Energy-exchange collisions of dark-bright-bright vector solitons

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We find a dark component guiding the practically interesting bright-bright vector one-soliton to two different parametric domains giving rise to different physical situations by constructing a more general form of three-component dark-bright-bright mixed vector one-soliton solution of the generalized Manakov model with nine free real parameters. Moreover our main investigation of the collision dynamics of such mixed vector solitons by constructing the multisoliton solution of the generalized Manakov model with the help of Hirota technique reveals that the dark-bright-bright vector two-soliton supports energy-exchange collision dynamics. In particular the dark component preserves its initial form and the energy-exchange collision property of the bright-bright vector two-soliton solution of the Manakov model during collision. In addition the interactions between bound state dark-bright-bright vector solitons reveal oscillations in their amplitudes. A similar kind of breathing effect was also experimentally observed in the Bose-Einstein condensates. Some possible ways are theoretically suggested not only to control this breathing effect but also to manage the beating, bouncing, jumping, and attraction effects in the collision dynamics of dark-bright-bright vector solitons. The role of multiple free parameters in our solution is examined to define polarization vector, envelope speed, envelope width, envelope amplitude, grayness, and complex modulation of our solution. It is interesting to note that the polarization vector of our mixed vector one-soliton evolves in sphere or hyperboloid depending upon the initial parametric choices.

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I. INTRODUCTION

The interactions between solitons are fascinating because they interact like particles. Specifically vector solitons (temporal or spatial) have richer interaction dynamics than their scalar counter parts due to their multicomponent nature [1]. They form when their components jointly induce a waveguide and populate its guided modes self-consistently. The induced index change does not include the interference terms between different modes in many cases, and as a result the system is governed by a Manakov-like model [2–4]. By solving the Manakov model bright-bright (BB) [5], dark-dark (DD) [6], and dark-bright (DB) [7] vector multisoliton solutions with more free parameters were recently derived to investigate the collision dynamics of such vector solitons.

Although certain spatial vector solitons were experimentally observed in the semiconductor quantum well wave guides [2] and the photo-refractives [3], the interactions between vector solitons were intensively studied only theoretically. However, a bound state between two DB solitons was experimentally demonstrated in Ref. [8]. Moreover, Anastassiou *et al.* [4] have demonstrated experimentally the collisions between two BB spatial vector one-solitons of the Manakov model in the photorefractive crystal that involve energy-exchange at large collision angles, for which scalar solitons simply pass through one another (i.e., practically unaffected). In addition, recently Rand *et al.* [9] have experimentally observed the energy-exchange collision dynamics of BB temporal vector one-solitons in a linearly birefringent optical fiber [9]. These experiments [4,9] confirm the energy-exchange collision property of BB vector two-soliton solution of the Manakov model [10]. The applications of such collisions were also studied in the Bose-Einstein condensates (BECs) [11], artificial meta-materials [12], and multiport devices [13]. Moreover the experimentally observed energy-exchange collision property of the BB vector solitons of the Manakov model [4] lays the foundation for optical computation [14,15]. Because Steiglitz [15] showed that such vector solitons have more than one distinct stable set of polarization states, and therefore these distinct equilibria can, in theory, be used to store and process information.

In this connection, in this paper we have theoretically predicted that a dark component guides the practically interesting BB vector one-soliton (supporting a specific physical situation) to two different parametric domains giving rise to different physical situations by constructing a more general form of a three-component DBB vector one-soliton solution (with nine real free parameters) of the generalized Manakov model

$$i\frac{\partial u_m}{\partial z}+\frac{\partial^2 u_m}{\partial t^2}+2\mu\left(\sum_{n=1}^{3}\mid u_n\mid^2\right)u_m=0,m=1,2,3. \quad (1)$$

Here u_m are the components of vector soliton, variables z and t are the normalized distance and time, respectively, and μ is the coefficient of nonlinearity. Here we retain the scaling parameter μ to define the physical situations of two distinct parametric regions of the DBB one-soliton. In the next section we have characterized the free parameters of this solution to define polarization vector, envelope speed, envelope width, envelope amplitude, grayness, and complex modulation of the solution. This solution actually couples two scalar bright solitons with $\mu>0$ (positive nonlinearity coefficient) and a scalar dark soliton with $\mu<0$ (negative nonlinearity coefficient) by using three complex free parameters α_1, $\alpha_2^{(1)}$, and $\alpha_3^{(1)}$ in their amplitude and complex modulation in z terms. Therefore we want to name these three complex free parameters as coupling parameters. It is well-known fact that if the minimum intensity of a scalar dark soliton is zero, then it is called a fundamental-dark (FD) soliton. Therefore in our studies we have referred to our general DBB vector one-soliton as a fundamental dark-bright-bright (FDBB) vector one-soliton if the minimum intensity of its dark component is zero. It is interesting to note that our general solution admits its dark component to take a minimum intensity value only under a parametric condition. Consequently our FDBB soliton has eight free real parameters. However, it supports a polarization vector while our general solution needs some other essential parametric condition to define its polarization vector. In the FDBB case the polarization vector evolves in hyperboloid, while in the general case (with a parametric condition) the polarization vector evolves in a sphere as explained in Sec. II (see Fig. 2). In addition it is interesting to note the role of a singularity in the general DBB vector soliton with nine free parameters and in the FDBB soliton with eight free real parameters as explained below.

In general if we look a sufficiently general form of a mixed vector soliton solution to the Manakov type of equations (which couple the components of equations with the same sign of nonlinearity coefficient), the resulting solution supports two different parametric domains (separated by a movable singularity) [7] giving rise to different physical situations. But if we look at a similar type of mixed solution to the modified Manakov type of equations (which couple the components of equations with different signs of nonlinearity coefficient) [16] then the resulting solution is simple and free from the singularity. Here our mixed DBB soliton solution with nine free real parameters is derived from the generalized Manakov model (1) by using the systematic procedure of Hirota technique as shown in Refs. [7,16]. Therefore it supports singularity as for as a dark soliton mixes its role with a bright component (DB case [7]) or two bright components (DBB case). This singularity separates two distinct parametric domains. Each domain has the same set of nine free real parameters of the general solution. Consequently our general DBB solution with nine free real parameters is valid in one parametric domain with $\mu < 0$ and in another parametric domain with $\mu > 0$. Hence we claim that our general solution admits its dark component to guide the BB pair of the Manakov model [5] to two different physical situations. Further the DBB one-solitons with distinct parametric domains for two different physical situations preserve the energy-exchange collision property of the BB vector soliton [10]. That is, during the collision between two DBB one-solitons, the energy-exchange occurs among the bright components of each colliding soliton without disturbing the energy of its dark component as shown in Sec. III. Therefore we trust that our investigations may promote the applications of BB vector solitons to composite nonlinear media [17–19], because they are composed of positive and negative index materials giving rise to different types of nonlinear coupling among the co-propagating fields.

In Sec. III we have derived a DBB multisoliton solution by using the Hirota method [20]. This Hirota method unfortunately restricts us to select a common background field for the dark component while deriving the multisoliton solution. This multisoliton solution explains the dynamics of two DBB one-solitons appearing at its asymptotic limits. Each asymptotic DBB one-soliton has a different set of nine free real parameters. However, among these nine free real parameters, each asymptotic soliton has two common free parameters due to the common field restriction. Consequently our resultant DBB two-soliton solution has 15 free real parameters. Moreover due to this common background field restriction, the speeds of its all interacting one-solitons become equal provided the minimum intensity of dark component of each interacting one-soliton (realized from our DBB two-soliton at its asymptotic limit) is zero. Our multisoliton solution with such nature has freedom to study (1) the collision between two DBB one solitons with nine free parameters, (2) the collision between a DBB one-soliton and a FDBB one-soliton, and (3) the interaction between two closely packed (a) DBB bound state solitons moving with a common velocity, (b) stationary DBB solitons, and (c) FDBB solitons. It is interesting to note that two bound state DBB one-solitons or two stationary solitons realized from our multisoliton solution set oscillations in their amplitudes if they are closely packed. This effect has the possibility to occur among the components of interacting solitons provided a dark component occurs in each interacting soliton. In the absence of any one of the bright components (i.e, in the resulting DB case [7,16]) although there is a chance to realize this effect, one can suppress or enhance it by adding a bright component using a coupling parameter ($\alpha_2^{(1)}$ or $\alpha_3^{(1)}$) with an appropriate value as shown in Sec. III with the help of our multisoliton solution. A similar kind of breathing effect was observed experimentally in the BECs [21–23]. Moreover as mentioned before the energy-exchange collision dynamics of BB vector solitons is also practically interesting. If we mix a dark component in the two colliding BB solitons, our investigation reveals that the dark component preserves the energy-exchange collision property of BB vector solitons. Moreover our colliding DBB solitons attain an ability not only to manage different physical situations by tuning their initial parametric values but also to suggest some possible ways (by using their coupling parameters without disturbing other soliton parameters) to manage beating, bouncing, jumping, and attraction effects in their colliding region. All our investigations are made by plotting our complicated multisoliton solution. Therefore we believe by following the work in the BB case [24,25] that the numerical technique as explained in Sec. III is more convenient to infer more about the collision dynamics of DBB vector solitons with different background fields in different parametric domains with $\mu > 0$ and $\mu < 0$. Work is in progress along this direction. Finally Sec. IV is used to give our overall conclusion.

II. DARK-BRIGHT-BRIGHT ONE-SOLITON SOLUTION AND ITS CHARACTERIZATION

By following the algorithmic steps of the Hirota method [7,16,20] (see the Appendix), we have obtained a sufficiently general form of three-component dark-bright-bright (DBB) mixed vector one-soliton solution as

$$\begin{aligned} u_1 &= \cos(\Phi) e^{-i\delta z} e^{2iA^2 z} A[i \sin\beta \\ &\quad + \cos\beta \tanh(\eta_{1R} + \Gamma/2)] e^{i(\psi' + \beta + \phi_1)}, \\ u_2 &= \sin(\Phi)\cos(\theta) e^{-i\delta z} e^{2iA^2 z} A[\operatorname{sech}(\eta_{1R} + \Gamma/2)] e^{i(\eta'_{1I} + \phi_2)}, \\ u_3 &= \sin(\Phi)\sin(\theta) e^{-i\delta z} e^{2iA^2 z} A[\operatorname{sech}(\eta_{1R} + \Gamma/2)] e^{i(\eta'_{1I} + \phi_3)}, \end{aligned} \tag{2}$$

where $\cos(\Phi) = 2|\alpha_1|/D_\Phi$, $\sin(\Phi) = N_\Phi/D_\Phi$, $\cos(\theta) = |\alpha_2^{(1)}|/N_\Phi$, $\sin(\theta) = |\alpha_3^{(1)}|/N_\Phi$, $\delta = (2k_R^{(1)2}/\mu\Delta)[\sin^2(\Phi) - \cos^2(\Phi)(\mu - 1)]$, $A = k_R^{(1)}(\mu\Delta)^{-1/2}$, $\beta = \arctan[(k_I^{(1)} - l)/k_R^{(1)}]$, $\phi_1 = \arctan(\alpha_{1I}/\alpha_{1R}) + \pi$, $\phi_m = \arctan(\alpha_{mI}^{(1)}/\alpha_{mR}^{(1)})$, $\psi' = lt - l^2 z + \psi^{(0)}$, $\eta_{1I}' = k_I^{(1)}t + (k_R^{(1)2} - k_I^{(1)2})z + \eta_{1I}^{(0)}$, $\eta_{1R} = k_R^{(1)}(t - 2k_I^{(1)}z) + \eta_{1R}^{(0)}$, and $\Gamma = \ln(R)$. Here $R = \mu\Delta D_\Phi^2/(4k_R^{(1)2})$ in which $\Delta = [\sin^2(\Phi) - \cos^2(\Phi)\cos^2\beta]$, $N_\Phi = (|\alpha_2^{(1)}|^2 + |\alpha_3^{(1)}|^2)^{1/2}$, and $D_\Phi = (4|\alpha_1|^2 + N_\Phi^2)^{1/2}$. The expression for Γ in the above equation is valid if $R > 0$ or in other words if $\mu\Delta > 0$.

The solution (2) with nine arbitrary real parameters $k_R^{(1)}$, $k_I^{(1)}$, l, α_{1R}, α_{1I}, $\alpha_{2R}^{(1)}$, $\alpha_{2I}^{(1)}$, $\alpha_{3R}^{(1)}$, and $\alpha_{3I}^{(1)}$ couples two bright scalar solitons with $\mu > 0$ [read one scalar bright from (2) if $\alpha_1 = \alpha_2^{(1)} = 0$ and other if $\alpha_1 = \alpha_3^{(1)} = 0$] and a scalar dark soliton with $\mu < 0$ [which one can read from (2) if $\alpha_m^{(1)} = 0$] by using three complex free parameters α_1, $\alpha_2^{(1)}$, and $\alpha_3^{(1)}$ in their amplitude and complex modulation in z terms as dictated by Eq. (1) (which decouples into a nonlinear Schrödinger equation with same sign of nonlinearity coefficient). That is, Eq. (1) couples its components u_1, u_2, and u_3 with same sign of μ, but its solution (2) couples scalar solitons with different possible signs of μ. Therefore we are referring here to the free parameters α_1, $\alpha_2^{(1)}$ and $\alpha_3^{(1)}$, as coupling parameters of the DBB mixed vector one-soliton solution. Here the suffixes R and I represent the real and imaginary parts of arbitrary complex parameters. In addition the solution (2) is composed of coupling three components with same envelope width ($k_R^{(1)}$), envelope speed ($k_I^{(1)}$), and envelope trough location ($\eta_{1R} + \Gamma/2 = 0$). Further the term β ($k_R^{(1)}$,$k_I^{(1)}$ and l) in (2) is useful to define the grayness. Particularly if $\beta = 0$ (i.e., $k_I^{(1)} = l$) then the u_1 component of Eq. (2) is fundamental dark (FD) (that is, the minimum intensity of u_1 is zero). Otherwise (if $k_I^{(1)} \neq l$ or $\beta \neq 0$) the u_1 component supports gray dark (GD). Therefore in the former case the general solution (2) defines a fundamental dark-bright-bright (FDBB) one-soliton with eight free real parameters, while in the later case it governs a gray dark-bright-bright (GDBB) one-soliton with nine real free parameters. Moreover one can read the BB one-soliton solution [5] and the DB one-soliton solution [7] of the Manakov model from (2) by setting $\alpha_1 = 0$ (i.e., $\Phi = \pi/2$) and $\alpha_2^{(1)} = 0$ (i.e., $\theta = \pi/2$) or $\alpha_3^{(1)} = 0$ (i.e., $\theta = 0$), respectively. Next it is important to characterize the role of Δ, because it sets the singularity in (2) and depends on the initial parametric choices of all the nine free real parameters of the solution (2) as explained below.

We have already mentioned that the DBB soliton (2) is valid if $\mu\Delta > 0$. It implies that μ follows the sign of the singularity term Δ. Therefore if $\Delta > 0/\Delta < 0$ [i.e., $\sin^2(\Phi)$ is greater or less than $\cos^2(\Phi)\cos^2\beta$ in Δ], then only our DBB solution (2) is valid with $\mu > 0/\mu < 0$ (positive or negative nonlinearity coefficient). These two DBB cases with nine free real parameters (each case has distinct parametric domain and different physical situations) are separated by a movable singularity as shown in Fig. 1. One can easily define this singularity by solving the equation Δ $(\Phi,\beta) = 0$ as $\Phi_c = \arctan\{k_R^{(1)2}/[k_R^{(1)2} + (k_I^{(1)} - l)^2]\}^{1/2}$. Therefore the DBB soliton (2) is valid if $\Phi > \Phi_c/\Phi < \Phi_c$ (i.e., $\mu\Delta > 0/\mu\Delta < 0$)

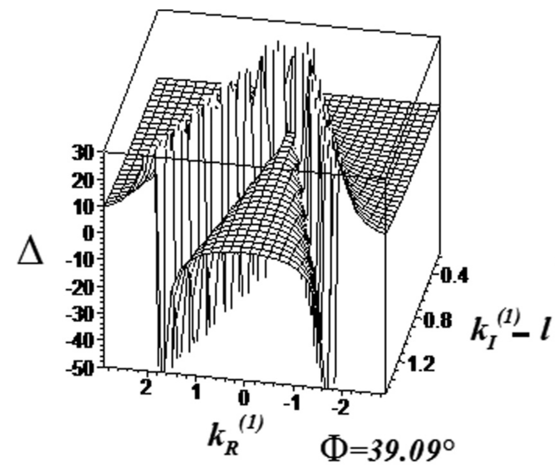

FIG. 1. The contour plot is obtained by plotting Δ in Eq. (2). (Note μ follows the sign of Δ and indicates the corresponding physical situation and its parametric domain for given Φ.)

and becomes extinct at $\Phi = \Phi_c$. When $\alpha_1 = 0$, this singularity disappears by leaving the BB case of the Manakov model with $\mu > 0$. But if $\alpha_2^{(1)} = 0$ or $\alpha_3^{(1)} = 0$ in (2), then the resultant also couples two scalar solitons (one with a positive and other with a negative nonlinearity coefficient) by admitting two different parametric domains (separated by a singularity) giving rise to different physical situations, because this is the mixed soliton solution of the Manakov model [7]. Therefore our DBB mixed vector one-soliton (2) of the generalized Manakov model (1) supports two distinct parametric domains with $\mu > 0$ and $\mu < 0$ until our three-component vector soliton has a dark component. It implies that the dark component guides the BB vector soliton to different physical situations and preserves its experimentally demonstrated energy-exchange collision property in the photorefractive crystals [4] (by using the spatial BB vector solitons) and in the birefringent fiber (by using the temporal BB vector solitons) based on the theoretical investigations of Radhakrishnan *et al.* [10,24] as shown in the next section. Therefore we believe that the DBB soliton (2) is helpful to promote the applications of experimentally observed BB vector soliton [4,9] to composite nonlinear media [17–19]

Although our solution generalizes earlier theoretical and experimental investigations the presence of singularity in A and δ through Δ sets an obscure structure for the amplitude part and the complex modulation in z. Some possible ways one can find to remove such complications in the terms A and δ are if we characterize the vector $[\cos(\Phi),\ \sin(\Phi)\cos(\theta),\ \sin(\Phi)\sin(\theta)]$. It is interesting to note that this vector is defined only in terms of the coupling parameters α_1, $\alpha_2^{(1)}$, and $\alpha_3^{(1)}$ as in the BB case [10]. However, it is not at all useful to vary the initial intensity distribution among the components without affecting the given total intensity by tuning θ and Φ with the help of coupling parameters as in the BB case [10,24]. Because here the amplitude term A depends on all the free coupling parameters. Therefore the total peak power (the sum of the maximum intensity of all the components) of (2) is not conserved against Φ as shown in Fig. 2(a) for given θ. Consequently the above vector is not able to play a polarization vector of (2) because the

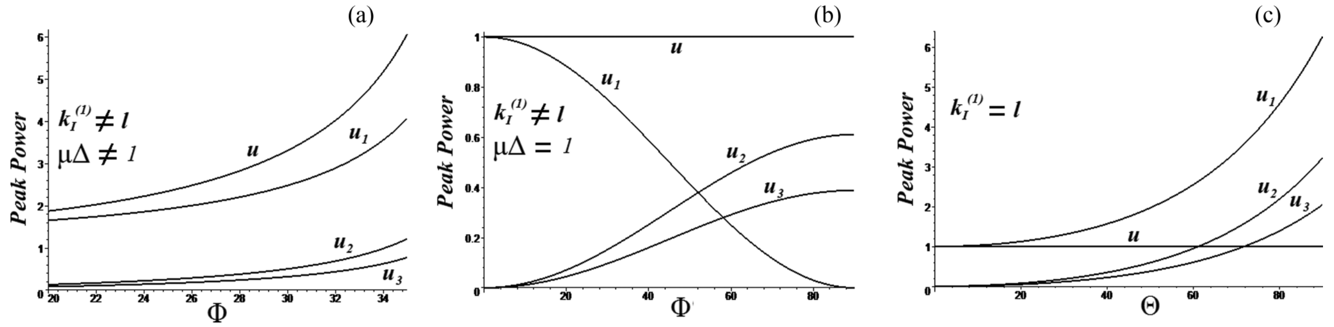

FIG. 2. The sum of the maximum intensity of all the components (a) is not conserved against Φ for given θ provided $k_I^{(1)} \neq l$ and $\mu\Delta \neq 1$ and (b) is conserved against Φ for given θ provided $\mu\Delta = 1$. (c) The difference between the maximum intensity of a dark component and the sum of the maximum intensity of two bright components is conserved against Θ for given θ under the parametric restriction $k_I^{(1)} = l$. For all these three plots $k_R^{(1)} = 1.0$ and $\theta = 38.58^\circ$.

role of A depends on the coupling parameters. To identify some possible ways to change such a nature of our general A we rewrite the general solution (2) under the parametric condition $\mu\Delta = 1$ as case (1) and under the parametric condition $k_I^{(1)} = l$ (i.e., $\beta = 0$) as case (2).

A. Case (1): DBB soliton with polarization vector ($\mu\Delta = 1$)

In this case, Eq. (2) becomes as

$$\begin{aligned}
u_1 &= \cos(\Phi) e^{i(2\mu k_R^{(1)2}\cos^2(\Phi))z} k_R^{(1)}[i\sin\beta \\
&\quad + \cos\beta\tanh(\eta_{1R} + \Gamma'/2)] e^{i(\psi'+\beta+\phi_1)}, \\
u_2 &= \sin(\Phi)\cos(\theta) e^{i(2\mu k_R^{(1)2}\cos^2(\Phi))z} k_R^{(1)}[\mathrm{sech}(\eta_{1R} + \Gamma'/2)] \\
&\quad \times e^{i(\eta'_{1I}+\phi_2)}, \\
u_3 &= \sin(\Phi)\sin(\theta) e^{i(2\mu k_R^{(1)2}\cos^2(\Phi))z} k_R^{(1)}[\mathrm{sech}(\eta_{1R} + \Gamma'/2)] \\
&\quad \times e^{i(\eta'_{1I}+\phi_3)},
\end{aligned} \tag{3}$$

where $\Gamma' = \ln(D_\Phi^2/4k_R^{(1)2})$. It is interesting to note that the above DBB soliton (3) realized from the general solution (2) under the condition $\mu\Delta = 1$ is free from the singularity. Further the amplitude is simply defined as $k_R^{(1)2}$ as in the BB case of the Manakov model [5]. Moreover the complex modulation in z has a simple mathematical form without singularity. Further Eq. (3) supports a fundamental dark component if $k_I^{(1)} = l$ without singularity. In addition from Eq. (3) one can easily understand that by tuning Φ and θ one can vary the initial intensity distribution among the components without affecting the total intensity as shown in Fig. 2(b). Therefore the vector $[\cos(\Phi), \sin(\Phi)\cos(\theta), \sin(\Phi)\sin(\theta)]$ defines the polarization vector of (3) and evolves on a spherical surface $\cos^2(\Phi) + \sin^2(\Phi)\cos^2(\theta) + \sin^2(\Phi)\sin^2(\theta) = 1$. It means that the sum of the maximum intensity of all the components is the total peak power that is conserved against Φ for given θ as shown in Fig. 2(b). If we set $\Phi = 90^\circ$ in this vector then it reduces to the BB case as $(\cos\theta, \sin\theta)$ [5] and is independent of Φ. In this BB case the complex modulation in z is independent of coupling parameters because $\cos(\Phi)$ is zero if $\Phi = 90^\circ$. Instead if we set $\theta = 0^\circ$ or $\theta = 90^\circ$ then the vector takes the form $[\cos(\Phi), \sin(\Phi)]$ and defines the polarization vector of DB [7] and is independent of θ. In this DB case the complex modulation in z supports the $\cos(\Phi)$ term. Consequently it depends on the coupling parameters.

It is interesting to note from (3) that if we mix a scalar dark soliton with $\mu < 0$ in the BB case of the Manakov model with $\mu > 0$, then the resulting DBB mixed vector soliton has a convenient mathematical form (with $\mu > 0$ or $\mu < 0$) without a singularity and supports a polarization vector provided $\Delta = 1/\mu$. In this case by tuning the parameter α_1 [which couples a dark component with the BB case as shown in (3)] one can change the initial intensity distribution among the bright components without disturbing any other soliton parameters. As mentioned before, our DBB multisoliton in the next section preserves the energy-exchange collision dynamics of BB vector solitons. Consequently by using this dark component coupling parameter, one can control the energy-exchange among the bright components during collision. Therefore our investigation using (3) may give further insight into the applications of BB vector solitons in the switching devices [9,11,13,14,26].

B. Case (2): FDBB soliton with polarization vector ($k_I^{(1)} = l$)

As mentioned before the minimum intensity of a dark component of our general solution (2) becomes zero provided $k_I^{(1)} = l$ (i.e., $\beta = 0$). Therefore a FDBB soliton with eight free real parameters directly appears from our general DBB soliton (2) with nine free real parameters under a parametric condition $k_I^{(1)} = l$ as

$$\begin{aligned}
u_1 &= \cosh(\Theta) e^{-i(2k_R^{(1)2}\cosh^2(\Theta))z} \frac{k_R^{(1)}}{\sqrt{\mu}}[\tanh(\eta_{1R} + \Gamma''/2)] \\
&\quad \times e^{i(\psi'+\phi_1)}, \\
u_2 &= \sinh(\Theta)\cos(\theta) e^{-i(2k_R^{(1)2}\cosh^2(\Theta))z} \frac{k_R^{(1)}}{\sqrt{\mu}}[\mathrm{sech}(\eta_{1R} + \Gamma''/2)] \\
&\quad \times e^{i(\eta'_{1I}+\phi_2)}, \\
u_3 &= \sinh(\Theta)\sin(\theta) e^{-i(2k_R^{(1)2}\cosh^2(\Theta))z} \frac{k_R^{(1)}}{\sqrt{\mu}}[\mathrm{sech}(\eta_{1R} + \Gamma''/2)] \\
&\quad \times e^{i(\eta'_{1I}+\phi_3)},
\end{aligned} \tag{4}$$

where $\cosh(\Theta) = 2|\alpha_1|/D_{\Phi F}$, $\sinh(\Theta) = N_\Phi/D_{\Phi F}$, and $\Gamma'' = \ln(\mu D_{\Phi F}^2/4k_R^{(1)2})$ in which $D_{\Phi F} = (4|\alpha_1|^2 - |\alpha_2^{(1)}|^2 -$

$|\alpha_3^{(1)}|^2)^{1/2}$. It is interesting to note that the amplitude of (4) is the same as in the previous case. However, the FDBB soliton (4) with eight free real parameters supports a singularity at $4|\alpha_1|^2 = |\alpha_2^{(1)}|^2 + |\alpha_3^{(1)}|^2$ and differs from the FDBB case of (3). [One can read the FDBB case of (3) from the DBB soliton (3) without singularity under the condition $k_I^{(1)} = l$.] The DBB soliton (3) has eight free real parameters if we solve the parametric restriction $\mu\Delta = 1$ in the case (1) for any one of the free parameters. Consequently the FDBB realized from this resultant eight-parameter DBB (3) under the condition $k_I^{(1)} = l$ supports seven free real parameters but without admitting any kind of singularities. Moreover the vector $[\cosh(\Theta), \sinh(\Theta)\cos(\theta), \sinh(\Theta), \sin(\theta)]$ in (4) is different from the corresponding vector in (3). Consequently it evolves on a hyperboloid surface $\cosh^2(\Theta) - \sinh^2(\Theta)\cos^2(\theta) - \sinh^2(\Theta)\sin^2(\theta) = 1/\mu$ (instead of sphere as in the previous case) and defines the polarization vector of (4). Therefore in this case the difference between the maximum intensity of a dark component and the sum of the maximum intensity of two bright components is conserved against Θ for given θ as shown in Fig. 2(c). Hence the polarization vector of (4) differs from that of (3) by supporting the singularity at $4|\alpha_1|^2 = |\alpha_2^{(1)}|^2 + |\alpha_3^{(1)}|^2$. Further the complex modulation in z differs from the previous case by supporting this singularity. One can also easily verify that if $\Theta = i\pi/2$ then the resulting vector $[\cos(\theta), \sin(\theta)]$ defines the polarization vector of the BB soliton of the Manakov model [5]. Suppose if $\theta = 0^\circ$ or $\theta = 90^\circ$ then the resulting vector reduces to the FDB soliton of the Manakov model [7]. It is interesting to note that the $\cosh(\Theta)$ term does not exist in the complex modulation in z of BB case but that appears in the FDB case. Therefore as in the previous case, the complex modulation in z is independent of the polarization vector in the BB case but depends on it in the mixed vector soliton cases. In addition the interactions between two FDBB one-solitons (4) are investigated in the next section by numerically plotting our two-soliton solution. This study reveals some breathing effect by setting oscillations in the amplitudes of interacting FDBB bound state solitons. A similar kind of effect was experimentally observed in the hyperfine states of ^{87}Rb BECs [21] as mentioned before.

In the next section we have derived a DBB multisoliton solution with a common background field for dark component. This solution has freedom to study the GDBB-GDBB collision dynamics (the collision between two GDBB one-solitons with singularity) and the GDBB-FDBB collision dynamics (the collision between a GDBB and a FDBB vector soliton with a singularity). Further our multisoliton solution sets a common speed for its asymptotic solitons under a condition at which the minimum intensity of dark component of each asymptotic soliton is zero. Therefore our multisoliton is able to let us to investigate the interaction between the two closely packed bound state FDBB vector solitons in addition to the above two investigations. All such investigations help us to realize the practically interesting energy-exchange collision property of DBB one-solitons, the experimentally observed breathing effect in the BECs [21–23], and some possible ways to control certain interesting interaction effects as explained below.

III. DARK-BRIGHT-BRIGHT TWO-SOLITON SOLUTION AND ITS COLLISION DYNAMICS

One can understand from the above discussions that if a dark component couples with the practically interesting BB case of the Manakov model, then the resulting solution (2) manages different physical situations by tuning its initial parametric values. Therefore next we want to investigate the collision dynamics of DBB vector solitons with this property by deriving the DBB two-soliton solution of (1) with the help of the Hirota algorithm [7,16] as shown in the Appendix. The complicated two-soliton solution (A.8) (see the Appendix) with seven arbitrary complex parameters (α_1, $\alpha_2^{(j)}$, $\alpha_3^{(j)}$, $k^{(j)}$, $j = 1,2$) and a real free parameter (l) numerically demonstrates the collision dynamics of two DBB one-solitons. By using the asymptotic analysis [5,16] one can define these two colliding solitons appearing before [i.e., soliton $(j)^-$ from Eq. (A.8) at the limit $t \to -\infty$] and after [i.e., soliton $(j)^+$ from Eq. (A.8) at the limit $t \to +\infty$] collision:

Before collision ($t \to -\infty$), soliton $(j)^-$:

$$\begin{aligned}
u_1^{(j)-} &= \cos(\Phi)^{(j)} e^{-i\delta_j z} e^{2iA_j^2 z} A_j[i\sin\beta_j + \cos\beta_j \tanh(\eta_{jR} + \Gamma^{(j)-}/2)] e^{i(\psi' + \beta_j + \phi_1^-)},\\
u_2^{(j)-} &= \sin(\Phi)^{(j)} \cos(\theta)^{(j)} e^{-i\delta_j z} e^{2iA_j^2 z} A_j[\operatorname{sech}(\eta_{jR} + \Gamma^{(j)-}/2)] e^{i(\eta'_{jI} + \phi_2^{(j)-})},\\
u_3^{(j)-} &= \sin(\Phi)^{(j)} \sin(\theta)^{(j)} e^{-i\delta_j z} e^{2iA_j^2 z} A_j[\operatorname{sech}(\eta_{jR} + \Gamma^{(j)-}/2)] e^{i(\eta'_{jI} + \phi_3^{(j)-})}.
\end{aligned}$$

After collision ($\to \infty$), soliton $(j)^+$:

$$\begin{aligned}
u_1^{(j)+} &= \cos(\Phi)^{(j)} e^{-i\delta_j z} e^{2iA_j^2 z} A_j[i\sin\beta_j + \cos\beta_j \tanh(\eta_{jR} + \Gamma^{(j)+}/2)] e^{i(\psi' + \beta_j + \phi_1^{(j)+})},\\
u_2^{(j)+} &= \sin(\Phi)^{(j)} \cos(\theta)^{(j)} \big(C_2^{(j)}/B^{1/2}\big) e^{-i\delta_j z} e^{2iA_j^2 z} A_j[\operatorname{sech}(\eta_{jR} + \Gamma^{(j)+}/2)] e^{i(\eta'_{jI} + \phi_2^{(j)+})},\\
u_3^{(j)+} &= \sin(\Phi)^{(j)} \sin(\theta)^{(j)} \big(C_3^{(j)}/B^{1/2}\big) e^{-i\delta_j z} e^{2iA_j^2 z} A_j[\operatorname{sech}(\eta_{jR} + \Gamma^{(j)+}/2)] e^{i(\eta'_{jI} + \phi_3^{(j)+})},
\end{aligned} \tag{5}$$

where $\cos(\Phi)^{(j)} = 2|\alpha_1| |e^{(\rho_{jj} - \rho)/2}/D_{\Phi c}$, $\sin(\Phi)^{(j)} = N_{\Phi c}/D_{\Phi c}$, $\cos(\theta)^{(j)} = |\alpha_2^{(j)}|/N_{\Phi c}$, $\sin(\theta)^{(j)} = |\alpha_3^{(j)}|/N_{\Phi c}$, $\delta_j = (2k_R^{(j)2}/\mu\Delta_j)(\sin^2(\Phi)^{(j)} - \cos^2(\Phi)^{(j)}(\mu - 1))$, $A_j = k_R^{(j)}/(\mu\Delta_j)^{1/2}$, $\beta_j = \arctan[(k_I^{(j)} - l)/k_R^{(j)}]$, $\eta'_{jI} = k_I^{(j)} t + (k_R^{(j)2} - k_I^{(j)2}) z + \eta_{jI}^{(0)}$, $\eta_{jR} = k_R^{(j)}(t - 2k_I^{(j)} z) + \eta_{jR}^{(0)}$, $\Gamma^{(j)-} = \ln(R_{jj})$, $\Gamma^{(j)+} = \ln(R_{1212}/R_{j'j'})$, $\phi_1^- = \arctan(\alpha_{1I}/\alpha_{1R}) + \pi$, $\phi_m^{(j)-} = \arctan(\alpha_{mI}^{(j)}/\alpha_{mR}^{(j)})$, $\phi_1^{(j)+} = \phi_1^- + \beta_{j'} - \pi$, $\phi_m^{(j)+} = \phi_m^{(j)-} + 2\zeta^{(j)}$, and $e^{2i\zeta^{(j)}} = \xi^{(j)}/\xi^{(j)*}$ in which $N_{\Phi c} = (|\alpha_2^{(j)}|^2 +$

$|\alpha_3^{(j)}|^2)^{1/2}$, $D_{\Phi c} = (4|\alpha_1|^2 e^{(\rho_{jj}-\rho)} + N_{\Phi c}^2)^{1/2}$, $C_m^{(j)} = [\alpha_m^{(j)}(k^{(j')} + k^{(j')*})e^{\rho_{j'j'}} - \alpha_m^{(j')}(k^{(j)} + k^{(j')*})e^{\rho_{jj'}}][\frac{(e^{\rho_{jj}} - e^{\rho_{j'j'}})/2}{|\alpha_m^{(j)}|})(\frac{k^{(j)}+k^{(j)*}}{k^{(j')}+k^{(j')*}}]^{1/2}$, $B = [(k^{(1)} + k^{(1)*})(k^{(2)} + k^{(2)*})e^{\rho_{11}+\rho_{22}} - (k^{(1)} + k^{(2)*})(k^{(2)} + k^{(1)*})e^{\rho_{12}+\rho_{21}}]$, $\Delta_j = [\sin^2(\Phi)^{(j)} - \cos^2(\Phi)^{(j)} \cos^2 \beta_j e^{(\rho_{jj}-\rho)}]$, $\xi^{(j)} = [(k^{(1)} - k^{(2)})(k^{(j)} + k^{(j')*})]^{1/2}$, $m = 2,3$, $j = 1,2$, and $j' = j \pm 1$ if $j = 1$ or $j = 2$.

It is interesting to note that the asymptotic solitons (5) realized from the two-soliton solution at the limits $t \to -\infty$ and $t \to +\infty$ have the mathematical form of the DBB vector one-soliton (2) and admit the free parameters α_1, $\alpha_2^{(1)}$, $\alpha_3^{(1)}$, $k^{(1)}$, and l to define one of the colliding solitons [i.e., soliton $(1)^{\mp}$ appearing before (−) and after (+) the collision at the asymptotic limits] and α_1, $\alpha_2^{(2)}$, $\alpha_3^{(2)}$, $k^{(2)}$, and l to define the other colliding soliton [i.e., soliton $(2)^{\mp}$ appearing before (−) and after (+) the collision at the asymptotic limits]. However, each colliding soliton has common free parameters α_1 and l, because the Hirota method restricts us to select the common background field for the dark components of two asymptotic solitons as in our earlier investigations about the collision dynamics of DB [7,16] and DD [6] vector solitons. Here the parameter α_1 is used to couple a dark component with the two bright components of each DBB asymptotic soliton, and l is used to define the grayness of such dark component. Therefore our DBB vector two-soliton is not more general and not able to govern the collision dynamics of DBB vector solitons with different background fields. Further the terms $\cos(\theta)^{(j)}$, $\sin(\theta)^{(j)}$, η_{jR}, η'_{jI}, ψ', β_j, ϕ_1^-, and $\phi_m^{(j)-}$ in (5) have the same mathematical expressions as the corresponding terms $\cos(\theta)$, $\sin(\theta)$, η_{1R}, η'_{1I}, ψ', β, ϕ_1, and ϕ_m in (2), but with a suitable superfix on the free parameters in their expressions to distinguish their role in different solitons. However, the terms $\cos(\Phi)^{(j)}$, $\sin(\Phi)^{(j)}$, δ_j, A_j, $\Gamma^{(j)\pm}$, and $\phi_m^{(j)+}$ in (5) include the additional terms like $e^{(\rho_{jj}-\rho)}$ and differ from the corresponding terms $\cos(\Phi)$, $\sin(\Phi)$, δ, A, Γ, and ϕ_m in (2). Here the additional terms like $e^{(\rho_{jj}-\rho)}$ in the asymptotic solitons appear from the two-soliton solution used to govern their collision dynamics. Therefore we refer to such terms as interaction terms in the following discussion.

Due to the presence of interaction terms, the vectors $[\cos(\Phi)^{(j)}, \sin(\Phi)^{(j)} \cos(\theta)^{(j)}, \sin(\Phi)^{(j)} \sin(\theta)^{(j)}]\ A_j$ of two colliding solitons (5) appearing before (−) and after (+) the collision and the phase-shift terms ($\Gamma^{(j)+} - \Gamma^{(j)-}$, $\phi_1^{(j)+} - \phi_1^-$, $\phi_m^{(j)+} - \phi_m^{(j)-}$) and the complex modulation in z (δ_j) depend on all the free parameters of the two-soliton and hence gain the mathematical complication. However, one can easily verify that $\cos^2(\Phi)^{(j)} + \sin^2(\Phi)^{(j)} \cos^2(\theta)^{(j)} + \sin^2(\Phi)^{(j)} \sin^2(\theta)^{(j)} = 1$. Moreover the asymptotic solitons (5) reduce to the BB case of the Manakov model [5] if $\alpha_1 = 0$. But in this BB case, the resultant asymptotic solitons [5] also have the complicated polarization vectors, because they depend on all the free parameters of BB two-soliton solution of the Manakov model [10] through some interaction terms obtained while deriving the BB two-soliton solution. However, the amplitude terms of BB asymptotic solitons are proportional to the pulse-width parameter k_{jR} and preserve the relation $\cos^2(\theta) + \sin^2(\theta) = 1$. But in the Eq. (5) the amplitude parts A_j support a singularity through the terms Δ_j even though we have $\cos^2(\Phi)^{(j)} + \sin^2(\Phi)^{(j)} \cos^2(\theta)^{(j)} + \sin^2(\Phi)^{(j)} \sin^2(\theta)^{(j)} = 1$. Unfortunately the interaction term $e^{(\rho_{jj}-\rho)}$ also appears in the expression for Δ_j. Consequently A_j becomes the function of all the free parameters of the two-soliton. It sets some mathematical complications while one can try to identify the distinct parametric domains for different physical situations by using the numerical plots describing the collision dynamics of two-soliton solution described below. In addition one can easily verify that our colliding solitons reduce to the asymptotic forms of the DB vector two-soliton solution of the Manakov model [7,16] if $\alpha_2^{(j)} = 0$ or $\alpha_3^{(j)} = 0$. But this DB case of the Manakov model [7,16] does not support the practically interesting energy-exchange collision while the BB case of the Manakov model [10] does support it. One can ask that when the DB and BB cases of the Manakov model are coupled together as dictated by the multisoliton solution, what does happen? Our investigations reveal that the resultant asymptotic solitons of multisoliton solution (1) preserve the energy-exchange collision property of the BB vector solitons, (2) support two parametric domains corresponding to different physical situations separated by a singularity, (3) exhibit some practically examined interaction effects, and (4) are able to control such interaction effects by tuning their coupling parameters without affecting other soliton parameters as numerically demonstrated in the following discussion.

By analyzing the colliding solitons which have the properties of (2) as mentioned before at $t \to \pm\infty$, one can find the following results: (1) The terms $T_m^{(j)} = C_m^{(j)}/\sqrt{B}$ in (5) define the changes in the amplitudes of two bright components of each colliding soliton due to collision. Here it is easy to verify that $|T_m^{(j)}| = 1$ if $\alpha_2^{(1)} : \alpha_3^{(1)} = \alpha_2^{(2)} : \alpha_3^{(2)}$ and restricts the DBB multisoliton solution to support the energy-preserving collision with a phase shift. Otherwise it supports the energy-exchange collision with the well-defined phase shift [namely, $\Gamma^{(j)+} - \Gamma^{(j)-} = \ln(R_{1212}/R_{j'j'}) - \ln(R_{jj})$, $\phi_1^{(j)+} - \phi_1^- = \beta_{j'} - \pi$, $\phi_m^{(j)+} - \phi_m^{(j)-} = 2\zeta^{(j)}$] as shown in Fig. 5. It may provide a theoretical framework to observe such a kind of phase shift due to collision [27]. (2) After a collision, the amplitudes of two bright components of each colliding soliton are transformed by the matrix ($T_2^{(j)}$, $T_3^{(j)}$, $j = 1,2$) without disturbing its dark component. Moreover the value of this matrix depends on the values of the 15 free real parameters of our two-soliton solution. Consequently depending upon the parametric choices the initial energy distribution among the bright components of each colliding soliton changes without disturbing the initial energy of its dark component during collision as shown in Fig. 5. One must remember that all the components of each colliding soliton gain a phase shift during the collision as defined in the previous point. (3) Under the condition $k_I^{(1)} = k_I^{(2)} = l$ (i.e., $\beta_j = 0$) as explained in the one-soliton case, our asymptotic solitons are two FDBB vector one-solitons. It is obvious to note that under this condition the speed parameters $k_I^{(1)}$ and $k_I^{(2)}$ of these two asymptotic solitons take the value of a common parameter l. Therefore if the dark components of two FDBB one-solitons have a common background field, then they move with a common speed l. Therefore our DBB multisoliton solution with a common background field is not helpful to investigate the collision dynamics of FDBB vector solitons.

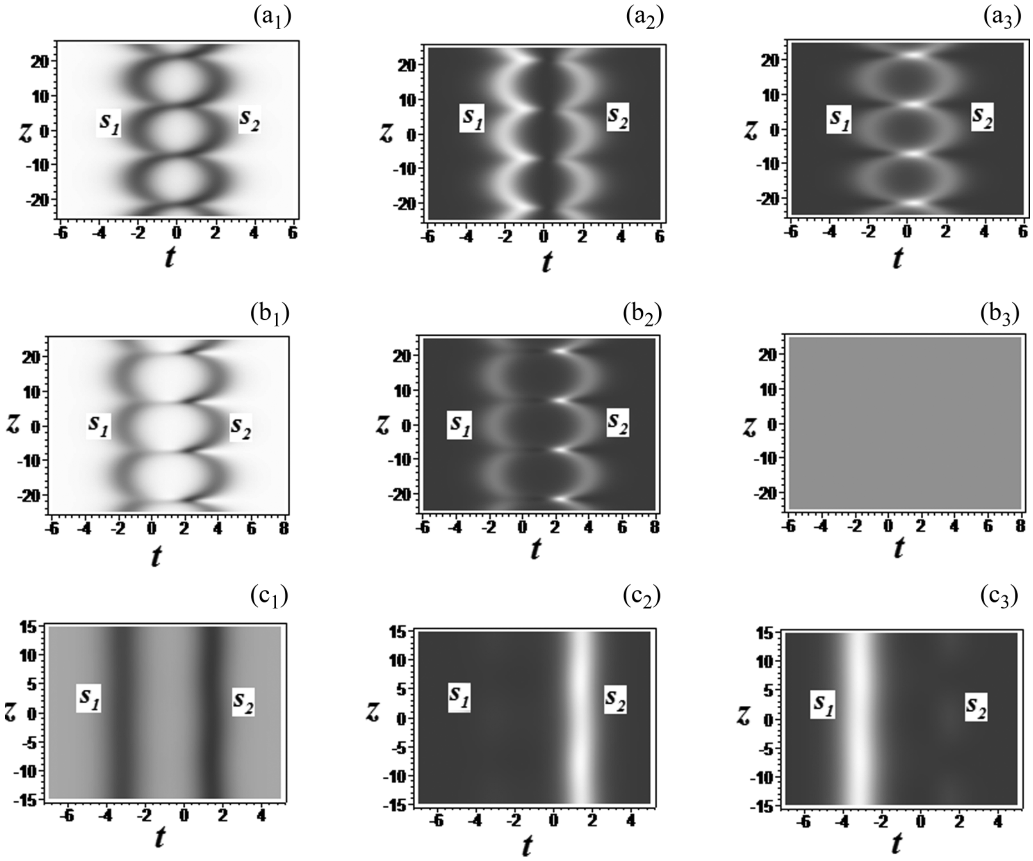

FIG. 3. Breathing effect appears due to the interaction between two (a) bound state DBB one-solitons and (b) bound state DB one-solitons. (c) Breathing effect in (b) is nullified by adding a bright component.

However, if we closely pack two such DBB bound state vector solitons under the parametric choices $\alpha_1 = 1.0167$, $\alpha_2^{(1)} = 8.4 + 0.89i, \alpha_2^{(2)} = 5 + 0.9i, \alpha_3^{(1)} = 5 + 3i, \alpha_3^{(2)} = 8 + 5i$, $k_R^{(1)} = 1$, $k_R^{(2)} = 1.2$, and $l = 0.1$ they interact. Because of this interaction in between the closely packed two FDBB one-solitons (i.e., FDBB-FDBB interaction) the amplitudes of interacting solitons oscillate and generate a breathing effect as shown in Fig. 3(a). By setting $\alpha_2^{(j)} = 0$ or $\alpha_3^{(j)} = 0$ $(j = 1,2)$ [in order to eliminate any one of the bright component in each interacting soliton in Fig. 3(a)] one can also see this breathing effect in the resultant bound state DB vector solitons as shown in Fig. 3(b). Suppose if the dark component is absent (i.e., $\alpha_1 = 0$), then a such breathing effect in Fig. 3(a) disappears and never appears again. Hence our investigations reveal that the role of the dark component is an essential one to realize the breathing effect during the interaction between two bound state DBB solitons. However, one can control this breathing effect in Fig. 3(a) (FDBB-FDBB interaction) (a) by tuning the coupling parameters $\alpha_2^{(j)}$ (or) (b) by tuning $\alpha_3^{(j)}$ (or) (c) by adding a bright component in Fig. 3(b) with a suitable value for $\alpha_3^{(j)}$ (note that $\alpha_3^{(j)}$ couples a bright component with the two-component DB soliton). For example, in Fig. 3(c) by adding a bright component with $\alpha_3^{(1)} = 45 + 13i$ and $\alpha_3^{(2)} = 0.2 + i$ the breathing effect in Fig. 3(b) is eliminated. Similarly one can also investigate such interaction effects under the conditions (a) $k_I^{(1)} = k_I^{(2)} = 0,\ l \neq 0$ (stationary GDBB-GDBB case) and (b) $k_I^{(1)} = k_I^{(2)} \neq l$ (bound state GDBB-GDBB case). A similar kind of interaction effect was also examined in the hyperfine states of ^{87}Rb BECs experimentally [21]. However, we have analytically demonstrated some possible ways to control this interaction by adding a bright component in the FDB case. All such investigations reveal that the presence of a breathing effect between the bound state DBB solitons depends on the choice of coupling parameters $(\alpha_1,\ \alpha_2^{(j)},\ \alpha_3^{(j)},\ j = 1,2)$ even though they are closely packed by using their pulse-width parameters $(k_R^{(j)})$. (4) In general under the conditions $k_I^{(1)} \neq k_I^{(2)} \neq 0$ and $\alpha_2^{(1)} : \alpha_3^{(1)} \neq \alpha_2^{(2)} : \alpha_3^{(2)}$, the two-soliton solution governs the energy-exchange collision dynamics of GDBB vector solitons as shown in Fig. 5(a) with the parametric choices $\alpha_1 = 0.0807, \alpha_2^{(1)} = 0.86 + 0.5i, \alpha_2^{(2)} = 3.5 + 3.86i, \alpha_3^{(1)} = 0.7 + 0.72i, \alpha_3^{(2)} = 0.44 + 0.9i, k^{(1)} = 1 + 0.1i$, $k^{(2)} = 1 - 0.1i$, $l = 1$ and $\mu = 1$. Moreover in this case if $k_I^{(1)} = l$ (say $k_I^{(1)} = l = 0.1$) but $k_I^{(2)} \neq l$ (i.e., $k_I^{(2)} = -0.1$) then the energy-exchange collision occurs in between the FDBB and GDBB vector one-solitons as shown in Fig. 5(b). In the colliding region of Fig. 5(b), FDBB and GDBB solitons beat each other. Such a beating effect occurs if we allow the colliding solitons to overlap for a long duration by decreasing the speed of colliding solitons (i.e., decreasing $k_I^{(j)}$ values) and also setting approximately equal values for the pulse-width parameters of colliding solitons $(k_R^{(j)})$. Further the strength of this beating effect depends on the choices of the coupling

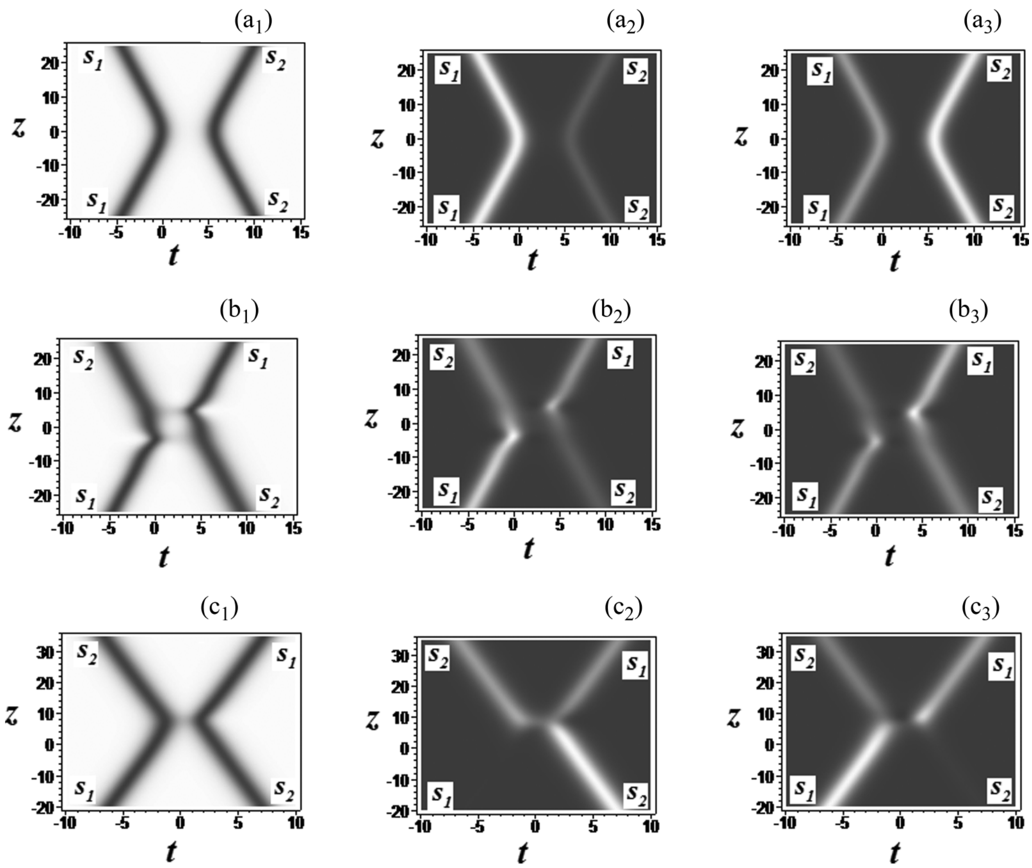

FIG. 4. Two DBB one-solitons repel each other without energy-exchange; (b) DBB solitons in (a) overcome the bouncing effect and make a jump in the colliding region if we decrease the value of pulse-width parameter $k_R^{(2)}$ to 0.7. (c) DBB solitons in (a) overcome the bouncing effect and make the smooth collision if we take the value of coupling parameters $\alpha_3^{(1)} = 15.7 + 10.72i$ and $\alpha_3^{(2)} = 0.94 + 0.001i$ without disturbing other parametric values of (a).

parameters. If we increase the beating effect by tuning the value of coupling parameter $\alpha_2^{(2)}$ then at one stage (i.e., at $\alpha_2^{(2)} = 0.5 + 0.86i$) the colliding solitons start to repel each other as shown in Fig. 4(a). Therefore in this bouncing case asymptotic solitons never collide. However, one can induce collision with energy-exchange as shown in Fig. 4(b) by tuning the parametric value of pulse-width parameter $k_R^{(2)}$ to 0.7. Figure 4(b) reveals that one-soliton crosses the other by making a jump in the colliding region. One can also set a smooth energy-exchange collision without beating and jumping effects in the colliding region as shown in Fig. 4(c) by tuning the coupling parameters $\alpha_3^{(j)}$ to the value $\alpha_3^{(1)} = 15.7 + 10.72i$ and $\alpha_3^{(2)} = 0.94 + 0.001i$ without disturbing other parametric values of Fig. 4(a). (5) All the solitons at $t \to \pm\infty$ are valid if $R_{jj} > 0$ or $\mu\Delta_j > 0$. But here the singularity terms $\Delta_j (j = 1,2)$ are defined in terms of all 15 real free parameters of two-soliton solution, because they support the complicated term $e^{(\rho_{jj}-\rho)}$ as mentioned before. Further each colliding soliton has nine free real parameters with a common field restriction. Moreover our multisoliton solution permits us to analyze the collision dynamics of DBB solitons by numerically plotting it. Therefore it is difficult to systematically identify a parametric domain for each colliding soliton in the numerical plot of a mathematically complicated two-soliton solution with a specific physical situation. However, if one can analyze such collision dynamics numerically by taking the linear combination of two general one-solitons as an initial condition (by following the earlier work in the BB case [24,25]) then one can set easily any one physical situation for all colliding solitons by tuning the free parameters suitably. Moreover such possible investigations may reveal the possibility for switching the energy-exchange collision dynamics of DBB solitons in one parametric domain with $\mu > 0$ into another parametric domain with $\mu < 0$ by selecting suitable initial parametric values, because our colliding solitons have the freedom to manage different physical situations by tuning their values of free parameters.

IV. CONCLUSION

In general if we look a fairly general form of mixed vector one-soliton solution to the Manakov-like equation or vector soliton solution (but not mixed type) to the modified Manakov model, the resultant solution supports two parametric domains (separated by a moveable singularity) giving rise to different physical situations [7,16]. In this connection it is interesting to note from our present investigation that if mix a dark component with the practically interesting the BB vector soliton by using the three-component Manakov model (1), the resultant three-component DBB vector soliton solution (2) with nine free real parameters gains the ability (a) to

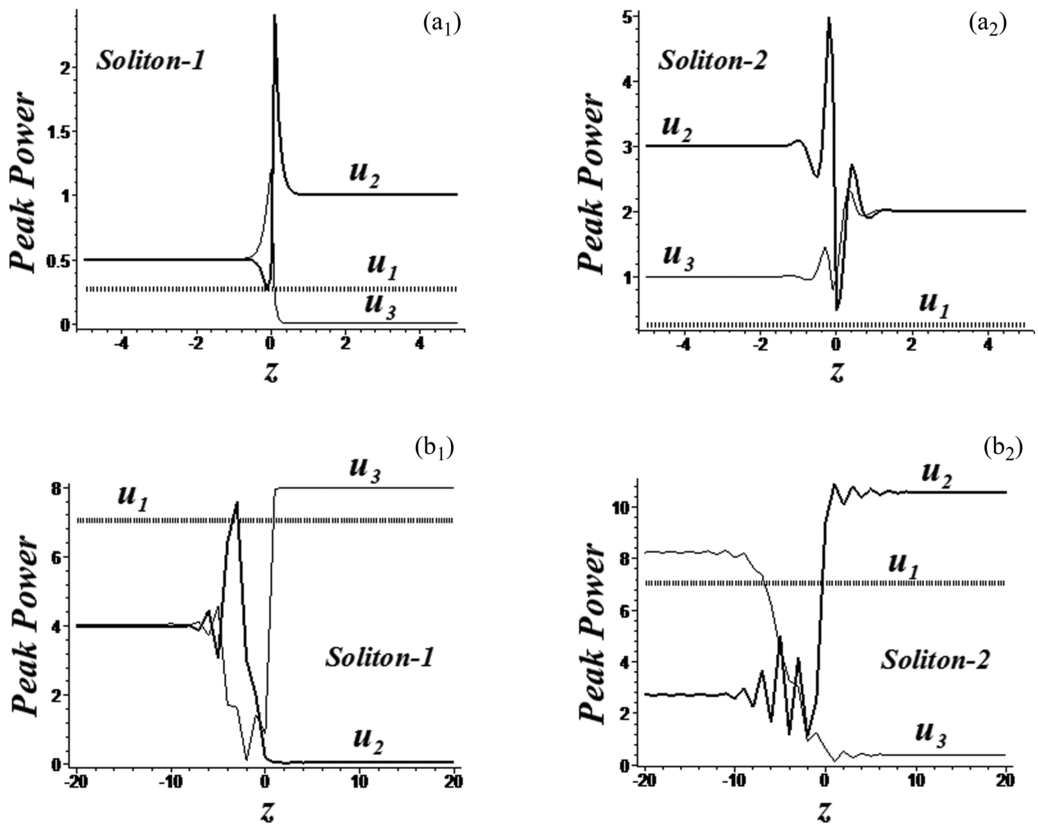

FIG. 5. Initial peak-power distributions in the two GDBB colliding one-solitons vary due to collision. (b) Head-on collision appears in between FDBB and GDBB one-solitons. Note the beating effect in the colliding region and change in the initial peak-power distribution among the bright components of each colliding soliton without disturbing its dark component.

manage the different physical situations by varying its initial parametric choices as shown in Fig. 1, (b) to preserve the experimentally observed energy-exchange collision dynamics of BB vector solitons in the photorefractive crystal [3] and also in the birefringent fiber [9] as shown in Fig. 5, (c) to identify the initial parametric choices needed to suppress or enhance the oscillations in the amplitudes of the closely packed bound state mixed vector solitons (i.e., the breathing effect due to the interaction between the closely packed stationary DBB solitons) as shown in Fig. (3), and (d) to overcome the bouncing effect by suppressing the beating effect in a colliding region of two DBB one-solitons as shown in Figs. 4 and 5.

It is interesting to note that the amplitude of the DBB soliton (2) with nine free real parameters is proportional to the pulse-width parameter ($k_R^{(1)}$) and free from the other free parameters as in the BB case [5] if the minimum intensity of its dark component is zero or if $\Delta = 1/\mu$. In the former case to define its polarization vector we need hyperbolic functions [see Eq. (4)] with a singularity. Consequently in this case the difference between the maximum intensity of a dark component and the sum of the maximum intensity of two bright components is conserved against Θ and θ as shown in Fig. 2(c). But in the later case the polarization vector is free from the singularity and admits only the circular functions to define it. Consequently in this $\Delta = 1/\mu$ case, the sum of the intensity distribution among its all components is conserved against Φ and θ as shown in Fig. 2(b). In these mixed vector soliton cases the complex modulation in the z term depends on the polarization vector (or) coupling parameters for it supports a dark component whereas in the BB case it is free from the polarization parameters.

In Sec. III we have numerically demonstrated the GDBBB-GDBB collision dynamics in Fig. 5(a) and the GDBB-FDBB collision dynamics in Fig. 5(b). Here each colliding soliton (with a common background field for its dark component) supports a singularity and the energy-switching properties (which are robust phenomena which can survive even in the presence of both loss and cross-phase perturbation terms [25]) of BB vector solitons. Therefore our investigations may promote the applications of BB vector solitons to composite nonlinear media and the artificial metamaterials [12]. It is also interesting to note from our multisoliton that the breathing effect due to the interaction between bound state DBB solitons has the possibility to occur because the interacting solitons have a dark component. One such possibility as shown in Fig. 3(b) by using the two DB one-solitons is eliminated by adding a bright component in this interaction picture as shown in Fig. 3(c). Although a similar kind of breathing effect was experimentally observed in the hyperfine states of ^{87}Rb BECs [21], we are able to predict a mechanism to control this kind of interaction effect by appropriately involving more free coupling parameters (α_1, $\alpha_2^{(j)}$, $\alpha_3^{(j)}$, $j = 1,2$) in the polarization vector of our interacting solitons. We have also defined each colliding soliton in our numerical plot by using the asymptotic analysis. Such solitons (5) support some complicated interaction terms. Such terms depend on all the free parameters of two colliding solitons as shown in Eq. (5). It restricts us to identify different parametric domains corresponding to different physical situations. However, work is in progress to overcome this kind of difficulty in our analytical solution by using the suitable numerical technique as explained in Sec. III.

Recently the transformation equation connecting the Manakov model and the coupled equation with four-wave mixing terms has been found [28]. By using our mixed solution in this transformation and in the other transformation equation used to include the effects of linear self- and cross-coupling terms in the Manakov model [29] we have noted a breather vector soliton solution. Such details will be focused elsewhere. In addition we believe that the present study will also give further insight into the mixed pair multicomponent solitons, collision in boson-fermion mixtures, and nonlinear left-handed materials and their applications in switching devices [9,11,13,14].

APPENDIX

1. DBB One-soliton solution

By substituting $u_1 = g/f$ and $u_m = h_m/f$, $m = 2,3$ in (1), it can be rewritten as a set of homogeneous equations involving only quadratic terms (f^2, fg, g^2) acted on by the Hirota bilinear operator $D_z^p D_t^q (g\cdot f) = (\partial_z - \partial_{z'})^p(\partial_t - \partial_{t'})^q [g(z,t)\, f(z',t')]|_{z=z',\, t=t'}$ as

$$f\big[(iD_z + D_t^2)g\cdot f\big] - g\left[D_t^2 f\cdot f - 2\left(\mu_1 g g^* + \sum_{m=2}^{3}\mu_m h_m h_m^*\right)\right] = 0,$$

$$f\big[(iD_z + D_t^2)h_m\cdot f\big] - h_m\left[D_t^2 f\cdot f - 2\left(\mu_1 g g^* + \sum_{m=2}^{3}\mu_m h_m h_m^*\right)\right] = 0. \tag{A1}$$

The next step is to decouple (A1) into a set of bilinear equations of which there are many possibilities to decouple. To find the mixed vector soliton solution, we decouple Eq. (A1) by introducing an unknown decoupling constant λ as

$$\mathcal{B}_1 g\cdot f = 0;\ \mathcal{B}_1 h_m\cdot f = 0;\ \mathcal{B}_2 f\cdot f = 2\left(\mu_1 g g^* + \sum_{m=2}^{3}\mu_m h_m h_m^*\right), \tag{A2}$$

where $\mathcal{B}_1$ and $\mathcal{B}_2$ are defined as $\mathcal{B}_1 = (iD_z + D_t^2 - \lambda)$ and $\mathcal{B}_2 = (D_t^2 - \lambda)$ in which λ is a constant to be determined. To find the DBB one-soliton solution, we assume a formal power series for g, h, and f as

$$g = g_0(1+\chi^2 g_2),\quad h_m = (\chi h_{1m})\quad \text{and}\quad f = (1+\chi^2 f_2). \tag{A3}$$

One can collect a set of linear partial differential equations (PDEs) at various powers of χ by using (A3) in (A2) as

$$\begin{aligned}
&\chi^0:\ \mathcal{B}_1(g_0\cdot 1) = 0,\ \mathcal{B}_2(1\cdot 1) = 2\mu(g_0 g_0^*),\\
&\chi^1:\ \mathcal{B}_1(h_{12}\cdot 1) = 0,\ \mathcal{B}_1(h_{13}\cdot 1) = 0,\\
&\chi^2:\ \mathcal{B}_1\cdot g_0(1\cdot f_2 + g_2\cdot 1) = 0,\quad \mathcal{B}_2(1\cdot f_2 + f_2\cdot 1) = 2\mu[g_0 g_0^*(g_2 + g_2^*) + h_{12}h_{12}^* + h_{13}h_{13}^*)],\\
&\chi^3:\ \mathcal{B}_1(h_{12}\cdot f_2) = 0,\ \mathcal{B}_1(h_{13}\cdot f_2) = 0,\\
&\chi^4:\ \mathcal{B}_1(g_2\cdot f_2) = 0,\ \mathcal{B}_2(f_2\cdot f_2) = 2\mu[g_0 g_0^*(g_2 g_2^*)].
\end{aligned} \tag{A4}$$

To solve the above set of PDEs with $\chi^r, (r = 2,3,4)$ for $g_2 = Z.Re^{\eta_1+\eta_1^*}$ and $f_2 = Ce^{\eta_1+\eta_1^*}$, we take $g_0 = \alpha_1 R^{-1/2} e^{(i\psi)}$ and $h_{1m} = \alpha_m^{(1)} e^{\eta_1}$, where $\psi = lt - (l^2+\lambda)z + \psi^{(0)}$ and $\eta_1 = k^{(1)}t - i(k^{(1)2} - \lambda)z + \eta_1^{(0)}$ as the plane wave solutions of equations with $\chi^{(0)}$ and $\chi^{(1)}$. Here the real constants ω, R and C and the complex constants $\omega^{(1)}$ and Z are fixed by the above set of linear PDEs without disturbing the arbitrary nature of complex parameters α_1, $\alpha_m^{(1)}$, and $k^{(1)}$ and a real parameter l. One can obtain the exponential form of DBB one-soliton after using these solutions in the bilinear transformations as

$$u_1 = \frac{\tau}{\sqrt{R}}\left[\frac{1 + ZRe^{\eta_1+\eta_1^*}}{1 + Ce^{\eta_1+\eta_1^*}}\right]e^{i(\psi)},\quad u_m = \left[\frac{\alpha_m^{(1)} e^{\eta_1}}{1 + Ce^{\eta_1+\eta_1^*}}\right]. \tag{A5}$$

The above solution is rewritten in the trigonometric form [see Eq. (2)] in Sec. II.

2. DBB Two-soliton solution

The calculation of this state is a tedious but straightforward algebraic exercise. To find the DBB two-soliton solution we assume the formal power series for g, h_m, and f as

$$g = g_0(1+\chi^2 g_2 + \chi^4 g_4), h_m = \chi h_{1m} + \chi^3 h_{3m}\ \text{and}\ f = 1 + \chi^2 f_2 + \chi^4 f_4, m = 2,3 \tag{A6}$$

By using (A6) in (A2), we collect the set of linear PDEs at various powers of χ as

$$
\begin{aligned}
&\chi^0: \ \mathcal{B}_1(g_0\cdot 1)=0,\ \mathcal{B}_2(1\cdot 1)=2\mu(g_0g_0^*),\\
&\chi^1: \ \mathcal{B}_1(h_{12}\cdot 1)=0,\ \mathcal{B}_1(h_{13}\cdot 1)=0,\quad \chi^2:\mathcal{B}_1\cdot g_0(1\cdot f_2+g_2\cdot 1)=0,\\
&\qquad \mathcal{B}_2(1\cdot f_2+f_2\cdot 1)=2\mu[g_0g_0^*(g_2+g_2^*)+h_{12}h_{12}^*+h_{13}h_{13}^*)],\\
&\chi^3: \ \mathcal{B}_1(h_{12}\cdot f_2+h_{32}\cdot 1)=0,\ \mathcal{B}_1(h_{13}\cdot f_2+h_{33}\cdot 1)=0,\\
&\chi^4: \ \mathcal{B}_1g_0(1\cdot f_4+g_2\cdot f_2+g_4\cdot 1)=0,\\
&\qquad \mathcal{B}_2(1\cdot f_4+f_2\cdot f_2+f_4\cdot 1)=2\mu[g_0g_0^*(g_4+g_2g_2^*+g_4^*)+(h_{12}h_{32}^*+h_{32}h_{12}^*)+(h_{13}h_{33}^*+h_{33}h_{13}^*)],\\
&\chi^5: \ \mathcal{B}_1(h_{12}\cdot f_4+h_{32}\cdot f_2)=0,\ \mathcal{B}_1(h_{13}\cdot f_4+h_{33}\cdot f_2)=0,\\
&\chi^6: \ \mathcal{B}_1g_0(g_2\cdot f_4+g_4\cdot f_2)=0,\\
&\qquad \mathcal{B}_2(f_2\cdot f_4+f_4\cdot f_2)=2\mu[g_0g_0^*(g_2g_4^*+g_4g_2^*)+(h_{32}h_{32}^*+h_{33}h_{33}^*)],\\
&\chi^7: \ \mathcal{B}_1(h_{32}\cdot f_4)=0,\ \mathcal{B}_1(h_{33}\cdot f_4)=0,\\
&\chi^8: \ \mathcal{B}_1g_0(g_4\cdot f_4)=0,\ \mathcal{B}_2(f_4\cdot f_4)=2\mu(g_0g_0^*g_4g_4^*).
\end{aligned}
\tag{A7}
$$

Now in order to find DBB two-soliton solution, we take $g_0=\alpha_1R^{-1/2}e^{(i\psi)}$ and $h_{1m}=\alpha_m^{(1)}e^{\eta_1}+\alpha_m^{(2)}e^{\eta_2}$ as the solution of equations with χ^0 and χ^1, respectively. After using these plane wave solutions for g_0 and h_{1m} in the PDEs with χ^r $(r=2,3,\ldots,8)$ we take $g_2=\sum_{j,n=1}^2 e^{\eta_j+\eta_n^*+\rho_{jn}+z_{jn}+i\pi}$, $f_2=\sum_{j,n=1}^2 e^{\eta_j+\eta_n^*+\rho_{jn}}$, $g_4=e^{(\sum_{j=1}^2(\eta_j+\eta_j^*)+\rho_{1212}+z_{1212})}$, and $f_4=e^{(\sum_{j=1}^2(\eta_j+\eta_j^*)+\rho_{1212})}$ as the solutions of the resulting equations. Therefore we like to name g_0 and h_{1m} as the judicious input ansatz for the above manipulations and have taken care to define them by considering their role in the required solution. It is important to note that the Hirota method allows us to take the input functions h_{1m} for two-soliton as the linear combination of two plane wave solutions while it restricts g_0 to take its form as in the one-soliton case. Consequently our dark component in the resultant two-soliton solution

$$
\begin{aligned}
u_1&=(\alpha_1e^{i\psi+\rho/2})/D\left[1+\sum_{j,n=1}^{2}e^{\eta_j+\eta_n^*+\rho_{jn}+z_{jn}+i\pi}+e^{(\sum_{j=1}^2(\eta_j+\eta_j^*)+\rho_{1212}+z_{1212})}\right],\\
u_m&=(1/D)\left[\sum_{j=1}^{2}\alpha_m^{(j)}e^{\eta_j}+\sum_{j=1}^{2}e^{\eta_1+\eta_2+\eta_j^*+\sigma_{mj}}\right],
\end{aligned}
\tag{A8}
$$

where $D=1+\sum_{j,n=1}^2 e^{\eta_j+\eta_n^*+\rho_{jn}}+e^{(\sum_{j=1}^2(\eta_j+\eta_j^*)+\rho_{1212})}$. Here $R=e^{\rho}=\sum_{j,n=1}^2 e^{\rho_{jn}}$, $\rho_{jn}=\ln(R_{jn})$, $e^{z_{jn}}=\frac{cg_j}{cg_n^*}$, $e^{z_{1212}}=\frac{cg_1cg_2}{cg_1^*cg_2^*}e^{\sigma_{mj}}=(k^{(1)}-k^{(2)})[\frac{\alpha_m^{(1)}e^{\rho_{2j}}}{(k^{(1)}+k^{(j)*})}-\frac{\alpha_m^{(2)}e^{\rho_{1j}}}{(k^{(2)}+k^{(j)*})}]$, $e^{\rho_{1212}}=|k^{(1)}-k^{(2)}|^2[\frac{e^{\rho_{11}+\rho_{22}}}{(k^{(1)}+k^{(2)*})(k^{(2)}+k^{(1)*})}-\frac{e^{\rho_{12}+\rho_{21}}}{(k^{(1)}+k^{(1)*})(k^{(2)}+k^{(2)*})}]$ in which $R_{jn}=[\frac{\mu(\alpha_2^{(j)}\alpha_2^{(n)*}+\alpha_3^{(j)}\alpha_3^{(n)*})}{(k^{(j)}+k^{(n)*})^2}-\frac{\mu|\alpha_1|^2}{cg_jcg_n^*e^{(\rho-\rho_{jn})}}]$, $\psi=lt-(l^2+\lambda)z+\psi^{(0)}$, $\eta_j=k^{(j)}t-i(k^{(j)2}-\lambda)z+\eta_j^{(0)}$, $cg_j=k^{(j)}-il$, $cg_n^*=k^{(n)*}+il$, and $\lambda=2\mu|\alpha_1|^2/R$ has a common background field.

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