi)e^{i\phi_i}, \tag{22}$$

where $S(\xi)$ is the slowly varying function of the acoustic pulse, $\phi_i = k_i z - \omega_i t$ are the phase functions, A_i, k_i, and ω_i are real constants, $i = 1,2$; $\xi = t - \frac{z}{V_0}$, where V_0 is the surface acoustic vector pulse velocity. The functions $e^{i\phi_i}$ are slow in comparison with the oscillations of the pulse and consequently, the inequalities $k_i \ll Q_{\pm1}$, $\omega_i \ll \Omega_{\pm1}$ are satisfied.

Substituting Eqs. (22) into Eqs. (20), we obtain after integration the steady-state solutions:

$$q_{1,2} = \frac{A_{1,2}}{bT}\,\mathrm{sech}\left(\frac{t - \frac{z}{V_0}}{T}\right)e^{i\phi_{1,2}}, \tag{23}$$

where

$$T^{-2} = V_0^2\frac{v_1 k_1 + k_1^2 p_1 - \omega_1}{p_1},$$
$$b^2 = V_0^2\frac{A_1^2 g_1 + A_2^2 r_1}{2p_1}. \tag{24}$$

Substituting Eq. (23) into (15) and (9), we obtain for the ε_{xx} component of the deformation tensor the acoustic vector soliton solution of the Rayleigh wave (at $x = 0$):

$$\varepsilon_{xx} = \frac{1}{bT}\,\mathrm{sech}\left(\frac{t - \frac{z}{V_0}}{T}\right)\{(\Omega_{+1} + \omega_+)A_1 \times \sin[(k + Q_{+1} + k_1)z - (\omega + \Omega_{+1} + \omega_1)t] - (\Omega_{-1} - \omega_2)A_2\sin[(k - Q_{-1} + k_2)z - (\omega - \Omega_{-1} + \omega_2)t]\}, \tag{25}$$

where the relations between the parameters A_i, ω_i, and k_i have the form

$$A_1^2 = \frac{p_1 g_2 - p_2 r_1}{p_2 g_1 - p_1 r_2}A_2^2, \quad k_i = \frac{V_0 - v_i}{2p_i},$$
$$\omega_1 = \frac{p_1}{p_2}\omega_2 + \frac{V_0^2(p_2^2 - p_1^2) + v_2^2 p_1^2 - v_1^2 p_2^2}{4p_1 p_2^2}. \tag{26}$$

Equation (25) is a two-component vector soliton solution for the ε_{xx} component of the deformation tensor of the Rayleigh pulse at $x = 0$. In expression (25), the functions $\sin[(k + Q_{+1} + k_+)z - (\omega + \Omega_{+1} + \omega_+)t]$ and $\sin[(k - Q_{-1} + k_-)z - (\omega - \Omega_{-1} + \omega_-)t]$ indicate the two different frequencies of oscillation. The Rayleigh wave is a two-dimensional wave with transverse structure in the semispace $x < 0$. Along the x axis, the vector soliton amplitude decays exponentially as one moves away from the surface at $x = 0$, which is determined by Eqs. (1).

IV. CONCLUSION

We have shown that during the propagation of the Rayleigh wave on the surface of an elastic medium with a transition resonance monolayer containing impurity paramagnetic atoms under the condition of SIT, a vector soliton of the Rayleigh wave can arise. We consider the situation when the surface acoustic pulse exhibits "forbidden" transitions in the impurity paramagnetic atoms.

The explicit profile and parameters of the acoustic two-component vector solitons of the Rayleigh wave are given by Eqs. (25), (19), (21), (24), and (26). The dispersion law and the relations between the quantities $\Omega_{\pm1}$ and $Q_{\pm1}$ are given by Eqs. (5) and (18), respectively.

In the present work, we have used the reduction perturbation expansion for the Rayleigh wave under the condition of acoustic SIT to obtain a resonance surface acoustic vector soliton with two different oscillation frequencies (sum $\omega + \Omega_{+1}$ and difference $\omega - \Omega_{-1}$). With this, we have also extended the theory of acoustic SIT for surface solitons and breathers to two-component resonance surface acoustic vector solitons, wherein it is essential to take into account the "forbidden" transitions in the resonance impurity paramagnetic atoms. It should be noted that the system of equations (10) also are valid in the case when the ϵ_{xx} component of the Rayleigh wave at the frequency ω_S excites only "allowed" transitions in paramagnetic impurities with an effective electron spin $S = 1$ [24].

We consider here the properties of the vector soliton with two different oscillation frequencies of the Rayleigh wave, but we can analogously spread this theory to other types of surface acoustic waves, for instance, to Stoneley and Love waves.

We investigate the processes of the formation of the resonance vector solitons of the Rayleigh wave in dielectric crystals with a small concentration of paramagnetic impurities with electron and nuclear spins. Although the electron-nuclear interaction in quantum dots is qualitatively different from that in paramagnetic crystals, the approach which we have presented here is of a rather general type and, after simple modification, can be used also for the consideration of the formation of acoustic vector solitons in coupled electron-nuclear spin systems in quantum dots [17].

Using typical parameters for the pulse, the materials, and the paramagnetic impurities [25], we can construct a plot of the ε_{xz} component of the deformation tensor for a two-component acoustic vector soliton of the Rayleigh wave (shown in Fig. 1 for a fixed value of the z coordinate).

The vector solitons and the scalar solitons and breathers may arise, for instance, in dielectrics coated with thin layers containing paramagnetic impurities or quantum dots. Methods of excitation of the nonlinear wave Eq. (25) are typical methods which are usually used for the excitation of soliton-like surface acoustic waves (see, for instance, Ref. [8]). The simplest way to excite a nonlinear surface wave Eq. (25) is to excite an initial wave at $z = 0$, the profile of which is as close as possible to Eq. (25) with the sum $\omega + \Omega_{+1}$ and difference $\omega - \Omega_{-1}$ oscillation frequencies.

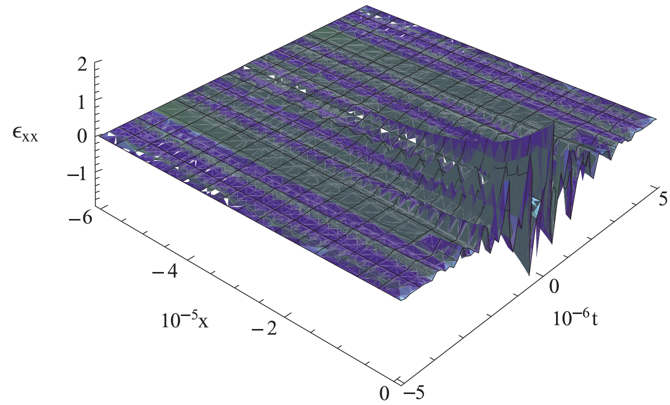

FIG. 1. Plot of the ε_{xx} (in arbitrary units) component of the deformation tensor at a fixed value of the z coordinate showing the two-dimensional vector soliton of the Rayleigh wave and the transverse structure of the pulse when the resonant "forbidden" transition monolayer containing the ensemble of resonance paramagnetic atoms is present. Along the x axis, the vector soliton amplitude decays exponentially as one moves away from the surface at $x = 0$. At $x = 0$, the profile of the vector soliton corresponds to the solution of Eq. (25).

These results are expected to stimulate new experiments. Such work will be informative not only for the study of the properties of acoustic surface resonance vector solitons, but also for applications in paramagnetic crystals and doped nanostructures to give us wider possibilities of applications of nonlinear surface acoustic waves in different acoustic devices.

[1] G. T. Adamashvili and D. J. Kaup, Phys. Lett. A **376**, 432 (2012).
[2] G. T. Adamashvili, N. T. Adamashvili, G. N. Motsonelidze, and M. D. Peikrishvili, Semiconductors **40**, 257 (2006).
[3] S. L. McCall and E. L. Hahn, Phys. Rev. **183**, 457 (1969).
[4] G. T. Adamashvili, Phys. Rev. E **85**, 067601 (2012).
[5] A. A. Kolomenskii, V. A. Lioubimov, S. N. Jerebtsov, and H. A. Schuessler, Rev. Scien. Instrum. **74**, 448 (2003).
[6] P. Hess and A. M. Lomonosov, Ultrasonics **50**, 167 (2010).
[7] C. Eckl, A. P. Mayer, and A. S. Kovalev, Phys. Rev. Lett. **81**, 983 (1998).
[8] C. Eckl, A. S. Kovalev, A. P. Mayer, A. M. Lomonosov, and P. Hess, Phys. Rev. E **70**, 046604 (2004).
[9] T. Sakuma and Y. Kawanami, Phys. Rev. B **29**, 869 (1984).
[10] G. T. Adamashvili, Phys. Lett. A **130**, 350 (1988).
[11] G. T. Adamashvili, Solid State Comm. **47**, 497 (1983).
[12] D. M. Barnett, J. Lothe, S. D. Gavazza, and M. J. P. Musgrave, Proc. R. Soc. London A **402**, 1822 (1985).
[13] P. J. S. van Capel, E. Peronneb, and J. I. Dijkhuis, Ultrasonics **56**, 36 (2015).
[14] A. Devos, Ultrasonics **56**, 90 (2015).
[15] V. V. Kozhushko and P. Hess, Ultrasonics **48**, 488 (2008).
[16] G. T. Adamashvili, Sov. Phys. JETP **65**, 1242 (1987).
[17] B. Urbaszek, X. Marie, T. Amand, O. Krebs, P. Voisin, P. Maletinsky, A. Hogele, and A. Imamoglu, Rev. Mod. Phys. **85**, 79 (2013).
[18] G. T. Yokoyama and M. Eto, in *IEEE International Symposium on Access Spaces (ISAS)* (IEEE, Keio University, Yokohama, Japan, 2011), p. 36.
[19] L. D. Landau and E. M. Lifshitz, *Theory of Elasticity* (Pergamon Press, Oxford, 1980), p. 200.
[20] J. A. Weil, J. R. Bolton, and J. E. Wertz, *Electron Paramagnetic Resonance: Elementary Theory and Practical Applications* (Wiley-Interscience, New York, 2001).
[21] A. I. Maimistov, A. M. Basharov, S. O. Elyutin, and Y. M. Sklyarov, Phys. Rep. **191**, 1 (1990).
[22] L. Allen and J. Eberly, *Optical Resonance and Two Level Atoms* (Dover, Mineola, NY, 1975).
[23] T. Taniuti and N. Iajima, J. Math. Phys. **14**, 1389 (1973).
[24] N. S. Shiren, Phys. Rev. B **2**, 2471 (1970).
[25] Parameters for the numerical simulation: $\omega = 2\pi \times 10^{10}$ Hz, $T = 2.35 \times 10^{-6}$ s, $n_0 = 10^{18}$ cm^{-3}, $\Omega_{+1} = 1.26 \times 10^9$ Hz, $\Omega_{-1} = 1.19 \times 10^9$ Hz, $c_t = 5 \times 10^5$ cm/s, $c_l = 7 \times 10^5$ cm/s, $H_0 = 2050\ G$, $F_{xxxx} = 725$ cm^{-1}, full-width half-maximum inhomogeneous broadening $T_2^* = \pi g(0) = 3 \times 10^{-9}$ s, $A = 44\ \mu$eV.