Continuous and discontinuous absorbing-state phase transitions on Voronoi-Delaunay random lattices

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We study absorbing-state phase transitions (APTs) in two-dimensional Voronoi-Delaunay (VD) random lattices with quenched coordination disorder. Quenched randomness usually changes the criticality and destroys discontinuous transitions in low-dimensional nonequilibrium systems. We performed extensive simulations of the Ziff-Gulari-Barshad model, and verified that the VD disorder does not change the nature of its discontinuous transition. Our results corroborate recent findings of Barghathi and Vojta [H. Barghathi and T. Vojta, Phys. Rev. Lett. **113**, 120602 (2014)], stating the irrelevance of topological disorder in a class of random lattices that includes VD, and raise the interesting possibility that disorder in nonequilibrium APT may, under certain conditions, be irrelevant for the phase coexistence. We also verify that the VD disorder is irrelevant for the critical behavior of models belonging to the directed percolation and Manna universality classes.

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I. INTRODUCTION

Nonequilibrium phase transitions from an active (fluctuating) to an inactive (absorbing) phase in spatially extended systems is a topic of broad interest [1–3]. The so-called absorbing-state phase transitions (APTs) arise in a wide variety of problems as, for example, heterogeneous catalysis [4], interface growth [5], population dynamics, and epidemiology [6]. Recent experimental realizations in turbulent liquid crystals [7], driven suspensions [8], and superconducting vortices [9] highlight the importance of this kind of transition.

In analogy with equilibrium phase transitions, it is expected that *continuous* APTs can be classified in universality classes [1,2]. Generically, single-component systems with short-range interactions exhibiting a continuous APT, in the absence of extra symmetries or conservation laws, belong to the directed percolation (DP) universality class [10,11], but other robust classes emerge when multiple absorbing states and conservation laws are included [1,2].

Of particular interest is how spatially quenched disorder affects the critical behavior of an APT. In real systems, quenched disorder appears in the form of impurities and defects [12]. On a regular lattice, quenched disorder can be added in the forms of random deletion of sites or bonds [13–18] or of random spatial variation of the control parameter [19–21]. In all the cases above, quenched randomness produces rare regions which are locally supercritical even when the whole system is subcritical. The lifetime of active rare regions is exponentially long in the domain size. The convolution of rare region and exceedingly large lifetimes can lead to a slow dynamics, with nonuniversal exponents, for some interval of the control parameter $\lambda_c^{(0)} < \lambda < \lambda_c$, where $\lambda_c^{(0)}$ and λ_c are the critical points of the clean and disordered systems, respectively. This interval of singularities is called Griffiths phase (GP) [22]. This GP behavior was verified in DP models with uncorrelated disorder irrespective of the disorder strength and corresponds to the universality class of the random transverse Ising model [18,23–26].

These findings are in agreement with the heuristic Harris' criterion [27], which states that uncorrelated quenched disorder is a relevant perturbation if

$$d\nu_\perp < 2, \tag{1}$$

where d is the dimensionality and $\nu_\perp$ is the correlation length exponent of the clean model. Note that in DP, this inequality is satisfied for all dimensions $d < 4$, since $\nu_\perp = 1.096854(4)$, 0.734(4), and 0.581(5), for $d = 1$, 2, and 3, respectively [28–30]. In the opposite way, simulations of the continuous APT in models with a conserved field in the Manna universality class [31], considering uncorrelated lattice dilution below the lattice percolation threshold, provide strong evidence that this kind of disorder is irrelevant although the Harris criterion is satisfied for $d < 4$ [32–34].

For equilibrium *discontinuous* phase transitions, the Imry-Ma criterion [35,36] governs the stability of macroscopic phase coexistence and disorder destroys phase coexistence by domain formation in dimensions $d \leqslant 2$. If the distinct phases are related by a continuous symmetry, the marginal dimension is $d = 4$ [36]. Therefore, first-order phase transitions become rounded in the presence of disorder for $d \leqslant 2$.

Recent numerical results provide evidence that the Imry-Ma argument for equilibrium systems can be extended to nonequilibrium APTs: Irrespective of the uncorrelated disorder strength, Buendia and Rikvold [37–39] reported that the absorbing discontinuous transition in the Ziff-Gulari-Barshad (ZGB) model for heterogeneous catalysis turns to a continuous one (see also the discussion in [40]). Analogous behavior was observed more recently by Martín *et al.* [41] for a two-dimensional quadratic contact process [42].

Another important question is the role played by disorder inherent in the underlying connectivity in a nonperiodic,

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random structure of integer dimension as the random lattice generated by the Voronoi-Delaunay (VD) triangulation [43]. This random lattice can be generated from a random (uniform) distribution of N points in a unitary square region. The triplets that can be circumscribed by a circle that does not overlap with any other point form a triangulation. The result is a two-dimensional connected graph with a Poissonian distribution of connectivity with average degree $\bar{q} = 6$ [44]. This lattice plays an important role in the description of idealized statistical geometries such as planar cellular structures, soap throats, etc. [43,44].

Recently, it was found that such a kind of VD disorder does not alter the character of the APT exhibited by the clean contact process (CP) [45], which is a prototypical model in the DP universality class. These results are in evident contrast with those for uncorrelated disorder which lead to an infinite-randomness critical point and strong GPs [18,26]. In order to determine the relevance of the disorder in these cases, we can apply the heuristic Harris-Luck criterion [46], in which the regular critical behavior remains unchanged when the wandering exponent[1] ω does not exceed a threshold value given by

$$\omega_c = 1 - \frac{1}{d\nu_\perp}. \tag{2}$$

For independent dilution, $\omega = 1/2$, and Luck's expression reduces to the Harris criterion.

Former numerical estimates of wandering exponents for VD triangulations indicated a value close to independent dilution $\omega = 1/2$ [47]. So, the clean critical behavior observed for CP on VD lattices posed doubts on the validity of the Harris-Luck criterion for the DP class [45]. This inconsistency was recently unfolded [48] with the determination of the correct wandering exponent of VD lattices as $\omega = 1/4$ in $d = 2$, implying a criterion $\nu_\perp > 2/3$, not $\nu_\perp > 1$, for a clean critical behavior.

In the present work, we investigate the role played by the disorder of VD lattice on the phase coexistence of the ZGB model. We provide evidence that the VD topological disorder does not destroy the phase coexistence and thus permits discontinuous phase transitions. We complement the paper with more evidence for the irrelevance of VD disorder for continuous APTs belonging to the DP [2] and Manna [31,49] universality classes.

The reminder of this paper is organized as follows. In the next section, we review the model's definitions and the details of the simulation methods we used. In Sec. III, we present our results and discussions. Section IV is devoted to summarize our conclusions.

II. MODELS AND METHODS

We constructed the Voronoi-Delaunay lattice with periodic boundary conditions, following the method described in [50]. For the sake of simplicity, the length of the domain where N nodes are randomly distributed will be expressed in terms of $L = \sqrt{N}$.

[1]The wandering exponent is associated with the decay of deviations from the average as a function of patch sizes where the averages are computed.

A. Discontinuous APT

The ZGB model [4], a lattice gas model introduced to investigate the reaction of CO oxidation on a catalytic substrate, follows the Langmuir-Hinshelwood mechanism,

$$\mathrm{CO_{gas}} + * \rightarrow \mathrm{CO_{ads}},$$
$$\mathrm{O_{2gas}} + 2* \rightarrow 2\mathrm{O_{ads}},$$
$$\mathrm{CO_{ads}} + \mathrm{O_{ads}} \rightarrow \mathrm{CO_2} + 2*,$$

where $*$ denotes an empty site, and subscripts indicate the state (gaseous or adsorbed) of each species. The O_{2gas} dissociates at the surface and requires two empty sites to adsorb, while CO requires only one site to adsorb (the model is also called the monomer-dimer model). The product CO_2 desorbs immediately on formation. CO_{gas} molecules arrive at rate Y per site, while O_2 arrives at rate $(1 - Y)$, with $0 \leqslant Y \leqslant 1$. Varying the control parameter Y, the model exhibits phase transitions between an active steady state and one of the two absorbing or "poisoned" states, in which the surface is saturated either by oxygen (O) or by CO. The first transition (O poisoned) is found to be continuous, while the second transition (CO poisoned) is strongly discontinuous.

The computer implementation is the following: With a probability Y, a CO adsorption attempt takes place, and with a complementary probability $(1 - Y)$, an O_2 adsorption attempt takes place. In the former case, one site is randomly chosen. If the site is occupied, either by O or CO, the attempt fails. If it is empty but one of its nearest neighbors is occupied by one O, both sites become empty (O and CO react instantaneously). Otherwise, the site becomes occupied by an adsorbed CO molecule. An analogous procedure is followed for an O_2 adsorption attempt, but in this case we have to choose at random a *pair* of nearest-neighbor sites, and check for the opposite species in all remaining nearest neighbors of the target pair.

B. Continuous APT

The CP [51] is the prototypical model of the DP class, and is defined on a lattice with each site either active ($\sigma_i = 1$) or inactive ($\sigma_i = 0$). Transitions from active to inactive occur spontaneously at a rate of unity. The transition from inactive to active occurs at rate $\sigma_j \lambda / k_j$, for each edge between active nearest neighbors j of site i. The computer implementation of CP in graphs with arbitrary connectivity is as follows [1,18]: An occupied site is chosen at random. With probability $p = 1/(1 + \lambda)$, the chosen particle is removed. With the complementary probability $1 - p = \lambda/(1 + \lambda)$, a nearest neighbor of the selected particle is randomly chosen and, if empty, is occupied; otherwise, nothing happens and the simulation runs to the next step. Time is incremented by $\delta t = 1/n$, where n is the number of particles. So, the creation mechanism in CP effectively compensates the local connectivity variation with a reduction of the spreading rate through a particular edge inversely proportional to the connectivity of the site that transmits a new particle. If we modify these rules to create offspring in *all* empty nearest neighbors of the randomly chosen occupied site, we obtain the A model [52,53] (in the A model, occupied sites become empty at unitary rate, as in the CP). This means that sites with higher coordination number produce more activity when compared with the CP,

enhancing the possible "rare region effect" [22,54]. Since contagion occurs more readily in the A model than in the CP, the critical creation rate λ_c is smaller, but the two models share the same critical behavior of the DP universality class [55].

The Manna model [31], a prototypical model of the Manna class and introduced to investigate the dynamic of sandpiles in the context of self-organized criticality, is defined on a lattice where each site assumes integer values (mimicking the number of "sand grains" deposited on the substrate). In the version we investigate, an unlimited number of particles per site is permitted. Sites with a number below a threshold $N_c = 2$ are inactive, while those where this number is equal to or larger than N_c are active. The active sites redistribute their particles among its nearest neighbors chosen at random, generating a dynamics that conserves the number of particles when considering the periodic boundary condition. The Manna model exhibits a continuous phase transition from an active to an inactive state depending on the control parameter p that is given by the density of particles on the lattice [56]. The absorbing stationary state, where all sites have a number of particles below N_c, is characterized by an infinite number of configurations. The computer implementation is analogous to that of the CP: one active site i ($N_i \geq N_c$) is randomly chosen. Each of the N_i particles is sent to a randomly chosen nearest neighbor, irrespective of its state. The site i becomes empty (inactive) and the nearest neighbors with $N_j \equiv N_c - 1$ particles that received a new one are activated.

C. Simulation methods

The central method we used involves the quasistationary state, in which averages are restricted to samples that did not visit an absorbing state [1]. To perform the quasistationary (QS) analysis, we applied the simulation method of Ref. [57]. The method is based on maintaining, and gradually updating, a set of configurations visited during the evolution; when a transition to the absorbing state is imminent, the system is instead placed in one of the saved configurations. Otherwise, the evolution is exactly that of a conventional simulation [58]. Each realization of the process is initialized in an active state and runs for at least 10^8 Monte Carlo time steps. Averages are taken in the QS regime after discarding an initial transient of 10^7 or more time steps. This procedure is repeated for each realization of disorder. The number of disorder realizations ranged from 20 (for the largest size used, $L = 2048$) to 10^3. Another important quantity is the lifetime in the QS regime, τ. In QS simulations, we take τ as the mean time between successive attempts to visit the absorbing state.

For discontinuous APTs, we estimated the transition point through the jump in the order parameter and the finite-size scaling of the maximum of the susceptibility. In the DP class, the spreading analysis starting from a single active site (a preabsorbing configuration) is a very accurate and computationally efficient method [1]. For the Manna class, spreading analysis is more cumbersome [2] due to infinitely many preabsorbing configurations. So, we proceeded using dimensionless moment ratios analysis in the QS state [53], which are size independent at criticality. Here, we analyze the critical moment ratio $m = \langle \rho^2 \rangle / \langle \rho \rangle^2$, which assumes a universal value m_c at the clean critical point.

III. RESULTS

A. Discontinuous APT

First-order transitions are characterized by a discontinuity in the order parameter and thermodynamic densities, with an associated δ-peak behavior in the susceptibility [2]. However, at finite volume, thermodynamic quantities become continuous and rounded. According to the finite-size theory, rounding and shifting of the coexistence point scale inversely proportional to the system volume L^d [59]. Although there is no established similar scaling theory for nonequilibrium systems, some studies show evidence of an analogous behavior for APTs [60–62].

Quasistationary analysis remains useful in the context of discontinuous APTs [62]. Considering the QS simulations, we observe a discontinuous phase transition from a low-density to a poisoned (absorbing) CO state, as shown in Fig. 1, instead of a rounded (continuous) transition expected for APTs in the presence of relevant disorder [41]. The inset of Fig. 1 shows the QS probability distribution for the density of active sites near the transition. We clearly observe a bimodal distribution, which is a hallmark of discontinuous phase transition [62].

In analogy to an equilibrium first-order phase transition, where at the transition point a thermodynamical potential (such as the free energy) is equal for both phases [59], we can define the coexistence value of the order parameter in which the areas under the peaks of the QS distribution related to each phase (active and absorbing) are equal [62]. The intercept of the linear fit from this equal histogram method yields $Y_c = 0.55928(3)$. Such a value is very close to the coexistence value $Y_c = 0.5596(5)$ that we found for the regular triangular lattice (see inset of Fig. 2).

The location of the maximum of the susceptibility χ, defined as variance of the order parameter $\chi = L^d(\langle \rho^2 \rangle - \langle \rho \rangle^2)$, scales as L^d in a discontinuous APT [60–62]. Figure 2 shows the finite-size scaling of the transition point, which clearly scales inversely to the volume, confirming again that

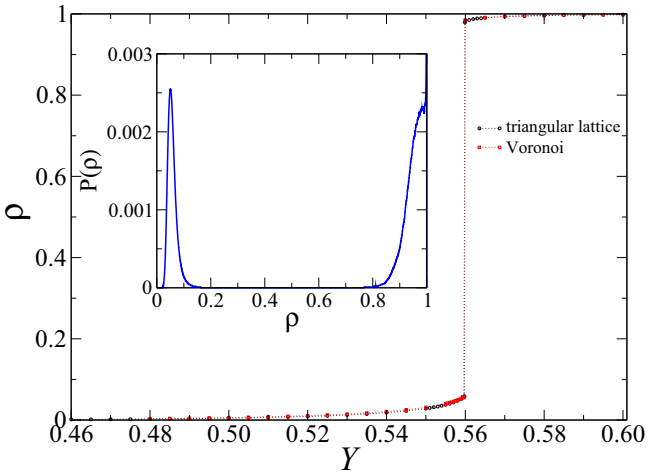

FIG. 1. QS density of CO sites in the ZGB model on triangular and Voronoi lattices of linear system size $L = 100$, showing a discontinuous phase transition to the CO-poisoned absorbing state close to $Y = 0.56$. Inset shows the QS distribution for $Y = 0.5590$ where the APT takes place.

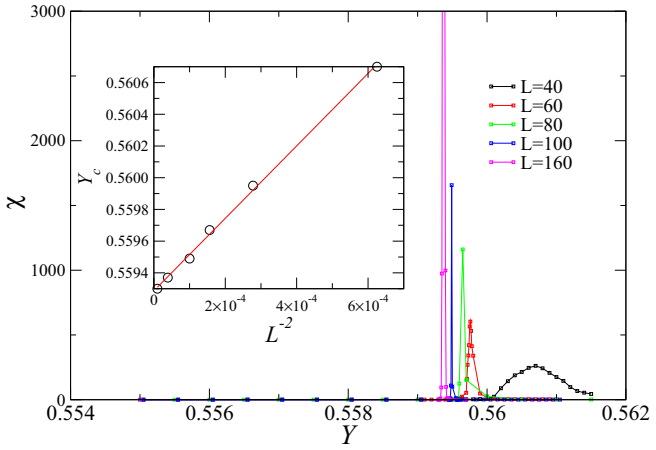

FIG. 2. Quasistationary susceptibility on the ZGB model on VD lattices as a function of Y for different sizes. Inset: Finite-size scaling for the susceptibility maxima in the range $L = 40$ to 320.

the disordered lattice does not alter the discontinuous nature of the transition.

Further evidence of discontinuity of the phase transition in the presence of quenched coordination disorder is shown in Fig. 3. Using conventional simulations, we observe the system bistability around the transition point: depending on the initial density, a homogeneous steady state may converge either to a stationary active state of high CO_2 production (and small CO density) or to the CO-poisoned (absorbing) state. These results contrast with those for uncorrelated disorder, for which no matter its strength, the discontinuous transition is replaced by a continuous one.

B. Continuous APT

The spreading analysis for the A model on VD lattices using the mean number of active sites against time, with a single occupied site as the initial condition, provides a critical value

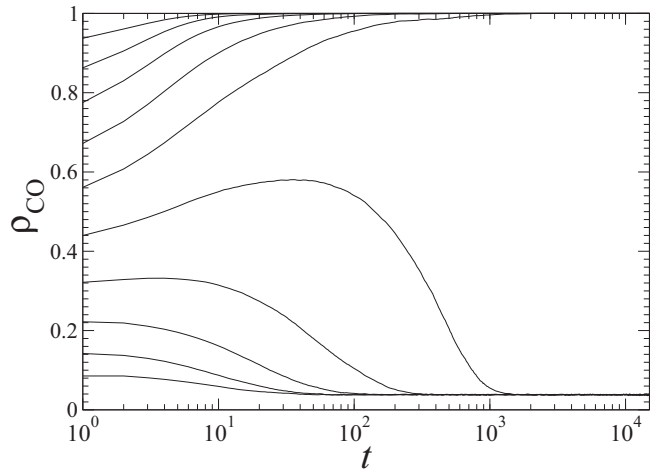

FIG. 3. Density of CO as a function of time for distinct initial conditions close to the transition point. Initial densities $\rho_{CO} = 0.0, 0.1, 0.2, \ldots, 0.9$, from bottom to top. Linear system size $L = 100$ and $Y = 0.5560$.

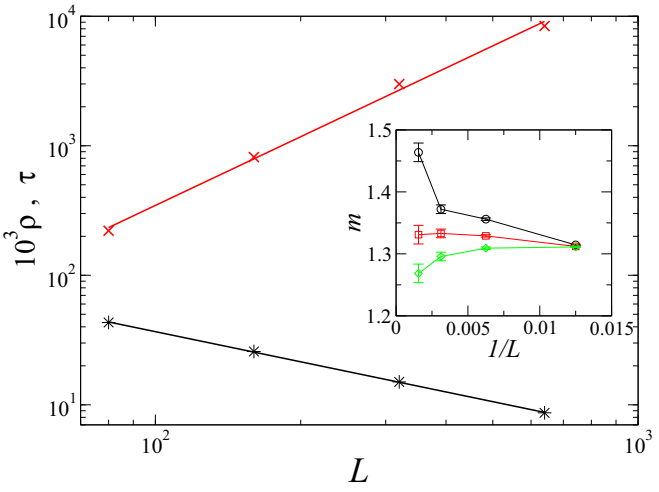

FIG. 4. Finite-size scaling of the critical A model. Quasistationary density of active sites ρ (stars) and lifetime of the QS state τ (crosses) as a function of the system sizes L for $\lambda = 0.32243$. Solid lines are power-law regressions. Inset: Quasistationary moment ratio m vs $1/L$, for $\lambda = 0.32238$, $\lambda = 0.32242$, and $\lambda = 0.32246$, from top to bottom.

$\lambda_c = 0.322430(5)$, which is smaller than $\lambda_c = 0.34047(1)$ found for the regular triangular lattice with $q = 6$. This difference is more significant than that obtained for CP for these same lattices [45], showing that the effect of disorder in the A model is stronger than in CP. However, the critical behavior remains that of the clean system, exhibiting power laws with spreading exponents very consistent with the DP class (results not shown).

Figure 4 shows that at the critical point we found, the QS density ρ decays as a power law, $\rho \sim L^{-\beta/\nu_\perp}$, with $\beta/\nu_\perp = 0.79(1)$. Besides, we observe that the lifetime of the QS state also follows a power law at criticality, with $\tau \sim L^z$, $z = 1.73(5)$. Both values of the exponents are close to the DP ones of $\beta/\nu_\perp = 0.797(3)$ and $z = 1.7674(6)$ [2]. The inset of Fig. 4 shows the ratio $m = \langle\rho^2\rangle/\langle\rho\rangle^2$ around the criticality for varying system sizes. From these data, we found $m_c = 1.33(1)$, in agreement with the value $m_c = 1.3257(5)$ found for the DP class in two dimensions [53]. All of the results presented here confirm the irrelevance of disorder of the VD lattice for the critical behavior of the A model.

Let us now turn our attention to the Manna class. The correlation length exponent $\nu_\perp = 0.799$ [63], which is larger than the DP value 0.7333, makes the modified Harris-Luck modified criterion $(d+1)\nu_\perp < 2$ still not fulfilled for VD lattices [48]. The critical point determination using moment ratios is shown in the inset of Fig. 5, resulting in the estimate $p_c = 0.688808(2)$ that is smaller than the triangular lattice threshold $p_c = 0.69375(5)$. The critical moment ratio is $m_c = 1.35(1)$, which agrees with the value we found for square lattices[2] $m_c = 1.348(7)$ at the threshold $p_c = 0.716957(2)$. The critical exponents we obtained using $L \geq 256$ were

[2]Our estimate of m does not agree with that of Ref. [64], where a restricted version of the Manna model, in which $N_i > 2$ is forbidden, was considered.

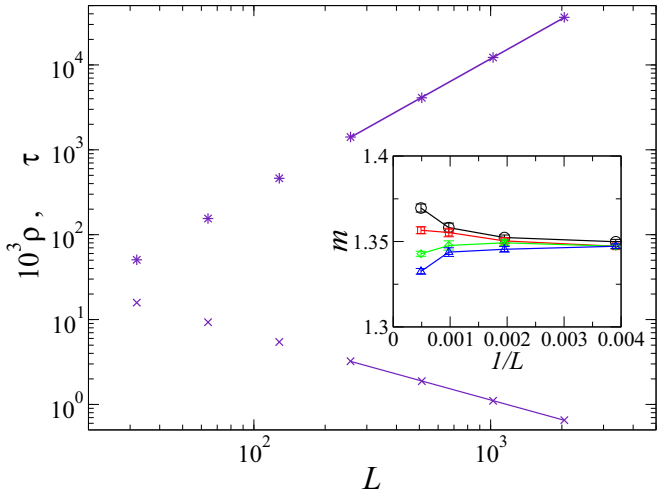

FIG. 5. Critical Manna model on VD lattices. Critical density of active sites (crosses) and lifetime (stars) against lattice size. Solid lines are power-law regressions. Inset: Moment ratio $m = \langle\rho^2\rangle/\langle\rho\rangle^2$ against inverse of size for $p = 0.688800, 0.688805, 0.688810$, and 0.688815, from top to bottom.

$\beta/\nu_\perp = 0.78(1)$ and $\nu_\parallel/\nu_\perp = 1.54(2)$ and are also in striking agreement with the Manna class exponents $\beta/\nu_\perp = 0.80(2)$ and $\nu_\parallel/\nu_\perp = 1.53(5)$.

It is known that critical exponents and moment ratios of the Manna class in $d = 2$ obtained via QS analysis are hardly distinguishable from the DP class [2,65]. In order to provide a more incisive verification that the Manna model on the VD lattice has exponents different from DP, we considered density around the critical point, which scales as [66]

$$\rho(\Delta, L) = \frac{1}{L^{\beta/\nu_\perp}} \mathcal{F}_\rho(L^{1/\nu_\perp}\Delta), \tag{3}$$

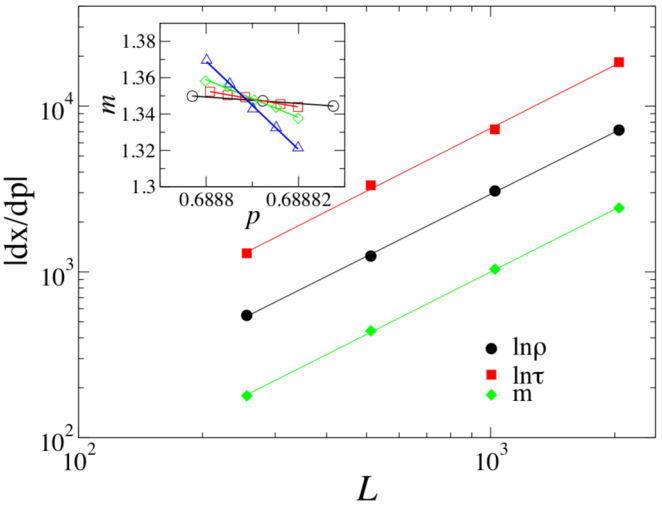

FIG. 6. Determination of critical exponent $\nu_\perp$ for the Manna model on VD lattices using different quantities $x = \ln\rho, \ln\tau$, and m. Inset: Moment ratio against control parameter around the critical point for $L = 256$, 512, 1024, and 2048 (the steeper the curve, the larger the size).

where $\Delta = p - p_c$. This implies that

$$\left|\frac{\partial \ln\rho}{\partial p}\right| \sim L^{1/\nu_\perp} \tag{4}$$

can be used to obtain the exponent $\nu_\perp$ explicitly. Similarly, for the moment ratio, we have $m(\Delta, L) = \mathcal{F}_m(L^{1/\nu_\perp}\Delta)$, implying that $\nu_\perp$ can also be directly obtained from

$$\left|\frac{\partial m}{\partial p}\right| \sim L^{1/\nu_\perp}. \tag{5}$$

A similar scaling law is expected for τ. The inset of Fig. 6 shows the moment ratios around the critical point where the slope clearly increases (in absolute values) with size. The main plot shows the derivatives against size. Using the three methods, we estimate a critical exponent $1/\nu_\perp = 1.252(10)$, which is remarkably close to the exponent for the Manna class $1/\nu_\perp = 1.250(18)$ [2] and definitely rules out the DP value $1/\nu_\perp = 1.364(10)$ [1].

IV. CONCLUSIONS

We investigate the effects of quenched coordination disorder in continuous and discontinuous absorbing-state phase transitions. Our extensive simulations of the ZGB model on the VD lattices reveal that the discontinuous nature of the absorbing-state transition featured by this model remains unchanged under this kind of disorder. Recently, it was shown that the Imry-Ma argument can be extended to nonequilibrium situations, including absorbing states, and, in addition, it was conjectured that first-order phase transitions cannot appear in low-dimensional disordered systems with an absorbing state. We show that this is not always true: Our results for the ZGB model raise the interesting possibility that disorder in nonequilibrium APT may, under certain conditions, be irrelevant for the phase coexistence. The underlying reason for this is that the fluctuations induced by correlated coordination disorder exhibited by the VD lattice decay faster and are not able to preclude phase coexistence.

In the case of continuous APT, we performed large-scale simulations of the A and Manna models on VD lattices. Our results confirm, as expected, that this kind of disorder does not alter the universality of the continuous transitions, supporting that strong anticorrelations present in the VD random lattice make topological disorder less relevant than uncorrelated randomness.

Our findings corroborate a recent work of Barghathi and Vojta [48], which shows systematically that the disorder fluctuations of the VD lattice are featured by strong anticorrelations and decay faster than those of random uncorrelated disorder. In particular, it was shown that the random VD lattice has wandering exponent $\omega = 1/4$ [48]. Hence, in this case, the Harris-Luck criterion yields that random connectivity is irrelevant at a clean critical point for $\nu_\perp > 2/3$ that is satisfied for both the Manna and DP universality classes. It is important to mention that in contrast to the A model, which belongs to the DP class, even the strong disorder of uncorrelated lattice dilution (below the lattice percolation threshold) was found to be irrelevant for the Manna class [32–34]. Therefore, our results are consistent with these

findings, since the coordination disorder of the VD lattice is weaker than lattice dilution. In addition, we determined that the exponent $1/\nu_\perp = 1.252(10)$ for the Manna class on the VD lattice definitely rules out the DP value $1/\nu_\perp = 1.364(10)$.

Further work should include the study of absorbing phase transitions on a three-dimensional random VD lattice, since it does not belong to the class of lattices with constrained total coordination [54]. In particular, according to the Harris criterion, the disorder might be relevant for the Manna class at least in three dimensions and there might be a dimensional difference between two and three dimensions. It would also be interesting to investigate if other kinds of correlated disorder are irrelevant for phase coexistence.

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