Multiplicative noise effects on electroconvection in controlling additive noise by a magnetic field

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We report multiplicative noise-induced threshold shift of electroconvection (EC) in the presence of a magnetic field $\boldsymbol{H}$. Controlling the thermal fluctuation (i.e., additive noise) of the rodlike molecules of nematic liquid crystals by $\boldsymbol{H}$, the EC threshold is examined at various noise levels [characterized by their intensity and cutoff frequency (f_c)]. For a sufficiently strong $\boldsymbol{H}$ (i.e., ignorable additive noise), a modified noise sensitivity characterizing the shift problem is in good agreement with experimental results for colored as well as white noise ($f_c \to \infty$); until now, there was a large deviation for (sufficiently) colored noises. The present study shows that $\boldsymbol{H}$ provides us with ideal conditions for studying the corresponding Carr-Helfrich theory considering pure multiplicative noise.

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I. INTRODUCTION

Ac-driven electroconvection (EC) in nematic liquid crystals (NLCs) has been intensively investigated for understanding pattern formation in nonequilibrium systems for the last five decades [1]. Recently, particular attention has been paid to nontrivial noise effects on the EC such as threshold-shift problems, order (or pattern) induction, and structure variation [2–4]. We have intensively studied the EC-threshold shifts (i.e., stabilization or destabilization effects) by controlling *multiplicative* external noise [4]. It was found that the characteristics of noise such as its intensity and correlation time τ_N [or cutoff frequency $f_c = 1/(2\pi\tau_N)$] play an important role in the variation of the EC threshold [5]. Very recently, it has been discovered that in addition to the *external time scale* of noise (τ_N), a certain *internal time scale* (τ_σ, the charge relaxation time) for EC plays a crucial role in the threshold problem (and pattern structures) [5].

Generally, noise-related phenomena in spatially extended systems have been dealt with in a Swift-Hohenberg stochastic model [6]:

$$\dot{\phi}(r,t) = a\phi - \phi^3 - \left[\nabla^2 + k_0^2\right]^2\phi + \xi(t)f(\phi) + \eta(r,t). \quad (1)$$

Here, ϕ is a certain order parameter (such as the angle of the director $\boldsymbol{n}$ against the x axis in Fig. 1), and ξ and η are multiplicative and additive noises, respectively. a indicates a control parameter for the pattern formation, which corresponds to an ac field $\boldsymbol{E}(t)$ for EC, on which $\xi(t)$ is superposed in the present experiment. k_0 means a characteristic wave number for patterns. In the usual thermoconvective system (i.e., Rayleigh-Bénard cells) the important roles of both types of noise have already been pointed out [7].

In the present study, we introduce a magnetic field $\boldsymbol{H}$ to control the fluctuations of rodlike NLC molecules [i.e., thermally originated *additive* noise $\eta(r,t)$]. Additive noise can serve as a prominent factor for multiplicative noise-induced pattern formations [8]. Our aim is to investigate the (external) multiplicative noise-induced EC threshold in controlling (internal) additive noise [7–9]. This approach will contribute to revealing the effects of both types of noise on EC more clearly. In particular, a considerable discrepancy between experimental results and theoretical analysis on the threshold-shift problems can be successfully resolved in a sufficiently strong $\boldsymbol{H}$ that can negate the effect of additive noise (at least for the EC thresholds).

II. EXPERIMENT

To induce EC in the system, a sinusoidal ac field $\boldsymbol{E}(t)$ was applied across a thin slab ($d = 50\,\mu$m) of an NLC [p-methoxybenzylidene-p'-n-butylaniline (MBBA)] sandwiched between two parallel, transparent electrodes. Two typical types of sample cells were used: planar alignment cells (the initial director of the NLCs $\boldsymbol{n}_0 \perp \boldsymbol{E}$) and homeotropic alignment cells ($\boldsymbol{n}_0 // \boldsymbol{E}$). Then, to examine noise effects on the EC, a Gaussian-type electric noise $\boldsymbol{\xi}(t)$ was additionally superposed on $\boldsymbol{E}(t)$; as a result, a fluctuating sinusoidal field was applied across the NLC cells. In our study, cutoff-frequency (f_c)-dependent *colored noises* were used, which were generated by the low-pass filters of a synthesizer (7075, Hioki). In addition, to control the fluctuations of the director $\boldsymbol{n}$ (a unit vector characterizing locally averaged orientation of the molecules), a magnetic field $\boldsymbol{H}$ generated by an electromagnet system (EMID-6, EMIC) was applied to the EC system ($\boldsymbol{H} \perp \boldsymbol{E}$), as shown in Fig. 1.

In this study, the (stochastic) noise intensity $V_N = d\sqrt{\langle \xi^2(t) \rangle}$ and cutoff frequency f_c (or the correlation time τ_N), and the (deterministic) sinusoidal ac intensity $V = d\sqrt{\langle E^2(t) \rangle}$ [with a fixed ac frequency $f = 30$ Hz, i.e., a fixed k_0 in Eq. (1)] were the main control parameters. All measurements were carried out at a stable temperature ($T = 25 \pm 0.2\,^\circ$C) using an electrothermal control system (TH-99, Japan Hightech). The details of our experiments were described in our previous papers [4,5].

III. RESULTS AND DISCUSSION

First, let us consider the shift problem of onset of EC (V_c) with respect to the V_N and f_c of noise in our previous study [5]. The threshold V_c for a typical EC [Williams domain (WD)] against noise was found as a function of V_N [10]:

$$V_c^2 = V_{c0}^2 + bV_N^2, \quad (2)$$

$$b = \frac{1 + (2\pi f \tau_\sigma)^2}{\zeta^2 - [1 + (2\pi f \tau_\sigma)^2]}. \quad (3)$$

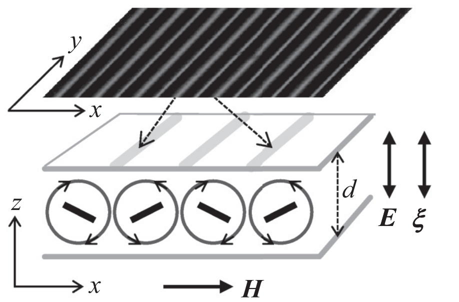

FIG. 1. Experimental scheme for electroconvection (EC) in liquid crystals. $\boldsymbol{E}$, $\boldsymbol{\xi}$, and $\boldsymbol{H}$ indicate an ac field for EC, an external multiplicative noise, and a magnetic field, respectively. The thick bars in vortices represent the directors $\boldsymbol{n}$ of liquid crystals. A typical pattern (Williams domain) in the xy plane can be observed in a polarizing optical microscope.

Here, V_{c0} denotes a threshold voltage at $V_N = 0$, and ζ^2 the dimensionless Helfrich parameter determined by the material constants of the NLCs (usually, $1.5 < \zeta^2 < 4$ for MBBA) [11]; Eq. (2) can be seen for $H = 0$ in Fig. 3.

The response sensitivity b of the WD against noise [i.e., Eq. (3)] is presented in Fig. 2. In the earlier studies [10,12], Eqs. (2) and (3) were well established for the variation of the threshold V_c (for the WD) against *white noise* ($f_c \to \infty$ or $\tau_N \to 0$); also, this linear relationship between V_c and V_N was repeatedly found in experiment (as shown in Fig. 3) [4,5]. In Fig. 2, b is given as a function of *colored noise* (characterized by f_c) for three different ζ^2 cells. Basically, b smoothly decreases with a decrease of f_c. In the case of extremely high f_c noise (i.e., $f_c \to \infty$ for the white-noise limit), however, b appears to be saturated, which is in good agreement with Eq. (3) considering white noise; the values of b reported in the earlier studies [4,11,12] are well consistent with the saturation value ($b \sim 0.85$). Unfortunately, however, such results are limited for $b > 0$.

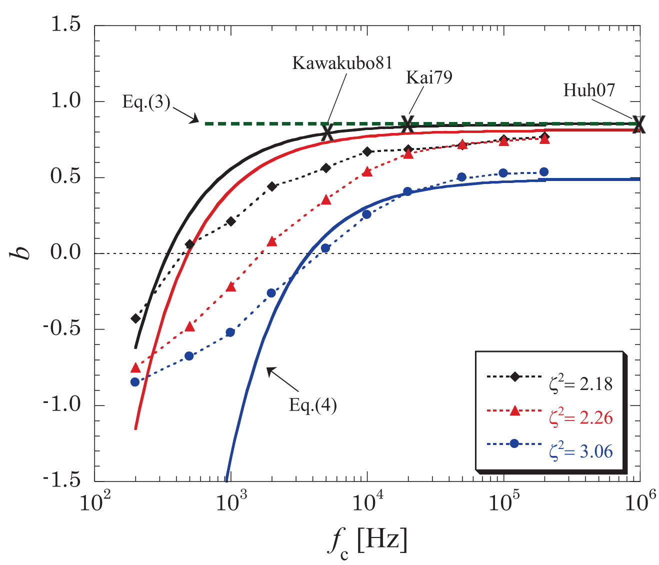

FIG. 2. (Color online) The noise cutoff frequency (f_c)-dependent sensitivity b of EC against noise. The solid lines for $b(f_c)$ were calculated from Eq. (4) (see the slopes of the fitting lines in Fig. 3). $b = 0$ means that the threshold V_c remains constant with increasing noise intensity V_N (i.e., no effect on V_c); $b > 0$ and $b < 0$ denote stabilization and destabilization effects of noise on EC, respectively. $b(f_c)$ was measured for three LC cells with different dimensionless Helfrich parameters ζ^2. Kai79, Kawakubo81, and Huh07 indicate the previous results for whitelike noise from Refs. [12,10,4], respectively. All of the results were obtained in planar alignment cells.

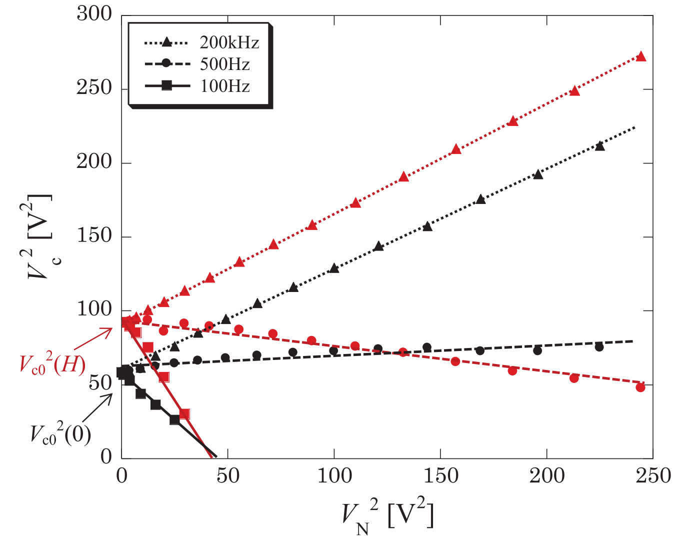

FIG. 3. (Color online) The dependence of the EC threshold (V_c) on the noise intensity V_N (for different noise cutoff frequencies f_c) in the absence (black symbols and lines) and presence [red (gray) ones] of a magnetic field ($H = 3000$ G). On applying $\boldsymbol{H}$, the slope (i.e., the sensitivity b) is noticeably changed for low f_c noise (e.g., $f_c = 100, 500$ Hz), but it remains nearly unchanged for high f_c noise (e.g., $f_c = 200$ kHz). The lines were fitted by Eq. (2). A homeotropic alignment cell ($f_{cd} \sim 600$ Hz dividing conduction and dielectric regimes) was used, because the fluctuation of the director $\boldsymbol{n}$ could be effectively controlled by $\boldsymbol{H}$.

As reported in our recent study [5], the impact of the f_c-dependent colored noise has been better understood with a modified b_{mod} covering the full range of b ($b_{\mathrm{mod}} < 0$ as well as $b_{\mathrm{mod}} > 0$):

$$b_{\mathrm{mod}} = b\left(1 - h\frac{\tau_N}{\tau_\sigma^m}\right), \tag{4}$$

$h = 8.1 \times 10^{-3}$ and $m = 1.6$. However, although b_{mod} may explain the behavior of $b(f_c)$ including $b < 0$ in Fig. 2, its (large) deviation from the experimental data still remains for extremely low f_c noise (i.e., abnormally colored noises, e.g., see b for $f_c \leqslant 1$ kHz for $\zeta^2 = 3.06$) [5]. In fact, this has motivated us to carry out the present study.

Next, we investigated the sensitivity b in a magnetic field $\boldsymbol{H}$. Due to the magnetic interaction energy of NLCs [e.g., $f_{\mathrm{mag}} = -(1/2)\mu_0 \Delta\chi H^2 \cos^2\phi$] [13], the director $\boldsymbol{n}$ tends to be parallel to $\boldsymbol{H}$ ($\Delta\chi > 0$ for MBBA). Namely, it contributes to suppressing the thermal fluctuation of NLCs; usually, the thermal fluctuation $\tilde{\phi} = \tilde{\phi}(T,H)$ is determined by competition between thermal randomness and magnetic orderliness ($\tilde{\phi} \propto H^{-2}$) [13]. Consequently, additive noise [i.e., $\eta = \eta(\tilde{\phi})$ in Eq. (1)] can be controlled, which was not considered in the

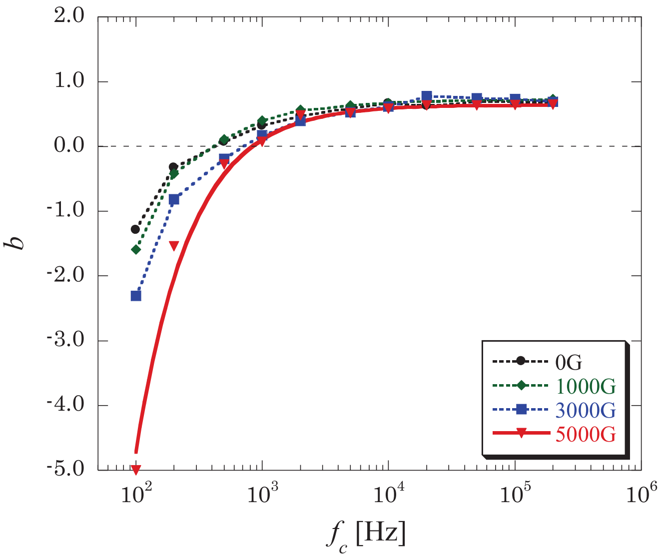

FIG. 4. (Color online) The sensitivity $b(f_c)$ for various values of the magnetic intensity H. On increasing H, the absolute value $|b|$ deviates markedly from that for $H = 0$; this is prominent for low cutoff-frequency noises, whereas it is slight for high f_c noises. Such a deviation is not found for white noise ($f_c \to \infty$). The solid fitting line for $H = 5000$ G was calculated by using Eq. (4).

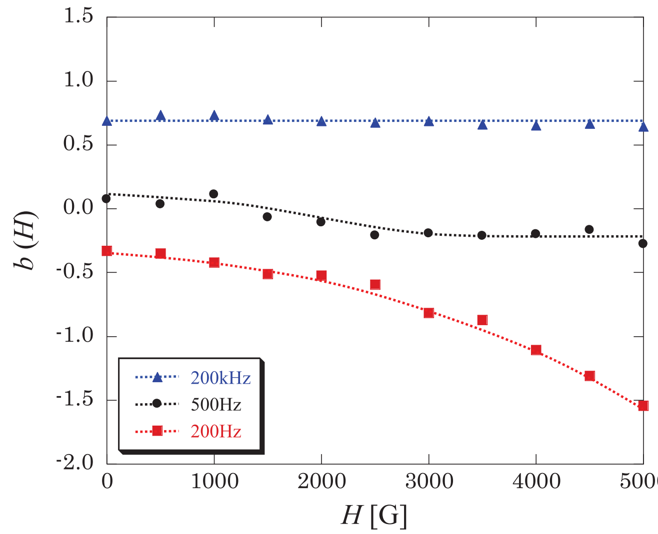

FIG. 5. (Color online) The sensitivity b as a function of magnetic intensity H for fixed noise cutoff frequency $f_c = 200$ Hz, 500 Hz, and 200 kHz. See also Fig. 4.

previous theory and experiment [Eqs. (2)–(4)]. A sufficiently strong $\boldsymbol{H}$ can give us a similar situation for the theoretical conditions [$\eta \to 0$]; this may explain the large deviation of b_{mod} from the experimental data for low f_c noise (in Fig. 2).

In this way, we examined the linear relationship between V_c and V_N in the absence and presence of a magnetic field $H = 3000$ G, as shown in Fig. 3. A typical $V_c(V_N)$ [Eq. (2)] was reproduced in the absence of H ($H = 0$); but $b < 0$ and $b > 0$ appear depending on f_c. Most importantly, applying H, $V_c\,(V_N)$ is varied with respect to $f_c(= 100{-}200\,\mathrm{k\,Hz})$, in addition to the shift from $V_{c0}(H = 0)$ to $V_{c0}(H = 3000\,\mathrm{G})$ for $V_N = 0$ due to the suppressing effect of the magnetic field on EC [14]. In other words, H gives rise to a variation of b, retaining the linear relationship. In comparison with $b(H = 0)$, the decrease of $b(H = 3000\,\mathrm{G})$ is noticeable for low f_c noise ($f_c = 100$, 500 Hz), whereas it is very slight for high f_c noise ($f_c = 200$ kHz). For $f_c = 500$ Hz, moreover, $b > 0$ (a stabilization effect) is dramatically changed to $b < 0$ (a destabilization effect) by application of H.

Finally, to represent the variation of b in H more clearly, $b(H)$ was measured as a function of f_c for different intensities of H. As shown in Fig. 4, the variation of $b(H)$ from $b(H = 0)$ becomes larger with a decrease of f_c and with an increase of H. However, no dependence of H on b is found for extremely high f_c noise (i.e., in the white-noise limit). In the case of $H = 5000$ G, Eq. (4) is well fitted with the experimental data, whereas it shows a large deviation from the data for $H = 0$. More conclusive evidence of the variation of $b(H)$ is shown in Fig. 5. For three selected levels of f_c noise (for $b > 0$, $b \sim 0$, and $b < 0$ at $H = 0$), b was measured as a function of H. The sensitivity b for extremely colored noise ($f_c = 200$ Hz) smoothly decreases with an increase of H, whereas it shows no dependence on H for whitelike noise ($f_c = 200$ kHz).

The degree of thermal fluctuation of the director $\boldsymbol{n}$ (i.e., the additive noise η) becomes weakened by an increase of magnetic intensity H (i.e., a suppression effect on $\boldsymbol{n}$) [13]. Applying strong H (sufficiently strong to suppress the thermal fluctuation), the EC system may be realized for *ignorable additive noise* conditions [i.e., $\eta \to 0$ in Eq. (1)] (at least for the EC-thresholds). Therefore, Eq. (4) in which η is not considered can be well fitted with $b(H = 5000\,\mathrm{G})$; notice again that Eqs. (2)–(4) in the previous studies were derived from the *pure multiplicative* noise condition [$\xi(t) \neq 0$ and $\eta = 0$]. Moreover, the present results show that additive noise η is buried in multiplicative whitelike noise $\xi(t)$; see the constant $b(f_c \to \infty)$ in Figs. 4 and 5. Therefore, b in Eq. (3) [$= b_{\mathrm{mod}}$ for $f_c \to \infty$ and $\tau_N \to 0$ in Eq. (4)] is quantitatively in good agreement with the experimental data (see b for $f_c > 10^4$ Hz in Fig. 4) with no dependence on H. On the other hand, the effect of additive noise (i.e., the fluctuation of the directors) [7,9] appears to be prominent in low f_c multiplicative noises (i.e., sufficiently colored noises) [14]. The reason why $b(f_c)$ for low f_c colored noises is strongly dependent on H (Figs. 4 and 5) is not clear in the present state; the correlation between the two noises [$\langle \xi(t)\eta(t')\rangle$] seems to need to be taken into account, as dealt with in the studies of stochastic resonance [15].

IV. SUMMARY AND CONCLUSION

Generally, noise is known to be associated with disordered fluctuations against the steady states of deterministic systems. Thus, it is intuitively understood that multiplicative noise can play a role in increasing the EC threshold [i.e., the sensitivity $b > 0$ in Eq. (2)] because it negates the periodic ac effect for EC due to its random oscillation. Therefore, for whitelike noise with high cutoff frequency $f_c(\to \infty)$, $b > 0$; the Carr-Helfrich theory considering white noise can explain $b > 0$ successfully [10,12]. For colored noise with low $f_c(\to 0)$, however, the theory is invalid for explaining $b < 0$ (i.e., the decrease of the EC threshold); such $b < 0$

seems to be counterintuitive. Thus, to explain $b < 0$ as well as $b > 0$, we modified b by using a relationship between internal and external time scales (i.e., the charge relaxation time τ_σ of EC and the correlation time τ_N of the noise) [5]; it was discovered that sufficiently colored noises ($\tau_N \gg \tau_\sigma$) can decrease the threshold ($b < 0$) [5]. In other words, a stabilization effect ($b > 0$) of the noise can be dramatically changed into a destabilization effect ($b < 0$) by colored noise satisfying the relationship [Eq. (4)].

However, despite our successful explanation of the transition between the effects, a considerable deviation between the modified b_{mod} and experimental b still remained [5]. To resolve this, we have taken into account additive noise (corresponding to a thermal fluctuation of the director $\boldsymbol{n}$ of NLCs). It was known that *in the presence of multiplicative noise*, additive noise plays a counterintuitive role in disorder-order (and reverse) transitions, and even for stochastic resonance [8]. In practice, the suppression of the fluctuation by $\boldsymbol{H}$ provides us with ideal conditions to test the above-mentioned theory considering *pure* multiplicative noise. In the presence of $\boldsymbol{H}$, the unignorable deviation can be successfully removed. Moreover, it is found that the theory is still valid for multiplicative whitelike noise (burying the effect of additive noise on EC), and does not depend on the magnetic intensity H. The present results may be useful for investigating the functional roles of (multiplicative and additive) noises in application fields such as nanotechnology, biotechnology, and environmental sciences [8,15].

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