Collision of ion acoustic solitary waves in a magnetized plasma: Effect of dust grains and trapped electrons

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The head-on collision of two ion acoustic solitary waves is investigated in a magnetized plasma containing trapped electrons and dust grains. For completeness, the fluctuations in dust grain charge are taken into account. By using the extended Poincaré-Lighthill-Kuo (PLK) perturbation method, an analytical expression is obtained for the phase shift that takes place due to the collision of the waves. How the phase shift behaves under the combined effect of trapped electrons and dust grains along with the finite temperature of ions is examined. A focus is given to uncover the situations of fluctuating charge and fixed charge on the dust grains in the plasma. Interestingly, the solitary waves acquire a larger phase shift and are delayed more in the case of dust grains having a fluctuating charge.

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I. INTRODUCTION

Solitary waves can evolve from a localized disturbance in a medium which is nonlinear and dispersive. Such types of waves occur only when there is a dynamical balance between the effects of the nonlinearity and the dispersion of the medium. These waves have been an important topic of research because of their existence in laboratory, space, and astrophysical plasmas. The excitation, propagation, and interaction of the solitary waves are important issues in theoretical plasma physics. The interaction of two electrostatic solitons has been studied in multi-ion plasma [1] and electron-positron (e-p) plasma [2]. The head-on collision of two electron acoustic solitary waves in an unmagnetized plasma has been investigated by taking into account the cold electrons fluid, stationary ions, and hot electrons that obey a nonextensive distribution [3]. Such a collision of an ion acoustic solitary wave and a shock wave in a two-electron-temperature plasma has also been reported [4]. The interaction of two counterpropagating ion acoustic solitary waves has been studied in different plasma models [4–12] including the e-p-ion plasma [7] and a plasma having q-distributed electrons. In addition, the interaction of two solitary waves has been investigated in different environments such as in fluid-filled elastic tubes, blood, water, and a nonlinear transmission line [13–16].

The interaction of two counterpropagating solitons of equal amplitudes has been studied experimentally and numerically in a monolayer strongly coupled complex plasma by suspending microparticles in usual ion-electron plasma [17,18]. In these investigations, the solitons were found to endure a delay after their collision. The solitons with higher amplitude have been found to experience longer delays after collision [19]. On the other hand, the head-on collisions of two concentric cylindrical ion acoustic solitary waves have been discussed [7]. The head-on collision of two ion-thermal solitary waves has also been investigated in a two-fluid plasma consisting of positive and negative ions as well as a fraction of stationary ($+ve/-ve$) charged dust impurities [20].

It is clear that a considerable amount of work has been done related to the head-on collision of two solitary waves in different plasmas. However, in most of the cases, the dust grains were neglected; these are available in realistic situations and become charged due to the ion and electron currents flowing into them. In some investigations [20–24] these grains were considered, but their charge was taken to be fixed. Since in a realistic situation, the charge on the dust grains fluctuates, it is desirable to realize the exact behavior of the solitary waves after their collision in a plasma having dust particles of fluctuating or variable charge. In this paper, we consider the same and also include the effect of the magnetic field in view of space-related plasmas.

To solve the above problem, we employ the extended Poincaré-Lighthill-Kuo method. The Poincaré-Lighthill-Kuo method [25–27], abbreviated as the PLK method, was developed based on the Poincaré-Lindstedt technique, which is useful for uniformly approximating periodic solutions to ordinary differential equations when regular perturbation approaches fail. The Poincaré-Lindstedt or Lindstedt-Poincaré technique of strained parameters removed secular terms (the terms that grow without bound) raised in the straightforward application of the perturbation theory to weakly nonlinear problems with finite oscillatory solutions. Here a perturbation of frequency was made that was equivalent to introducing a linear transformation of the time coordinate. This technique was generalized by Lighthill [28] in the case of an aerodynamics problem, where he introduced nonlinear transformations of coordinates for the successful elimination of the singularities in perturbation solutions; actually the ordinary differential equations were generalized to hyperbolic partial differential equations. Then Kuo's contribution to the above generalized technique was the addition of the boundary layer method [29]. Finally, the whole technique, developed based on the combination of the Poincaré-Lindstedt, Lighthill, and Kuo methods, was given the name as the Poincaré-Lighthill-Kuo method or the PLK method. The extended PLK method [7,14,20,24,30–35] is a combination of the standard reductive perturbation method and the technique of the strained coordinates. Here asymptotic expansions of both the flow field variables and the spatial or time coordinates are used in the limit of the long wavelength approximation, which is the case with the solitary waves. With the help of this, one gets a uniformly valid asymptotic expansion (removal of the secular terms) and at the same time the change of the trajectories of solitary waves after the collisions.

II. MATHEMATICAL FORMULATION

We consider a magnetized plasma containing charge-fluctuating stationary dust grains, inertial warm positive ions, isothermal electrons, and nonisothermal (trapped) electrons that follow the vortex-like distribution. A static magnetic field $\vec{B}_0$ is taken to be applied in the z direction at an angle of θ with respect to the direction of the wave propagation in the (x,z) plane. The ions are assumed to be singly charged and the dust grains are assumed to be massive so that their motion can be neglected. The nonisothermality [36–38] of the plasma is considered through the electron density n_{el}. Hence, the normalized basic fluid equations are written as

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial x}(nv_x)+\frac{\partial}{\partial z}(nv_z) = 0, \tag{1}$$

$$n\frac{\partial v_x}{\partial t}+nv_x\frac{\partial v_x}{\partial x}+nv_z\frac{\partial v_x}{\partial z}+n\frac{\partial \phi}{\partial x}-nAv_y+2\sigma\frac{\partial n}{\partial x}=0, \tag{2}$$

$$n\frac{\partial v_z}{\partial t}+nv_x\frac{\partial v_z}{\partial x}+nv_z\frac{\partial v_z}{\partial z}+n\frac{\partial \phi}{\partial z}+2\sigma\frac{\partial n}{\partial z}=0, \tag{3}$$

$$\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_z\frac{\partial v_y}{\partial z}+Av_x=0, \tag{4}$$

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}+n-n_{el}-n_{eh}+n_{d0}Z_d=0, \tag{5}$$

$$n_{el}=n_{el0}\left\{1+\frac{T_{\rm eff}}{T_{el}}\phi-\frac{4}{3}b_l\left(\frac{T_{\rm eff}}{T_{el}}\phi\right)^{3/2}+\frac{1}{2}\left(\frac{T_{\rm eff}}{T_{el}}\phi\right)^2+\dots\right\}, \tag{6}$$

$$n_{eh}=n_{eh0}\exp\left(\frac{T_{\rm eff}\phi}{T_{eh}}\right). \tag{7}$$

The densities of the electron species at lower temperature (trapped) and higher temperature (isothermal) are given by n_{el} and n_{eh}, respectively, together with their respective unperturbed values as n_{el0} and n_{eh0}. The ion density, dust grain density, and dust charge number are given by n, n_{d0}, and Z_d. All the densities are normalized by the unperturbed plasma density n_0 at an arbitrary reference point (say $x=z=0$). $T_{\rm eff}$ is the effective temperature [38,39] of the plasma, defined by $T_{\rm eff}=(n_{elo}+n_{eho})T_{el}T_{eh}/(n_{elo}T_{eh}+n_{eho}T_{el})$, and b_l is the nonisothermal parameter defined as $b_l={(T_{\rm eff}-T_{el})}/{T_{\rm eff}\sqrt{\pi}}$. The ion flow velocities v_x, v_y, and v_z are normalized by the ion acoustic speed $(T_{\rm eff}/m_i)^{1/2}$, where m_i is the mass of the ion, and the electric potential ϕ is normalized by $T_{\rm eff}/e$. The space coordinates x and z are normalized by the Debye length $(\varepsilon_0T_{\rm eff}/n_0e^2)^{1/2}$ and the time t by the inverse of the ion plasma frequency $\omega_{pi}=(n_0e^2/\varepsilon_0m_i)^{1/2}$.

The basic processes that lead to the charging of the dust grains are quiet complex and depend mainly on the environment around the dust grains. When the dust grains are immersed in gaseous plasma, the plasma particles (electrons and ions) are collected by the dust grains and hence, they are charged by the collection of the plasma particles flowing onto their surfaces. This happens due to the difference in the surface potential of the dust grains and the plasma potential. The charge on the dust grains will fluctuate if the plasma potential ϕ fluctuates or oscillates. In this context, we define $\mathbf{\Phi}$ as the dust grain surface potential relative to the plasma potential ϕ, and $\mathbf{\Phi}_i(0)=\mathbf{\Phi}_i$ corresponding to the case of zero plasma potential ($\phi=0$). Considering the dust grains to be spherical, we can write the charge on each grain as $Q=C\mathbf{\Phi}$ together with C as the capacity of the dust grain. Taking Z_d as the normalized dust charge, i.e., $Z_d=\frac{\mathbf{\Phi}}{\mathbf{\Phi}_i(0)}$ (C gets cancelled), we expand $\mathbf{\Phi}$ near $\mathbf{\Phi}_i(0)$ and obtain $Z_d=1+\gamma_1\phi+\gamma_2\phi^2+\dots$, where $\gamma_1=\frac{\mathbf{\Phi}'_i(0)}{\mathbf{\Phi}_i(0)}=\frac{\mathbf{\Phi}'_{i0}}{\mathbf{\Phi}_{i0}}$ and $\gamma_2=\frac{\mathbf{\Phi}''_i(0)}{2\mathbf{\Phi}_i(0)}=\frac{\mathbf{\Phi}''_{i0}}{2\mathbf{\Phi}_{i0}}$. This will be understood as per the following description.

Considering finite-sized neutral dust particles immersed in a magnetized plasma, the charging currents due to the electrons and ions are written as [40]

$$I_e=-e\pi r^2\left(\frac{8T_e}{\pi m_e}\right)^{1/2}n_e\exp\left(\frac{e\mathbf{\Phi}}{T_e}\right), \tag{8}$$

$$I_i=e\pi r^2\left(\frac{8T_i}{\pi m_i}\right)^{1/2}n_i\left(1-\frac{e\mathbf{\Phi}}{T_i}\right). \tag{9}$$

The dust grains always rest at a floating potential with respect to the plasma potential, which means the electron and ion currents towards the dust grains are equal in this equilibrium state. By using $\mathbf{\Phi}_i=\frac{e\mathbf{\Phi}}{T_i}$ and $n_e=n_{el}+n_{eh}$ in Eqs. (8) and (9) in view of the two types of the electrons [Eqs. (6) and (7)], we obtain

$$\sqrt{\frac{\sigma}{\mu_i}}n(1-\mathbf{\Phi}_i)-(n_{el0}+n_{eh0})\begin{pmatrix}1+\phi+\\ \phi^2T'_{\rm eff}\end{pmatrix}\exp\left(\frac{e\mathbf{\Phi}}{T_{\rm eff}}\right)+\frac{4}{3}b_ln_{el0}\left(\frac{T_{\rm eff}}{T_{el}}\phi\right)^{3/2}\exp\left(\frac{e\mathbf{\Phi}}{T_{\rm eff}}\right)=0, \tag{10}$$

where $T'_{\rm eff}=\frac{1}{2}\left(\frac{n_{el0}T_{eh}{}^2+n_{eh0}T_{el}{}^2}{n_{el0}T_{eh}+n_{eh0}T_{el}}\right)\frac{T_{\rm eff}}{T_{el}T_{eh}}$ and $\mu_i=\frac{m_i}{m_e}$. The normalized dust charge Z_d is given by $\frac{\mathbf{\Phi}_i}{\mathbf{\Phi}_i(0)}$ together with $\mathbf{\Phi}_i(0)=\mathbf{\Phi}_i$ at $\phi=0$. The expression for $\mathbf{\Phi}_i(0)$ is obtained from Eq. (10) as $\mathbf{\Phi}_i(0)=\left[\frac{n_{ilh}\sqrt{\frac{\sigma}{\mu_i}}-1}{n_{ilh}\sqrt{\frac{\sigma}{\mu_i}}+\sigma}\right]$ along with the following condition:

$$\sqrt{\frac{\sigma}{\mu_i}}n[1-\mathbf{\Phi}_i(0)]-(n_{el0}+n_{eh0})\exp{[\sigma\mathbf{\Phi}_i(0)]}=0. \tag{11}$$

Here n_{ilh} is defined as $n_{ilh}=\frac{n_0}{n_{el0}+n_{eh0}}$. From Eq. (11) we obtain

$$\left(\frac{d\mathbf{\Phi}_i}{d\phi}\right)_{\phi=0}\equiv\mathbf{\Phi}'_i(0)=-\frac{1-\mathbf{\Phi}_{i0}}{1+\sigma(1-\mathbf{\Phi}_{i0})}, \tag{12}$$

$$\left(\frac{d^2\mathbf{\Phi}_i}{d\phi^2}\right)_{\phi=0}\equiv\mathbf{\Phi}''_i(0)=\left[2T'_{\rm eff}+2\sigma\left(\frac{d\mathbf{\Phi}_i}{d\phi}\right)_{\phi=0}+\sigma^2\left(\frac{d\mathbf{\Phi}_i}{d\phi}\right)^2_{\phi=0}\right]\left(\frac{d\mathbf{\Phi}_i}{d\phi}\right)_{\phi=0}. \tag{13}$$

Since we are considering the fluctuating charge on the dust grains [34], the charge variation is governed by $Z_d = 1 + \gamma_1\phi + \gamma_2\phi^2 + \ldots$.

III. DYNAMICS OF SOLITONS' HEAD-ON COLLISION

We assume that two solitons A and B, which are initially far apart, travel towards each other at an angle θ with respect to the direction of the external magnetic field. After some time, they collide and depart from each other after having a weak interaction. Here we expect that the collision will be quasi-elastic so that it will only cause shifts of the postcollision trajectories (phase shift). To analyze the effect of the collision, we employ the extended PLK perturbation method (as described in Sec. I) and introduce the following stretched coordinates:

$$\xi = \varepsilon(x - \lambda t) + \varepsilon^2 P_0(\eta,\tau) + \varepsilon^3 P_1(\xi,\eta,\tau) + \ldots, \quad (14)$$

$$\eta = \varepsilon(x + \lambda t) + \varepsilon^2 Q_0(\xi,\tau) + \varepsilon^3 Q_1(\xi,\eta,\tau) + \ldots, \quad (15)$$

$$\tau = \varepsilon^3 t. \quad (16)$$

Here ξ and η denote the trajectories of the two solitons traveling towards each other. The wave velocity λ and the variables P_0, P_1, Q_0, and Q_1 are to be determined later for investigating their trajectories and the phase change. For this, the dependent variables are expanded as

$$d = d_o(x,z) + \varepsilon^2 d_1(x,z,t) + \varepsilon^3 d_2(x,z,t) + \varepsilon^4 d_3(x,z,t) + \cdots \text{ together with } d \equiv n_i, n_{el}, n_{eh}, \phi, \quad (17a)$$

$$v_x = \varepsilon^4 v_{x1} + \varepsilon^5 v_{x2} + \varepsilon^6 v_{x3} + \ldots, \quad (17b)$$

$$v_y = \varepsilon^3 v_{y1} + \varepsilon^4 v_{y2} + \varepsilon^5 v_{y3} + \ldots, \quad (17c)$$

$$v_z = \varepsilon^3 v_{z1} + \varepsilon^4 v_{z2} + \varepsilon^5 v_{z3} + \ldots. \quad (17d)$$

The power of ε determines the magnitude of perturbation, and the lower power of ε means the perturbation of larger magnitude and the higher power of ε means the perturbation of lower magnitude. In Eqs. (14) to (16), ξ and η are the space like coordinates and τ is the time-like coordinate. It has been established that the solitons evolve in the homogeneous plasma if the physical quantities vary slowly with time in comparison to their variation in space. It means the balance between the effects of nonlinearity and dispersion arises only under this situation in homogeneous plasmas. Hence, the power of ε is taken to be higher in the time-like coordinate. It is taken as the cubic of the power of ε in a space-like coordinate, which is a general practice and is well-established in the literature. Moreover, the expansion of physical quantities like densities, velocity components, and potential is also responsible for achieving the balance between the effects of nonlinearity and dispersion. Here, the powers of velocity components parallel and perpendicular to the direction of the magnetic field in Eq. (17) are taken to be different in view of the fact that the Lorentz force acts only in the perpendicular direction to the direction of the magnetic field. In view of the magnetic field in the z direction, this is also obvious that the magnitude of the perturbation in v_x and v_y will be lower than the one in v_z. Finally, the types of perturbations taken in Eqs. (14)–(17) also cause the various terms in the KdV equation to carry the same powers in ε, which happens to be the confirmative test.

After employing the stretched coordinates (14)–(16) and the expansion of the dependent variables [(Eq. (17)] in the basic fluid equations, we obtain the following set of equations corresponding to different orders of the expansion parameter

$$\lambda\left(-\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)n_1 + \cos\theta\left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)v_{z1} = 0, \quad (18)$$

$$2\sigma\sin\theta\left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)n_1 + \sin\theta\left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)\phi_1 - v_{y1} = 0, \quad (19)$$

$$2\sigma\cos\theta\left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)n_1 + \cos\theta\left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)\phi_1 + \lambda\left(-\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)v_{z1} = 0. \quad (20)$$

Based on the above equations and taking $N = n_{el0} + n_{eh0} + n_{d0}\gamma_1$, we obtain

$$[(1+2\sigma N)\cos^2\theta - N\lambda^2]\left[\frac{\partial^2\phi_1}{\partial\xi^2} + \frac{\partial^2\phi_1}{\partial\eta^2}\right] + [(1+2\sigma N)\cos^2\theta + N\lambda^2]\frac{\partial^2\phi_1}{\partial\xi\partial\eta} = 0. \quad (21)$$

Equation (21) can be classified as hyperbolic, the solution of which can be written as a combination of two variables or functions $\phi_{11}(\xi,\tau)$ and $\phi_{12}(\eta,\tau)$. Hence

$$\phi_1 = \phi_{11}(\xi,\tau) + \phi_{12}(\eta,\tau). \quad (22)$$

The phase velocity relation is given by $\lambda = \pm\sqrt{(2\sigma + \frac{1}{N})}\cos\theta$.

Based on the first-order equations (18)–(21) and the solution (22), we obtain the relations of n_1 and v_{z1} in terms of ϕ_{11} and ϕ_{12}

$$n_1 = N[\phi_{11}(\xi,\tau) + \phi_{12}(\eta,\tau)], \quad (23)$$

$$v_{z1} = \frac{1}{4\lambda\cos\theta}[(1+2\sigma N)\cos^2\theta + N\lambda^2][\phi_{11}(\xi,\tau) - \phi_{12}(\eta,\tau)]. \quad (24)$$

In addition to the above first-order equations, we obtain higher-order equations also, as depicted in the Appendix. The integration of Eq. (A15) of the Appendix yields

$$\frac{4\cos\theta}{3\lambda}v_{z3} = -\frac{(1+4\lambda^2)}{3\lambda}\int\left(\frac{\partial\phi_{11}}{\partial\tau} - a_{11}\phi_{11}\frac{\partial\phi_{11}}{\partial\xi} + b_{11}\frac{\partial^3\phi_{11}}{\partial\xi^3}\right)d\eta - \frac{(1+4\lambda^2)}{3\lambda}\int\left(\frac{\partial\phi_{12}}{\partial\tau} - a_{11}\phi_{12}\frac{\partial\phi_{12}}{\partial\eta} + b_{11}\frac{\partial^3\phi_{12}}{\partial\eta^3}\right)d\xi$$
$$- \int\left(\frac{\partial P_0}{\partial\eta} + c_{11}\phi_{12}\right)\frac{\partial^2\phi_{11}}{\partial\xi^2}d\xi d\eta + \int\left(\frac{\partial Q_0}{\partial\xi} + c_{11}\phi_{11}\right)\frac{\partial^2\phi_{12}}{\partial\eta^2}d\xi d\eta, \quad (25)$$

together with

$$a_{11} = \frac{4}{9}\lambda\left[\frac{4b_l\left(\frac{T_{\rm eff}}{T_{el}}\right)^{3/2} + 3n_{d0}\gamma_1}{(1+4\lambda^2)(n_{el0}+n_{eh0}+n_{d0}\gamma_1)}\right], \quad b_{11} = \frac{\lambda(2+\lambda\sin\theta\tan\theta)}{(1+4\lambda^2)(n_{el0}+n_{eh0}+n_{d0}\gamma_1)}, \quad c_{11} = \frac{16\sigma}{3}\left[\frac{4}{3}b_l\left(\frac{T_{\rm eff}}{T_{el}}\right)^{3/2} + n_{d0}\gamma_2\right].$$

In obtaining Eq. (25), we have not taken into account the term ϕ_3, which carries the lowest magnitude in view of the fact that it is accompanied by the highest power of ε. The first term of the right-hand side of Eq. (25) will be proportional to η whereas the second term will be proportional to ξ, as the corresponding integrated functions are independent of η and ξ, respectively. It means these terms are all secular terms [31,34], which must be eliminated to avoid spurious resonances. Hence, we have

$$\frac{\partial\phi_{11}}{\partial\tau} - a_{11}\phi_{11}\frac{\partial\phi_{11}}{\partial\xi} + b_{11}\frac{\partial^3\phi_{11}}{\partial\xi^3} = 0, \tag{26}$$

$$\frac{\partial\phi_{12}}{\partial\tau} + a_{11}\phi_{12}\frac{\partial\phi_{12}}{\partial\eta} - b_{11}\frac{\partial^3\phi_{12}}{\partial\eta^3} = 0. \tag{27}$$

The third and fourth terms of the right-hand side of Eq. (25) are not secular terms in this order, but they will become secular in the next order, which can be explained as follows. This is clear that these terms carry double integration with regard to (w.r.t.) ξ and η. The integrations yield $(\frac{\partial\phi_{11}}{\partial\xi})[P_0 + \frac{16N_b\sigma}{3}\int\phi_{12}d\eta]$ and $(\frac{\partial\phi_{12}}{\partial\eta})[Q_0 + \frac{16N_b\sigma}{3}\int\phi_{11}d\xi]$ for the third and fourth terms. This is due to the fact that P_0 is a function of η and Q_0 is a function of ξ. At higher order these terms will contain P_1 and Q_1, respectively, which are functions of ξ and η both. Then the integration shall cause them to become secular due to the multiplication of P_0 term with ξ and Q_0 term with η. Hence, we have

$$\frac{\partial P_0}{\partial\eta} = -c_{11}\phi_{12}, \tag{28}$$

$$\frac{\partial Q_0}{\partial\xi} = -c_{11}\phi_{11}. \tag{29}$$

Equations (26) and (27) are the KdV equations for the two travelling waves in the reference frame of ξ and η, respectively. Their corresponding solutions are

$$\phi_{11} = \varphi_A \sec h^2\left[\frac{(\xi - U\tau)}{\sqrt{4b_{11}/U}}\right], \tag{30}$$

$$\phi_{12} = \varphi_B \sec h^2\left[\frac{(\eta + U\tau)}{\sqrt{4b_{11}/U}}\right]. \tag{31}$$

Here $\varphi_A = (\frac{3U}{a_{11}})$ and $\varphi_B = (\frac{3U}{a_{11}})$ are the amplitudes of the solitons A and B in their initial positions, and their width is $\sqrt{\frac{4b_{11}}{U}}$ together with U as a constant determining the velocity of the solitons.

The leading phase changes due to the collision can be calculated from Eqs. (28) and (29) along with the use of soliton solutions ϕ_{11} and ϕ_{12}. Hence, we get

$$P_0 = -c_{11}\left(\frac{36b_{11}U}{{a_{11}}^2}\right)^{1/2}\left\{\tanh\left[\left(\frac{U}{4b_{11}}\right)^{1/2}(\eta + U\tau)\right]\right\}, \tag{32}$$

$$Q_0 = -c_{11}\left(\frac{36b_{11}U}{{a_{11}}^2}\right)^{1/2}\left\{\tanh\left[\left(\frac{U}{4b_{11}}\right)^{1/2}(\xi - U\tau)\right]\right\}. \tag{33}$$

After having calculated the values of P_0 and Q_0, we can evaluate the trajectories of the two solitary waves for their weak head-on interactions. These are obtained from Eqs. (14) and (15), as follows:

$$\xi = \varepsilon(x - \lambda t) - \varepsilon^2 c_{11}\left(\frac{36b_{11}U}{{a_{11}}^2}\right)^{1/2} \times \left\{\tanh\left[\left(\frac{U}{4b_{11}}\right)^{1/2}(\eta + U\tau)\right] + 1\right\} + O(\varepsilon^3), \tag{34}$$

$$\eta = \varepsilon(x + \lambda t) - \varepsilon^2 c_{11}\left(\frac{36b_{11}U}{{a_{11}}^2}\right)^{1/2} \times \left\{\tanh\left[\left(\frac{U}{4b_{11}}\right)^{1/2}(\xi - U\tau)\right] - 1\right\} + O(\varepsilon^3). \tag{35}$$

To obtain the phase shifts of the two solitons after the collision, we assume that the solitons A and B are initially far from each other at the initial time ($t = -\infty$) i.e. the soliton A is at $\xi = 0$, $\eta = -\infty$ and the soliton B is at $\eta = 0$, $\xi = +\infty$. After the collision ($t = +\infty$), the soliton A is far to the right of the soliton B, i.e., the soliton A is at $\xi = 0$, $\eta = +\infty$ and the soliton B is at $\eta = 0$, $\xi = -\infty$. Based on these conditions [9,31,34,41] and using Eqs. (34) and (35), we obtain the corresponding phase shifts Δ_A and Δ_B as follows:

$$\Delta_A = \varepsilon(x - \lambda t)|_{\xi=0,\eta=+\infty} - \varepsilon(x - \lambda t)|_{\xi=0,\eta=-\infty} = 2\varepsilon^2 c_{11}\left(\frac{36b_{11}U}{{a_{11}}^2}\right)^{1/2}, \tag{36}$$

$$\Delta_B = \varepsilon(x - \lambda t)|_{\xi=-\infty,\eta=0} - \varepsilon(x - \lambda t)|_{\xi=\infty,\eta=0} = -2\varepsilon^2 c_{11}\left(\frac{36b_{11}U}{{a_{11}}^2}\right)^{1/2}. \tag{37}$$

The above phase shifts can also be obtained in terms of the width and amplitude of the soliton, which has not so far been explored by other investigators. Hence, $\Delta_A = 2\varepsilon^2 c_{11}$ (Width Amplitude) and $\Delta_B = -2\varepsilon^2 c_{11}$ (Width Amplitude). Since the soliton energy E depends on its amplitude and width as [41–44] $E = \frac{4}{3}$ (Width Amplitude^2), the phase shift can be expressed in terms of the soliton energy as

$$\Delta_A = -\Delta_B = \frac{3}{2}c_{11}\left(\frac{\text{Energy}}{\text{Amplitude}}\right). \tag{38}$$

In view of the propagation of the soliton A towards the right side and that of the soliton B towards the left side, Eqs. (36)

and (37) reveal that each soliton has a positive phase shift in its travelling direction due to the collision and the magnitudes of the phase shifts are related to the physical parameters.

IV. RESULTS AND THEIR COMPARISON TO PREVIOUS FINDINGS

We examine the phase shift of the solitons under the effect of various parameters, i.e., dust density, dust charge fluctuation, magnetic field, ion temperature and density, and the temperature of the trapped electrons.

Figure 1 shows that the phase shift increases linearly with the amplitude of the soliton. Since the soliton's energy is also increased for the higher amplitude, it can be concluded that the solitons with larger energy experience a larger phase shift after the collision. Similar results of enhanced phase shift of the solitons have been observed in an experimental investigation of a monolayer strongly coupled complex plasma, as it was found that the solitons are delayed after the collision [19]. On the other hand, we find that the solitons undergo a greater phase shift if the dust grains have a fluctuating charge. Such a behavior of the dust acoustic solitons has been confirmed in a complex plasma [19]. It means the behavior of the ion acoustic soliton after its collision with another ion acoustic soliton in a dusty plasma remains the same as that of the dust acoustic soliton. In another case of head-on collision of dust acoustic solitary waves in an unmagnetized dusty plasma, the phase shift was found to increase under the effect of the dust particles; a similar effect has also been observed in a magnetized warm dusty plasma [34]. However, a reduction in the phase shift with the increase of the dust charge has been obtained in a magnetized dusty plasma containing two-temperature ions [21].

This is clear from Fig. 2 that the phase shift increases for the higher values of the dust density. The shift is higher as well as more sensitive in the case of the fluctuating charge on the dust grains. This is as per variation of the amplitude of the solitons. Since the solitons acquire a larger amplitude in the case of a higher dust density, a larger phase shift is expected after their collision. The same has been observed in the case of ion thermal solitary waves in a pair-ion plasma containing stationary charged dust grain density [20]. A higher phase shift of the dust acoustic solitary waves has also been observed in a strongly coupled dusty plasma system consisting of negatively charged dust grains, electrons, and ions [22]. It means the presence of the dust grains in the plasma enforces the phase shifting of the solitons after their collision and delays the solitons, but the presence of the negative ions in the dusty plasma does not influence the collision mechanism significantly [22].

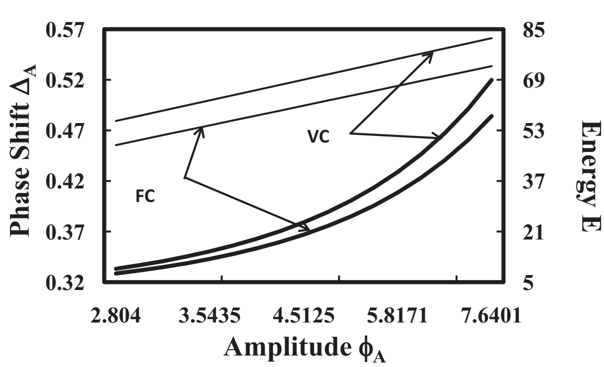

FIG. 1. Variation of phase shift (thin lines) and energy (thick lines) with amplitude ($T_{el} = 1.1$–1.4 eV) for fluctuating charge (marked as VC) and fixed charge (FC) dust grains. The parameters are $n_0 = 0.8$, $T_i = 0.02$ eV, $T_{eh} = 3$ eV, $n_{el0} = 0.20$, $n_{d0} = 0.28$, $B = 0.05$ T, and $\theta = 40^\circ$.

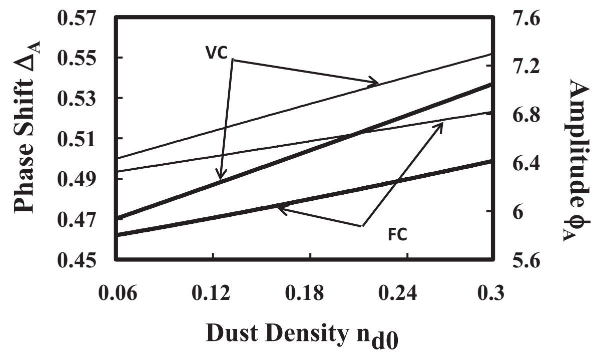

FIG. 2. Variation of phase shift (thin lines) and soliton amplitude (thick lines) with the dust density for a fluctuating charge (VC) and a fixed charge (FC) dust grains. Here $T_{el} = 1.2$ eV and other parameters are the same as in Fig. 1.

Figure 3 shows that the solitons acquire a greater phase shift after the collision if the magnetic field is applied at a larger angle from the direction of the wave propagation. The greater phase shift for the larger obliqueness has also been observed in the case of two colliding dust acoustic solitary waves in hot dusty plasmas [21,34]. On the other hand, a greater phase shift for the case of a higher ion temperature in our case is consistent with the observation made by the authors of Refs. [20] and [34]. In our case, the greater phase shift is attributed to the enhanced amplitude of the solitons in the presence of ions having higher temperature. However, in the case of an adiabatic hot dusty plasma containing two temperature ions, the collisional phase shift was found to reduce as the temperature ratio (low to high) of the ions was increased [21]. Hence, it appears that the nonlinearity of the plasma (in view of the larger shift or amplitude of the solitons) is significantly enhanced in the case

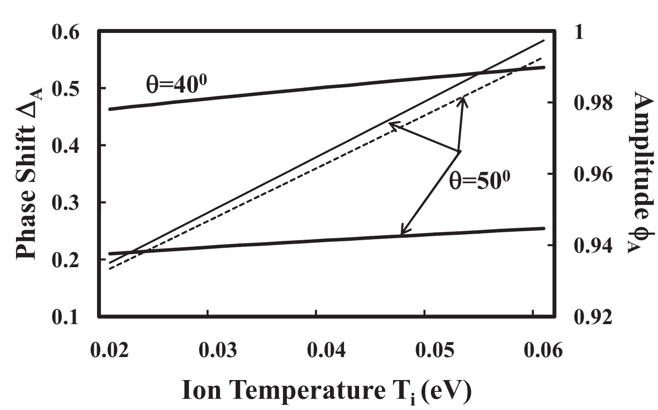

FIG. 3. Variation of phase shift (thin lines) and soliton amplitude (thick lines) with ion temperature for fluctuating charge (solid lines) and fixed charge (dashed line) dust grains. Here the parameters are the same as in Fig. 1.

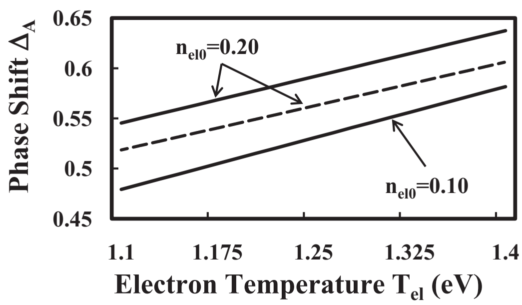

FIG. 4. Variation of phase shift with the temperature of trapped electrons for the cases of fluctuating charge (solid lines) and fixed charge (dashed line) dust grains. Here the parameters are taken as the same as in Fig. 1.

of two temperature electrons but the opposite is realized in the case of two temperature ions. A reduction in phase shift of the dust ion acoustic solitary waves with the ion temperature is also found in unmagnetized dusty plasma [35] and the plasma having nonthermal ions [23]. The same was also obtained for the ion acoustic solitary waves in unmagnetized plasma containing nonthermal electrons and warm adiabatic ions [8,22,45].

From Fig. 4, the phase shift has been found to increase with the temperature of the trapped electrons for both the cases of dust grains, i.e., the dust having a fixed charge or variable charge. The change in the phase shift takes place at a very significant rate when the dust grains carry variable charge. Moreover, a higher phase shift is attained by the solitons in a plasma containing dust with variable charge; the same is the case with the higher density of the trapped electrons. In an unmagnetized nonplanar plasma consisting of warm adiabatic ions and nonthermally distributed electrons, a greater phase shift of ion acoustic solitary waves after their head-on collision was found for the increasing nonthermality parameter and temperature of the electrons [45]. On the other hand, the nonplanar dust ion acoustic solitary waves were found to attain a greater shift for the larger nonthermality parameter in an unmagnetized plasma containing nonthermal electrons distribution; however, the opposite was obtained for the larger electron density [35].

V. CONCLUSIONS

Our analytical calculations concerning the head-on collision of the ion acoustic solitary waves in magnetized plasma reveal that the greater amplitude waves acquire a larger phase shift and hence are delayed more after the collision. The phase shift was found to be larger under the effect of a higher density of the dust grains and higher temperature of the ions. The solitary waves were also found to delay further for the higher concentration and temperature of the trapped electrons. The phase shift changes significantly in the plasma if the dust grains carry fluctuating charge.

APPENDIX

The next order yields

$$n_2 = N[\phi_{22}(\xi,\tau) + \phi_{23}(\eta,\tau)], \tag{A1}$$

$$v_{z2} = \frac{1}{4\lambda\cos\theta}[(1+2\sigma N)\cos^2\theta + N\lambda^2][\phi_{22}(\xi,\tau) - \phi_{23}(\eta,\tau)], \tag{A2}$$

$$v_{y2} = \frac{\sin\theta}{4\cos^2\theta}[(1+2\sigma N)\cos^2\theta + N\lambda^2]\left[\frac{\partial\phi_{22}}{\partial\xi} - \frac{\partial\phi_{23}}{\partial\eta}\right], \tag{A3}$$

$$\lambda\left(-\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)n_3 + \cos\theta\left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)v_{z3} + \sin\theta\left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)v_{x2} + \left(\frac{\partial}{\partial\tau} + \lambda P_{0\eta}\frac{\partial}{\partial\xi} - \lambda Q_{0\xi}\frac{\partial}{\partial\eta}\right)n_1$$
$$+ \cos\theta\left(P_{0\eta}\frac{\partial}{\partial\xi} + Q_{0\xi}\frac{\partial}{\partial\eta}\right)v_{z1} = 0, \tag{A4}$$

$$2\sigma\sin\theta\left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)n_3 + \sin\theta\left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)\phi_3 - v_{y3} + \lambda\left(-\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)v_{x2} + \sin\theta\left(P_{0\eta}\frac{\partial}{\partial\xi} + Q_{0\xi}\frac{\partial}{\partial\eta}\right)\phi_1 = 0, \tag{A5}$$

$$2\sigma\cos\theta\left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)n_3 + \cos\theta\left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)\phi_3 + \lambda\left(-\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)v_{z3} + 2\sigma\cos\theta\left(P_{0\eta}\frac{\partial}{\partial\xi} + Q_{0\xi}\frac{\partial}{\partial\eta}\right)n_1$$
$$+ \cos\theta\left(P_{0\eta}\frac{\partial}{\partial\xi} + Q_{0\xi}\frac{\partial}{\partial\eta}\right)\phi_1 + \left(\frac{\partial}{\partial\tau} + \lambda P_{0\eta}\frac{\partial}{\partial\xi} - \lambda Q_{0\xi}\frac{\partial}{\partial\eta}\right)\phi_1 = 0, \tag{A6}$$

$$A v_{x3} + \lambda\left(-\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)v_{y2} = 0, \tag{A7}$$

$$\sin\theta\left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)v_{x3} + \left(\frac{\partial}{\partial\tau} + P_{0\xi}\frac{\partial}{\partial\xi} - Q_{0\eta}\frac{\partial}{\partial\eta}\right)n_2 + \cos\theta\left(P_{0\xi}\frac{\partial}{\partial\xi} + Q_{0\eta}\frac{\partial}{\partial\eta}\right)v_{z2} = 0, \tag{A8}$$

$$2\sigma\cos\theta\left(P_{0\xi}\frac{\partial}{\partial\xi} + Q_{0\eta}\frac{\partial}{\partial\eta}\right)n_2 + \cos\theta\left(P_{0\xi}\frac{\partial}{\partial\xi} + Q_{0\eta}\frac{\partial}{\partial\eta}\right)\phi_2 + \left(\frac{\partial}{\partial\tau} + P_{0\xi}\frac{\partial}{\partial\xi} - Q_{0\eta}\frac{\partial}{\partial\eta}\right)v_{z2} = 0, \tag{A9}$$

$$2\sigma\sin\theta\left(P_{0\xi}\frac{\partial}{\partial\xi}+Q_{0\eta}\frac{\partial}{\partial\eta}\right)n_2+\sin\theta\left(P_{0\xi}\frac{\partial}{\partial\xi}+Q_{0\eta}\frac{\partial}{\partial\eta}\right)\phi_2+\lambda\left(-\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)v_{x3}+\left(\frac{\partial}{\partial\tau}+P_{0\xi}\frac{\partial}{\partial\xi}-Q_{0\eta}\frac{\partial}{\partial\eta}\right)v_{x1}=0, \tag{A10}$$

$$\sin\theta\left(P_{0\xi}\frac{\partial}{\partial\xi}+Q_{0\eta}\frac{\partial}{\partial\eta}\right)v_{x2}+\cos\theta\left(P_{0\xi}\frac{\partial}{\partial\xi}+Q_{0\eta}\frac{\partial}{\partial\eta}\right)v_{z3}=0, \tag{A11}$$

$$\left(\frac{\partial}{\partial\tau}+P_{0\xi}\frac{\partial}{\partial\xi}-Q_{0\eta}\frac{\partial}{\partial\eta}\right)v_{x2}+\sin\theta\left(P_{0\xi}\frac{\partial}{\partial\xi}+Q_{0\eta}\frac{\partial}{\partial\eta}\right)\phi_3+2\sigma\sin\theta\left(P_{0\xi}\frac{\partial}{\partial\xi}+Q_{0\eta}\frac{\partial}{\partial\eta}\right)n_3=0, \tag{A12}$$

$$\left(\frac{\partial}{\partial\tau}+P_{0\xi}\frac{\partial}{\partial\xi}-Q_{0\eta}\frac{\partial}{\partial\eta}\right)v_{z3}+\cos\theta\left(P_{0\xi}\frac{\partial}{\partial\xi}+Q_{0\eta}\frac{\partial}{\partial\eta}\right)\phi_3+2\sigma\cos\theta\left(P_{0\xi}\frac{\partial}{\partial\xi}+Q_{0\eta}\frac{\partial}{\partial\eta}\right)n_3=0, \tag{A13}$$

$$\left[\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}+2\frac{\partial^2}{\partial\xi\partial\eta}\right]\phi_1+n_3-N\phi_3-N_b{\phi_1}^2=0. \tag{A14}$$

In the above equation $N_b=\frac{4}{3}b_l(\frac{T_{\rm eff}}{T_{el}})^{3/2}+n_{d0}\gamma_2$. Now by differentiating Eq. (A14) w.r.t. ξ and η simultaneously, using Eq. (22) and eliminating higher-order terms using Eqs. (A1)–(A13), we get

$$\begin{aligned}\frac{4\cos\theta}{3\lambda}\frac{\partial^2 v_{z3}}{\partial\xi\partial\eta}=&-\frac{(1+4\lambda^2)}{3\lambda}\frac{\partial}{\partial\xi}\left[\frac{\partial\phi_{11}}{\partial\tau}-2\frac{N_b}{N_\lambda}\phi_{11}\frac{\partial\phi_{11}}{\partial\xi}+\frac{(2+\lambda\sin\theta\tan\theta)}{2N_\lambda}\frac{\partial^3\phi_{11}}{\partial\xi^3}\right]\\&-\frac{(1+4\lambda^2)}{3\lambda}\frac{\partial}{\partial\eta}\left[\frac{\partial\phi_{12}}{\partial\tau}+2\frac{N_b}{N_\lambda}\phi_{12}\frac{\partial\phi_{12}}{\partial\eta}-\frac{(2+\lambda\sin\theta\tan\theta)}{2N_\lambda}\frac{\partial^3\phi_{12}}{\partial\eta^3}\right]-\left[\frac{\partial P_0}{\partial\eta}+\frac{16N_b\sigma}{3}\phi_{12}\right]\frac{\partial^2\phi_{11}}{\partial\xi^2}\\&+\left[\frac{\partial Q_0}{\partial\xi}+\frac{16N_b\sigma}{3}\phi_{11}\right]\frac{\partial^2\phi_{12}}{\partial\eta^2}+\left(N+\frac{1}{4\sigma}\right)\left[\frac{\partial^2}{\partial\xi^2}-\frac{\partial^2}{\partial\eta^2}\right]\phi_3=0.\end{aligned} \tag{A15}$$

Here $N_\lambda=\frac{N(1+4\lambda^2)}{2\lambda}$, $P_{0\xi}=\frac{\partial P_0}{\partial\xi}$, $P_{0\eta}=\frac{\partial P_0}{\partial\eta}$, and so on.

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