Lloyd-model generalization: Conductance fluctuations in one-dimensional disordered systems

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We perform a detailed numerical study of the conductance G through one-dimensional (1D) tight-binding wires with on-site disorder. The random configurations of the on-site energies ϵ of the tight-binding Hamiltonian are characterized by long-tailed distributions: For large ϵ, $P(\epsilon) \sim 1/\epsilon^{1+\alpha}$ with $\alpha \in (0,2)$. Our model serves as a generalization of the 1D Lloyd model, which corresponds to $\alpha = 1$. First, we verify that the ensemble average $\langle -\ln G \rangle$ is proportional to the length of the wire L for all values of α, providing the localization length ξ from $\langle -\ln G \rangle = 2L/\xi$. Then, we show that the probability distribution function $P(G)$ is fully determined by the exponent α and $\langle -\ln G \rangle$. In contrast to 1D wires with standard white-noise disorder, our wire model exhibits bimodal distributions of the conductance with peaks at $G = 0$ and 1. In addition, we show that $P(\ln G)$ is proportional to G^β, for $G \to 0$, with $\beta \leqslant \alpha/2$, in agreement with previous studies.

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I. INTRODUCTION AND MODEL

The recent experimental realizations of the so-called Lévy glasses [1] as well as "Lévy waveguides" [2] has refreshed the interest in the study of systems characterized by Lévy-type disorder (see, for example, Refs. [3–15]), that is, disorder characterized by random variables $\{\epsilon\}$ whose density distribution function exhibits a slow decaying tail:

$$P(\epsilon) \sim \frac{1}{\epsilon^{1+\alpha}}, \tag{1}$$

for large x, with $0 < \alpha < 2$ (this kind of probability distributions are known as α-stable distributions [16]). In fact, the study of this class of disordered systems dates back to Lloyd [17], who studied spectral properties of a three-dimensional (3D) lattice described by a 3D tight-binding Hamiltonian with Cauchy-distributed on-site potentials [which corresponds to the particular value $\alpha = 1$ in Eq. (1)]. Since then, a considerable number of works have been devoted to the study of spectral, eigenfunction, and transport properties of the Lloyd model in its original 3D setup [18–27] and in lower-dimensional versions [26–43].

Of particular interest is the comparison between the one-dimensional (1D) Anderson model (1DAM) [44] and the 1D Lloyd model, since the former represents the most prominent model of disordered wires [45]. Indeed, both models are described by the 1D tight-binding Hamiltonian:

$$H = \sum_{n=1}^{L} [\epsilon_n \mid n\rangle\langle n \mid -v_{n,n+1} \mid n\rangle\langle n+1 \mid -v_{n,n-1} \mid n\rangle\langle n-1 \mid], \tag{2}$$

where L is the length of the wire given as the total number of sites n, ϵ_n are random on-site potentials, and $v_{n,m}$ are the hopping integrals between the nearest neighbors (which are set to a constant value $v_{n,n\pm1} = v$). However, while for the standard 1DAM (with white-noise on-site disorder $\langle \epsilon_n \epsilon_m \rangle = \sigma^2 \delta_{nm}$ and $\langle \epsilon_n \rangle = 0$) the on-site potentials are characterized by the finite variance $\sigma^2 = \langle \epsilon_n^2 \rangle$ (in most cases the corresponding probability distribution function $P(\epsilon)$ is chosen as a box or a Gaussian distribution), in the Lloyd model the variance σ^2 of the random on-site energies ϵ_n diverges since they follow a Cauchy distribution.

It is also known that the eigenstates Ψ of the *infinite* 1DAM are exponentially localized around the site position n_0 [45]:

$$|\Psi_n| \sim \exp\left(-\frac{|n - n_0|}{\xi}\right), \tag{3}$$

where ξ is the eigenfunction localization length. Moreover, for weak disorder ($\sigma^2 \ll 1$), the only relevant parameter for describing the statistical properties of the transmission of the *finite* 1DAM is the ratio L/ξ [46], a fact known as single-parameter scaling. The above exponential localization of eigenfunctions makes the transmission or dimensionless conductance G exponentially small [47], i.e.,

$$\langle -\ln G \rangle = \frac{2L}{\xi}; \tag{4}$$

thus, this relation can be used to obtain the localization length. Remarkably, it has been shown that Eq. (4) is also valid for the 1D Lloyd model [41], implying a single-parameter scaling (see also Ref. [38]).

It is also relevant to mention that studies of transport quantities through 1D wires with Lévy-type disorder, different from the 1D Lloyd model, have been reported. For example, wires with scatterers randomly spaced along the wire according to a Lévy-type distribution were studied in Refs. [3,4,48,49]. Concerning the conductance of such wires, a prominent result reads that the corresponding probability distribution function $P(G)$ is fully determined by the exponent α of the power-law decay of the Lévy-type distribution and the average (over disorder realizations) $\langle -\ln G \rangle$ [48,49]; i.e., all other details of the disorder configuration are irrelevant. In this sense, $P(G)$ shows *universality*. Moreover, this fact was already verified experimentally in microwave random waveguides [2] and tested numerically using the tight-binding model of Eq. (2) with $\epsilon_n = 0$ and off-diagonal Lévy-type disorder [50] (i.e., with $v_{n,m}$ in Eq. (2) distributed according to a Lévy-type distribution).

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It is important to point out that 1D tight-binding wires with power-law distributed random on-site potentials, characterized by power-laws different from $\alpha = 1$ (which corresponds to the 1D Lloyd model), have been scarcely studied; for a prominent exception see Ref. [41]. Thus, in this paper we undertake this task and study numerically the conductance though disordered wires defined as a generalization of the 1D Lloyd model as follows. We study 1D wires described by the Hamiltonian of Eq. (2) having constant hopping integrals, $v_{n,n\pm1} = v = 1$, and random on-site potentials ϵ_n which follow a Lévy-type distribution with a long tail, like in Eq. (1) with $0 < \alpha < 2$. We name this setup the 1DAM with Lévy-type on-site disorder. We note that when $\alpha = 1$ we recover the 1D Lloyd model.

Therefore, in the following section we show that (i) the conductance distribution $P(G)$ is fully determined by the power-law exponent α and the ensemble average $\langle -\ln G\rangle$; (ii) for $\alpha \leqslant 1$ and $\langle -\ln G\rangle \sim 1$, bimodal distributions for $P(G)$ with peaks at $G \sim 0$ and $G \sim 1$ are obtained, revealing the coexistence of insulating and ballistic regimes; and (iii) the probability distribution $P(\ln G)$ is proportional to G^{β}, for vanishing G, with $\beta \leqslant \alpha/2$.

II. RESULTS AND DISCUSSION

Since we are interested in the conductance statistics of the 1DAM with Lévy-type on-site disorder we have to define first the scattering setup we shall use: We open the isolated samples described above by attaching two semi-infinite single channel leads to the border sites at opposite sides of the 1D wires. Each lead is also described by a 1D semi-infinite tight-binding Hamiltonian. Using the Heidelberg approach [51] we can write the transmission amplitude through the disordered wires as $t = -2i\,\sin(k)\,\mathcal{W}^T(E - \mathcal{H}_{\rm eff})^{-1}\mathcal{W}$, where $k = \arccos(E/2)$ is the wave vector supported in the leads and $\mathcal{H}_{\rm eff}$ is an effective non-Hermitian Hamiltonian given by $\mathcal{H}_{\rm eff} = H - e^{ik}\mathcal{W}\mathcal{W}^T$. Here, $\mathcal{W}$ is a $L \times 1$ vector that specifies the positions of the attached leads to the wire. In our setup, all elements of $\mathcal{W}$ are equal to zero except $\mathcal{W}_{11}$ and $\mathcal{W}_{L1}$, which we set to unity (i.e., the leads are attached to the wire with a strength equal to the intersite hopping amplitudes: $v = 1$). Also, we have fixed the energy at $E = 0$ in all our calculations, although the same conclusions are obtained for $E \neq 0$. Then, within a scattering approach to the electronic transport, we compute the dimensionless conductance as [52] $G = |t|^2$.

First, we present in Fig. 1(a) the ensemble average $\langle -\ln G\rangle$ as a function of L for the 1DAM with Lévy-type disorder for several values of α. It is clear from this figure that $\langle -\ln G\rangle \propto L$ for all the values of α we consider here. Therefore, we can extract the localization length ξ by fitting the curves $\langle -\ln G\rangle$ vs L with Eq. (4); see dashed lines in Fig. 1(a). This behavior should be contrasted to the case of 1D wires with off-diagonal Lévy-type disorder [53] which shows the dependence $\langle -\ln G\rangle \propto L^{1/2}$ when $\alpha = 1/2$ at $E = 0$ [50].

Also, we have confirmed that the cumulants $\langle\langle(-\ln G)^k\rangle\rangle$ obey a linear relation with the wire length [41,54], i.e.,

$$\lim_{L\to\infty} \frac{\langle\langle(-\ln G)^k\rangle\rangle}{L} = 2^k c_k, \tag{5}$$

where the coefficients c_k, with $c_1 \equiv \xi^{-1}$, characterize the Lyapunov exponent of a generic 1D tight-binding wire with on-site disorder. We have verified the above relation, Eq. (5), for $k = 1$, 2, and 3; as an example in Fig. 1(b) we present the results for $\langle\langle(-\ln G)^2\rangle\rangle$ as a function of L for different values of α. The dashed lines are fittings of the numerical data (open dots) with the function $\langle\langle(-\ln G)^2\rangle\rangle = 4c_2L$ [see Eq. (5)], which can be used to extract the higher-order coefficient c_2.

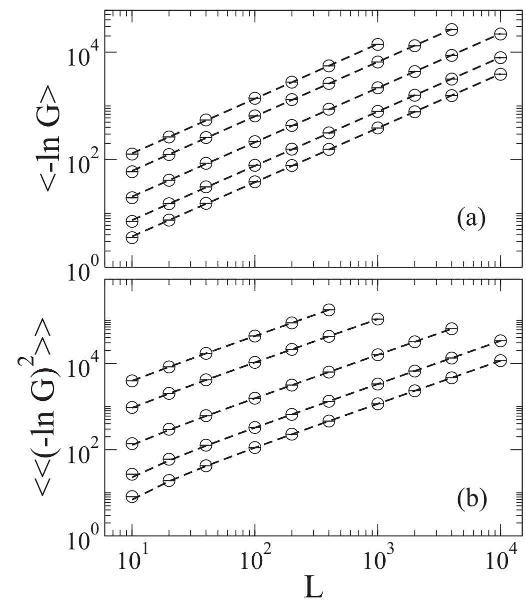

FIG. 1. (a) Average logarithm of the conductance $\langle -\ln G\rangle$ as a function of L for the 1DAM with Lévy-type on-site disorder (symbols). Dashed lines are the fittings of the data with Eq. (4) used to extract ξ. (b) $\langle\langle(-\ln G)^2\rangle\rangle$ as a function of L (symbols). Dashed lines are fittings of the data with the function $\langle\langle(-\ln G)^2\rangle\rangle = 4c_2L$ [see Eq. (5)]. In both panels $\alpha = 1/10$, 1/5, 1/2, 1, and 3/2 (from top to bottom). Each point was calculated using 10^4 disorder realizations. $E = 0$ was used.

Now, in Fig. 2 we show different conductance distributions $P(G)$ for the 1DAM with Lévy-type on-site disorder for fixed values of $\langle -\ln G\rangle$; note that fixed $\langle -\ln G\rangle$ means fixed ratio L/ξ. Several values of α are reported in each panel. We can observe that for fixed $\langle -\ln G\rangle$, by increasing α the conductance distribution evolves towards the $P(G)$ corresponding to the 1DAM with white noise disorder, $P_{\rm WN}(G)$, as expected. The curves for $P_{\rm WN}(G)$ are included as a reference in all panels of Fig. 2 as red dashed lines [55,56]. In fact, $P(G)$ already corresponds to $P_{\rm WN}(G)$ once $\alpha = 2$.

We recall that for 1D tight-binding wires with off-diagonal Lévy-type disorder $P(G)$ is fully determined by the exponent α and the average $\langle -\ln G\rangle$ [50]. It is therefore pertinent to ask whether this property also holds for *diagonal* Lévy-type disorder. Thus, in Fig. 3 we show $P(G)$ for the 1DAM with Lévy-type on-site disorder for several values of α, where each panel corresponds to a fixed value of $\langle -\ln G\rangle$. For each combination of $\langle -\ln G\rangle$ and α we present two histograms (in red and black) corresponding to wires with on-site random potentials $\{\epsilon_n\}$ characterized by two *different* density distributions [57], but with the same exponent α of their corresponding power-law tails. We can see from Fig. 3 that for each value of α the histograms (in red and black) fall on top of each other, which is evidence that the conductance distribution $P(G)$ for the 1DAM with Lévy-type

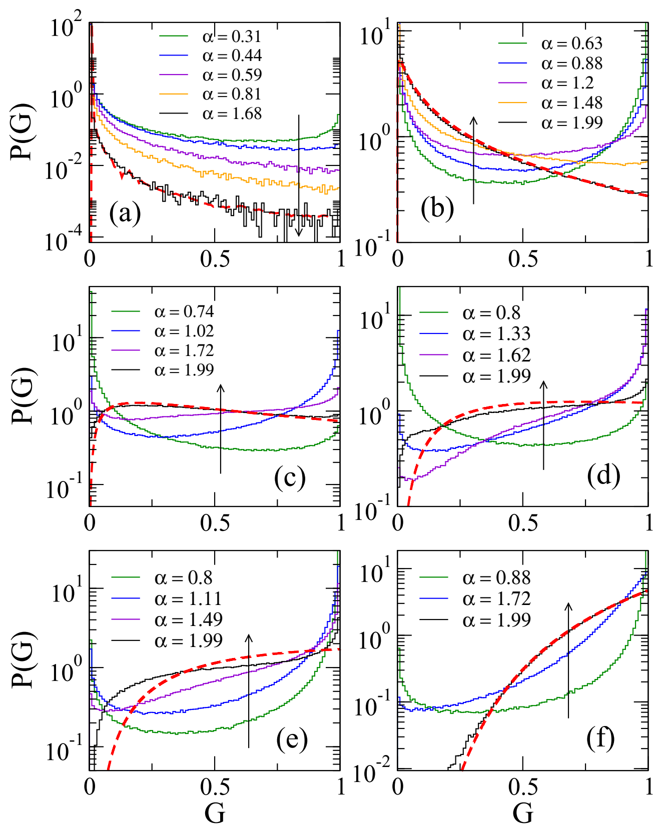

FIG. 2. Conductance distribution $P(G)$ for the 1DAM with Lévy-type disorder (histograms). Each panel corresponds to a fixed value of $\langle -\ln G\rangle$: (a) $\langle -\ln G\rangle = 20$, (b) $\langle -\ln G\rangle = 2$, (c) $\langle -\ln G\rangle = 1$, (d) $\langle -\ln G\rangle = 2/3$, (e) $\langle -\ln G\rangle = 1/2$, and (f) $\langle -\ln G\rangle = 1/5$. In each panel we include histograms for several values of α, where α increases in the arrow direction. $E = 0$ was used. Each histogram was calculated using 10^6 disorder realizations. The red dashed lines are the theoretical predictions of $P(G)$ for the 1DAM with white noise disorder $P_{\rm WN}(G)$ corresponding to the particular value of $\langle -\ln G\rangle$ of each panel.

on-site disorder is invariant once α and $\langle -\ln G\rangle$ are fixed; i.e., $P(G)$ displays a universal statistics.

Moreover, we want to emphasize the coexistence of insulating and ballistic regimes characterized, respectively, by the two prominent peaks of $P(G)$ at $G = 0$ and $G = 1$. This behavior, which is more evident for $\langle -\ln G\rangle \sim 1$ and $\alpha \leqslant 1$ (see Figs. 2 and 3), is not observed in 1D wires with white-noise disorder (see, for example, the red dashed curves in Fig. 2). This coexistence of opposite transport regimes has been already reported in systems with anomalously localized states: 1D wires with obstacles randomly spaced according to Lévy-type density distribution [48,50] as well as in the so-called random-mass Dirac model [58].

Finally, we study the behavior of the tail of the distribution $P(\ln G)$. Thus, using the same data of Fig. 3, in Fig. 4 we plot $P(\ln G)$. As expected, since $P(G)$ is determined by α and $\langle -\ln G\rangle$, we can see that $P(\ln G)$ is invariant once those two quantities (α and $\langle -\ln G\rangle$) are fixed (red and black histograms fall on top of each other). Moreover, from Fig. 4 we can deduce a power-law behavior,

$$P(\ln G) \propto G^{\beta}, \tag{6}$$

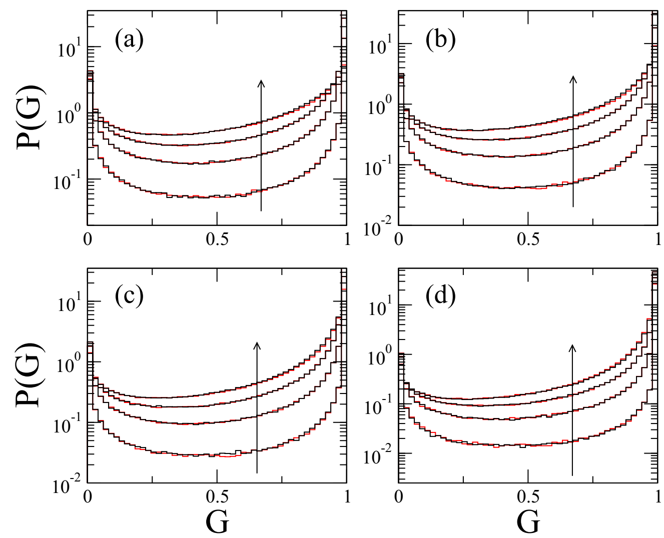

FIG. 3. Conductance distribution $P(G)$ for the 1DAM with Lévy-type on-site disorder. Each panel corresponds to a fixed value of $\langle -\ln G\rangle$: (a) $\langle -\ln G\rangle = 1$, (b) $\langle -\ln G\rangle = 3/4$, (c) $\langle -\ln G\rangle = 1/2$, and (d) $\langle -\ln G\rangle = 1/4$. In each panel we include histograms for $\alpha = 1/4$, 1/2, 3/4, and 1, where α increases in the arrow direction. $E = 0$ was used. For each value of α we present two histograms using different Lévy-type density distributions of on-site disorder: $\rho_1(\epsilon)$ in red and $\rho_2(\epsilon)$ in black; see Ref. [57]. Each histogram was calculated using 10^6 disorder realizations.

for $G \to 0$ when $\alpha < 2$. For $\alpha = 2$, $P(\ln G)$ displays a log-normal tail (not shown here), expected for 1D systems in the presence of Anderson localization. Actually, the behavior (6) was already anticipated in Ref. [41] as $P(G) \sim G^{-(2-\lambda)/2}$ for $G \to 0$ with $\lambda < \alpha$, which in our study translates as $P(\ln G) \propto G^{\lambda/2}$ [since $P(\ln G) = G\,P(G)$] with $\lambda/2 \equiv \beta \leqslant \alpha/2$. Indeed, we have validated the last inequality in Fig. 5 where we report

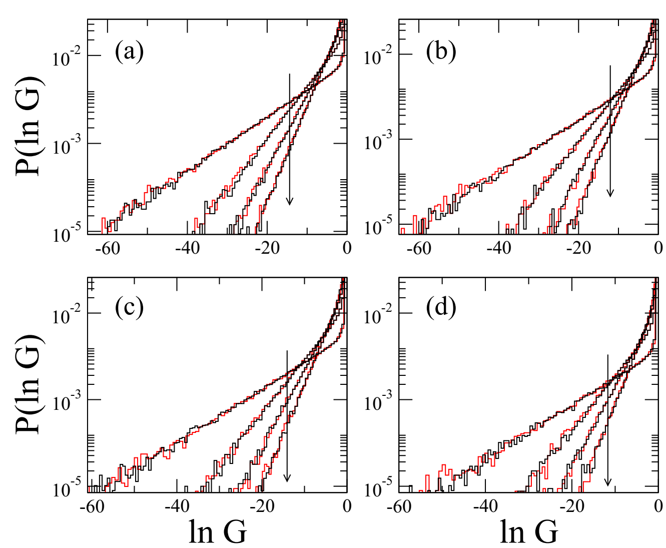

FIG. 4. Probability distribution functions $P(\ln G)$ for the 1DAM with Lévy-type on-site disorder. Same parameters as in Fig. 3. Recall that in each panel we included histograms for $\alpha = 1/4$, 1/2, 3/4, and 1. Here, α increases in the arrow direction.

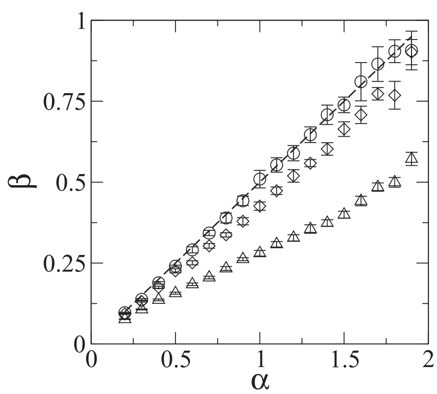

FIG. 5. The exponent β [see Eq. (6)] as a function of α for $\langle -\ln G\rangle = 1/10$ (circles), 1 (diamonds), and 10 (triangles). The dashed line corresponds to $\beta = \alpha/2$. β was obtained from power-law fittings of the tails of the histograms of $P(\ln G)$ in the interval $P(\ln G) \in [10^{-5}, 10^{-3}]$.

the exponent β obtained from power-law fittings of the tails of the histograms of $P(\ln G)$. In addition, we have observed that the value of β depends on the particular value of $\langle -\ln G\rangle$ characterizing the corresponding histogram of $P(\ln G)$. Also, from Fig. 5 we note that $\beta \approx \alpha/2$ as the value of $\langle -\ln G\rangle$ decreases.

III. CONCLUSIONS

In this work we have studied the conductance G through a generalization of the Lloyd model in one dimension: We consider 1D tight-binding wires with on-site disorder following a Lévy-type distribution [see Eq. (1)] characterized by the exponent α of the power-law decay. We have verified that different cumulants of the variable $\ln G$ decrease linearly with the length wire L. In particular, we were able to extract the eigenfunction localization length ξ from $\langle -\ln G\rangle = 2L/\xi$. Then, we have shown some evidence that the probability distribution function $P(G)$ is invariant, i.e., fully determined, once α and $\langle -\ln G\rangle$ are fixed; in agreement with other Lévy-disordered wire models [2,48–50]. We have also reported the coexistence of insulating and ballistic regimes, evidenced by peaks in $P(G)$ at $G = 0$ and $G = 1$; these peaks are most prominent and commensurate for $\langle -\ln G\rangle \sim 1$ and $\alpha \leqslant 1$. Additionally we have shown that $P(\ln G)$ develops power-law tails for $G \to 0$, characterized by the power-law β (also invariant for fixed α and $\langle -\ln G\rangle$) which, in turn, is bounded from above by $\alpha/2$. This upper bound of β implies that the smaller the value of α the larger the probability of finding vanishing conductance values in our Lévy-disordered wires.

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$$P_{\rm WN}(G) = C\sqrt{\frac{\mathrm{acosh}\left(1/\sqrt{G}\right)}{G^3\sqrt{1-G}}}\exp\left[-\frac{1}{s}\mathrm{acosh}^2\left(\frac{1}{\sqrt{G}}\right)\right],$$

where C is a normalization constant and $s = L/\ell$, with ℓ being the mean free path. The parameter s can the obtained numerically from the ensemble average $\langle \ln G\rangle = -L/\ell$.
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$$\rho_1(\epsilon) = \frac{1}{\Gamma(\alpha)}\left(\frac{1}{2}\right)^{\alpha}\frac{1}{\epsilon^{1+\alpha}}\exp\left(-\frac{1}{2\epsilon}\right)$$

and

$$\rho_2(\epsilon) = \frac{\alpha}{(1+\epsilon)^{1+\alpha}},$$

where Γ is the Euler gamma function.
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