Landauer's blowtorch effect as a thermodynamic cross process: Brownian cooling

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The local heating of a selected region in a double-well potential alters the relative stability of the two wells and gives rise to an enhancement of population transfer to the cold well. We show that this Landauer's blowtorch effect may be considered in the spirit of a thermodynamic cross process linearly connecting the flux of particles and the thermodynamic force associated with the temperature difference and consequently ensuring the existence of a reverse cross effect. This reverse effect is realized by directing the thermalized particles in a double-well potential by application of an external bias from one well to the other, which suffers cooling.

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I. INTRODUCTION

Transport plays a very important role in systems that lie away from equilibrium [1–12]. By transport, we refer to both the movement of mass as well as energy. The transfer of particles can be affected by different means. The most straightforward way to cause mass transfer is to apply directional force on the particles. Even when the applied force on an average does not provide any bias to a particular direction, the asymmetry of the potential field in which the particles are moving, may bring about passage of particles to a given direction by rectification of thermal fluctuations, a phenomenon known as ratchet effect. This has attracted wide attention over the past two decades [13–24]. Another interesting mode of transfer of mass was discussed by Landauer [25–29]. When a selected portion of a bistable potential is heated, the relative stability of the two wells differs as a result of which some proportion of mass is transferred from the heated well to the other. The origin of this population transfer lies on the extra kinetic energy gained by the particles at the high temperature region, which supplies the extra energy for barrier crossing. This process is now known as Landauer's blowtorch effect [30–33].

Blow-torch effect has serious implication in explaining transport process as it stresses on the detailed kinetics of the path along which the transport takes place [26]. It has played an interesting role for modeling Brownian motors [34–46]. The significant point to note about this effect is that here the difference in temperature between two parts of a body causes a mass transfer. From a thermodynamic point of view, the variation in chemical potential between two systems causes flow of mass between them and the difference in temperature is responsible for the heat flow to occur. In the blowtorch effect, however, while one maintains two different zones at two different temperatures, mass transfer takes place between these two domains. It would therefore seem that the process may be characterized as a thermodynamic cross effect where the difference in temperature gives rise to transport of mass. This ensures that a reverse process must also exist as an immediate consequence. In what follows, we explore this reverse effect in a model bistable potential where mass transfer from one well to the other is affected by an external bias and inquire whether a temperature difference is generated among different spatial regions of the potential well as a consequence of the flow of particles. In other words, we build up a general setup to understand whether the reverse process of blowtorch effect can occur as a thermodynamic cross effect causing thermal asymmetry between the two wells as a result of mass transfer by an external force.

The paper is organized as follows. In Sec. II, we briefly describe Landauer's blowtorch effect, a qualitative argument to view it as a thermodynamic cross effect and discuss our model to investigate the reverse process as a support of this assertion. In Sec. III, we try to establish our argument in a more rigorous way using the Fokker-Planck equation for the system. In Sec. IV, the numerical simulation on our model regarding the study of the temperature of different parts of the bistable potential in the reverse mechanism is presented and discussed. The paper is concluded in Sec. V.

II. CHARACTERIZING LANDAUER'S BLOW-TORCH EFFECT: MODEL OF DIRECT AND REVERSE PROCESSES

A. Landauer's blowtorch effect

We consider an overdamped Brownian particle characterized by a position coordinate x moving in a bistable potential $V(x)$ as shown in Fig. 1(a). To start with, the temperature of the overall system is denoted by T. A part of the left potential well, between the x range x_l and x_r, is heated to temperature T_1, i.e., within this interval the system is in contact with a heat bath at temperature T_1. When the particle crosses the hot domain, it withdraws $V(x_r) - V(x_l)$ equivalent of heat from the high-temperature bath. The entropy decrease for the heat bath at temperature T_1 is $-\frac{V(x_r)-V(x_l)}{T_1}$. The heat bath at temperature T receives the same amount of energy and the entropy increase of this bath is $\frac{V(x_r)-V(x_l)}{T}$. Therefore, due to the passage of the particle through the hot domain, essentially, $V(x_r) - V(x_l)$ amount of energy is transferred from the high-temperature to the low-temperature bath and the overall entropy increase for the process is $[V(x_r) - V(x_l)](\frac{1}{T} - \frac{1}{T_1})$. Therefore, the probability to find the particle at the right side of x_r enhances by the exponential factor of this entropy increase. The relative population density of the two wells in the steady

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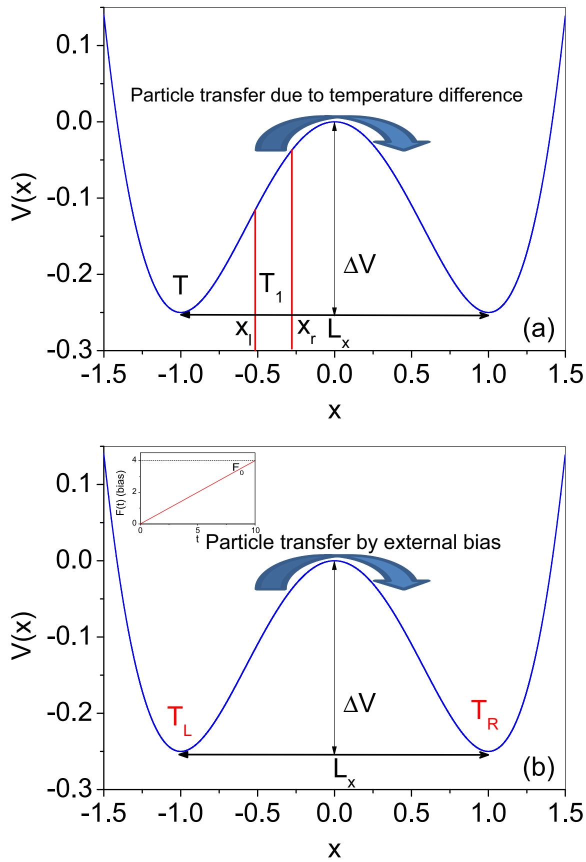

FIG. 1. (Color online) (a) The setup for Landauer's blowtorch effect; the region between x_l and x_r is kept at a higher temperature T_1 compared to the rest. (b) The setup to realize the reverse process of blowtorch effect. A time-dependent linear external bias as depicted in the inset is applied for the transfer of particles to the right well.

state gets modified and can be represented as

$$\frac{P_R^{\rm st}}{P_L^{\rm st}} = \frac{P_R^{\rm eq}}{P_L^{\rm eq}}\exp\left\{[V(x_r)-V(x_l)]\left(\frac{1}{T}-\frac{1}{T_1}\right)\right\}. \quad (2.1)$$

Here, $P_i^{\rm st}$ and $P_i^{\rm eq}$ refer to the integrated probability of residence of the particles in the ith state ($i = L,R$ for the left and the right well, respectively) at the nonisothermal steady state and isothermal equilibrium state, respectively. The above relation Eq. (2.1) implies that the probability to find the particle in the right well or the population density of that well increases in the steady state due to heating at the left potential well. This blowtorch effect has been illustrated schematically in Fig. 1(a).

B. Characterization of Landauer's blowtorch effect as a thermodynamic cross effect

In the blowtorch effect, the entropy increase due to the passage of the Brownian particle through the hot layer implies increased rate of barrier crossing, which manifests itself in the altered relative population density in the two wells. Therefore, the rate also changes over its transition state value by the exponential nonequilibrium factor of the entropy increase. The modified rate constant κ can be expressed as

$$\kappa = \kappa_{\rm TST}\exp\left\{[V(x_r)-V(x_l)]\left(\frac{1}{T}-\frac{1}{T_1}\right)\right\}. \quad (2.2)$$

(The Boltzmann constant k_B has been taken to be equal to unity.) Here, $\kappa_{\rm TST}$ is the transition state value of the rate constant. It is thermodynamic in its content and is represented as $\kappa_{\rm TST} = (2\pi/\omega_0)\exp(-\Delta V/T)$. ΔV and ω_0 correspond to the activation energy and frequency of the left potential well, respectively. If we define, $V(x_r) - V(x_l)$ as ΔV_{rl} and T_1 as $T + \Delta T$, for $\Delta T \ll T$, Eq. (2.2) reduces to

$$\kappa = \kappa_{\rm TST}\exp(\Delta V_{rl}\Delta T/T^2). \quad (2.3)$$

For small value of ΔT, the difference of the modified rate and the transition state value of the barrier-crossing rate can be expressed as follows:

$$\kappa - \kappa_{\rm TST} = \kappa_{\rm TST}\Delta V_{rl}\Delta T/T^2. \quad (2.4)$$

κ can be considered as the average velocity over the barrier top. Therefore, the difference can be represented as [from Eq. (2.4)]

$$\kappa - \kappa_{\rm TST} = \langle v\rangle_{\rm noneq} - \langle v\rangle_{\rm TST} = J_1/n_0. \quad (2.5)$$

Here J_1 refers to the flow of particles and n_0 denotes the population density at the left potential well. It is evident from Eq. (2.5) that the flow is zero at equilibrium. Under nonequilibrium steady-state condition, the flow of particles persists. If we compare Eq. (2.4) with Eq. (2.5), we immediately obtain the following form of J_1 described as

$$J_1 = \kappa_{\rm TST}n_0(\Delta V_{rl}\Delta T/T^2). \quad (2.6)$$

Equation (2.6) clearly signifies that the temperature difference ΔT induces mass flow. In the language of irreversible thermodynamics, the flow J_1 is linearly related to the thermodynamic force $X_2(=\Delta T/T^2)$ associated with the temperature difference as $J_1 = L_{12}X_2$, where $L_{12} = n_0\kappa_{\rm TST}\Delta V_{rl}$. We therefore suggest that the Landauer's blowtorch effect is a cross process typically in the spirit of Peltier, Seebeck, and Thompson effects. This characterization of Landauer's blowtorch effect as a thermodynamic cross process signifies that there should exist a reverse mechanism that gives rise to temperature difference in the system due to mass transfer. We, therefore, try to explore this reverse cross effect. The flow of matter is generally affected by a difference in chemical potential. It is well-known [47] that the chemical potential difference may be effected by the application of a mechanical force. Consequently, to study the reverse process, we need to generate a flow of particles to a particular well of a bistable potential by a time-linear mechanical force and examine the temperature of the two wells.

C. The reverse process: The model

We return to the model of overdamped Brownian particle moving in a bistable potential. A time-linear force is applied on the particle in such a way that the particle experiences a bias toward the right potential well as illustrated in Fig. 1(b). The dynamics of the particle is represented by the following Langevin equation,

$$\Gamma\frac{dx}{dt} = -V'(x) + F(t) + \sqrt{\Gamma T}\xi(t). \quad (2.7)$$

Here Γ represents the damping coefficient. $V'(x)$ corresponds to the force gradient generated from the bistable potential of the form, $V(x) = \frac{a}{4}x^4 - \frac{b}{2}x^2$. $F(t)$ is the external forcing and at any arbitrary time t during the forcing time period can be expressed as $F(t) = F_0\frac{t}{t_f}$, where F_0 stands for the amplitude of the bias and t_f denotes the time at which it is switched off. The Boltzmann constant has been taken to be equal to unity as mentioned earlier. T is the temperature of the bath. $\xi(t)$ is the zero mean, Gaussian, white noise, i.e.,

$$\begin{aligned} \langle \xi(t) \rangle &= 0, \\ \langle \xi(t)\xi(t') \rangle &= 2\delta(t - t'). \end{aligned} \tag{2.8}$$

The prefactor of the noise term in Eq. (2.7) signifies that the noise present in the system obeys the fluctuation-dissipation relation. To make the system and the dynamics dimensionless, we use the following characteristic quantities. The length scale of the dynamics is made dimensionless by L_x representing the distance between the two minima of the double-well potential and time t by $\tau = \frac{\Gamma L_x^2}{T_{\rm ref}}$ where $T_{\rm ref}$ is a reference temperature. τ is essentially twice the time required by the Brownian particle to diffuse distance L_x at temperature $T_{\rm ref}$. The scaled length and time can be represented as, $\tilde{x} = x/L_x$ and $\tilde{t} = t/\tau$, respectively. The force $F(t)$ is scaled by $\Gamma L_x/\tau$, i.e., the dimensionless force can be denoted as, $\tilde{F}(\tilde{t}) = F(t)\tau/\Gamma L_x$. In order to keep brevity and notational convenience, we will omit the tilde description from now on and use the untilde symbols to denote the dimensionless quantities. In dimensionless form, the Langevin equation can be presented as follows,

$$\frac{dx}{dt} = -ax^3 + bx + F(t) + \sqrt{D}\xi(t). \tag{2.9}$$

a and b are dimensionless parameters. D is the rescaled temperature and has the form, $D = T/T_{\rm ref}$. $\xi(t)$ represents properly scaled fluctuating term.

As we are interested to explore the temperature of the different parts of the system, it is necessary to consider a large ensemble of particles. The kinetic energy averaged over this large ensemble bears the notion of the temperature. As the initial condition, both the wells are kept equally populated. The external bias directs the particles gradually toward the right lobe, i.e., a net flow of particles has been induced from the left to the right well. Our object is to evaluate the average kinetic energy of the particles in the left as well as in the right well, separately, so that any thermal asymmetry between the two wells can be monitored. The numerical simulations are carried out for the entire forcing time period in Sec. IV for the study of the reverse blowtorch effect.

III. FORMULATION OF THE EFFECT USING THE FOKKER-PLANCK DESCRIPTION

To establish our argument to depict the Landauer's blowtorch effect as a thermodynamics cross process employing the Fokker-Planck description of the dynamics, we follow the procedure described in Ref. [48]. We start with the Gibb's equation for the system as follows:

$$\delta\rho(x,t)s = \frac{1}{T}\delta(\rho e) - \frac{m}{T}\int \mu(x,t)\delta P(x,t)dx. \tag{3.1}$$

Here δ represents the total differential of a quantity. m corresponds to the mass of the Brownian particle moving in the potential field. e and s are the energy and entropy per unit mass, respectively. $\rho(x,t)$ is the mass density related to the probability density function and is given by $\rho(x,t) = P(x,t)m$. $\mu(x,t)$ denotes the chemical potential per unit mass. To get the rate of change of entropy per unit volume, we differentiate Eq. (3.1) with respect to time:

$$\frac{\partial}{\partial t}(\rho s) = \frac{1}{T}\frac{\partial(\rho e)}{\partial t} - m\int \frac{\mu(x,t)}{T}\frac{\partial P(x,t)}{\partial t}dx. \tag{3.2}$$

We describe below the essential quantities to evaluate Eq. (3.2), such as, $\frac{\partial(\rho e)}{\partial t}$, $\mu(x,t)$, and $\frac{\partial P(x,t)}{\partial t}$. The law of conservation of energy can be interpreted as follows,

$$\frac{\partial(\rho e)}{\partial t} = -\vec{\nabla}\cdot\vec{J_q}, \tag{3.3}$$

where J_q is the associated heat flux. $\mu(x,t)$ can be written as

$$\mu(x,t) = \mu^{{\rm eq}m} + \frac{T}{m}\ln\frac{P(x,t)}{P_{{\rm eq}m}}. \tag{3.4}$$

To get $\frac{\partial P(x,t)}{\partial t}$, we consider the Fokker-Planck equation suggested by van Kampen, which is applicable for systems with spatially varying temperature [31]. The Fokker-Planck equation, which takes care of the temperature variation along the direction of transport, is written as

$$\begin{aligned} \frac{\partial P(x,t)}{\partial t} &= \frac{\partial}{\partial x}\epsilon(x)\left[V'(x)P(x,t) + \frac{\partial}{\partial x}T(x)P(x,t)\right] \\ &= -\vec{\nabla}\cdot\vec{J}. \end{aligned} \tag{3.5}$$

Here, $\epsilon(x)$ refers to the mobility of the system and $T(x)$ represents space-dependent temperature. The equilibrium solution of Eq. (3.5), where the probability flow vanishes, is given by

$$P_{{\rm eq}m}(x) = \frac{B}{T(x)}\exp\left[-\int_{-\infty}^{x}\frac{V'(x')}{T(x')}dx'\right]. \tag{3.6}$$

van Kampen has demonstrated [31] that the exponent in brackets of the above expression for stationary distribution [Eq. (3.6)] is responsible for mass transfer due to local temperature gradient of the blowtorch effect. This has also formed the basis of our discussion in Secs. II A and II B. Substituting the quantities according to Eqs. (3.3)–(3.5) in Eq. (3.2) and using the vector identity,

$$\vec{\nabla}\cdot A_1\vec{A_2} = A_1\vec{\nabla}\cdot\vec{A_2} + \vec{A_2}\cdot\vec{\nabla}A_1 \tag{3.7}$$

(where A_1 is a scalar and $\vec{A_2}$ is a vector quantity), we can calculate the entropy production σ for the process. Force and flux can be identified and the linear relations appear to be

$$\begin{aligned} J_q &= -L_{qq}\frac{\vec{\nabla}T}{T^2} - \int L_{qx}\frac{\partial}{\partial x}\ln\frac{P}{P_{{\rm eq}m}}dx, \\ J_x &= -L_{xq}\frac{\vec{\nabla}T}{T^2} - \int L_{xx}\frac{\partial}{\partial x}\ln\frac{P}{P_{{\rm eq}m}}dx. \end{aligned} \tag{3.8}$$

Considering Eq. (3.6), we obtain

$$\ln\frac{P}{P_{{\rm eq}m}} = \int_{-\infty}^{x}\frac{V'(x')}{T(x')}dx'. \tag{3.9}$$

Substituting $\ln \frac{P}{P_{eqm}}$ in Eq. (3.8) according to Eq. (3.9) for small interval, the expressions for the flux turns out to be

$$\begin{aligned} J_q &= -L_{qq}\frac{\Delta T}{T^2} - L_{qx}\frac{\Delta V}{T}, \\ J_x &= -L_{xq}\frac{\Delta T}{T^2} - L_{xx}\frac{\Delta V}{T}. \end{aligned} \tag{3.10}$$

Here for a small interval, we have considered $T(x') \sim T$ and $\int V'(x')dx' = \Delta V$. Keeping in view of Onsager symmetries, we have as usual $L_{qx} = L_{xq}$. The expression for J_x suggests that the mass transfer, which is a direct effect of application of external forcing to overcome the potential energy barrier in the reverse protocol of the Landauer's blowtorch effect, causes a temperature variation ΔT as a cross process. This supports our interpretation to view Landauer's blowtorch effect as a thermodynamics cross phenomenon that directly follows from the consideration of the Fokker-Planck description of the dynamics.

Coupling between flux of particles and temperature difference known as Soret effect and Duffor effects has been investigated extensively in many situations [49–51]. For example, the effect of temperature difference on an activated process like protein crystal growth has been studied in Ref. [52]. The direct bearing of Soret effect on the blowtorch effect may be traced to Eq. (3.5), where the factor $T(x)$ appears following the derivative on its right-hand side. This gives rise to a preexponential factor T^{-1} in the stationary distribution function [Eq. (3.6)]. Physically, this implies that independent of phase-space considerations, the particles in the hotter region will spend less time on an average and the population will be transferred more toward the colder region. Our study on the reverse of Landauer's blowtorch effect shows that controlled forcing in a given domain of phase space may lead to cooling of molecules.

IV. PROBING THE REVERSE EFFECT: NUMERICAL SIMULATIONS AND DISCUSSIONS

As discussed in the Sec. II, we calculate the average kinetic energy for the particles in the left and in the right potential well of the double-well potential during the action of the external force. To obtain the velocity of the overdamped Brownian particle, we solve the Langevin Eq. (2.7) using improved Euler algorithm. The noise term has been generated employing standard Box-Muller algorithm. The time step has been set as $\Delta t = 10^{-2}$ and the values of a and b have been kept fixed at unity for the entire numerical simulation. The calculated kinetic energy is averaged over 10^9 trajectories. We consider such a large ensemble to make sure that a substantial amount of particles remain in the left potential well even after the transfer of the mass from the left to the right potential well by the external bias. This is to ensure that the average kinetic energy obtained from the numerical simulation becomes a reliable measure of temperature for the left potential well. We have plotted in Fig. 2(a) the temperature for the right and the left potential well (T_R and T_L, respectively) against time for a given bath temperature and forcing amplitude. T_R and T_L are, actually, the dimensionless temperatures. The variation of the normalized integrated population density

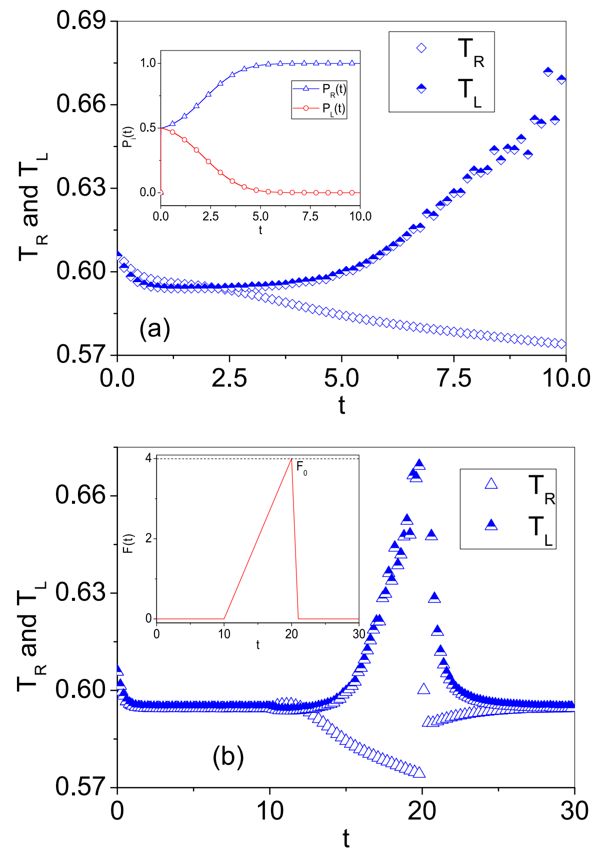

FIG. 2. (Color online) (a) Variation of T_R and T_L with time during population transfer. The bath temperature (T_{bath}) is 0.6 and the forcing amplitude F_0 is 4.0. P_R and P_L have been plotted against time for the forcing time period in the inset. (b) Variation of T_R and T_L with time plotted for three steps: initial thermalization, population transfer, and removal of the bias with $T_{bath} = 0.6$ and $F_0 = 4.0$. $F(t)$ vs. t plot has been shown in the inset.

$(P_i(t), i = R, L)$ with respect to time has been presented in the inset of Fig. 2(a). It is observed that at the beginning, both the wells remain equally occupied and the time linear external bias initiates population transfer from the left to the right potential well. With increased value of the magnitude of forcing, the probability density in the right potential well increases as expected. The important observation is that the temperature of the right well decreases, whereas that of the left potential well increases with time. It should be noted that the forcing amplitude has been kept optimally large to get a noticeable difference in temperature for both the wells. As a result, the population density in the left potential well gets depleted to a considerable extent (here, $P_L(t_f) \sim 10^{-5}$). However, consideration of very large ensemble (10^9 trajectories) ensures that even at the final stage, the averaging of the kinetic energy has been done over 10^4 trajectories for the left potential well. The study thus reveals that the transfer of particles from the left to the right potential well leaves the left well to a higher temperature compared to that of the right one. This process as anticipated in our theoretical considerations essentially produces the reverse of Landauer's blowtorch effect, where heating at one potential well causes mass transfer to the other. Our depiction of the effect as a thermodynamic cross process is

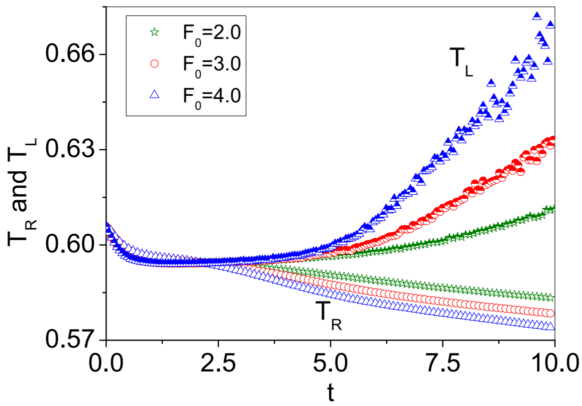

FIG. 3. (Color online) Variation of T_R and T_L against time during the population transfer process for three different values of F_0 and $T_{\mathrm{bath}} = 0.6$. The open symbols stand for T_R and the half-filled symbols with same sign signify the corresponding T_L.

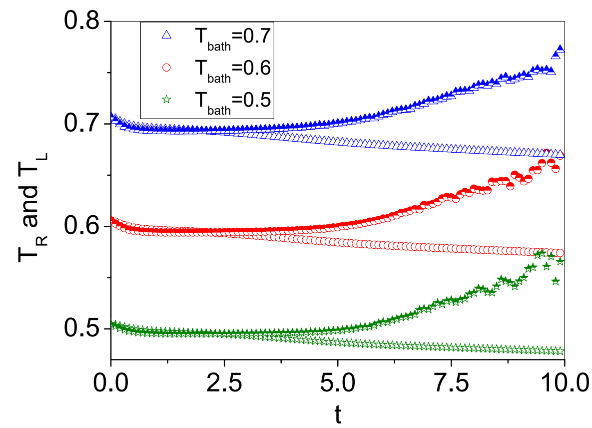

FIG. 4. (Color online) Variation of T_R and T_L against time during the population transfer process for three different values of T_{bath} and $F_0 = 4.0$. The open symbols stand for T_R and the half-filled symbols with same sign signify the corresponding T_L.

therefore corroborated. To understand the effect of population transfer by external bias on the temperature of the two wells of the bistable potential more clearly, we have studied the time evolution of the temperature for both the wells where three steps are involved in the external forcing. Initially, all the particles are allowed to get thermalized with the bath for sufficient time followed by transport of particles to the right potential well by switching on the external bias and finally the forcing is turned off and the system returns to its initial state. During the initial thermalization period, both the wells adopt the same temperature as that of the bath. The flow of particles from the left to the right well causes increase of temperature of the left well, whereas that of the right potential well decreases. When the external bias is removed, the left potential well starts getting colder and the temperature of the right well begins to rise and, ultimately, both the wells return to their initial temperature. The variation of temperature with time for the two wells as obtained has been represented in Fig. 2(b). The details of external forcing has been shown in the inset of this plot.

The variation of T_R and T_L with time during the mass transfer process has been examined for different forcing amplitude. It is observed that for higher value of F_0, the difference between T_R and T_L becomes higher. This is because the larger amplitude of external forcing transfers particles in greater proportion. In Fig. 3, we have presented the results for three different values of F_0 and the same bath temperature.

Finally, for different values of the bath temperature, the time evolution of T_R and T_L is followed keeping F_0 fixed. Similar effect is recovered for all the cases. The variation of temperature of both the wells against time for the reverse blowtorch effect has been presented in Fig. 4 for three different values of T_{bath} and the same value of F_0.

V. CONCLUSION

In summary, we have shown that Landauer's blowtorch effect can be viewed as a thermodynamic cross process. Taking into account the entropy increase for the process of crossing the hot zone of a bistable potential by the diffusive particles, we obtain an expression for the flux of particles that is related linearly to the small temperature difference between the hot and cold regions or, more precisely, the thermodynamic force associated with the temperature difference. The linear relationships have been derived on the basis of a Fokker-Planck equation, which takes care of a temperature gradient as an essential component of the associated process. To establish our description, we suggest that there must exist a reverse effect where mass transfer will lead to temperature difference between different parts of the system. To examine this effect, we consider transport of overdamped Brownian particles from one well of a bistable potential to the other by mechanical forcing. Our study reveals that the well from which the particles are transferred attains higher temperature, and the well where the particles get accumulated cools down. Thus, we conclude that the reverse blowtorch effect, which is a thermodynamic cross process, may serve as a mechanism of Brownian cooling.

ACKNOWLEDGMENT

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[1] D. H. Zanette, Phys. Rev. E **50**, 1171 (1994); J. G. Kirkwood, J. Chem. Phys. **15**, 72 (1947); Y. Kobatake and H. Fujita, *ibid.* **40**, 2219 (1964); K. B. Bota, P. Ortoleva, and J. Ross, *ibid.* **60**, 3124 (1974).

[2] F. Zhong and H. Meyer, Phys. Rev. E **51**, 3223 (1995); A. Chaudhuri, D. Chaudhuri, and S. Sengupta, *ibid.* **76**, 021603 (2007); T. D. Swinburne, *ibid.* **88**, 012135 (2013).

[3] S. Jagannathan, J. S. Dahler, and W. Sung, J. Chem. Phys. **83**, 1808 (1985); E. Fried, A. Q. Shen, and M. E. Gurtin, Phys. Rev. E **73**, 061601 (2006); S. K. Das, Jürgen Horbach, and K. Binder, J. Chem. Phys. **119**, 1547 (2003).

[4] A. Di Vita, Phys. Rev. E **81**, 041137 (2010); P. Muratore-Ginanneschi and K. Schwieger, *ibid.* **90**, 060102(R) (2014); J. Ray Chaudhuri, S. Chattopadhyay, and S. K. Banik, J. Chem. Phys. **127**, 224508 (2007).

[5] J. F. Gwan and A. Baumgaertner, J. Chem. Phys. **127**, 045103 (2007); B. Q. Ai and Y. F. He, *ibid.* **132**, 094504 (2010).

[6] S. Savel'ev, F. Marchesoni, A. Taloni, and F. Nori, Phys. Rev. E **74**, 021119 (2006); P. K. Ghosh and F. Marchesoni, J. Chem. Phys. **136**, 116101 (2012); P. K. Ghosh, P. Hänggi, F. Marchesoni, F. Nori, and G. Schmid, Europhys. Lett. **98**, 50002 (2012).

[7] M. Borromeo and F. Marchesoni, Phys. Rev. E **78**, 051125 (2008); M. Borromeo, F. Marchesoni, and P. K. Ghosh, J. Chem. Phys. **134**, 051101 (2011); P. K. Ghosh and F. Marchesoni, Eur. Phys. J. Special Topics **187**, 41 (2010).

[8] B. Lindner and L. Schimansky-Geier, Phys. Rev. Lett. **89**, 230602 (2002); B. Lindner, M. Kostur, and L. Schimansky-Geier, Fluct. Noise Lett. **1**, R25 (2001).

[9] P. Romanczuk, F. Müller, and L. Schimansky-Geier, Phys. Rev. E **81**, 061120 (2010).

[10] S. Martens, G. Schmid, L. Schimansky-Geier, and P. Hänggi, Phys. Rev. E **83**, 051135 (2011).

[11] P. K. Ghosh, P. Hänggi, F. Marchesoni, S. Martens, F. Nori, L. Schimansky-Geier, and G. Schmid, Phys. Rev. E **85**, 011101 (2012).

[12] X. Guo, D. Singh, J. Murthy, and A. A. Alexeenko, Phys. Rev. E **80**, 046310 (2009); K. Kundu, *ibid.* **61**, 5839 (2000).

[13] T. E. Dialynas, K. Lindenberg, and G. P. Tsironis, Phys. Rev. E **56**, 3976 (1997).

[14] P. Reimann, Phys. Rep. **361**, 57 (2002).

[15] F. Jülicher and J. Prost, Phys. Rev. Lett. **75**, 2618 (1995); F. Jülicher, A. Ajdari, and J. Prost, Rev. Mod. Phys. **69**, 1269 (1997).

[16] M. O. Magnasco, Phys. Rev. Lett. **71**, 1477 (1993); **72**, 2656 (1994).

[17] J. A. Freund and L. Schimansky-Geier, Phys. Rev. E **60**, 1304 (1999); B. Lindner, L. Schimansky-Geier, P. Reimann, P. Hanggi, and M. Nagaoka, *ibid.* **59**, 1417 (1999).

[18] S. Savel'ev, F. Marchesoni, and F. Nori, Phys. Rev. E **70**, 061107 (2004); S. Savel'ev, F. Marchesoni, P. Hänggi, and F. Nori, *ibid.* **70**, 066109 (2004).

[19] S. Savel'ev, F. Marchesoni, and Franco Nori, Phys. Rev. E **71**, 011107 (2005); A. Pototsky, A. J. Archer, M. Bestehorn, D. Merkt, S. Savel'ev, and F. Marchesoni, *ibid.* **82**, 030401(R) (2010).

[20] F. Marchesoni, S. Savel'ev, and F. Nori, Phys. Rev. E **73**, 021102 (2006).

[21] P. K. Ghosh, V. R. Misko, F. Marchesoni, and Franco Nori, Phys. Rev. Lett. **110**, 268301 (2013).

[22] P. K. Ghosh, D. Barik, and D. S. Ray, Phys. Rev. E **71**, 041107 (2005); P. K. Ghosh and D. S. Ray, *ibid.* **73**, 036103 (2006); D. Mondal, P. K. Ghosh, and D. S. Ray, J. Chem. Phys. **130**, 074703 (2009).

[23] W. L. Reenbohn and M. C. Mahato, Phys. Rev. E **91**, 052151 (2015); J. Spiechowicz and J. Luczka, *ibid.* **91**, 062104 (2015).

[24] P. Malgaretti, I. Pagonabarraga, and J. M. Rubi, J. Chem. Phys. **138**, 194906 (2013); B. Q. Ai and J. C. Wu, *ibid.* **140**, 094103 (2014).

[25] R. Landauer, Phys. Lett. A **68**, 15 (1978).

[26] R. Landauer, Physica Scripta **35**, 88 (1987).

[27] R. Landauer, J. Stat. Phys. **53**, 233 (1988).

[28] R. Landauer, Phys. Rev. A **15**, 2117 (1977).

[29] R. Landauer, J. Stat. Phys. **13**, 1 (1975).

[30] R. Landauer, Physica A **194**, 551 (1993).

[31] N. G. van Kampen, IBM J. Res. Dev. **32**, 107 (1988); J. Math. Phys. **29**, 1220 (1988).

[32] N. G. van Kampen, Superlat. Microstruct. **23**, 559 (1998).

[33] M. Bekele, S. Rajesh, G. Ananthakrishna, and N. Kumar, Phys. Rev. E **59**, 143 (1999).

[34] R. D. Astumian, Science **276**, 917 (1997); R. D. Astumian and M. Bier, Phys. Rev. Lett. **72**, 1766 (1994).

[35] I. Derenyi and T. Vicsek, Phys. Rev. Lett. **75**, 374 (1995).

[36] Y. A. Makhnovskii, V. M. Rozenbaum, S. Y. Sheu, D. Y. Yang, L. I. Trakhtenberg, and S. H. Lin, J. Chem. Phys. **140**, 214108 (2014); L. Machura, M. Kostur, P. Hänggi, P. Talkner, and J. Luczka, Phys. Rev. E **70**, 031107 (2004); L. Machura, M. Kostur, P. Talkner, J. Luczka, F. Marchesoni, and P. Hänggi, *ibid.* **70**, 061105 (2004).

[37] A. Gomez-Marin and J. M. Sancho, Phys. Rev. E **71**, 021101 (2005).

[38] R. Benjamin and R. Kawai, Phys. Rev. E **77**, 051132 (2008); M. Santillán and M. C. Mackey, *ibid.* **78**, 061122 (2008).

[39] C. Touya, T. Schwalger, and B. Lindner, Phys. Rev. E **83**, 051913 (2011); D. Wu and S. Zhu, *ibid.* **85**, 061101 (2012).

[40] I. Goychuk and V. Kharchenko, Phys. Rev. E **85**, 051131 (2012).

[41] K. J. Challis and M. W. Jack, Phys. Rev. E **88**, 042114 (2013); M. Asfaw, *ibid.* **89**, 012143 (2014).

[42] F. Kohler and A. Rohrbach, Phys. Rev. E **91**, 012701 (2015).

[43] P. Hänggi and F. Marchesoni, Rev. Mod. Phys. **81**, 387 (2009); P. Hänggi, F. Marchesoni, and F. Nori, Ann. Phys. **14**, 51 (2005).

[44] L. Machura, M. Kostur, F. Marchesoni, P. Talkner, P. Hänggil, and J. Luczka, J. Phys.: Condens. Matter **17**, S3741 (2005).

[45] J. Spiechowicz, P. Hänggi, and J. Luczka, Phys. Rev. E **90**, 032104 (2014); A. Fiasconaro, E. Gudowska-Nowak, and W. Ebeling, *ibid.* **87**, 032111 (2013).

[46] A. Igarashi, S. Tsukamoto, and H. Goko, Phys. Rev. E **64**, 051908 (2001).

[47] C. Van den Broeck and R. Kawai, Phys. Rev. Lett. **96**, 210601 (2006).

[48] A. Pérez-Madrid, J. M. Rubí, and P. Mazur, Physica A **212**, 231 (1994).

[49] L. Onsager, Phys. Rev. **37**, 405 (1931); R. G. Mortimer and H. Eyring, Proc. Natl. Acad. Sci. USA **77**, 1728 (1980).

[50] W. Hort, S. J. Linz, and M. Lücke, Phys. Rev. A **45**, 3737 (1992); S. Duhr and D. Braun, Proc. Natl. Acad. Sci. USA **103**, 19678 (2006).

[51] R. Piazza and A. Guarino, Phys. Rev. Lett. **88**, 208302 (2002); J. K. Platten, J. Appl. Mech. **73**, 5 (2006).

[52] I. Santamaría-Holek, A. Gadomski, and J. M. Rubí, J. Phys.: Condens. Matter **23**, 235101 (2011).