Fermionic thermocoherent state: Efficiency of electron transport

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On the basis of the fermionic coherent state of Cahill and Glauber [Phys. Rev. A **59**, 1538 (1999)], we have introduced here the fermionic thermocoherent state in terms of the quasiprobability distribution which shows the appropriate thermal and coherent limits as in the bosonic case or the Glauber-Lachs state. It is shown that the fermionic thermocoherent state can be realized as a displaced thermal state of fermions. Its relation with the fermionic displaced number state and the fermion-added coherent state are explored in the spirit of the bosonic case. We have investigated the nature of the average current and the suppression of noise due to the thermocoherent character of the source. The theory is applied to the problem of electronic conduction. A modification of the Landauer conductance formula is suggested which reflects the role of nonzero coherence of the source in electron transport.

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I. INTRODUCTION

Recently the electron transport [1–3] properties of small systems with a discrete energy level structure have attracted much interest where the quantum dynamics are dominated by coherent effects. For example, quantum tunneling in a system of self-assembled quantum dot array reveals antibunching [4,5] and near lifetime limited linewidths [6]. Furthermore, in an effort to achieve coherent control of single electron or electron spin, quantum transport has been used to detect the quantized motion of electrons in nanostructures [7–9]. A great deal of theoretical [10] and experimental [11] results are already available which involve a coherent transfer of electrons from the source to sink reservoirs maintained at different chemical potentials [2,12]. The availability of a coherent source of electrons [13] can be utilized to inject coherence [13,14] into the thermal electron source for the suppression of noise [15,16] as observed in terms of the interference effects [3,5,9] and the correlation function of electron sources [17,18].

The characteristics of fully or partially coherent sources are well understood for bosonic systems [19–25] and the thermocoherent state [26–29] in the case of light was conceived long ago by Lachs [27]. Here our main objective is to construct a fermionic thermocoherent state similar to the bosonic thermocoherent state to understand more about the coherent characteristics of electronic conduction problems [15,16]. Although fermionic coherent states were studied in many contexts [25,30] such as the spin coherent state [31] and quantum states of many-electron systems [32], however, Cahill and Glauber [33] in 1999 first systematically introduced the fermionic coherent state and the corresponding quasiprobability distribution functions similar to the bosonic coherent state [21]. In contrast to the bosonic case, the vacuum state of the fermionic field is the only physically realizable eigenstate of the annihilation operator which makes the formal definition of the fermionic coherent state little special [33]. Again since fermion field variables anticommute, the eigenvalues are anticommuting numbers obeying rules of Grassmann algebra which in the corresponding bosonic case are in general complex. However, even with such mathematical differences, one can find close resemblance in the quasiprobability distributions and various moments in both cases.

Before proceeding further about the fermionic thermocoherent state let us first briefly outline about the thermocoherent state of light. In their pioneering works, Glauber [19] and Sudarshan [20] have independently introduced the expansion of the density operator in terms of the coherent states of a harmonic oscillator to obtain the quantum mechanical description of the superposition of electromagnetic fields. Thereby the quasiprobability distribution function [22] of the position and momentum of a harmonic oscillator are found by this expansion and the connection between these with electric and magnetic fields are established. This approach is extended further [21,26,27] to derive some quantum properties of radiation which is a mixture of thermal and coherent radiation. The resultant field thus can be expressed in terms of the probability densities, the average photon numbers, and the average photon number fluctuations of the constituent fields in a particular mode by the expansion of the density operator.

The coherent state [21] $|\alpha\rangle$, which is an eigenstate of the photon annihilation operator, is utilized to compute the quantum mechanical description of the superposition of the electromagnetic field like in the classical case. The density operator is expanded in coherent state space [19,20] as

$$\rho = \int d^2\alpha |\alpha\rangle\langle\alpha| P(\alpha), \tag{1}$$

where the weight factor $P(\alpha)$ is known as the quasiprobability distribution function or P distribution in the sense that although it bears a classical analog by the normalization relation, $\int d^2\alpha\, P(\alpha) = 1$, the distribution function can take negative values due to the intrinsic quantum character of the coherent state or more precisely the coherent state is becoming an overcomplete set of states. Besides this P-distribution function in the coherent-state representation other quasiprobability distributions including the Wigner function and the Q function [22] play similar convenient roles in representing the density operator. Glauber [21,26] and Lachs [27] have shown that such a distribution, $P(\alpha)$, can also be defined for a superposition of two fields with P distributions $P_1(\alpha)$ and $P_2(\alpha)$ with a resultant P distribution as a convolution integral,

$$P(\alpha) = \int P_1(\alpha^{'}) P_2(\alpha - \alpha^{'}) d^2\alpha^{'}. \tag{2}$$

Using the P distribution of the corresponding coherent and thermal fields [21,27], one can thus obtain the P distribution of the thermocoherent state. Now going back to the number state representation the photon distribution of a single mode field with frequency ω in the thermocoherent or Glauber-Lachs state is given by [26,27,29]

$$P^{\rm GL}(n) = \rho^{\rm GL}_{nn} = \frac{\bar{n}_T^n}{(1+\bar{n}_T)^{1+n}} \times \exp\left(-\frac{\bar{n}_c}{1+\bar{n}_T}\right) L_n\left(-\frac{\bar{n}_c}{\bar{n}_T(\bar{n}_T+1)}\right), \quad (3)$$

where $\bar{n}_T = e^{-\frac{\hbar\omega}{kT}}(1-e^{-\frac{\hbar\omega}{kT}})^{-1}$ is the thermal average photon number corresponding to the Bose-Einstein distribution and $\bar{n}_c = |\alpha|^2$ is the coherent-average photon number for the Poisson distribution of the coherent state. L_n is the Laguerre polynomial [29,34,35] expressed as $L_n(x) = \sum_{i=0}^{n}(-x)^i \frac{n!}{(i!)^2(n-i)!}$. When the coherent-average excitation number $\bar{n}_c$ vanishes, the distribution becomes a thermal one corresponding to an equilibrium state at temperature T, i.e.,

$$\text{Lim}_{\bar{n}_c\to 0}\rho^{\rm GL}_{nn} = \frac{\bar{n}_T^n}{(1+\bar{n}_T)^{n+1}}. \quad (4)$$

Similarly when the thermal-average excitation number $\bar{n}_T$ vanishes the state assumes a coherent state with the distribution becoming Poissonian, i.e.,

$$\text{Lim}_{\bar{n}_T\to 0}\rho^{\rm GL}_{nn} = e^{-\bar{n}_c}\frac{\bar{n}_c^n}{n!}. \quad (5)$$

The mean and variance of the average photon number distribution has also been calculated [21], respectively, as

$$\bar{n} = \bar{n}_T + \bar{n}_c, \quad (6)$$

and

$$\sigma_n^2 = \langle n^2\rangle_{\rm TC} - \langle n\rangle^2_{\rm TC} = 2\bar{n}_T\bar{n}_c + \bar{n}_c + \bar{n}_T + \bar{n}_T^2. \quad (7)$$

For the coherent state the variance in the photon number is equal to the average photon number, however, for the thermocoherent state the excess variance of the photon number over the Poisson distribution of coherent radiation causes photon bunching [23]. Another remarkable property of P distribution is the superposition of the constituent mixture of quantized fields which is like the addition of corresponding classical electromagnetic waves above the common zero-point field. The Glauber-Lachs state can be prepared if an external Gaussian pulse of light excites a cavity mode initially in a thermal distribution and subsequently thermalization takes place before the mode exchanges its energy with any other mode or bath [28]. The probability of photon number distribution for the thermocoherent field interpolates between the thermal and coherent limits [21,28,29]. This can be mathematically treated either by convolution of quasiprobability P-distribution function or by unitarily displacing a thermal state (see Appendix A for a brief outline).

In the bosonic case the study of the mixture of quantized fields is basically motivated by the classical superposition of electromagnetic fields. However, for the fermionic case such superposition of fields may not be so straightforward as there is no such classical correspondence. Here we would like to study the fate of such mixing of thermal and coherent fermionic fields using the approach of the P distribution function. Then we have also shown quite extensively the utility of such a mixture of fermionic fields in the conductance problem. More specifically in this work we have derived here the fermionic thermocoherent state using the definition of the fermionic coherent state and the corresponding quasiprobability distribution functions of Cahill and Glauber [33]. Then we have investigated the alternative physical features of the fermionic thermocoherent state by showing it as the fermionic displaced thermal state and explored its relation with the fermionic displaced number state and the fermion-added coherent state as in the bosonic case. As an immediate application of the fermionic thermocoherent state, we have examined its role in electron transport through a quantum system if the source reservoir is maintained in a thermocoherent state while the sink reservoir in the usual thermal state. The motivation is to study the modification of the current noise [24,36,37] due to the coherent character in the source reservoir. Here, we have constructed the relevant equation of the density operator [24,36] to obtain the current through the system in both the steady state and transient regime. The steady-state behavior of the current is monitored in terms of the current noise spectrum [38,39] and the possible modification of conductance [15,16,40] is shown in this context.

The rest of the paper is organized as follows: After a brief outline of the bosonic thermocoherent state in the introduction in Sec. II we have provided a derivation of the fermionic thermocoherent state as a result of the superposition of the thermal and coherent P-distribution functions for fermions. In the next section, we have established how to relate the thermocoherent state with the displaced thermal state and displaced number states. In Sec. IV we have discussed an application of the fermionic thermocoherent state showing the results of electron transport in the steady state and dynamic regime and the noise spectrum through a single-level quantum system connected to a thermal sink but with a thermocoherent source. Finally the article is concluded in Sec. V.

II. FERMIONIC THERMOCOHERENT STATE

In this section our aim is to formulate a fermionic thermocoherent state using the definition of the fermionic coherent state by Cahill and Glauber [33]. Here it is shown that the mathematical methods that have been used to analyze the properties of the bosonic thermocoherent field and the thermal and coherent limits in quantum optics, have their counterparts for the fermionic field. In particular, using the close analogs of the bosonic coherent states, the displacement operators, the P representation, and the other operator expansions we have described, the quantum statistical features of the fermionic thermocoherent coherent state are based upon the Grassmann calculus of anticommuting variables.

The normalized fermionic coherent state [33] is defined as

$$|\xi\rangle = D(\xi)|0\rangle, \quad (8)$$

with $D(\xi)$ being the spin displacement operators defined as

$$D(\xi) = e^{a^\dagger\xi - \xi^* a} = 1 + (a^\dagger\xi - \xi^* a) + \left(a^\dagger a - \tfrac{1}{2}\right)\xi^*\xi, \quad (9)$$

where a and $a^\dagger$ are the fermionic step-down and step-up operators defined as $a|1\rangle = |0\rangle$ and $a^\dagger|0\rangle = |1\rangle$, respectively, with the anticommutaion relation $\{a,a^\dagger\}_+ = 1$. For any mode i, ξ_i and ξ_i^* are Grassmann numbers for the corresponding mode obeying the following anticommutation relations:

$$\{\xi_i,\xi_j\}_+ = 0, \tag{10}$$

$$\{\xi_i^*,\xi_j\}_+ = 0, \tag{11}$$

$$\{\xi_i^*,\xi_j^*\}_+ = 0. \tag{12}$$

We also assume [33] the anticommutation of Grassmann numbers ξ_i and ξ_i^* with the operators a and $a^\dagger$, for example,

$$\{\xi_k,a\}_+ = 0. \tag{13}$$

For the single mode fermion, the s-ordered characteristic function $\chi(\xi,s)$ can be expressed as

$$\chi(\xi,s) = \mathrm{Tr}[\rho(1 + (\xi a^\dagger - a\xi^*) + \xi^*\xi\{a^\dagger a\}_s)], \tag{14}$$

with $\{a^\dagger a\}_s = a^\dagger a + \frac{1}{2}(s-1)$, where $s = 1$ means normal ordering, $s = -1$ means antinormal ordering, and $s = 0$ means symmetrically ordered product, and ρ is the density operator.

The s-ordered quasiprobability distribution function is again defined as

$$W(\alpha,s) = \int d^2\xi\, e^{(\alpha\xi^* - \xi\alpha^*)}\chi(\xi,s), \tag{15}$$

where α and α^* are also Grassmann numbers.

By analogy with boson, the normally ordered quasiprobability distribution function, $P(\alpha) = W(\alpha,1)$ is defined as

$$P(\alpha) = -\int d^2\beta\, e^{-(\alpha-\beta)(\alpha^*-\beta^*)}Q(\beta), \tag{16}$$

where the antinormal ordered quasiprobabilty distribution function Q is defined as

$$Q(\beta) = \langle\beta|\rho| - \beta\rangle. \tag{17}$$

While deducing Eq. (16), we have considered that $a|\beta\rangle = \beta|\beta\rangle$ and $\langle\beta|a^\dagger = \langle\beta|\beta^*$ along with the trace formula,

$$\mathrm{Tr}B = \int d^2\beta\langle\beta|B| - \beta\rangle = \int d^2\beta\langle -\beta|B|\beta\rangle, \tag{18}$$

and the completeness relation,

$$\int d^2\beta|\beta\rangle\langle\beta| = I. \tag{19}$$

The single mode fermion density operator for a thermal state can be expressed as [33]

$$\rho = (1-\bar{n}_T)\left(\frac{\bar{n}_T}{1-\bar{n}_T}\right)^{a^\dagger a}, \tag{20}$$

where $\bar{n}_T$ is the mean occupation number defined as the Fermi-Dirac distribution function,

$$\bar{n}_T = \frac{1}{e^{\frac{\hbar\omega-\mu}{kT}} + 1}, \tag{21}$$

where μ is the chemical potential per particle and $\hbar\omega$ is the energy for the respective mode. After having a little algebraic manipulation using Eqs. (17)–(21), we have the Q-distribution function for a single mode thermal fermionic state as

$$Q(\beta) = \exp\left(\frac{\beta\beta^*}{1-\bar{n}_T}\right). \tag{22}$$

The P-distribution function of the thermal state from Eq. (16) becomes

$$P(\alpha) = -\bar{n}_T e^{-\frac{\alpha\alpha^*}{\bar{n}_T}}. \tag{23}$$

Now we evaluate the P-distribution function for a fermionic coherent state defined by the density operator $\rho = |\alpha_0\rangle\langle -\alpha_0|$ as

$$\begin{aligned} P(\alpha) &= -\int d^2\beta\, e^{-(\alpha-\beta)(\alpha^*-\beta^*)}\langle\beta|\alpha_0\rangle\langle -\alpha_0| - \beta\rangle \\ &= \delta(\alpha-\alpha_0). \end{aligned} \tag{24}$$

Construction of the above Eq. (23) utilizes the formula [33],

$$\langle\beta|\alpha\rangle\langle\alpha|\beta\rangle = e^{-(\beta^*-\alpha^*)(\beta-\alpha)}, \tag{25}$$

and the definition of the delta function [33] as

$$\delta(\xi-\zeta) = \int d^2\alpha\, e^{\alpha(\xi^*-\zeta^*)-(\xi-\zeta)\alpha^*}. \tag{26}$$

We are now in a position to construct the thermocoherent state for a fermionic system by the superposition of the thermal and coherent P-distribution functions $P_1(\alpha_1)$ and $P_2(\alpha_2)$ in terms of the density operator χ as

$$\chi = \int d^2\alpha_2 P_2(\alpha_2)D(\alpha_2)\rho_1 D^\dagger(-\alpha_2), \tag{27}$$

where

$$\rho_1 = \int d^2\alpha_1 P_1(\alpha_1)|\alpha_1\rangle\langle -\alpha_1|, \tag{28}$$

where $P_1(\alpha)$ and $P_2(\alpha)$, respectively, represent the P distributions of the thermal and coherent states. Considering the properties of the displacement operators [33],

$$D(\alpha_1)D(\alpha_2) = D(\alpha_1+\alpha_2)e^{\alpha_1^*\alpha_2-\alpha_2^*\alpha_1}, \tag{29}$$

and

$$D^\dagger(-\alpha_1)D^\dagger(-\alpha_2) = D^\dagger(-\alpha_2-\alpha_1)e^{\alpha_2^*\alpha_1-\alpha_1^*\alpha_2}, \tag{30}$$

the resultant P-distribution function comes out as

$$P(\alpha) = \int d^2\alpha_2 P_1(\alpha-\alpha_2)P_2(\alpha_2), \tag{31}$$

which gives

$$P(\alpha) = -\bar{n}_T\exp\left(-\frac{(\alpha-\alpha_0)(\alpha^*-\alpha_0^*)}{\bar{n}_T}\right), \tag{32}$$

with the corresponding density operator,

$$\chi = -\bar{n}_T\int d^2\alpha\exp\left(-\frac{(\alpha-\alpha_0)(\alpha^*-\alpha_0^*)}{\bar{n}_T}\right)|\alpha\rangle\langle -\alpha|. \tag{33}$$

In what follows to evaluate the integral we use the following Fourier transform formula involving Grassmann calculus [33] as

$$\int d^2\xi\exp(\alpha\xi^* - \xi\beta^* + \lambda\xi\xi^*) = \lambda\exp\left(\frac{\alpha\beta^*}{\lambda}\right). \tag{34}$$

Using Eq. (34), we calculate the diagonal matrix elements of the thermocoherent density matrix as

$$\langle 1|\chi|1\rangle = \chi_{11} = 1 - (1-\bar{n}_T)\exp\left(\frac{\alpha_0\alpha_0^*}{1-\bar{n}_T}\right), \quad (35)$$

and

$$\langle 0|\chi|0\rangle = \chi_{00} = -\chi_{11} -\bar{n}_T\int d^2\alpha \exp\left(-\frac{(\alpha-\alpha_0)(\alpha^*-\alpha_0^*)}{\bar{n}_T}\right), \quad (36)$$

so that

$$\chi_{00} + \chi_{11} = 1. \quad (37)$$

At this point, to have a correspondence of a fermionic thermocoherent state with that of a bosonic one, we evaluate the mean occupation number, $\langle n\rangle_{\rm TC}$ for a fermionic thermocoherent state as

$$\langle n\rangle_{\rm TC} = \sum_{n=0,1} n\chi_{nn} = 1-(1-\bar{n}_T)e^{-\frac{\bar{n}_c}{1-\bar{n}_T}} = \bar{n}_T + \bar{n}_c, \quad (38)$$

and the mean square occupation number $\langle n^2\rangle_{\rm TC}$ as

$$\langle n^2\rangle_{\rm TC} = \sum_{n=0,1} n^2\chi_{nn} = 1-(1-\bar{n}_T)e^{\frac{-\bar{n}_c}{1-\bar{n}_T}}, \quad (39)$$

where $\bar{n}_c = \alpha_0^*\alpha_0$. Hence the variance becomes

$$(\Delta n)^2_{\rm TC} = \langle n^2\rangle_{\rm TC} - \langle n\rangle^2_{\rm TC} = (1-\bar{n}_T)e^{-\frac{\bar{n}_c}{1-\bar{n}_T}}\left[1-(1-\bar{n}_T)e^{-\frac{\bar{n}_c}{1-\bar{n}_T}}\right]. \quad (40)$$

In appropriate thermal and coherent limits, Eq. (40) gives the following result:

$$\mathrm{Lim}_{\bar{n}_T\to 0}(\Delta n)^2_{\rm TC} = \bar{n}_c, \quad (41)$$

and

$$\mathrm{Lim}_{\bar{n}_c\to 0}(\Delta n)^2_{\rm TC} = \bar{n}_T(1-\bar{n}_T). \quad (42)$$

Equation (41) is exactly similar to the thermocoherent bosonic field in the coherent limit [see Eq. (7)]. On the contrary, the fermionic thermocoherent field corresponding to the thermal limit $\bar{n}_c \to 0$ [see Eq. (42)] is different from the bosonic case by a minus sign [see Eq. (7)]. This is due to the fermionic anticorrelation between the particles and the expression is already deduced by Cahill and Glauber [33] for the thermally chaotic fermion field. Such a remarkable similarity in the fermionic domain is essentially a major outcome of the crucial roles played by the Grassmann numbers. This is due to the fact that any polynomial in the Grassmann number should be linear making the higher order terms in the exponential function appearing in Eqs. (38) and (40) vanish which produces a similar mathematical structure of the moments in the bosonic and fermionic thermocoherent states.

III. FERMIONIC DISPLACED THERMAL STATE, DISPLACED NUMBER STATE, AND FERMION-ADDED COHERENT STATES USING DISPLACEMENT OPERATOR APPROACH

In the mid-1960s Glauber [26] and Lachs [27] had introduced the bosonic thermocoherent state by the approach of superposition of quasiprability distributions; Filipowicz [28] has also defined it in terms of the displaced thermal state by coherently driving a cavity mode in a thermal equilibrium state. However, in fermionic system these two definitions appear very different as the eigenvalues of the annihilation operator are anticommuting Grassmann numbers and the vacuum state is the only eigenstate of the annihilation operator. Here we have studied the properties of the fermionic thermocoherent state by the approach of the fermionic displacement operator. Using Grassmann algebra we arrive at the fermionic thermocoherent state by unitarily displacing the thermal state and we have shown their relation with the fermionic displaced number states and the fermion-added coherent state. This approach is subsequently used to study the effect of coherence in the thermal electron transport [15] which is purely quantum mechanical in origin. Here we have shown that it is possible to introduce coherence in the thermal fermion bath modes resulting in a thermocoherent bath by coherently driving a thermal bath followed by equilibration.

A. Fermionic thermocoherent state as displaced thermal state

The density operator for the fermionic thermal state can be obtained as a steady-state solution of the quantum master equation for a fermionic mode coupled to a thermal fermion bath. The master equation in the Markov limit for the reduced density operator $\rho_T(t)$ is given by

$$\dot{\rho_T}(t) = \hat{L}\rho_T(t), \quad (43)$$

where

$$\hat{L}\rho_T(t) = -\Big\{\frac{\gamma}{2}(1-\bar{n}_T)[a^\dagger a\rho_T(t) - a\rho_T(t)a^\dagger] + \frac{\gamma}{2}\bar{n}_T[aa^\dagger\rho_T(t) - a^\dagger\rho(t)a] - \frac{\gamma}{2}\bar{n}_T[a^\dagger\rho_T(t)a - \rho_T(t)aa^\dagger] - \frac{\gamma}{2}(1-\bar{n}_T)[a\rho_T(t)a^\dagger - \rho_T(t)a^\dagger a]\Big\}. \quad (44)$$

The unitarily displaced density operator ρ_d can be expressed as

$$\rho_d = D(\alpha)\rho_T D^\dagger(-\alpha), \quad (45)$$

where $D(\alpha)$ is the single mode fermionic displacement operator as $D(\alpha) = e^{a^\dagger\alpha - \alpha^* a}$. Equation (45) bears a minor dissimilarity of a minus sign with that of the displaced thermal state of a harmonic oscillator [41]. The reason for such a structural difference can be traced in the intricate nature of the Grassmann numbers which makes its presence felt in the density operator for the fermionic coherent state as $\rho = |\alpha\rangle\langle -\alpha|$, different by a minus sign in the bosonic case.

The density operator for the thermal state for a single mode fermion can be obtained from the steady-state solution of Eq. (44) as

$$\rho_T = (1-\bar{n}_T)|0\rangle\langle 0| + \bar{n}_T|1\rangle\langle 1|, \quad (46)$$

where $\bar{n}_T$ is the mean occupation number [see Eq. (21)].

Keeping the following standard relations [33,42],

$$D(\alpha)|1\rangle\langle 1|D^\dagger(-\alpha) = -\alpha^*\alpha|0\rangle\langle 0| - \alpha^*|0\rangle\langle 1| + \alpha|1\rangle \times\langle 0| + |1\rangle\langle 1|(1+\alpha^*\alpha), \quad (47)$$

and

$$D(\alpha)|0\rangle\langle 0|D^{\dagger}(-\alpha) = -\alpha^{*}|0\rangle\langle 1| + \alpha|1\rangle\langle 0| + |0\rangle \times\langle 0|(1-\alpha^{*}\alpha) + \alpha^{*}\alpha|1\rangle\langle 1|, \quad (48)$$

we can get the matrix elements of the density operator for the displaced thermal state as

$$\langle 0|\rho_d|0\rangle = (1-\bar{n}_T)(1-\bar{n}_c) - \bar{n}_T\bar{n}_c, \quad (49)$$

and

$$\langle 1|\rho_d|1\rangle = (1-\bar{n}_T)\bar{n}_c + \bar{n}_T(1+\bar{n}_c), \quad (50)$$

with $\bar{n}_c = \alpha^{*}\alpha$. Now, using Eqs. (49) and (50), we can evaluate the average occupation number for the displaced thermal state for a single mode fermion as

$$\langle n\rangle = \sum_{n=0,1} n\langle n|\rho_d|n\rangle = \bar{n}_T + \bar{n}_c. \quad (51)$$

Here, we see that Eq. (51) gives the same result as that of Eq. (38) which means the fermionic thermocoherent state can be called a displaced thermal state. This is very much similar to the idea of coherently driving a thermal bosonic field to prepare a bosonic thermocoherent state. Here again we must emphasize that the key role lies in Grassmann algebra which gives such a close one-to-one correspondence.

B. Displaced number states and fermion-added coherent state

Here we establish the connection between the fermionic thermocoherent state with the displaced number states and the fermion-added coherent state as often defined in the bosonic case. The primary idea is to start from the definition of the unitarily displaced fermionic thermal state which is equivalent to the fermionic thermocoherent state and to express the latter as a linear combination of displaced number states.

Following Ref. [33], the displaced number state can conveniently be expressed as

$$|n,\alpha\rangle = D(\alpha)|n\rangle, \quad (52)$$

and

$$\langle n,-\alpha| = \langle n|D^{\dagger}(-\alpha), \quad (53)$$

with $n = 0,1$. Hence, the displaced thermal state can be expressed as

$$\rho_d = (1-\bar{n}_T)D(\alpha)\left(\frac{\bar{n}_T}{1-\bar{n}_T}\right)^{a^{\dagger}a} D^{\dagger}(\alpha) \times\left\{\sum_{m=0,1}|m,\alpha\rangle\langle m,-\alpha|\right\}, \quad (54)$$

where $a(a^{\dagger})$ is the fermionic annihilation (creation) operator. Defining $\bar{n}_T = 1/(e^{\lambda}+1)$, we obtain

$$\left(\frac{\bar{n}_T}{1-\bar{n}_T}\right)^{a^{\dagger}a} = 1 - \lambda a^{\dagger}a. \quad (55)$$

The simplified form of Eq. (55) is markedly due to the fact that the higher orders of the fermionic operators always vanish which stems from the Pauli exclusion principle.

Inserting the operator identity $D(\alpha)D^{\dagger}(\alpha) = 1$ twice in Eq. (55), we get

$$\left(\frac{\bar{n}_T}{1-\bar{n}_T}\right)^{a^{\dagger}a} = 1 - \lambda(D(\alpha)aD^{\dagger}(\alpha))^{\dagger}D(\alpha)aD^{\dagger}(\alpha). \quad (56)$$

We are now in a position to define displaced fermionic operators as

$$D(\alpha)aD^{\dagger}(\alpha) = a - \alpha, \quad (57)$$

and

$$D(\alpha)a^{\dagger}D^{\dagger}(\alpha) = a^{\dagger} - \alpha^{*}. \quad (58)$$

Hence, Eq. (54) can be re-expressed as

$$\rho_d = (1-\bar{n}_T)\left(\frac{\bar{n}_T}{1-\bar{n}_T}\right)^{(a^{\dagger}-\alpha^{*})(a-\alpha)}\left\{\sum_{m=0,1}|m,\alpha\rangle\langle m,-\alpha|\right\}. \quad (59)$$

Therefore, the fermionic thermocoherent state can be expressed as a mixture of orthogonal states which are basically the displaced number states like in the bosonic case [41]. Hence, for a multimode fermionic thermocoherent reservoir, the density operator can be expressed as

$$\rho^{\mathrm{TC}}_{\mathrm{bath}} = \Pi_k\left[(1-\bar{n}_k)\left(\frac{\bar{n}_k}{1-\bar{n}_k}\right)^{(a_k^{\dagger}-\alpha^{*})(a_k-\alpha)} \times\left\{\sum_{m_k=0,1}|m_k,\alpha\rangle\langle m_k,-\alpha|\right\}\right]. \quad (60)$$

It is be noted that although a reservoir is generally considered as intrinsically incoherent, however, here we have shown that it is possible to make a reservoir which is a partially coherent source of particles. This motivates us to study a simple transport problem involving a single-level system connected to two reservoirs, one of which is in a thermocoherent state.

IV. APPLICATION TO ELECTRON TRANSPORT

In this section, we discuss the role of the source reservoir which is in a fermionic thermocoherent state in the context of the electron transport process through a single-level quantum system. For the model of quantum current we consider an arrangement of source-system-sink where the source and the sink are fermionic reservoirs. After giving a brief discussion about the master equation for transport we apply it to build the equation of motion for current. In the next subsection we show the steady-state limit to calculate the Fano factor to probe the current noise spectrum.

A. Steady state and dynamic regimes of quantum transport: Modification of conductance formula

Here we consider a quantum system coupled to both the source and sink of electrons as two fermionic reservoirs. We consider the source bath is in a thermocoherent state and the sink is in a thermal state.

To start with, we consider the total Hamiltonian H_T as

$$H_T = H_s + H_E + H_C + H_{\mathrm{int}} \quad (61)$$

where H_s is the system Hamiltonian expressed as

$$H_s = \hbar\omega_0 c^\dagger c, \tag{62}$$

H_E is the Hamiltonian of the source or emitter expressed as

$$H_E = \hbar \sum_k \omega_k^E a_k^\dagger a_k, \tag{63}$$

H_C is the Hamiltonian of the sink or collector expressed as

$$H_C = \hbar \sum_p \omega_p^C b_p^\dagger b_p, \tag{64}$$

and the interaction Hamiltonian H_{int} is expressed as

$$H_{\text{int}} = \hbar \sum_k (T_{Ek} c^\dagger a_k + c a_k^\dagger T_{Ek}^*) + \hbar \sum_p (T_{Cp} b_p^\dagger c + c^\dagger b_p T_{Cp}^*). \tag{65}$$

In Eqs. (62)–(65), $c(c^\dagger)$, $a_k(a_k^\dagger)$, and $b_p(b_p^\dagger)$ are the step-down(step-up) operators for the system, kth emitter mode and pth collector mode, respectively. The system-lead coupling coefficients $T_{Ek}(T_{Ek}^*), T_{Cp}(T_{Cp}^*)$ are considered to obey Grassmann algebra [see Eqs. (10)–(13)].

If $\rho_I(t)$ is the density operator for the system in the interaction picture and ρ_E and ρ_C represent the density operators for the emitter and collector, respectively, then the coarse-grained equation of motion [24] can be expressed as

$$\frac{d\rho_I(t)}{dt} = -\frac{1}{\hbar^2}\int_0^t dt' Tr_R \times [H_{\text{int}}(t),[H_{\text{int}}(t'),\rho_I(t')\otimes\rho_E\otimes\rho_C]]. \tag{66}$$

Putting the reduced density operator of the system after tracing over the emitter and the collector modes for the fermionic bath from Eq. (33)and after performing the Markovian approximation in the Schrodinger picture for weak system-bath coupling, we get

$$\begin{aligned}\frac{d\rho}{dt} = & -i\omega_0[c^\dagger c,\rho] - \frac{1}{2}\big[(c^\dagger c\rho - c\rho c^\dagger)\big(\gamma_c\big(1-\bar{n}_{\text{TC}}^e\big) \\ & - \gamma_e\big(1-\bar{n}_{\text{TC}}^e\big)\big) - (c^\dagger\rho c - \rho c c^\dagger)\big(\gamma_e\bar{n}_{\text{TC}}^e - \gamma_c\bar{n}_{\text{TC}}^c\big)\big] \\ & - \frac{1}{2}\big[(cc^\dagger\rho - c^\dagger\rho c)\big(\gamma_e\bar{n}_{\text{TC}}^e - \gamma_c\bar{n}_{\text{TC}}^c\big) \\ & - (c\rho c^\dagger - \rho c^\dagger c)\big(\gamma_c\big(1-\bar{n}_{\text{TC}}^c\big) - \gamma_e\big(1-\bar{n}_{\text{TC}}^e\big)\big)\big],\end{aligned} \tag{67}$$

where we have assumed that γ_e and γ_c are the corresponding rate constants for the emitter and collector, respectively [24]. In what follows we assume that the collector is in the thermal state with $\bar{n}_T^c = 0$, which physically signifies that the chemical potential of the collector is much smaller than that of the system energy scale, i.e., $\hbar\omega - \mu_c \gg kT$, where μ_c is the chemical potential of the collector (sink). From the emitter side, we consider that the transport of electrons through the system is controlled by both the thermal as well as the coherent parameter of the emitter (source) bath. In the following analysis, we will show the dependence of the current through the system in terms of the scaled parameter $\frac{\hbar\omega-\mu_e}{kT}$, as well as the coherent parameter $\bar{n}_c$, of the source reservoir.

First of all, we define the average current as

$$\langle\hat{i}\rangle(t) = \tfrac{1}{2}[\gamma_c\langle c^\dagger c\rangle + \gamma_e\langle cc^\dagger\rangle], \tag{68}$$

where the occupation number operator averages are calculated as

$$\langle c^\dagger c\rangle(t) = \frac{\gamma_e\bar{n}_{\text{TC}}^e}{2\gamma_e\bar{n}_{\text{TC}}^e - \gamma_e + \gamma_c}\big(1 - e^{-(2\gamma_e\bar{n}_{\text{TC}}^e - \gamma_e + \gamma_c)t}\big) + e^{-(2\gamma_e\bar{n}_{\text{TC}}^e - \gamma_e + \gamma_c)t}, \tag{69}$$

with

$$\langle cc^\dagger\rangle(t) = 1 - \langle c^\dagger c\rangle(t). \tag{70}$$

Hence the steady-state value of the average current by using Eqs. (68)–(70) comes out as

$$\langle i\rangle_{ss} = \frac{1}{2}\left[\frac{\gamma_e\big(\bar{n}_T^e + \bar{n}_C^e\big)(\gamma_e+\gamma_c) - \gamma_e(\gamma_e-\gamma_c)}{2\gamma_e\big(\bar{n}_T^e+\bar{n}_C^e\big) - \gamma_e + \gamma_c}\right], \tag{71}$$

as $\bar{n}_{\text{TC}}^e = \bar{n}_T^e + \bar{n}_C^e$ and here we have considered that the sink is at zero temperature, i.e., $\bar{n}_T^c = 0$. For the usual thermal source $\bar{n}_{\text{TC}}^e = \bar{n}_T^e$ and when the difference in the thermal occupation number of source and sink is unity, i.e., if $\bar{n}_T^e = 1$ and making $\bar{n}_T^e - \bar{n}_T^c = 1$ from the above equation one obtains the Landauer conductance formula [3,16,40],

$$\langle i\rangle_{ss} = \frac{\gamma_e\gamma_c}{\gamma_e+\gamma_c}. \tag{72}$$

However, for the thermocoherent source, i.e., $\bar{n}_C^e \neq 0$, the modified formula of the steady-state current becomes

$$\langle i\rangle_{ss} = \frac{1}{2}\left[\frac{\gamma_e\bar{n}_C^e(\gamma_e+\gamma_c) + 2\gamma_e\gamma_c}{2\gamma_e\bar{n}_C^e + \gamma_e + \gamma_c}\right]. \tag{73}$$

The steady-state current through the system is expressed here in terms of γ_e and γ_c, the rate constants of the flow of electrons from the source and to the sink. The Landauer conductance formula [Eq. (72)] for the system connected with the traditional thermal electron source and sink is modified here by the nonzero coherent average fermion number introduced into the source bath $\bar{n}_C^e$. The main result in this section is the effective modification of the conductance formula coming in Eq. (73) due to the thermocoherent state of the emitter through the coherent and thermal population terms.

A few comments are relevant here regarding the formula of quantum electron transport through nanostructures for which various methods have been developed. Foremost is the Landauer-Buttiker formalism [43], which establishes a basic relationship between scattering amplitudes and currents through nanostructures where the conductance is proportional to the transmission coefficient. Secondly, the nonequilibrium Green's function (NEGF) scheme deals with many-body interaction effects in quantum transport. Haug and Jauho [44] applies NEGF formalism to arrive at the Landauer formula of conductance [3,44,45]. However, recently approaches of quantum optics [23,36] are also applied to study time-dependent transport processes through solid-state structures. In the present context, the Landauer formula for conductance which is given here expresses the average of the time-dependent current without considering any inelastic scattering processes. Equation (71) carries a signature of coherence in the electron conductance formula coming from the source electron

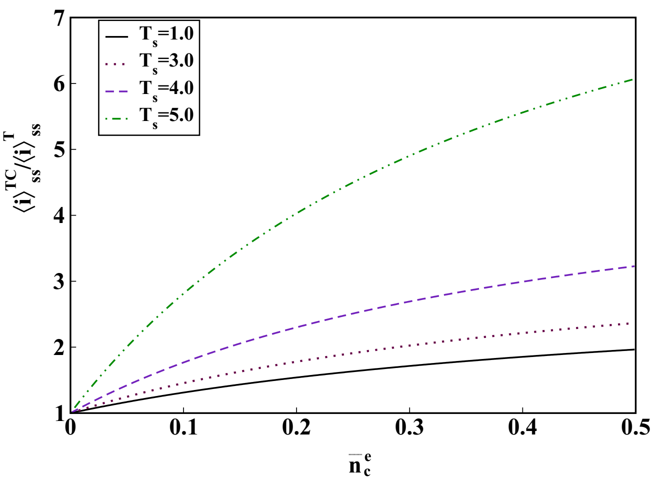

FIG. 1. Plot of $\frac{\langle i\rangle_{ss}^{\mathrm{TC}}}{\langle i\rangle_{ss}^{T}}$ versus the coherent population of the emitter $\bar{n}_c^e$ for various scaled temperatures $T_s = \frac{kT}{\hbar\omega_0-\mu_e}$. Increase in the value of $\frac{kT}{\hbar\omega_0-\mu_e}$ causes an enhancement of the steady-state current with the coherent population of the emitter.

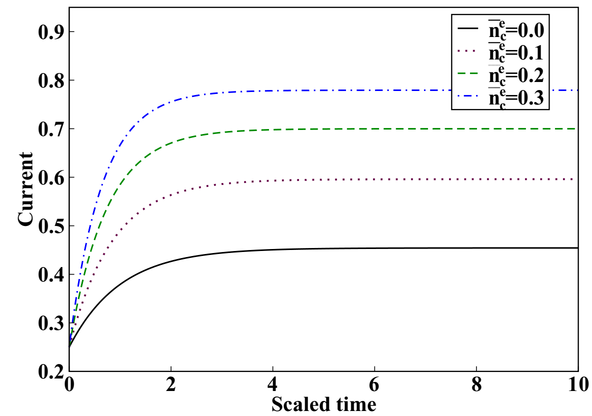

FIG. 2. Plot of $\langle i\rangle^{\mathrm{TC}}(t)$ against scaled time [in units of $(\gamma_e+\gamma_c)^{-1}$] for different parametric values of $\bar{n}_c^e$ at a fixed scaled temperature of $T_s = \frac{kT}{\hbar\omega_0-\mu_e} = 0.01$. Increase in $\bar{n}_c^e$ increases the steady value and also decreases the delay time for the current to reach the magnitude of the steady state.

reservoir. This has significant implications for the nature of the noise in the current unlike the thermal one in usual tunneling conductance of the Landauer type.

In order to investigate the effect of the thermocoherent state of the emitter in the electron transport process, we evaluate the steady-state current and the transient behavior of the current through the quantum system.

In Fig. 1, we study the steady-state behavior of the current in terms of the ratio of the steady-state thermocoherent current to the thermal one $\frac{\langle i\rangle_{ss}^{\mathrm{TC}}}{\langle i\rangle_{ss}^{T}}$ against the coherent population parameter of the emitter $\bar{n}_c^e$ for different scaled temperatures $\frac{kT}{\hbar\omega_0-\mu_e}$. For a fixed value of the scaled temperature, the ratio $\frac{\langle i\rangle_{ss}^{\mathrm{TC}}}{\langle i\rangle_{ss}^{T}}$ increases nonlinearly with the increase in the value of the coherence parameter $\bar{n}_c^e$. It is found that with increase in scaled temperature the ratio with $\bar{n}_c^e$ shows enhancement. Here, we particularly note that the assigned values of the scaled temperature does not necessarily signify a high temperature limit. This actually suggests that the difference in electrochemical potential between the source and the sink is very high with respect to the thermal energy kT. Figure 1 therefore suggests that, as the energy difference between the system and the Fermi level of the emitter is decreased in comparison to the thermal energy, the steady-state value increases and the increment is more prominent for the higher range of the coherent parameter. We ascribe this phenomenon to the thermal efficiency of electron transport for the coherent character of the source bath.

Next we consider the current through the quantum system in its dynamical or transient regime. Figure 2 shows the transient current for a fixed value of scaled temperature, say $\frac{kT}{\hbar\omega_0-\mu_e} = 0.01$, for different parametric values of $\bar{n}_c^e$. We see that the steady current depends on $\bar{n}_c^e$ and in addition to it, the increment in its value causes an increase in the magnitude of steady current. But for a fixed value of the scaled temperature, the time delay, which is defined as the characteristic time needed for the system to reach the steady-state value, decreases. In other words, introduction of coherence in the emitter causes a decrease in the delay time. This is purely a consequence of the quantum nature of the emitter.

We therefore conclude here that the efficiency of transport through the quantum system can be characterized by the magnitude of the delay time which depends strongly on the value of the coherence parameter of the emitter. The more the delay time, the less efficient the transport will be. Thus, the introduction of the coherent character in the emitter enhances the transport efficiency through the device.

B. Current noise spectrum and Fano factor

For the dynamical information about the current and its noise characteristics here we have calculated the noise spectrum and Fano factor from the current-current correlation function. Following Eq. (68), we have calculated the equation for the average value of current as

$$\frac{d\langle i\rangle(t)}{dt} = \frac{1}{2}\left[\gamma_c\gamma_e\bar{n}_c^e + \gamma_e\left(\gamma_e\bar{n}_c^e - \gamma_e + \gamma_c\right)\right] - \left(2\gamma_e\bar{n}_c^e - \gamma_e + \gamma_c\right)\langle i\rangle(t). \tag{74}$$

We use the steady-state solution of Eq. (74) along with the quantum regression theorem [36] (see Appendix B) to arrive at the current-current correlation function as

$$\langle i(0)i(\tau)\rangle = \frac{\gamma_e\gamma_c\left(\bar{n}_c^e+1\right) + \gamma_e^2\left(\bar{n}_c^e-1\right)}{2\gamma_e\bar{n}_c^e - \gamma_e + \gamma_c} \times \left[1 - \frac{\gamma_e\gamma_c\left(\bar{n}_c^e+1\right) + \gamma_e^2\left(\bar{n}_c^e-1\right)}{2\gamma_e\bar{n}_c^e - \gamma_e + \gamma_c}\right] \times \exp\left(-\left(2\gamma_e\bar{n}_c^e - \gamma_e + \gamma_c\right)\tau\right) + \left[\frac{\gamma_e\gamma_c\left(\bar{n}_c^e+1\right) + \gamma_e^2\left(\bar{n}_c^e-1\right)}{2\gamma_e\bar{n}_c^e - \gamma_e + \gamma_c}\right]^2. \tag{75}$$

The Fano factor is defined as

$$F(\omega) = \frac{S(\omega)}{2\langle i(0)\rangle}, \tag{76}$$

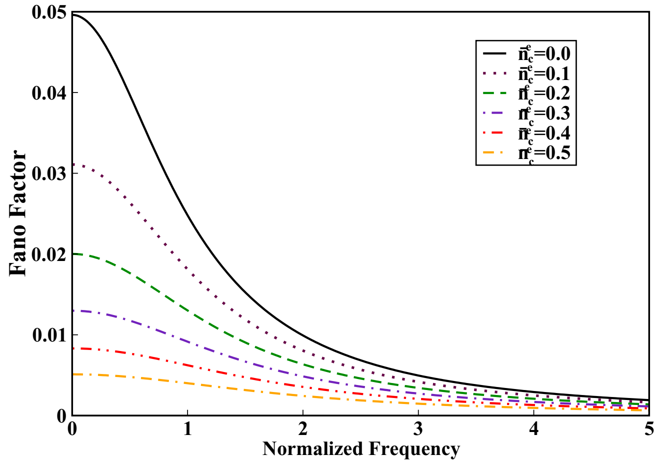

FIG. 3. Plot of the current Fano factor $F(\omega)$ against normalized frequency $\frac{\omega}{\gamma_e+\gamma_c}$ for a fixed value of the scaled temperature $T_s = \frac{kT}{\hbar\omega_0-\mu_e} = 0.01$ for different parametric values of the coherent parameter of the emitter $\bar{n}_c^e$. The figure clearly reveals that an increase in the value of the coherent parameter results in a substantial suppression in noise at a fixed temperature of the emitter.

where $S(\omega)$ is the Fourier transform of the current-current correlation function defined as

$$S(\omega) = \int_{-\infty}^{+\infty} e^{i\omega\tau}\langle i(0)i(\tau)\rangle d\tau. \tag{77}$$

In what follows, we plot the Fano factor $F(\omega)$ against the scaled frequency $\frac{\omega}{\gamma_e+\gamma_c}$, for different parametric values of the coherence parameter of the emitter $\bar{n}_e^c$ and scaled temperature $\frac{kT}{\hbar\omega_0-\mu_e}$.

In Fig. 3, we plot the current Fano factor against the scaled frequency $\frac{\omega}{\gamma_e+\gamma_c}$ for a fixed value of the scaled temperature $\frac{kT}{\hbar\omega_0-\mu_e} = 0.01$ for different parametric values of the coherent parameter of the emitter $\bar{n}_c^e$. As the coherent driving parameter for the emitter increases, the variation becomes more and more flat, revealing that the increase in the coherent population of the emitter results in a suppression of current noise. This can be explained on the basis of the fact that, as more and more coherently injected electrons from the emitter tunnel into the system, no further electrons can enter the well until it is drained into the collector. As the emitter as well as the collector are assumed to be at a very low temperature and no coherence is present in it, the time scale in which the draining of electrons from the system to the collector takes place will be principally guided by the rate constants γ_e and γ_c along with the thermocoherent state of the emitter. This suppresses the noise which is evident from a decrease in the Fano factor.

To conclude the section, we note that the results shown in Fig. 3 reveal a quite close resemblance with that obtained earlier [15], where an increase in the coherent coupling parameter between two quantum systems suppresses the current noise. This evidently gives an indication for an experimental realizability of the thermocoherent bath. An emitter with a suitable thermocoherent state can conveniently be prepared by coupling a system to a fermionic reservoir with a coherent driving followed by thermalizing the entire system at a given low temperature.

V. CONCLUSION

Following the fermionic coherent state formulation of Cahill and Glauber [33], we have introduced a fermionic thermocoherent state. In order to obtain a thermocoherent state, the key element in the derivation is to obtain the quasiprobability P-distribution functions for fermionic thermal and coherent states separately followed by the convolution integration. For the fermionic case the integration is over the anticommuting Grassmann variables which have no classical analog. We have shown that the corresponding average occupation number and the variance gives the appropriate coherent and thermal limits, which puts the fermionic thermocoherent state with the bosonic counterpart on the same footing which is particularly due to the treatment of the Grassmann algebra. We also show that the thermocoherent state can alternately be formulated as a displaced thermal state and subsequently we have shown its connection with the fermionic displaced number state and the fermion-added coherent state. This clearly gives a systematic procedure to introduce coherent character in a thermal fermionic reservoir, which is traditionally considered as an incoherent source of electron.

As an immediate application, we have studied electron transport characteristics of a quantum system connected to a source in a thermocoherent state and a traditional thermal sink. Here we have suggested a modification in the Landauer conductance formula in the present context as a function of thermal and coherent population of the source reservoir. It is found that with the introduction of the coherent character of the emitter, the steady-state current increases and the delay time to reach the steady state decreases which is solely a consequence of the coherent nature of the emitter. Then, we have calculated the current noise spectrum and the Fano factor to monitor the steady-state fluctuation of current. When the emitter is prepared in a thermocoherent state, the noise of the current is suppressed. The suppression of noise in the current reveals a quite close resemblance with the result obtained earlier [15] in the bosonic case using a prepared coherently coupled state of the system instead of introducing coherence in the source as in our case. This evidently gives an indication of an experimental implication of the fermionic thermocoherent source.

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APPENDIX A

Bosonic coherent state [21] $|\alpha\rangle$ is an eigenstate of the harmonic oscillator annihilation operator $\hat{A}$ defined as

$$\hat{A}|\alpha\rangle = \alpha|\alpha\rangle. \tag{A1}$$

Here α, the eigenvalue, is a complex number such that $\alpha = |\alpha|e^{i\phi}$ which practically corresponds to complex wave amplitude in classical optics. The coherent states are minimum uncertainty product states with the usual relation $\Delta p \Delta q = \frac{\hbar}{2}$ and it forms a so-called overcomplete set owing to the relation,

$$\int |\alpha\rangle\langle\alpha| \frac{d^2\alpha}{\pi} = \sum_{n=0}^{n=\infty} |n\rangle\langle n| = 1. \tag{A2}$$

On the other hand, thermal states are mixed states where the probability P_n for an oscillator that one mode with frequency ω is excited with n excitations is given by Boltzmann distribution at temperature T and the corresponding density operator [21] is

$$\rho^{\rm th} = \left(1 - e^{-\frac{\hbar\omega}{kT}}\right) e^{-\frac{\hbar\omega A^\dagger A}{kT}}. \tag{A3}$$

The P distribution for the corresponding coherent and thermal fields [21] is given by

$$P^{\rm coh}(\alpha) = \delta^2(\alpha - \alpha_0), \tag{A4}$$

and

$$P^{\rm th}(\alpha) = \frac{1}{\pi \bar{n}_T} e^{-\frac{|\alpha|^2}{\bar{n}_T}}. \tag{A5}$$

Using the convolution of quasiprobability functions of the thermal and coherent states in the introduction one can find the quasiprobability distribution function of the thermocoherent state $P(\alpha)$ as

$$P(\alpha) = \frac{1}{\pi \bar{n}_T} e^{-\frac{|\alpha-\alpha_0|^2}{\bar{n}_T}}. \tag{A6}$$

As a result, the probability of photon number distribution for the thermocoherent field $\rho_{nn}^{\rm GL}$ which interpolates between the thermal and coherent limits [21,28,29] is given in Eq. (3).

The probability of photon number distribution for the thermocoherent field $\rho_{nn}^{\rm GL}$ can also be mathematically obtained by unitarily displacing a thermal equilibrium state of a cavity mode, a thermal state characterized by the density operator given by Eq. (A5). The displacement operator $D(\alpha)$ is defined as

$$D(\alpha) = e^{\alpha A^\dagger - \alpha^* A}, \tag{A7}$$

and the density matrix operator of the Glauber-Lachs state [27,28] can be expressed as

$$\begin{aligned} \rho^{\rm GL} &= D(\alpha)\rho^{\rm th} D^\dagger(\alpha) \\ &= \left(1 - e^{-\frac{\hbar\omega}{KT}}\right) \sum_n e^{-\frac{n\hbar\omega}{KT}} D(\alpha)|n\rangle\langle n| D^\dagger(\alpha). \end{aligned} \tag{A8}$$

The displaced number state finds its expression [21,46] as

$$\begin{aligned} D(\alpha)|n\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{k=0}^{\infty} \frac{\alpha^k}{k!} \sum_{j=0}^{n} \frac{(-\alpha^*)^j}{j!} \\ \times \sqrt{\frac{(n-j-k)!n!}{(n-j)!(n-j)!}} |n-j+k\rangle, \end{aligned} \tag{A9}$$

which after substitution in Eq. (A8) followed by considering the diagonal matrix element gives Eq. (3).

APPENDIX B

Here we give a short note on calculating the current-current correlation function using the quantum master equation [36]. In general, a two-time average of two operators $A_1(t)$ and $A_2(t')$ in the interaction picture can be expressed as

$$\langle A_1(t) A_2(t')\rangle = {\rm Tr}_{S+B}[\chi(0) A_1(t) A_2(t')], \tag{B1}$$

where the total density operator $\chi(t)$ can be expressed as

$$\chi(0) = e^{\frac{iHt}{\hbar}} \chi(t) e^{-\frac{iHt}{\hbar}}, \tag{B2}$$

with the equations of motion for the operators being expressed as

$$\dot{A}_i(t) = \frac{1}{i\hbar}[A_i, H], \tag{B3}$$

along with the solution,

$$A_i(t) = e^{\frac{iHt}{\hbar}} A_i e^{-\frac{iHt}{\hbar}}. \tag{B4}$$

Using Eqs. (B1)–(B3), we obtain

$$\langle A_1(t) A_2(t+\tau)\rangle = {\rm Tr}_s[A_2(0) Tr_B\{\chi_{A_1}(\tau)\}], \tag{B5}$$

where we define $t' - t = \tau$ and

$$\chi_{A_1}(\tau) = e^{-\frac{iH\tau}{\hbar}} (\chi(t) A_1(0)) e^{\frac{iH\tau}{\hbar}}. \tag{B6}$$

We now exclude the explicit reference of the reservoir by tracing over the reservoir variables thereby defining the reduced operator $\rho_{A_1}(\tau)$ as

$$\rho_{A_1}(\tau) = {\rm Tr}_B[\chi_{A_1}(\tau)], \tag{B7}$$

with

$$\rho_{A_1}(0) = {\rm Tr}_B[\chi_{A_1}(0)] = \rho(t) A_1(0). \tag{B8}$$

We can express an equation of motion for ρ_{A_1} as

$$\rho_{A_1}(\tau) = e^{L\tau}[\rho(t) A_1(0)], \tag{B9}$$

where L is a generalized Liouvillean superoperator. After having a little algebra, we can rewrite Eq. (B5) as

$$\langle A_1(t) A_2(t+\tau)\rangle = {\rm Tr}_s[A_2(0) e^{L\tau} \rho(t) A_1(0)]. \tag{B10}$$

For a complete set of system operators $\{K_\mu\}(\mu = 1,2,3,....)$ such that for any arbitrary operator O, one can write

$$ {\rm Tr}_s[K_\mu(LO)] = \sum_\theta M_{\mu\theta} Tr_s(K_\theta O), \tag{B11}$$

where $M_{\mu\theta}$ are constants. Using Eq. (B11), we obtain

$$\langle \dot{K}_\mu\rangle = \sum_\theta M_{\mu\theta}\langle K_\theta\rangle, \tag{B12}$$

which in the matrix form can be expressed as

$$\langle \dot{\mathbf{K}}\rangle = \mathbf{M}\langle \mathbf{K}\rangle. \tag{B13}$$

Now, using Eqs. (B10) and (B13), after a slight algebra we obtain

$$\frac{d}{d\tau}\langle A_1(t) K_\mu(t+\tau)\rangle = \sum_\theta M_{\mu\theta}\langle A_1(t) K_\theta(t+\tau)\rangle, \tag{B14}$$

a form of quantum regression theorem [36], which is used to calculate the current-current correlation function.

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