Correlation functions and correlation widths in quantum-chaotic scattering for mesoscopic systems and nuclei

J. G. G. S. Ramos,[1] A. L. R. Barbosa,[2] B. V. Carlson,[3] T. Frederico,[3] and M. S. Hussein[3,4]

[1]*Departamento de Física, Universidade Federal da Paraíba, 58051-970, João Pessoa, Paraíba, Brazil*

[2]*Departamento de Física, Universidade Federal Rural de Pernambuco, Dois Irmãos, 52171-900 Recife, Pernambuco, Brazil*

[3]*Departamento de Física, Instituto Tecnológico de Aeronáutica, CTA, São José dos Campos, S.P., Brazil*

[4]*Instituto de Estudos Avançados and Instituto de Física, Universidade de São Paulo, C.P. 66318, 05314-970 São Paulo, SP, Brazil*

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We derive analytical expressions for the correlation functions of the electronic conductance fluctuations of an open quantum dot under several conditions. Both the variation of energy and that of an external parameter, such as an applied perpendicular or parallel magnetic fields, are considered in the general case of partial openness. These expressions are then used to obtain the ensemble-averaged density of maxima, a measure recently suggested to contain invaluable information concerning the correlation widths of chaotic systems. The correlation width is then calculated for the case of energy variation, and a significant deviation from the Weisskopf estimate is found in the case of two terminals. The results are extended to more than two terminals. All of our results are analytical. The use of these results in other fields, such as nuclei, where the system can only be studied through a variation of the energy, is then discussed.

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I. INTRODUCTION

Chaotic quantum systems [1–3], such as the compound nucleus [3–5], open quantum dots [6], and graphene flakes [7–9], exhibit common universal features characterized by fluctuating observables, such as the electronic conductance in the latter four references, and the compound nucleus cross section in the former three references [3–5]. Useful information about the system in this regime of the dynamics are obtained through the average of the observables and through the correlation functions defined as the average of products of two fluctuating observables at two values of the independent variable, such as the energy or an external field. The correlation function is of paramount importance as it measures the degree of coherence present in the otherwise fully chaotic system. This measure resides in the correlation width, which specifies the shape of the correlation function. In the case of energy variation the shape is Lorentzian, as shown more than half a century ago by Ericson [10], as long as the resonances are strongly overlapping or in the opposite limit of isolated, while in the case of a varying external parameter such as a magnetic field, the correlation function first obtained by Efetov is a square Lorentzian [11–13]. In Ref. [13], the effect of temperature on the conductance correlation function was investigated.

Several papers have been published over the past 20 years aiming at a generalization of the correlation function of chaotic systems with important deviations from the Ericson-Efetov limits [3,14–20]. This was necessary as more precise and detailed data have become available and have shown a clear deviation from the classic Ericson and Efetov limits [17,21–26].

Notwithstanding the many works published since the 1980s, dedicated to the study of the correlation function and its deviation from the Ericson/Efetov limits, we give in this paper more discussion of this matter based on our application of the average density of maxima method [27,28], as we believe that further work is called for.

Before presenting our results, it is important to point out that the first exact analytical results for the two-point S-matrix correlation functions from the region of isolated to the region of strongly overlapping resonances were provided in the seminal work of Verbaarschot *et al.* [3] for chaotic scattering systems with preserved time-reversal symmetry (TRS) and extended to systems with completely violated TRS [15]. See also Ref. [17]. For the case of partially broken time-reversal symmetry exact analytical results are given in Ref. [20]. Several detailed experimental investigations have been published [17,21–24]. Furthermore, the deviations of the cross-section and/or conductance correlation functions from the Ericson or the Efetov limits, respectively, were investigated in detail numerically in Ref. [29], and both experimentally and numerically in Ref. [25].

It is therefore clear that further detailed investigation of the correlation functions and their deviations from the above-mentioned two extreme Ericson-Efetov limits and the resulting correlation widths, is still an important line of research. In this paper we calculate analytically the correlation functions for the case of partially open system (with a varying degree of transmission to the open channels). Three types of external magnetic fields are considered. The energy correlation function is discussed in details and the corresponding correlation width is extracted and found to be significantly different from the Weisskopf estimate. This last result is quite important as it affects the value of the dwell time in these systems.

II. QUANTUM CHAOTIC SCATTERING

Quantum chaotic scattering (QCS) is a widely occurring phenomenon in physics. It operates in a variety of systems, such as electronics [6], spintronics [30], biomolecules [31], disordered mesoscopic nanostructutes [2], and the compound nucleus [32]. The emergence of the phenomenon is, just as in other cases of quantum chaos, directly related to the intrinsic chaotic dynamics associated with quasibound states of a quantum system. Random matrix theory (RMT) supplies the formal S matrix of QCS, as it describes the intrinsic Hamiltonian that governs the dynamics and the scattering. The empirical manifestation of QCS is evidenced by universal fluctuations observed in electronic conductance in open quantum dots [6], in Graphene flakes [7–9], transmittance in microwave resonators [17], and in compound nucleus cross sections [3].

The S matrix describing QCS is given by

$$S(\varepsilon,X) = \mathbb{1} - 2\pi i W^{\dagger}(\varepsilon - H(X) + i\pi W W^{\dagger})^{-1} W, \quad (1)$$

where $H(X)$ is a random Hamiltonian matrix of dimension $M \times M$ that describes the resonant states in the chaotic system, which is subject to the influence of an external parameter, X, which can be an external magnetic field, or a change in the shape of the mesoscopic system. The number of resonances is very large ($M \to \infty$). The matrix W of dimension $M \times (N)$ contains the channel-resonance coupling matrix elements. Using the above S matrix, one is then able to calculate observables. The ensemble average $\langle S_{cc'}(\varepsilon) S^{\star}_{cc'}(\varepsilon')\rangle$ supplies the S-matrix autocorrelation function for $c \neq c'$, while for $c = c'$ we get $\langle S_{cc}\rangle^2 = 1 - T_c$, with T_c being the transmission coefficient into channel c. Thus, we get the average compound nucleus cross section if $S_{cc'}$ is constructed to describe nuclear scattering, while in the case of a quantum dot the S matrix is used to describe the average electronic conductance. The ensemble average of a product of four S matrices supplies the four-point function, $\langle S_{cc'}(\varepsilon_1) S^{\star}_{cc'}(\varepsilon_1) S_{cc'}(\varepsilon_2) S^{\star}_{cc'}(\varepsilon_2)\rangle$, namely the cross-section correlation function which is a function of the difference $\delta\varepsilon \equiv |\varepsilon_1 - \varepsilon_2|$. This correlation function is characterized by the correlation width, of great importance in the study of chaotic quantum systems.

In application to conductance statistics in quantum dots with three or more terminals (leads) it is convenient to represent the S matrix as

$$S = \begin{pmatrix} r_{11} & t_{12} & t_{13} \\ t'_{12} & r_{22} & t_{23} \\ t'_{13} & t'_{23} & r_{33} \end{pmatrix}, \quad (2)$$

where t_{ij} indicates the probability amplitude of transmission of the channel(s) contained in the terminal i for the channel(s) contained in the terminal j and r_{ii} denotes the probability amplitude of reflexions of the channels in the terminal i.

The calculation of the correlation function using the above RMT-based S matrix can be done both in the case of a variation of the energy or in the case of a variation of the external parameter, X (representing an applied magnetic field, change in the shape of the quantum dot, etc.) [11–13,18]. Analytical calculation of the ensemble averages, done using the method of supersymmetry, has been presented in several publications, see, e.g., Refs. [3,15,33–35]. The expressions obtained are rather complicated as they involve, among other things, the final evaluation of triple integrals. Most applications of the above S matrix in the calculation of the correlation functions for chaotic systems, especially for the purpose of the analysis of the experimental data [19,21,26,33] involve numerical simulations using a random matrix generator. Recently an alternative method has been devised to potentially supply a simpler way to obtain analytical results. This alternative method is based on the distribution of the S matrix itself, an idea pioneered by Mello and collaborators [36–38]. See also Refs. [33,35,39]. The stub model is such an alternative [40–42].

III. THE STUB MODEL AND CALCULATION OF THE CORRELATION FUNCTION AND WIDTH

We assume that the particles dynamics is ballistic and ergodic, and we model the system statistical properties using the random matrix theory (RMT)-based stub model. Following Refs. [40,42], for particles with spin, the scattering matrix S can be represented as a unitary matrix with quaternionic entries and is parameterized as

$$\mathcal{S}(\varepsilon,\mathcal{B}) = \bar{\mathcal{S}} + \mathcal{P}\mathcal{U}[1 - \mathcal{K}^{\dagger}\mathcal{R}(\varepsilon,\mathcal{X})\mathcal{K}\mathcal{U}]^{-1}\mathcal{P}^{\dagger}. \quad (3)$$

Here, $\mathcal{U}$ matrix, $M \times M$, is the scattering matrix counterpart of an isolated quantum system, while $\bar{\mathcal{S}}$ is the average of the scattering matrix of the system $\mathcal{S}$, which has dimension $N \times N$. The M stands for the number of resonances of the system, while $N = N_1 + N_2 + N_3$ is the total number of open channels. The universal regime requires $M \gg N$. The $\mathcal{K}$ matrix is a projection operator of order $(M - N) \times M$, while $\mathcal{P}$, of order $N \times M$, describes the channels-resonances couplings. Their explicit forms read $\mathcal{K}_{i,j} = \delta_{i+N,j}$, $\mathcal{P}_{i,j} = \mathrm{diag}(i\delta_{i,j}\sqrt{T_1}, i\delta_{i+N_1,j}\sqrt{T_2}, i\delta_{i+N_1+N_2,j}\sqrt{T_3})$ and $\bar{\mathcal{S}}_{i,j} = \mathrm{diag}(\delta_{i,j}\sqrt{1-T_1}, \delta_{i+N_1,j}\sqrt{1-T_2}, \delta_{i+N_1+N_2,j}\sqrt{1-T_3})$, we are assuming the equivalent coupling for channels in the same terminal. The $\mathcal{R}$ matrix (representing the external fields) is the stub model counterpart of order $(M - N) \times (M - N)$ as described by Ref. [42]. The parameter X represents, e.g., the type of external applied magnetic field employed. The quantities T_i are transmission coefficients of the equivalent channels in terminal i.

The calculation of the ensemble averages can be done in a closed form using the stub S matrix above. Details of such calculations can be found in the original references (see, e.g, Refs. [18,28]). The ensemble average of the conductance, $\mathcal{Q} = tr(t_{12}t'_{12})$, is easily calculated, [39], following the Ref. [42] of a chaotic QS. We obtain

$$\langle \mathcal{Q}\rangle = \frac{\mathcal{T}_1\mathcal{T}_2}{\mathcal{T}}\left[1 - \frac{2-\beta}{\beta}\frac{\mathcal{T}_1 T_2 + \mathcal{T}_2 T_1}{\mathcal{T}^2}\right], \quad (4)$$

where $\mathcal{T}_i = N_i T_i$ is the total transmission coefficient for the terminal i with equivalent channels and $\mathcal{T} = \mathcal{T}_1 + \mathcal{T}_2 + \mathcal{T}_3$, and the ε-dependence disappear in the ensemble average. In the case of the compound nucleus, the quantity Q becomes the cross section and the its ensemble average, similar to Eq. (4), is known as the Hauser-Feshbach cross section [43], containing the product of two $\mathcal{T}_i$ divided by their sum, $\mathcal{T} = \mathcal{T}_1 + \mathcal{T}_2 + \mathcal{T}_3$. This is a common feature of the average fluctuation cross section and also of the average conductance. The factor $[1 - \frac{2-\beta}{\beta}\frac{\mathcal{T}_1 T_2 + \mathcal{T}_2 T_1}{\mathcal{T}^2}]$ represents what is known as the elastic enhancement factor, which is known exactly both for orthogonal and unitary symmetry [34,35]. Note that the number of channels, N, in all the terminals is taken to be large, $N >> 1$, and thus we are in the semiclassical regime. Note further that the symmetry parameter β takes on the value 1 in the Gaussian orthogonal ensemble, 2 in the Gaussian unitary ensemble, and 4 in the Gaussian symplectic ensemble.

As for the correlation functions, the calculation using the stub model can be carried out as done in Ref. [18]. In the following we merely write down the correlation functions for a chaotic quantum dot in the case of only two terminals, specified by T_1 and T_2 and and consider $T_1 = T_2 = T$, we find for the energy variation,

$$\frac{C(\delta E)}{1/8\beta} = \frac{3T(2-T)-2}{1+(\delta E/\Gamma_E)^2} + \frac{4[1+T(T-2)]}{[1+(\delta E/\Gamma_E)^2]^2}, \tag{5}$$

where β is a measure of the universality class to which H pertains. It should be mentioned that magnetoconductance correlation functions were calculated using the Hamiltonian approach, Eq. (1), and the study of a relevant asymptotic expansion in inverse channel number was reported in Ref. [44].

For the variation of the external parameter, we consider three cases. An external perpendicular magnetic field, $B_\perp$, and external perpendicular magnetic filed acting on the spin-orbit interaction, the so-called Rashba-Drasselhaus field, $H_\perp$ [45,46], and a parallel magnetic field, $H_\parallel$, whose effect has been studied recently by Zumbúhl *et al.* [47],

$$\frac{C(\delta B_\perp)}{1/8\beta} = \frac{2T(1-T)}{1+(\delta B_\perp/\Gamma_{B_\perp})^2} + \frac{2+T(3T-4)}{[1+(\delta B_\perp/\Gamma_{B_\perp})^2]^2}, \tag{6}$$

$$\frac{C(\delta H_\perp)}{1/8\beta} = \frac{T(2-T)}{2+(\delta H_\perp/\Gamma_{H_\perp})^2} + \frac{4+T(3T-4)}{(2+(\delta H_\perp/\Gamma_{H_\perp})^2)^2}. \tag{7}$$

Finally, for the case of a parallel magnetic field considered recently in Ref. [47],

$$\frac{C(\delta H_\parallel)}{1/8\beta} = \frac{T(2-T)+2}{1+(\delta H_\parallel/\Gamma_{H_\parallel})^2}. \tag{8}$$

This last correlation function is new and deserves some discussion. It does depend on the openness parameter, the transmission coefficient, T, yet it has a pure Lorentzian shape, in contrast to the cases of applied perpendicular magnetic fields, Eqs. (6) and (7). The derivation of the above equations for the correlation functions relies on the assumption of equivalent channels in each terminal and large number of these channels.

It should be mentioned that a thorough study of the energy dependence and correlation length in quantum chaotic scattering with many channels was performed in Ref. [35]. A general relation was established there between fluctuations in scattering and the distribution of complex energies (poles of the S matrix). In particular, the correlation length was shown to be given by the spectral gap in the pole distribution, and the deviations of the gap from the (semiclassical) Weisskopf estimate [Eq. (14) of the present work] were analyzed in great detail there as well.

The important feature that characterizes these correlation functions is that they all (except the case of $H_\parallel$) deviate from pure Lorentzian or square Lorentzian shape, for an arbitrary value of the openness probability. The application to the case of the compound nucleus, Eq. (5), allows the investigation of the statistics of resonances both in the weak (isolated resonances) and strong (overlapping resonances) absorption cases, as well as in the intermediate cases. Several quantities can be obtained from the correlation functions. The average density of maxima in the fluctuating observable is one of them. In Ref. [28], this quantity was derived and analyzed for $C(E)$ and $C(X)$. For completeness we give below the main results and extend them to other types of applied magnetic fields:

$$\langle \rho_z \rangle = \frac{1}{2\pi}\sqrt{\frac{T_4}{T_2}}$$
$$T_2 = -\frac{d^2}{d(\delta z)^2}C(\delta z)\Big|_{\delta z=0}$$
$$T_4 = \frac{d^4}{d(\delta z)^4}C(\delta z)\Big|_{\delta z=0}. \tag{9}$$

Taking for the correlation function the general form suggested by our results Eqs. (5)–(8), $C(z) = A/(1+z^2) + B/(1+z^2)^2$, we obtain $< \rho_z > \Gamma_z = (\sqrt{3}/\pi)\sqrt{(A+3B)/(A+2B)}$.

Thus, for the energy variation,

$$\langle \rho_E \rangle = \frac{\sqrt{3}}{\pi\Gamma_E}\sqrt{\frac{9T^2-18T+10}{5T^2-10T+6}} \equiv \frac{\sqrt{3}}{\pi\Gamma_{\text{corr}}}, \tag{10}$$

which defines Γ_{corr} [27]. For the case of an external perpendicular magnetic field,

$$\langle \rho_{B_\perp} \rangle = \frac{\sqrt{3}}{\sqrt{2}\pi\Gamma_{B_\perp}}\sqrt{\frac{7T^2-10T+6}{2T^2-3T+2}}. \tag{11}$$

In the case of the Rashba-Dresselhaus field, the correlation function has the general form $C(z) = A'/(2+z^2) + B'/(2+z^2)^2$. The average density of maxima is then $< \rho_z > \Gamma_z = (1/\pi)\sqrt{(3A'+5B')/(A+2B)}$. Thus,

$$\langle \rho_{H_\perp} \rangle = \frac{1}{\pi\Gamma_{H_\perp}}\sqrt{\frac{6T^2-7T+10}{5T^2-6T+8}}. \tag{12}$$

Interestingly, for the case of a parallel magnetic field, the result is independent of the openness parameter and is identical to the Brink-Stephen [27] one,

$$\langle \rho_{H_\parallel} \rangle = \frac{\sqrt{3}}{\pi\Gamma_{H_\parallel}} \approx \frac{0.55}{\Gamma_{H_\parallel}}. \tag{13}$$

The important point to emphasize here is that both the correlation function and the average density of maxima are characterized, for a fixed value of the openness probability, by a single quantity, the correlation width. In the energy variation case, this width is the inverse of the dwell time and is usually estimated using the Weisskopf expression [48],

$$\Gamma_E \approx \Gamma_W = \frac{\overline{\Delta}}{2\pi}\sum_c T_c, \tag{14}$$

where, $\overline{\Delta}$ is the average spacing between the resonances in the chaotic system, and the sum extends over all the open channels

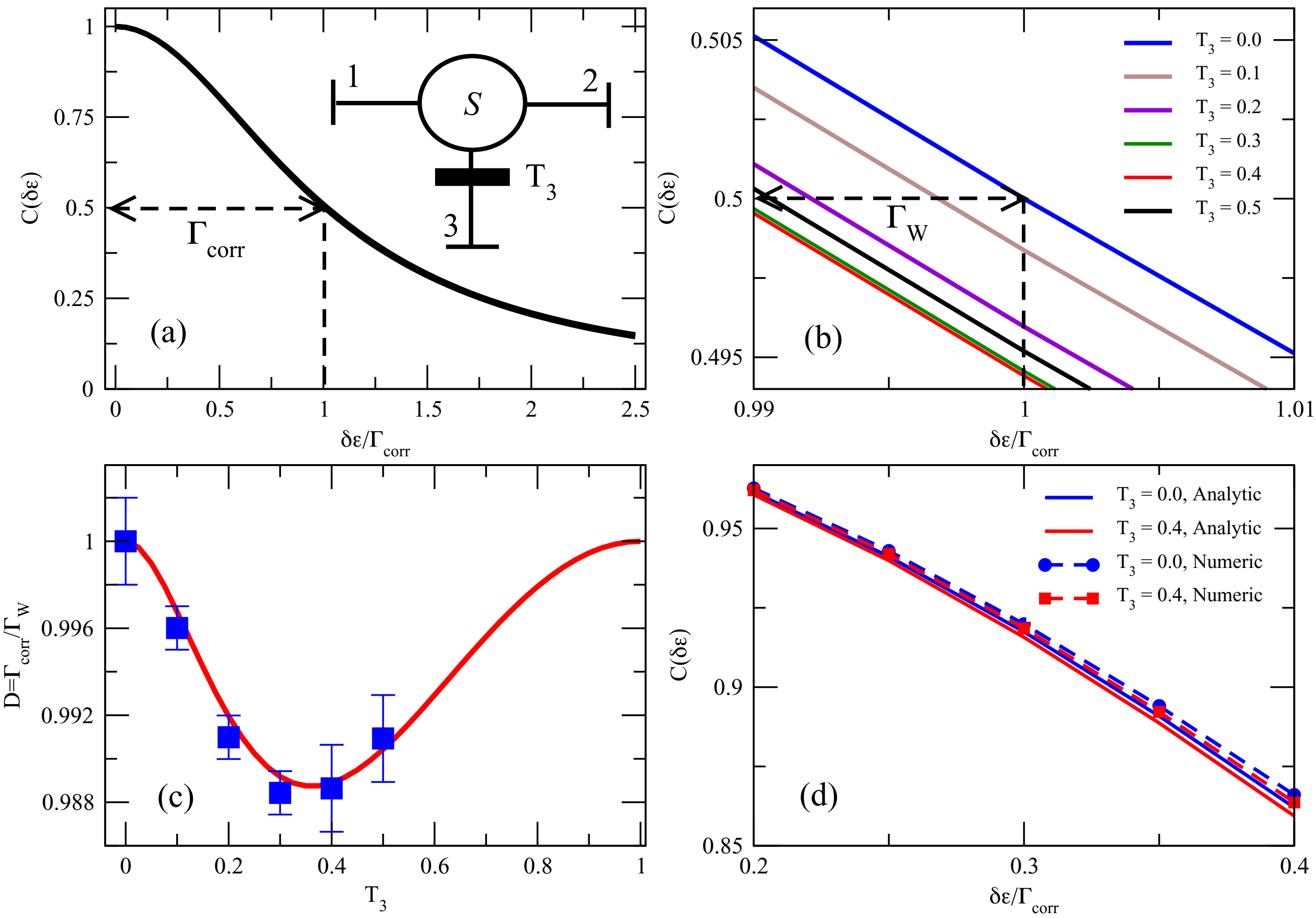

FIG. 1. (a) A typical Lorentzian behavior of the correlation function and the correlation width Γ_{corr} associated with a transport observable between terminals 1 and 2 influenced by another nonlinear terminal 3, as the inset indicates. (b) A deviation from the Weisskopf correlation width in the semiclassical regime as affected by the presence of other terminals lumped into an effective lead designated by 3, Eq. (18). The different lines correspond to different values of the effective transmission through this third terminal, and they show how the correlation function varies with $\varepsilon/\Gamma_{\text{corr}}$ in the interval [0.99, 1.01], a region of an almost linearity. By changing the value of T_3, one sees a gradual decrease of the correlation function indicating a changing of of the ratio $D = \Gamma_{\text{corr}}/\Gamma_W$ as a function of T_3. (c) The ratio $D = \Gamma_{\text{corr}}/\Gamma_W$ as a function of T_3 exhibiting the attainment of the Weisskopf length only for a completely open ($T_3 = 1$) or completely closed ($T_3 = 0$) extra terminals lumped into an effective one. See text for details. The continuous line is the analytical result and the dots are the results of the numerical Hamiltonian simulation using the S matrix of Eq. (1). (d) The correlation function vs. $\varepsilon/\Gamma_{\text{corr}}$ in the interval [0.2, 0.4] and exhibits the deviation of the correlation width from the classic Weisskopf estimate both analytically (full curves) and numerically (dashed curves). The “numerical” calculation was obtained as in (c). See text for more details.

reached through the transmission coefficient, T_c. Deviation from the Weisskopf estimate was calculated in Ref. [26] using the S matrix of Eq. (1). With the help of a random matrix generator, these authors calculated numerically the transmittance correlation function and tested their results using experimental data obtained with microwave resonators. They obtained the correlation width, Γ_{corr}, as the width at half maximum of the correlation function. They further found for the ratio $D = \Gamma_{\text{corr}}/\Gamma_W$ versus $\sum_c T_c$, values that reach up to 1.1. In our work here, we can read out the change in the correlation width as Eq. (10),

$$D = \frac{\Gamma_{\text{corr}}}{\Gamma_W} = \sqrt{\frac{5\,T^2 - 10\,T + 6}{9\,T^2 - 18\,T + 10}}. \tag{15}$$

For almost closed system, $T \ll 1$, the deviation reaches the value $D = \sqrt{3/5} = 0.78$. Of course in the other limit of a completely open quantum dot, $T \approx 1$, the ratio D attains the value of unity as expected.

It is interesting to generalize our result to the case of two terminals in the presence of other terminals lumped together as an effective terminal characterized by T_3 and study the variation of the correlation width as a function of T_3. Thus, we extend the stub method for the new calculation of the general correlation function, including the quantum interference terms. Using the same stub $\mathcal{S}$ matrix described above, together with the typical large number of diagrams for the correlation function [40], we calculate the averages of products of two observables, $Q(E)Q(E')$, in the limit $\mathcal{T}_1 = \mathcal{T}_2 = N$, and $N_1 = N_2 = N_3 = N$. Note that in this case of more than two terminals, the result we have obtained are valid for $T_1 = T_2 = 1$,

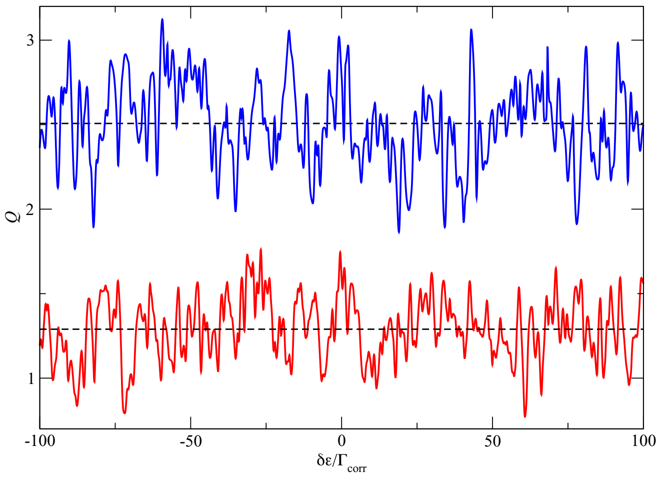

FIG. 2. Typical fluctuations, obtained through numerical simulations, in a transport observable $\mathcal{Q}$ as a function of $\varepsilon/\Gamma_{corr}$ for $N = 50$ open channels coupled to 5×10^3 resonances, for a single realization of H in Eq. (1). The top curve indicates a single measure in the absence of the extra effective terminal and the bottom one shows the effect of the extra effective terminal with $T_3 = 0.3$. These curves serve as numerical "data."

to get

$$\frac{\mathcal{C}(\delta E)}{1/\beta} = \frac{\left[\mathcal{A}_1(T_3) - 8N^2 T_3^2(1-T_3)^2\right]}{N^4(2+T_3)^6\left[1+(\delta E/\Gamma_W)^2\right]} + \delta_{2\beta}\frac{T_3\mathcal{A}_2(T_3)}{N^3(2+T_3)^6\left[1+(\delta E/\Gamma_W)^2\right]} + \frac{8T_3^2(1-2T_3+T_3^2)}{(2+T_3)^6\left[1+(\delta E/\Gamma_W)^2\right]^2}, \quad (16)$$

and

$$\frac{\mathcal{A}_1(T_3)}{N^4} \equiv A_1(T_3) = T_3^4 + 2(2p+T_3)(2+T_3)T_3^3 + \left[2T_3^2 - 4(2+T_3)^2 T_3 + 16(2+T_3)^2\right]T_3^2 + 2(2p+T_3)\left[-4T_3^2(2+T_3) + 12(2+T_3)^2\right]T_3 + 8(2+T_3)^4$$

$$\frac{\mathcal{A}_2(T_3)}{N^3} \equiv A_2(T_3) = T_3(2+T_3)^2[2(2+T_3)+1]. \quad (17)$$

Equation (16) shows convincingly that the correlation function as a function of the Weisskopf width is not a Lorentzian. Therefore, Γ_W is no longer the transport correlation width. Figure 1(b) shows the modification of the Weisskopf width as a function of the T_3. On the another hand, the amplitude of the universal fluctuations (variance) $\mathrm{var}[\mathcal{Q}] = \mathcal{C}(0) = 1/\beta$ is maintained unchanged regardless of the additional terminals. Equation (16) can be written in a Lorentzian form. After some algebra we find the following:

$$\frac{C(\delta E)}{1/\beta} = \frac{1}{1+(\delta E/\Gamma_{corr})^2}, \quad \Gamma_{corr} \equiv \Gamma_W D, \quad (18)$$

with

$$D \equiv \frac{\sqrt{64T_3^4(1-T_3)^4 + [A_1(T_3) + \delta_{2\beta}T_3A_2(T_3)]^2}}{A_1(T_3) + \delta_{2\beta}T_3A_2(T_3)} - \frac{8T_3^2(1-T_3)^2}{A_1(T_3) + \delta_{2\beta}T_3A_2(T_3)}, \quad (19)$$

which is highly nonliner as a function of T_3. The forms of Eqs. (18) and (19) show analytically the effect of the presence of more terminals on the correlation properties of the other two terminals. It simulates absorption of the flux in the two-terminal subsystem. The nonlinear effect disappears in the limits $T_3 = 1$ (ideal) and $T_3 \to 0$ (opaque) for which $D \to 1$, and the correlation length approaches the Weisskopf length, as Fig. 1(c) shows clearly. These results were verified by numerical simulations with random matrix generator using the S matrix of Eq. (1).

For the purpose of verifying our analytical results, we compare them to those obtained using the RMT-based S matrix used in Ref. [3]. Thus, we have performed a numerical simulation using the Hamiltonian model of Eq. (1) with a configuration of 50 open channels and 5×10^3 resonances in an energy range $\Delta\varepsilon/\Gamma \in [-100, 100]$. As shown in Fig. 2, the transport observable $\mathcal{Q}$ is numerically obtained for the same scattering QS. For such single QS the $\mathcal{Q}$ is largely affected in the presence of the extra nonlinear terminals in two forms. First, the reference line of the fluctuations is lowered (from the top to the bottom plots of Fig. 2), as can be expected using Eq. (4). This implies that the average value of $\mathcal{Q}$ is reduced by the addition of a third terminal. Second, the number of maxima, calculated by merely counting them from the figures, increases from 110 to 111 in the same energy interval $[\Delta\varepsilon/\Gamma = 200]$, when a third terminal is added, reasonably confirming our analytical findings. Due to the very complicated numerical simulations when three leads are involved, as is the case here, we have made only one run. We are quite aware of the need to make several runs in order to get good statistics. However, we feel that we can still make a reasonable statement concerning our analytical results versus the random S-matrix simulation described above.

Thus, we find good agreement between our analytical calculation and the numerical one. In a way we are also confirming the results of Ref. [26] obtained for transmittance in a microwave cavity. See also Refs. [15,17,20–24,33].

IV. CONCLUSIONS

In this paper, we analytically calculate the correlation functions for chaotic systems using the quantum chaotic scattering theory. We show that in the case of energy variation and in the variation of an external magnetic field, these functions deviate significantly from the expected Lorentzian and square Lorentzian shapes and thus confirm earlier findings. The parameter that measures this deviation in our calculation is identified as the transmission coefficient, which acts as the "openness" probability. We further identify the deviation of the correlation width from the Weisskopf width for an arbitrary observable of the quantum transport. The stub calculation of the ensemble average of the products of four scattering matrices allows finding this deviation, of the order of 1.0%, and reveals the effects of adding more terminals (leads) on the chaotic quantum transport. The consequences of our results include the increase of the dwell time of an arbitrary chaotic scattering system. The results are general and applicable to any scattering amplitudes between two terminals. Therefore, the deviation of the correlation width from the Weisskopf estimate can occur in spin and/or charge channels for electronic nanostructures, the transmittance of antennas, sublattices and/or subvalleys channels for graphene flakes [49], etc.

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