Microscopic theory of heat transfer between two fermionic thermal baths mediated by a spin system

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In this paper we have presented the heat exchange between the two fermionic thermal reservoirs which are connected by a fermionic system. We have calculated the heat flux using solution of the *c*-number Langevin equation for the system. Assuming small temperature difference between the baths we have defined the thermal conductivity for the process. It first increases as a nonlinear function of average temperature of the baths to a critical value then decreases to a very low value such that the heat flux almost becomes zero. There is a critical temperature for the fermionic case at which the thermal conductivity is maximum for the given coupling strength and the width of the frequency distribution of bath modes. The critical temperature grows if these quantities become larger. It is a sharp contrast to the Bosonic case where the thermal conductivity monotonically increases to the limiting value. The change of the conductivity with increase in width of the frequency distribution of the bath modes is significant at the low temperature regime for the fermionic case. It is highly contrasting to the Bosonic case where the signature of the enhancement is very prominent at high temperature limit. We have also observed that thermal conductivity monotonically increases as a function of damping strength to the limiting value at the asymptotic limit. There is a crossover between the high and the low temperature results in the variation of the thermal conductivity as a function of the damping strength for the fermionic case. Thus it is apparent here that even at relatively high temperature, the fermionic bath may be an effective one for the strong coupling between system and reservoir. Another interesting observation is that at the low temperature limit, the temperature dependence of the heat flux is the same as the Stefan-Boltzmann law. This is similar to the bosonic case.

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I. INTRODUCTION

The technological advancement is accelerated very rapidly by the use of novel submicron and nanosize electrical devices. Molecular devices already demonstrated include molecular wire, field effect transistors, single electron transistor, molecular diodes, rectifiers, and switches [1,2]. Localized Joule heating would control the functionality and the reliability of such devices. Therefore, understanding of heat transport through the microscopic system is a focal theme in the field of nonequilibrium statistical mechanics [3–6]. Heat conduction in condensed matter is a long-standing problem which has attracted renewed interest recently both theoretically [7] and experimentally [8], especially in quantum systems, due to the growing interest in the energy transport at the nanoscale, whose understanding is an important step toward utilizing nanostructures for possible energy applications [9]. Recently heat transport at the microscopic level has been studied based on either the classical or the quantum Langevin equation of motion [10]. Most of these studies have considered heat transferred between the bosonic thermal baths mediated by a bosonic system [11]. The objective of the present study is investigation of the heat transfer between two fermionic thermal baths by a fermionic system based on the microscopic description of the thermal baths and the system. We have calculated heat flux using solution of the *c*-number Langevin equation for the system [12]. Assuming small temperature difference between the baths we have defined thermal conductivity for the process. It first increases as a nonlinear function of average temperature of the baths to a critical value then decreases to a very low value such that the heat flux almost becomes zero. A similar result has been demonstrated in Ref. [4]. Here the thermal conductivity has been calculated based on the Schrödinger dynamics. Subunits are coupled by an energy exchanging next-neighbor interaction, chosen to be a (normalized) random Hermitian matrix allowing for any possible transition such as to avoid any bias. Thus this method is not as fully microscopic a description as the present approach. Then one may say that the present study may give better understanding about the heat flux in the microscopic system as well as giving the justification of the approximation scheme [4]. The critical temperature grows if the width of the frequency distribution of the bath modes rises or the coupling between the system and reservoirs is enhanced. It is in sharp contrast to the bosonic case where the thermal conductivity monotonically increases to the limiting value. The change of the conductivity with increase in width of the frequency distribution of the bath modes is significant at the low temperature regime for the fermionic case. It is highly contrasting to the bosonic case where the signature of the enhancement is very prominent at the high temperature limit. We have also observed that the thermal conductivity monotonically increases as a function of damping strength to the limiting value at the asymptotic limit. There is a crossover between the high and the low temperature results in the variation of the thermal conductivity as a function of the damping strength for the fermionic case. Thus it is apparent here that even at relatively high temperature, the fermionic bath may be an effective one for the strong coupling between system and reservoir. Another point to be noted here is that at the low temperature limit, the temperature dependence of the heat flux is the same as the Stefan-Boltzmann law. This is similar to the bosonic case.

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The layout of the present paper is as follows: First, we have described the model in Sec. II. Then we have studied the temperature dependence of the heat flux in Sec. III. In the next section we have presented a comparative study on heat flux and the related quantities for both the fermionic and the bosonic cases, respectively. The paper is concluded in Sec. V.

II. MODEL

To study the present problem we consider two independent macroscopic fermionic thermal baths [13–20], 1 and 2, which are held at fixed temperatures T_1 and T_2, respectively. The two baths are connected by a two level system. The reservoirs are represented by the assembly of two level [21] independent systems at thermal equilibrium. In what follows we consider that the two level system is linearly coupled to these thermal environments. The Hamiltonian for the system [21] can be written as

$$\hat{H}_S = \hbar\omega_0 \hat{S}_z. \tag{1}$$

$\hat{S}_z$ in the above equation corresponds to the Pauli population difference operator. It is related to the other operators for the system through the following commutation relations:

$$[\hat{S}^\dagger, \hat{S}_z] = -\hat{S}^\dagger, \tag{2}$$

$$[\hat{S}, \hat{S}_z] = \hat{S}, \tag{3}$$

and

$$[\hat{S}^\dagger, \hat{S}] = 2\hat{S}_z. \tag{4}$$

Here we have used $\hat{S}^\dagger = \hat{S}_x + i\hat{S}_y$ and $\hat{S} = \hat{S}_x - i\hat{S}_y$. Thus $\hat{S}^\dagger$ and $\hat{S}$ are well known creation and annihilation operators, respectively. The anticommutation rules between the spin-$\frac{1}{2}$ operators are given by

$$\hat{S}^\dagger\hat{S} + \hat{S}\hat{S}^\dagger = 1. \tag{5}$$

The above anticommutation relation has the immediate consequence that spin-$\frac{1}{2}$ particles or two level atoms obey the Fermi-Dirac statistics. From the commutation relation (2) and the anticommutation relation (5) one can define a number operator, $\hat{N} = \hat{S}^\dagger\hat{S}$. It is related to $\hat{S}_z$ through the following relation:

$$\hat{S}_z = \hat{N} - \tfrac{1}{2}. \tag{6}$$

Then the system Hamiltonian takes the following form:

$$\hat{H}_S = \hbar\omega_0\big(\hat{N} - \tfrac{1}{2}\big). \tag{7}$$

The eigenvalue equation for the number operator may be written as $\hat{N}|n\rangle = n|n\rangle$ with $n = 0, 1$. The general state with no quanta is represented by $|0\rangle$, which satisfies $\hat{N}|0\rangle = 0$ and $\hat{H}_S|0\rangle = \hbar\omega_0(\hat{N} - \frac{1}{2})|0\rangle = -\frac{\hbar\omega_0}{2}|0\rangle$. The state with one quanta is denoted by $|1\rangle$ which obeys $\hat{N}|1\rangle = 1$ and $\hat{H}_S|1\rangle = \hbar\omega_0(\hat{N} - \frac{1}{2})|1\rangle = +\frac{\hbar\omega_0}{2}|1\rangle$. Thus $\hbar\omega_0$ is the energy difference between the two levels of the system. To avoid any confusion we should mention here the following point. Although mass does not appear in Eq. (1) it does not mean that the system has no mass. The signature of mass may be embedded in the energy difference. For example, the energy difference between the two levels (corresponding to the z component of the spin angular momentum of an electron) in the presence of a magnetic field depends on the mass of the electron. It is also true for the spin $-\frac{1}{2}$ nucleus. Here the mass of the nucleus is incorporated in the quantity of Bohr magneton. The energy difference between the two levels for the system depends on the Bohr magneton [21]. Finally, the above Hamiltonian may also correspond to the massless fermionic system [22] having two levels in the presence of a magnetic field.

We now consider the Hamiltonian for the thermal bath. Following Eq. (1) one can write the Hamiltonian for the νth thermal reservoir as

$$\hat{H}_{R\nu} = \hbar\sum_k \omega_{\nu k}\hat{S}_{z\nu k}; \quad \nu = 1 \text{ and } 2. \tag{8}$$

$\hbar\omega_{\nu k}$ in the above equation is the energy difference between the two levels of the kth independent mode in the νth bath. The operator for the z component of the spin angular momentum is $\hat{S}_{z\nu k}$. It has the same characteristics as described above for the system. Finally, the Hamiltonian for the system-reservoir interaction is given by [12]

$$\hat{H}_{SRI} = \hbar\sum_{\nu,k} g_{\nu k}\left\{\frac{g'_{\nu k}}{\omega_{\nu k}}\hat{I} - (\hat{S}^\dagger_{\nu k} + \hat{S}_{\nu k})(\hat{S}^\dagger + \hat{S})\right\}. \tag{9}$$

The coupling between the system and the kth mode in the νth thermal bath is represented by the coupling constant $g_{\nu k}$. $g'_{\nu k}$ is a frequency scaling factor which is a necessity for the correct normalization of the spin-coherent state (it will be used shortly to describe both the system and the thermal reservoirs in terms of c number). $\hat{I}$ in the above equation is the identity operator. To have the classical looking c-number Hamiltonian in terms of the coherent state variables, the identity operator has been chosen in terms of system operators as [12]

$$\hat{I} = \hat{S}^{\dagger 2} + \hat{S}^2 + \hat{S}^\dagger\hat{S} + \hat{S}\hat{S}^\dagger. \tag{10}$$

Here the relations $\hat{S}^{\dagger 2} = 0$ and $\hat{S}^2 = 0$ have been used. The consequence of the inclusion of the identity operator with the above expression in the interaction Hamiltonian will be clear after we derive the physically meaningful c-number Langevin equation of motion for the system in the presence of thermal baths. The remaining sum in the Hamiltonian (9) implies that in the system-reservoir interaction bilinear coupling has been assumed. With these the total Hamiltonian which may describe the heat transfer between the two fermionic thermal baths through a fermionic two level system is

$$\hat{H} = \hat{H}_S + \hat{H}_{R1} + \hat{H}_{R2} + \hat{H}_{SRI}. \tag{11}$$

The above Hamiltonian for the system-reservoir model is general in the sense that it is applicable both for the two level fermion with or without mass. Now we are in a position to write a c-number Hamiltonian from Eq. (11) by quantum mechanical averaging [12,23]. To make the present study self-sufficient, a brief note on the method is given here. We start considering the initial state to be product separable as

$$|\xi\rangle\prod_{\nu,k}|\mu_{\nu k}\rangle,$$

$|\xi\rangle$ and $|\mu_{\nu k}\rangle$ being the spin coherent states of the system and the kth component of the νth bath respectively, where a

normalized spin coherent state may be defined as

$$|\mu\rangle = (1+|\mu|^2)^{-S}\sum_{n=0}^{2S}\left(\frac{2S!}{n!(2S-n)!}\right)^{1/2}\mu^n|n\rangle$$
$$= (1+|\mu|^2)^{-S}\exp(\mu\hat{S})|0\rangle. \quad (12)$$

Here μ runs over the entire complex plane and $\{|n\rangle\}$,$n = 0,1,\ldots 2S$ are the eigenfunctions of $\hat{S}_z$ such that $\hat{S}_z|n\rangle = (S-n)|n\rangle$. The state $|0\rangle$ in Eq. (12) denotes the ground state with the maximum spin projection S. The next step is to perform the quantum mechanical averaging of Eq. (11) with the normalized product separable coherent state utilizing the relations [23,24]

$$\langle\mu|\hat{S}^\dagger|\mu\rangle = \tilde{C}(S,|\mu|)\mu, \quad \langle\mu|\hat{S}|\mu\rangle = \tilde{C}(S,|\mu|)\mu^\star,$$
$$\langle\mu|\hat{n}|\mu\rangle = \tilde{C}(S,|\mu|)|\mu^2. \quad (13)$$

Here $\tilde{C}(S,|\mu|) = \frac{2S}{1+|\mu|^2}$ and $\hat{n}$ is the spin deviation operator, $S-\hat{S}_z$, having the same set of eigenfunctions $[\{|n\rangle\}, n = 0,1,\ldots 2S]$ as $\hat{S}_z$. The set of c numbers $\{\mu,\mu^\star\}$ obtained from the matrix elements as shown in Eq. (13) would be useful in constructing some *fictitious* coordinates $[\{r\}, \{\alpha_{\nu k}\}]$ and momenta $[\{\rho\}, \{\beta_{\nu k}\}]$ like quantities as

$$\alpha_{\nu k} = \sqrt{\frac{\tilde{C}(S_{\nu k},|\mu_{\nu k}|)\hbar}{2\omega_{\nu k}}}(\mu^\star_{\nu k}+\mu_{\nu k}),$$
$$\beta_{\nu k} = i\sqrt{\frac{\tilde{C}(S_{\nu k},|\mu_{\nu k}|)\hbar\omega_{\nu k}}{2}}(\mu^\star_{\nu k}-\mu_{\nu k}),$$
$$r = \sqrt{\frac{\tilde{C}(S_{\nu k},|\xi|)\hbar}{2\omega_0}}(\xi^\star+\xi), \quad (14)$$
$$\rho = i\sqrt{\frac{\tilde{C}(S,|\xi|)\hbar\omega_0}{2}}(\xi^\star-\xi).$$

The c-number Hamiltonian may be expressed in terms of those *fictitious* coordinates and momenta variables as

$$H = \frac{\rho^2}{2}+\frac{1}{2}\omega_0^2r^2+\frac{1}{2}\sum_{\nu,k}\beta_{\nu k}^2+\frac{1}{2}\sum_{\nu,k}\omega_{\nu k}^2\left(\alpha_{\nu k}-\frac{c_{\nu k}r}{\omega_{\nu k}^2}\right)^2. \quad (15)$$

where $c_{\nu k}=\sqrt{g_{\nu k}g'_{\nu k}\,\omega_0\,\omega_{\nu k}\,\tilde{C}(S,|\xi|)}$ or $g'_{\nu k}\sqrt{\omega_0\,\omega_{\nu k}\frac{\tilde{C}(S,|\xi|)}{\tilde{C}(S_{\nu k},|\mu_{\nu k}|)}}$, the fictitious coupling constant between the system and the kth atom of the νth bath. Thus using the Radcliffe coherent states [23], we get a c-number Hamiltonian that is identical with the Hamiltonian for a bosonic particle in contact with two bosonic thermal baths.

A. *c*-number Langevin equation

From the Hamiltonian as described in Eq. (15), the equations of motion for the system and the bath variables may be written as

$$\dot{r} = \rho,$$
$$\dot{\rho} = -\omega_0^2r+\sum_{\nu,k}c_{\nu k}\alpha_{\nu k}-r\sum_{\nu,k}\frac{c_{\nu k}^2}{\omega_{\nu k}^2}, \quad (16)$$

and

$$\dot{\alpha}_{\nu k} = \beta_{\nu k},$$
$$\dot{\beta}_{\nu k} = -\omega_{\nu k}^2\alpha_{\nu k}+c_{\nu k}r. \quad (17)$$

It may be noted here that the above equations are quite similar to the classical equations of motion. The similarity is expected for the bosonic case (having the system and the bath modes with harmonic oscillator) according to the Ehrenfest theorem. Here the similarity appears even for the system-reservoir model with spin angular momentum by virtue of the Hamiltonian (15) which is a manifestation of coherent state representation both for the system and the reservoirs [23]. But the distribution of initial conditions for the state variables of the bath modes makes them different from a standard problem of coupled classical oscillators. Shortly we will introduce the relevant distribution function for initial conditions. Another point to be mentioned here is that if the number of accessible states for the bath mode is very very large (as happens for boson [24]) at the high temperature limit then the above equations exactly correspond to the coupled classical oscillators.

We now eliminate the bath variables form the equation of motion for the system following the standard procedure [11,25–27] which gives the Langevin equation of motion. Using the well known Laplace transformation and back transformation technique, we solve Eq. (17) to get

$$\alpha_{\nu k}(t) = \alpha_{\nu k}(0)\cos(\omega_{\nu k}t)+\frac{\beta_{\nu k}(0)}{\omega_{\nu k}}\sin(\omega_{\nu k}t)$$
$$+\frac{c_{\nu k}}{\omega_{\nu k}}\int_0^t dt'r(t')\sin[\omega_{\nu k}(t-t')] \quad (18)$$

and

$$\beta_{\nu k}(t) = -\omega_{\nu k}\alpha_{\nu k}(0)\sin(\omega_{\nu k}t)+\beta_{\nu k}(0)\cos(\omega_{\nu k}t)$$
$$+c_{\nu k}\int_0^t dt'r(t')\cos[\omega_{\nu k}(t-t')]. \quad (19)$$

Combining Eqs. (16) and (18), we derive the c-number Langevin equation:

$$\dot{\rho} = -\omega_0^2r-\gamma(t)r(0)-\int_0^t dt'\gamma(t-t')\rho(t')+\eta(t), \quad (20)$$

where

$$\gamma(t) = \sum_{\nu,k}\frac{c_{\nu k}^2}{\omega_{\nu k}^2}\cos(\omega_{\nu k}t) \quad (21)$$

and

$$\eta(t) = \sum_{\nu,k}c_{\nu k}\left[\alpha_{\nu k}(0)\cos(\omega_{\nu k}t)+\frac{\beta_{\nu k}(0)}{\omega_{\nu k}}\sin(\omega_{\nu k}t)\right] \quad (22)$$

are the time dependent damping and the noise, respectively. The Langevin equation implies that the deterministic force derived from the system is not affected by the interaction Hamiltonian and the damping is homogeneous. Thus the inclusion of the identity operator (10) in the interaction Hamiltonian leads us to avoid any nonphysical situations. We now consider the noise term. To have $\eta(t)$ as an effective c-number noise, it is necessary to assume a canonical distribution

of Gaussian form for statistical averaging, $\langle \dots \rangle_s$, over the c-number bath variables $\{\alpha_{\nu k}(0)$ and $\beta_{\nu k}(0)\}$ as

$$\mathcal{P}_{\nu k}(\alpha_{\nu k}(0),\beta_{\nu k}(0)) = \mathcal{N} \exp\left[-\frac{\beta_{\nu k}(0)^2 + \omega_{\nu k}^2 \alpha_{\nu k}(0)^2}{\hbar\omega_{\nu k} \tanh\left(\frac{\hbar\omega_{\nu k}}{2k_B T_\nu}\right)} \right], \tag{23}$$

where $\mathcal{N}$ is the normalization constant. Equation (23) is the counterpart [28] of the Wigner canonical distribution function [29] for the bosonic bath. The only difference between the two lies in the width of the distribution function. For the fermionic bath, it is $[\frac{\hbar\omega_{\nu k}}{2} \tanh(\frac{\hbar\omega_{\nu k}}{2k_B T_\nu})]$, whereas for the Wigner distribution it is $[\frac{\hbar\omega_{\nu k}}{2} \coth(\frac{\hbar\omega_{\nu k}}{2k_B T_\nu})]$. At zero temperature, both the distributions merge to a single value, the differences being noticeable at finite temperatures. At high T_ν, the bosonic bath reaches the classical limit, but not the other one. The width of the above distribution function for the fermionic bath mode becomes zero at this temperature limit. This is due to the following fact. The population difference between the two levels of each bath mode of the thermal bath is zero at very high temperature. Then no net transition occurs between the levels. It suggests only one allowed value of the fictitious coordinate. Similarly there would be only one value for the momentum also. Thus the bath becomes inert at this regime and the system loses its diffusive behavior. This is reflected in the above distribution function. Thus the present calculation is valid for the positive thermodynamic temperature. However, the above distribution in c number bath variables implies that $\eta(t)$ arising out of the two-level bath modes must satisfy the relations

$$\langle \eta(t) \rangle_s = 0 \tag{24}$$

and

$$\langle \eta(t)\eta(t') \rangle_s = \frac{1}{2} \sum_{\nu,k} \frac{c_{\nu k}^2}{\omega_{\nu k}^2} \hbar\omega_{\nu k} \tanh\left(\frac{\hbar\omega_{\nu k}}{2k_B T_\nu}\right) \cos[\omega_{\nu k}(t-t')]. \tag{25}$$

B. Solution of the c-number Langevin equation: Response function

The c-number generalized Langevin equation (20) may be solved to obtain the time dependent coordinate $r(t)$ of the particle as

$$r(t) = \rho(0)\chi(t) + r(0)\dot{\chi}(t) + \int_0^t dt' \eta(t-t')\chi(t'), \tag{26}$$

where

$$\chi(t) = \mathcal{L}^{-1}\{\tilde{\chi}(s)\} = \mathcal{L}^{-1}\left\{ \frac{1}{s^2 + \omega_0^2 + s\tilde{\gamma}(s)} \right\}. \tag{27}$$

Here $\mathcal{L}^{-1}$ denotes Laplace inversion and $\tilde{\gamma}(s)$ is the Laplace transform of $\gamma(t)$, i.e., $\tilde{\gamma}(s) = \int_0^\infty dt \exp(-st)\gamma(t)$. $\chi(t)$ in Eq. (26) is the response function or susceptibility having the properties $\chi(0) = 0$ and $\dot{\chi}(0) = 1$.

We now consider the following Drude-Ullersma model [30–32] for the frequency dependence of the coupling coefficients:

$$\omega_{\nu k} = k\Delta_\nu \qquad c_{\nu k} = \sqrt{\frac{2\gamma_\nu \omega_{\nu k}^2 D_\nu^2 \Delta_\nu}{\pi(\omega_{\nu k}^2 + D_\nu^2)}}. \tag{28}$$

where $k = 1,2,3\dots,N_\nu$ and Δ_ν are the mode spacing constant. In the present calculation we have considered the thermodynamic limit, i.e., $N_\nu \to \infty$ and $\Delta_\nu \to 0$. D_ν in the above equation is the characteristic cutoff frequencies for the baths. For simplicity, we consider that $D_1 = D_2 = D$. Another parameter γ_ν corresponds to the coupling constant between a given reservoir and the mediator. Putting these in Eq. (21) and replacing the summation by integration over all bath modes, we get the memory kernel as

$$\gamma(t) = \frac{2\gamma_0}{\pi} \int_0^\infty \frac{D^2}{\omega^2 + D^2} \cos(\omega t) d\omega = \gamma_0 D \exp(-D|t|) \tag{29}$$

with $\gamma_0 = \sum_\nu \gamma_\nu$. Thus the above equation suggests that the frequency distribution of the bath modes follows the Lorentzian form and the width of the distribution function grows with increase in cutoff frequency D. For the above form of time dependent friction, Eq. (27) becomes

$$\tilde{\chi}(s) = \frac{s+D}{(s^2 + \omega_0^2)(s+D) + \gamma_0 D s}. \tag{30}$$

We may now express the response function as

$$\chi(t) = \sum_n \chi_n \exp(-\mu_n t). \tag{31}$$

Here μ_n are the roots of the equation $(\mu - D)(\mu^2 + \omega_0^2) + \gamma_0 D\mu = 0$ and χ_n are defined as $\chi_n = [\frac{s+D}{3s^2+2Ds+\omega_0^2+\gamma_0 D}]_{s=-\mu_n}$.

In the limit $D \gg \omega_0$, we get

$$\mu_{1,2} = \frac{1}{2\tau_\rho}(1 \mp q), \quad \mu_3 = D - 1/\tau_\rho \gg \mu_{1,2} \tag{32}$$

with

$$\chi_1 = -\chi_2 \approx \frac{\tau_\rho}{q}, \quad \chi_3 \approx 0, \tag{33}$$

where $q = \sqrt{1 - 4\tau_\rho/\tau_r}$, $1/\tau_\rho = \gamma_0$, $1/\tau_r = \omega_0^2/\gamma_0$. Thus, μ_1 and μ_2 are real for $\tau_\rho/\tau_r = \omega_0^2\tau_\rho^2 \leqslant 4$. Here τ_r and τ_ρ characterizes the relaxation times for coordinate and momentum respectively, provided $\tau_r \gg \tau_\rho$. On the other hand, when $\tau_\rho/\tau_r > 4$, $\mu_{1,2} = 1/2\tau_\rho \mp i/\tau_0$. Here $\tau_0 = \sqrt{\omega_0^2 - \gamma_0^2/4}$ denotes the oscillation time in this case.

It may be mentioned here that one can also solve Eqs. (16) and (17) in the following way. These equations can be written together in matrix notation as

$$\frac{dR}{dt} = AR(t), \tag{34}$$

where

$$R = \begin{pmatrix} r \\ \rho \\ \alpha_{11} \\ \beta_{11} \\ . \\ . \\ \alpha_{1N} \\ \beta_{1N} \\ \alpha_{21} \\ \beta_{22} \\ . \\ . \\ \alpha_{2N} \\ \beta_{2N} \end{pmatrix} \tag{35}$$

and A is a $(2N+2)\times(2N+2)$ square matrix. Here N is the number of modes for each bath. We have assumed that both baths have the same number of modes. However, the element of the matrix A can be determined from Eqs. (16) and (17). For example, $A_{11}=1$, $A_{1j}=0$ when $j=2,\ldots,(2N+2)$. To solve the Eq. (34) we transform it into the following form:

$$\frac{d\underline{R}}{dt} = B\underline{R}(t), \tag{36}$$

where

$$\underline{R} = MR, \tag{37}$$

$$B = MAM^{-1}, \tag{38}$$

Thus Eq. (36) implies that the equations of motion are decoupled through the diagonal matrix B. The solution of Eq. (36) can be written as

$$\underline{R}_i(t) = \underline{R}_i(0)e^{\lambda_i t}, \quad i=1,\ldots,(2N+2). \tag{39}$$

$\underline{R}_i(t)$ in the above equation is the ith element in the column matrix $\underline{R}$ and λ_i corresponds to the matrix element B_{ii} of the diagonal matrix B. Using the above equation in Eq. (37) we get

$$R_i = \sum_{j=1}^{2N+2}(M^{-1})_{ij}\underline{R}_j(0)e^{\lambda_j t}, \quad i=1,\ldots,(2N+2). \tag{40}$$

Thus for $i=1$, one may write that

$$r = \sum_{k=1}^{2N+2}\sum_{j=1}^{2N+2}(M^{-1})_{1j}e^{\lambda_j t}M_{jk}R_k(0). \tag{41}$$

Comparing Eq. (26) with the above equation one may get the following correspondence between the two methods:

$$\dot{\chi}(t) = \sum_{j=1}^{2N+2}(M^{-1})_{1j}e^{\lambda_j t}M_{j1}, \tag{42}$$

$$\chi(t) = \sum_{j=1}^{2N+2}(M^{-1})_{2j}e^{\lambda_j t}M_{j2}, \tag{43}$$

and

$$\int_0^t dt'\eta(t-t')\chi(t') = \sum_{k=3}^{2N+2}\sum_{j=1}^{2N+2}(M^{-1})_{1j}e^{\lambda_j t}M_{jk}R_k(0). \tag{44}$$

Using the solution (26) of the Langevin equation of motion we will calculate the thermal conductivity of the system in the next section.

III. THERMAL CONDUCTIVITY

The rate of change of energy of the νth thermal bath may be given as

$$\frac{d}{dt}\sum_k\left\langle\frac{\beta_{\nu k}^2}{2}+\frac{1}{2}\omega_{\nu k}^2\alpha_{\nu k}^2\right\rangle = -\langle P_\nu\rangle, \tag{45}$$

where $\langle P_\nu\rangle$ is the power dispersed in the νth bath [11]. After the transient period the above equation becomes

$$\langle P_\nu\rangle = \langle P_\nu\rangle^{(1)} + \langle P_\nu\rangle^{(2)}, \tag{46}$$

where

$$\langle P_\nu\rangle^{(1)} = -\sum_k c_{\nu k}\Bigg[\cos(\omega_{\nu k}t)\int_0^t dt'\chi(t-t')\langle\eta(t')\beta_{\nu k}(0)\rangle - \omega_{\nu k}\sin(\omega_{\nu k}t)\int_0^t dt'\chi(t-t')\langle\eta(t')\alpha_{\nu k}(0)\rangle\Bigg] \tag{47}$$

and

$$\langle P_\nu\rangle^{(2)} = -\sum_k c_{\nu k}^2\Bigg[\int_0^t dt'\chi(t-t')\times\int_0^t dt''\cos[\omega_{\nu k}(t-t'')]\langle\eta(t')r(t'')\rangle\Bigg]. \tag{48}$$

To reach Eq. (46) we have used Eqs. (19) and (26) respectively. Performing the statistical averages using the distribution function (23), Eq. (47) may be simplified as

$$\langle P_\nu\rangle^{(1)} = \frac{\hbar}{2}\sum_k c_{\nu k}^2\tanh\left(\frac{\hbar\omega_{\nu k}}{2k_BT_\nu}\right)\times\int_0^t dt'\chi(t-t')\sin[\omega_{\nu k}(t-t')]. \tag{49}$$

Making use of Eq. (28) and replacing the summation over the bath frequency modes by integration we get

$$\langle P_\nu\rangle^{(1)} = \frac{\hbar\gamma_\nu D^2}{\pi}\sum_n\chi_n\int_0^\infty d\omega\frac{\omega^3\tanh\left(\frac{\hbar\omega_{\nu k}}{2k_BT_\nu}\right)}{(\mu_n^2+\omega^2)(D^2+\omega^2)}. \tag{50}$$

Similarly one can simplify Eq. (48) as

$$\langle P_\nu\rangle^{(2)} = -\gamma_\nu D^2\Bigg[\int_0^t dt'\chi(t-t')\times\int_0^t dt''S(t-t'')\langle\eta(t')r(t'')\rangle\Bigg], \tag{51}$$

where

$$S(t) = \frac{2}{\pi}\sum_k \frac{\omega_{\nu k}^2}{\omega_{\nu k}^2 + D^2} \equiv \frac{2}{\pi}\int_0^\infty d\omega \frac{\omega^2\cos(\omega t)}{\omega^2 + D^2} = 2\delta(t) - D\exp(-D|t|), \tag{52}$$

and

$$\langle \eta(t) r(t')\rangle = \int_0^{t'} d\tau \langle \eta(t)\eta(\tau)\rangle \chi(t'-\tau). \tag{53}$$

Making use of Eq. (25) into the above equation we obtain

$$\langle \eta(t) r(t')\rangle = \frac{\hbar}{2}\sum_{\nu,k,n} \frac{c_{\nu k}^2 \chi_n}{\mu_n^2 + \omega_{\nu k}^2} \tanh\left(\frac{\hbar\omega_{\nu k}}{2k_B T_\nu}\right) \times \left[\sin[\omega_{\nu k}(t'-t)] + \frac{\mu_n}{\omega_{\nu k}}\cos[\omega_{\nu k}(t'-t)]\right]. \tag{54}$$

Incorporating the above result in Eq. (51) and evaluating the integrals for steady-state values one can write that

$$\langle P_\nu\rangle^{(2)} \approx -\frac{\hbar\gamma_\nu D^4}{\pi}\sum_{m,n,\nu'} \gamma_{\nu'}\chi_m\chi_n \times \int_0^\infty d\omega \frac{\omega^3 \tanh\left(\frac{\hbar\omega_{\nu k}}{2k_B T_{\nu'}}\right)(\omega^2 + \mu_m\mu_n)}{\left(\mu_m^2+\omega^2\right)\left(\mu_n^2+\omega^2\right)(D^2+\omega^2)^2}. \tag{55}$$

Since at the steady-state regime the power acquired by one thermal bath is equal to that lost by the other, we may represent the steady-state heat current as $J_{st}^F = \langle P_1\rangle = -\langle P_2\rangle = \frac{1}{2}\langle P_1 - P_2\rangle$. Thus J_{st}^F may be written as

$$J_{st}^F = \frac{1}{2}[\langle P_1\rangle - \langle P_2\rangle] = \frac{1}{2}[\langle P_1\rangle^{(1)} - \langle P_2\rangle^{(1)}] + \frac{1}{2}[\langle P_1\rangle^{(2)} - \langle P_2\rangle^{(2)}]. \tag{56}$$

From Eqs. (50) and (55) one can write that

$$\frac{\gamma}{2\gamma_1\gamma_2}\langle P_1\rangle = -\frac{\hbar D^2}{\pi}\sum_n \chi_n \int_0^\infty d\omega \frac{\omega^3\left[n_1^F(\omega) - n_2^F(\omega)\right]}{\left(\mu_n^2+\omega^2\right)(D^2+\omega^2)}, \tag{57}$$

where $n_\nu^F(\omega) = \frac{1}{\exp(\frac{\hbar\omega}{k_B T_\nu})+1}$ is the average occupation number of mass less fermion in the ν-th thermal reservoir at temperature T_ν. Due to quantifying the average energy of a fermionic bath mode (having energy difference between the two level $\hbar\omega$) in terms of massless fermion with zero chemical potential, n_ν^F is free from the quantity, viz. the chemical potential. Zero chemical potential for the system having massless fermion may be evident from the fact that the number of this kind of particle only depends on the temperature like the number of photon or phonon. Because of the temperature dependence of the number, the chemical potential is zero for the thermodynamic system with assembly of photons or phonons. The mean phonon occupation number $[n^B(\omega)]$ for the bosonic thermal reservoir (with the assembly of independent harmonic oscillator) at temperature T is given by

$$n_\nu^B(\omega) = \frac{1}{\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1}. \tag{58}$$

This is the same as the mean photon occupation number in black body radiation. Thus the above expression is valid for both the reservoirs with massless boson (photon) and harmonic oscillator having mass. Similarly, the mean fermion occupation number, $n_\nu^F(\omega) = \frac{1}{\exp(\frac{\hbar\omega}{k_B T_\nu})+1}$, is also valid for the fermionic reservoirs having two level modes with mass or without mass. The validity is implied in the Hamiltonian for the system-reservoir model. Thus the outcome of the present study should be valid for both reservoirs.

We now come back to Eq. (57). From this equation the formula for the steady-state heat flux can be written as

$$J_{st}^F = -\frac{2\hbar D^2\gamma_1\gamma_2}{\pi\gamma}\sum_n \chi_n \int_0^\infty d\omega \frac{\omega^3\left[n_1^F(\omega) - n_2^F(\omega)\right]}{\left(\mu_n^2+\omega^2\right)(D^2+\omega^2)}. \tag{59}$$

For $\gamma_1 = \gamma_2$, the above equation becomes

$$J_{st}^F = -\frac{\hbar D^2}{2\pi\tau_\rho}\sum_n \chi_n \int_0^\infty d\omega \frac{\omega^3\left[n_1^F(\omega) - n_2^F(\omega)\right]}{\left(\mu_n^2+\omega^2\right)\left(D^2+\omega^2\right)}. \tag{60}$$

If we assume the temperature difference between the two baths to be smaller compared to their average temperature, i.e., $|T_1 - T_2| \ll (T_1+T_2)/2 \equiv T$, the thermal conductivity may be expressed as

$$K_F = -\lim_{\Delta T\to 0}\frac{J_{st}}{\Delta T} = -\frac{\tau_\hbar^2 k_B D^2}{8\pi\tau_\rho}\sum_n \chi_n \int_0^\infty d\omega \frac{\omega^4 \mathrm{sech}^2\left(\frac{\hbar\omega}{2k_B T}\right)}{\left(\mu_n^2+\omega^2\right)(D^2+\omega^2)}, \tag{61}$$

where $\Delta T = T_2 - T_1$ and $\tau_h = \hbar/k_B T$. Now one may compare the above results with the heat transfer between two bosonic thermal bath mediated by a harmonic oscillator. Following Ref. [11] the formulas for the heat current and the thermal conductivity can be written as

$$J_{st}^B = -\frac{\hbar D^2}{2\pi\tau_\rho}\sum_n \chi_n \int_0^\infty d\omega \frac{\omega^3\left[n_1^B(\omega) - n_2^B(\omega)\right]}{\left(\mu_n^2+\omega^2\right)(D^2+\omega^2)} \tag{62}$$

and

$$K_B = -\frac{\tau_\hbar^2 k_B D^2}{8\pi\tau_\rho}\sum_n \chi_n \int_0^\infty d\omega \frac{\omega^4 \mathrm{cosech}^2\left(\frac{\hbar\omega}{2k_B T}\right)}{\left(\mu_n^2+\omega^2\right)(D^2+\omega^2)}. \tag{63}$$

where $n_\nu^B(\omega) = \frac{1}{\exp(\frac{\hbar\omega}{k_B T_\nu})-1}$ is the mean phonon occupation number for the νth bosonic thermal reservoir at temperature T_ν. In the following section we will demonstrate differences (if any) between thermal conductivities of the two cases.

IV. A COMPARATIVE STUDY BETWEEN BOSONIC AND FERMIONIC CASES

A. Effect of average temperature of thermal baths on thermal conductivity of the system

In Fig. 1 we have demonstrated variation of thermal conductivity as a function of average temperature of the coupled thermal baths. It shows that the fermionic thermal bath (FTB) has a generic role to include optimum behavior in the thermal conductivity. At small T, the thermal conductivity first grows rapidly as a function of T for both bosonic and fermionic cases. If the average temperature is appreciably large then K_B converges to the limiting value but K_F decreases to zero value after passing through a maximum. One can account for all these quantitatively considering different temperature limits of the above analytical results.

At the low average temperature regime, we have $\frac{\hbar\omega}{k_B T} \gg 1$. Evaluating the integral in Eq. (62) at this limit we get

$$J_{st} \approx -\frac{7\pi^3 k_B^4}{240\hbar^3\tau_\rho}\left(T_1^4 - T_2^4\right)\sum_n \frac{\chi_n}{\mu_n^2}.$$

From the Laplace transform of Eq. (31) we have

$$\frac{d}{ds}[\tilde{\chi}(s)] = -\sum_n \frac{\chi_n}{(s+\mu_n)^2}. \tag{64}$$

$\frac{d}{ds}[\tilde{\chi}(s)]$ for $s = 0$ may be evaluated from Eq. (30) as

$$\frac{d}{ds}[\tilde{\chi}(s)]_{s=0} = \frac{\gamma_0}{\omega_0^4}. \tag{65}$$

Thus Eq. (64) may be simplified as

$$J_{st}^F = \frac{7\pi^3 k_B^4}{240\hbar^3\tau_\rho^2\omega_0^4}\left(T_1^4 - T_2^4\right) \tag{66}$$

and

$$K_F \approx \frac{7\pi^3 k_B^4}{60\hbar^3\tau_\rho^2\omega_0^4}T^3. \tag{67}$$

Similarly, for the bosonic case the heat flux and the thermal conductivity at low T limit can be written as

$$J_{st}^B \approx \frac{\pi^3 k_B^4}{30\hbar^3\tau_\rho^2\omega_0^4}\left(T_1^4 - T_2^4\right), \tag{68}$$

$$K_B \approx \frac{2\pi^3 k_B^4}{15\hbar^3\tau_\rho^2\omega_0^4}T^3. \tag{69}$$

Equation (68) implies that the temperature dependence of the heat flux at the low temperature regime for the bosonic case is the same as the Stefan-Boltzmann law. This result was established in Ref. [11]. Based on Eq. (66) it is to be noted here that at the low temperature limit, the temperature dependence of the heat flux for the fermionic case is also the same as the Stefan-Boltzmann law. Another point to be mentioned here is that an increase in thermal conductivity as a function of average temperature of the thermal baths follows the identical T dependence at the deep quantum regime for both cases. These can be understood in the following way. At low temperature, the transition probability to the excited states of the bath modes is very very small. Therefore, although the bosonic bath modes possess an infinite number of states still they behave similar to the fermionic bath modes which have only one excited state. The signature of the infinite number of excited states of the bosonic bath mode is implied through the appearance of a dimensionless higher numerical factor in both the heat flux and the thermal conductivity for the bosonic case compared to the other case.

We now consider the high temperature limit. At this regime, $\frac{\hbar\omega}{k_B T} \ll 1$, then Eq. (61) reduces to

$$K_F \approx \frac{\hbar^2 D^2}{8\pi k_B\tau_\rho T^2}\left[\sum_n -\chi_n\int_0^\infty d\omega \frac{\omega^4}{(\mu_n^2+\omega^2)(D^2+\omega^2)}\right]. \tag{70}$$

Similarly, the thermal conductivity at high temperature for the bosonic case can be written as

$$K_B \approx -\frac{k_B D^2}{4\tau_\rho}\sum_n \frac{\chi_n}{D+\mu_n}, \tag{71}$$

indicating that at this limit, K_B is independent of temperature. Since $\chi_1 = -\chi_2$, $\chi_3 = 0$, and $\mu_1 < \mu_2$, K_B is positive. Thus the above two equations suggest that the nature of the thermal conductivity is drastically different for the two cases at the high temperature limit. For the fermionic case, the heat conductance decreases as $K_F \propto 1/T^2$ with increase in the mean temperature of the thermal baths. This is a generic signature of the fermionic thermal bath. The present calculation is valid for positive thermodynamic temperature.

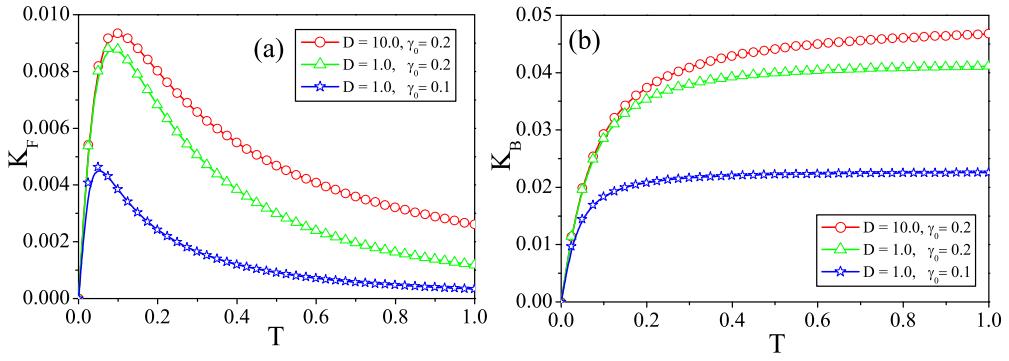

FIG. 1. (Color online) Variation of heat conductance with average temperature (T) of baths for the parameter set $\hbar = 1.0$, $k_B = 1.0$, $\omega_0 = 0.01$. (a) Fermionic baths, (b) bosonic baths (units are arbitrary).

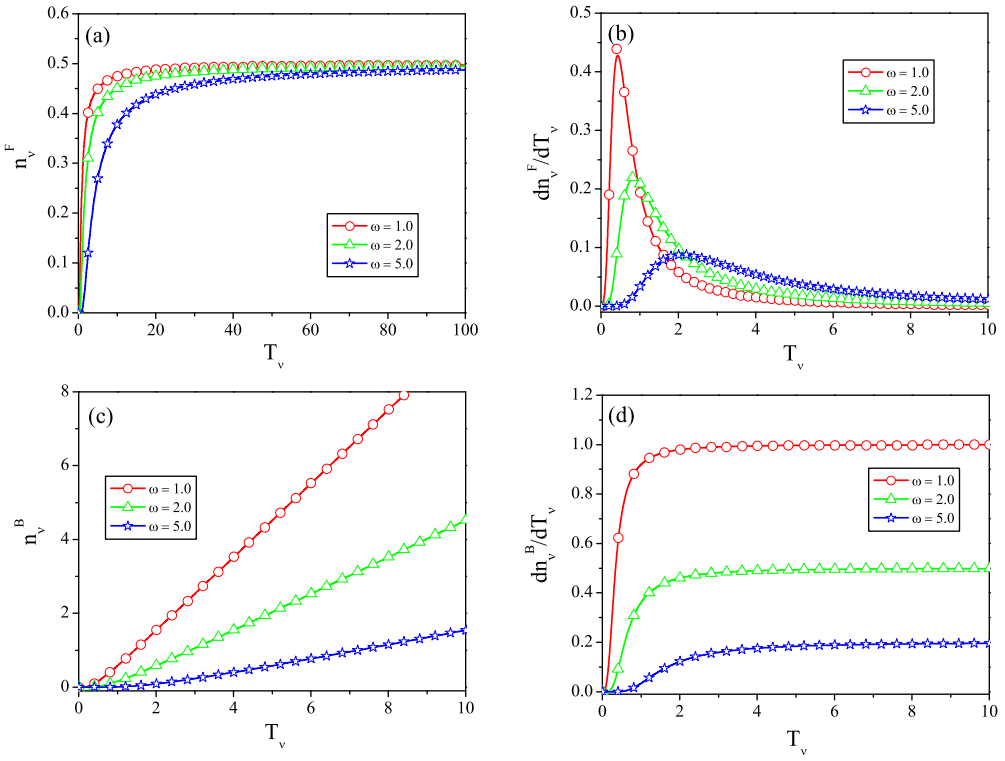

FIG. 2. (Color online) (a) Plot of massless average fermion occupation number vs bath temperature for fermionic bath. (b) Plot of the rate of change of massless average fermion occupation number with bath temperature vs bath temperature for fermionic bath. (c) Plot of average phonon occupation number vs bath temperature for bosonic bath. (d) Plot of the rate of change of average phonon occupation number number with bath temperature vs bath temperature for bosonic bath. In each case, the parameter set is $\hbar = 1.0$, $k_B = 1.0$ (units are arbitrary).

The population inversion between the two levels for each bath mode is not allowed. With increase of temperature the population difference decreases and it becomes zero at the high temperature limit. Then the thermal bath would be inert and the magnitude of the thermal conductivity of the system is zero. Thus for the fermionic case, there is a critical temperature. Below it, the thermal conductivity increases with an increase in T. To understand these aspects in further detail, the variation of average occupation number of massless fermion or boson at the excited state with the temperature has been demonstrated in Fig. 2. In the same figure we have also demonstrated their rate of change with temperature. At the low temperature regime, an increase of n_ν^F, n_ν^B and their rate of change with temperature is due to the enhancement of the transition probability between the respective levels with energy difference $\hbar\omega$. It suggests that at the low temperature regime, the difference between the n_ν^F for the high temperature thermal bath and the same which is for the low temperature thermal bath is enhanced with increase in temperature. Then the thermal bath with the higher temperature will be more active to transfer heat to the low temperature thermal bath through the mediator. This is the reason for the increase of thermal conductivity with increase in average temperature between the thermal baths. But if the temperature of the thermal bath is appreciably large then the population difference between the two levels of bath mode having an energy difference $\hbar\omega$ goes to zero with a limiting value of the transition probability as well as n_ν^F. This converging behavior suggests a maximum in the variation of $\frac{dn_\nu^F}{dT_\nu}$ as a function of T_ν. Thus at the high average temperature of the thermal baths, the fermionic bath becomes inert and the thermal conductivity of the system is zero. Because of these behaviors the frequency distribution of the bath modes and the coupling strength between the system and the thermal reservoir sets a critical temperature at which thermal conductivity of the system is maximum as shown in Fig. 1. As the frequency distribution becomes wider with increase in D [33] the enhancement of the rate of change of population with temperature sustains up to an appreciably high temperature for the bath modes having higher frequencies. Therefore the critical average temperature increases as the width of frequency distribution of the bath modes grows which is reflected in Fig. 1(a). It is also reflected in this figure that for a given D, with increase of coupling strength the contribution from the bath modes of higher frequency may become important to grow the critical temperature. Shortly we will demonstrate explicitly the role of these two quantities to control the thermal conductivity of the microscopic system. Before leaving this part we would explain the monotonic increase of thermal conductivity for the bosonic case. In the bosonic case, there is no restriction on the average phonon occupation number n_B, as shown in Fig. 2. At high temperature, the energy of the system becomes continuous as occurs in a classical system. Therefore, $\frac{dn_F}{dT_\nu}$ is independent on temperature at this regime. Then the difference between the two bosonic thermal baths is also temperature independent. Thus the thermal conductivity of the system which mediates the two bosonic thermal baths does not depend on the temperature at the high temperature limit as shown in

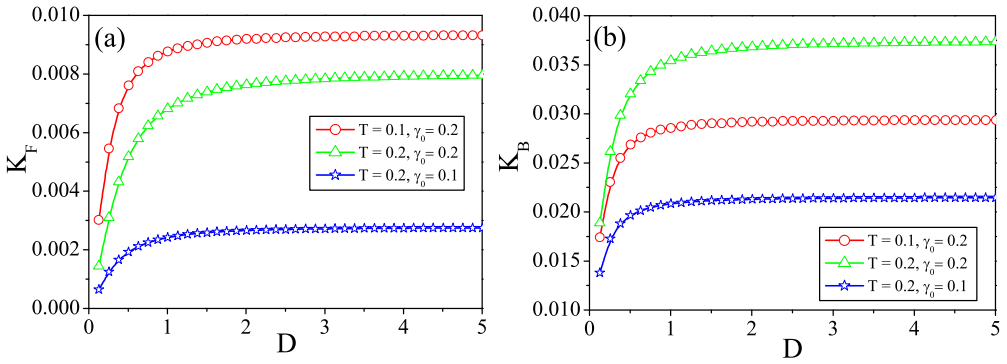

FIG. 3. (Color online) Variation of heat conductance with Debye cutoff frequency (D) for the parameter set $\hbar = 1.0$, $k_B = 1.0$, $\omega_0 = 0.01$. (a) Fermionic baths, (b) bosonic baths (units are arbitrary).

Fig. 1(b). Similarly, if there is no upper limit for the number of accesses of states to each bath mode of the fermionic thermal bath (as happens for the constituents of the Fermi gas) then the bath mode would behave as a classical system at the high temperature limit. Then the rate of change of energy of the thermal bath with temperature would be a constant and the thermal conductivity of the system would be similar to the bosonic case. We now come back to Fig. 1(b). It suggests that the converging behavior of the thermal conductivity depends on both D and γ_0. One can easily account for this based on the frequency distribution and contribution from the modes of higher frequency as discussed above.

Now we would like to mention that the unexpected difference in behavior of thermal conductivity of the fermionic system at the high temperature limit compared to the bosonic case can be verified experimentally considering systems like spin chains as demonstrated in Refs. [4,34]. The experimental verification may have very important consequences. People are thinking to develop a Si-based nuclear spin quantum computer [35,36]. A two level spin system here may have an important role to transfer the heat which is associated with the computer function. The failure of the two level system to transfer heat may result in local heating that may create an additional problem through the structural deformation of the constituents in the computer. It suggests that the quantum computer should be run carefully at very low temperature. A similar problem may happen in devices which contain magnetic materials. Another implication of the present study is that at high temperature, the contribution to the heat transfer from the free electron as a two level system in the metals and spintronics in the presence of a magnet is very very small.

B. Role of width of frequency distribution of bath modes on thermal conductivity of the system

Figure 1 implies that the width of the frequency distribution of the bath modes has an important role to control the thermal conductivity of the microscopic system. In Fig. 3 we have demonstrated how the thermal conductivity varies with D which measures the width of the frequency distribution of the bath modes. It shows that the thermal conductivity monotonically increases to a limiting value as a function of D for both the bosonic and the fermionic cases. This is a signature of involvement of bath modes having higher frequency in the system reservoir interaction with increase in the width of the distribution function. For very large D all the bath modes are included and therefore the thermal conductivity is independent of it at the asymptotic limit. It is to be noted here that the change of thermal conductivity with D is significant at the low temperature regime for the fermionic case as the thermal bath becomes inert in the high temperature situation. This is in contrast to the bosonic case where the transition between the two levels of the bath mode having higher frequency is favored by the increase in the temperature. It is a clear justification of the above mentioned argument which claims to explain the enhancement of the thermal conductivity

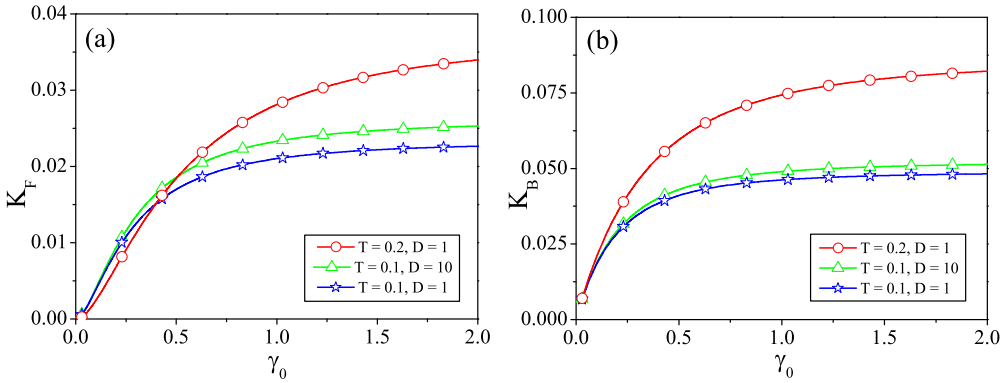

FIG. 4. (Color online) Variation of heat conductance with damping constant (γ_0) for fermionic bath for the parameter set $\hbar = 1.0$, $k_B = 1.0$, $\omega_0 = 0.01$. (a) fermionic baths, (b) bosonic baths (units are arbitrary).

(TC) with increase in the width of the frequency distribution of the bath modes. For any temperature regime, it is further enhanced by strengthening of coupling between the system and the thermal reservoir. Due to involvement of bath modes of higher frequencies with increase in D, the change of TC as a function of D is significant at high temperature for the bosonic thermal baths. This is implied in Fig. 3.

C. Effect of damping strength on thermal conductivity of the system

Finally, in Fig. 4 we have demonstrated the variation of thermal conductivity as a function of damping strength for both the fermionic and the bosonic cases. In both the cases, the conductivity monotonically increases to the limiting value at the asymptotic regime for the damping strength. The converging behavior appears as a signature of balance among the energy flow rate, the contribution from the bath modes of higher frequency, and the energy dissipation effect. The contribution from the bath modes of higher frequency is justified by the enhancement of the conductivity with the increase in width of the frequency distribution of the bath modes. Another point to be mentioned here is that there is a crossover between high and low temperature results for the fermionic case. Thus it is apparent in Fig. 4(a) that even at relatively high temperature the fermionic bath may be an effective one for the strong coupling between the system and the reservoir.

V. CONCLUSION

We have studied the dynamics of a fermionic system which is coupled to the two fermionic thermal baths. Here we have used the Radcliffe coherent states. It leads to getting a c-number Hamiltonian that is identical with the Hamiltonian for a bosonic particle in contact with two bosonic baths. Based on this Hamiltonian description we have calculated thermal conductivity of the system in a very simple way. We compare our results with the thermal conductivity of a bosonic system which is coupled to the two bosonic thermal baths. This includes the following important observation.

(i) At the low temperature limit, the temperature dependence of the heat flux for both the bosonic and the fermionic cases is the same as the Stefan-Boltzmann law.

(ii) For fermionic thermal baths, the thermal conductivity first rapidly grows as the average temperature of the baths rises up to a critical value and then it decreases ($K_F \propto 1/T^2$) with increase in the mean temperature of the thermal baths. But in the bosonic case, the conductivity monotonically increases to the asymptotic limiting value. At the low temperature regime, the variation of thermal conductivity with the temperature is quite similar for both cases. Thus the results are drastically different at the high temperature regime as a generic signature of the fermionic thermal bath. There is a critical temperature for the fermionic case at which the thermal conductivity is maximum for the given coupling strength and the width of the frequency distribution of bath modes. The critical temperature grows if these quantities become larger.

(iii) Thermal conductivity increases with increase in D which measures the width of the frequency distribution function for the bath modes. This is further enhanced by the strengthening coupling between the system and the thermal reservoir. This behavior is significant at the low temperature regime for the fermionic case. It is highly contrasting to the bosonic case where the signature of the enhancement is very prominent at the high temperature limit.

(iv) Thermal conductivity monotonically increases as a function of damping strength to the limiting vale at the asymptotic limit. There is a crossover between high and low temperature results for the fermionic case. Thus it is apparent that even at relatively high temperature the fermionic bath may be an effective one for the strong coupling between the system and the reservoir.

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