Microscopic nonlinear relativistic quantum theory of absorption of powerful x-ray radiation in plasma

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The microscopic quantum theory of plasma nonlinear interaction with the coherent shortwave electromagnetic radiation of arbitrary intensity is developed. The Liouville–von Neumann equation for the density matrix is solved analytically considering a wave field exactly and a scattering potential of plasma ions as a perturbation. With the help of this solution we calculate the nonlinear inverse-bremsstrahlung absorption rate for a grand canonical ensemble of electrons. The latter is studied in Maxwellian, as well as in degenerate quantum plasma for x-ray lasers at superhigh intensities and it is shown that one can achieve the efficient absorption coefficient in these cases.

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I. INTRODUCTION

The recent remarkable progress in x-ray free electron laser technology [1–3] allows production of electromagnetic (EM) wave pulses with intensity 10^{20} W/cm^2 [3] exceeding the peak brilliance of conventional synchrotron sources by many orders of magnitude. For such intensities the electron-wave field energy exchange over an x-ray wavelength is larger than the photon energy, and the laser-matter interaction has essentially multiphoton character [4]. Thus, in recent decades the wide research field is opened up, where common nonlinear effects are extended to high energy transitions for various systems—from atoms [5–9], through molecules [10–13], to plasma and solid samples [14–19]. In a solid sample or in plasma with partially ionized atoms the main mechanisms of x-ray absorption are connected with the deep bounded electrons. In particular, the photoionization with relatively long laser pulses from the K and L shells are responsible for x-ray absorption [14,19]. In completely ionized underdense plasma, there are several type of instabilities [20,21] which can be developed at the laser plasma interaction causing collisionless plasma heating for relatively long laser pulses. Besides, relativistic quantum effects may be essential for plasmas of high densities [22,23]. For x-ray lasers of ultrarelativistic intensities the stimulated Raman scattering and two-plasmon decay instabilities can cause strong collisionless plasma heating [23]. For the short enough laser pulses the inverse-bremsstrahlung absorption may become the dominant mechanism of the absorption of strong laser pulses in plasma [24]. Among the fundamental processes of laser-plasma interaction, the inverse bremsstrahlung absorption of an intense laser field in plasma is one of the contemporary problems that have applications ranging from plasma diagnostics to thermonuclear reactions and generation of intense x-ray radiation. Note that in the field of intense x-ray laser, an electron may gain considerable energies absorbing even a few quanta, which makes it an effective mechanism for laser-plasma heating. The theoretical description of this phenomenon in such superstrong radiation fields requires one to go beyond the scope of common quantum electrodynamics–Feynman diagrams corresponding to the perturbation theory.

With the advent of lasers many pioneering papers have been devoted to the theoretical investigation of the electron-ion scattering processes in gas or plasma in the presence of a laser field using nonrelativistic [24–35] as well as relativistic [36–40] considerations. The appearance of superpower ultrashort laser pulses of relativistic intensities has triggered new interest in stimulated bremsstrahlung (SB) in a relativistic domain, where investigations were carried out in the Born [41–43], eikonal [38], and generalized eikonal [44] approximations over the scattering potential. Beyond this approximation for the infrared and optical lasers, in the multiphoton interaction regime, one can apply classical theory and the main approximation in the classical theory is low frequency or impact approximation [24,30]. Low frequency approximations have been generalized for the relativistic case in Refs. [39,40], where the effect of an intense EM wave on the dynamics of SB and nonlinear absorption of intense laser radiation by a monochromatic electron beam and by relativistic plasma due to the SB have been carried out. Regarding the solid densities, the absorption coefficient of inverse-bremsstrahlung may reach considerably large values. As was shown in Ref. [45], where interaction of superstrong lasers with thin plasma targets of solid densities was investigated via particle-in-cell simulations, the inverse-bremsstrahlung absorption is dominant for electron densities above 10^{21} cm^{-3}.

For the infrared and optical lasers the quantum effects in the SB process are smeared out due to smallness of the photon energy. Meanwhile, for a intense x-ray radiation the nonlinear over the field quantum effects will be considerable. Thus it is of interest to consider x-ray multiphoton absorption via inverse bremsstrahlung in ultradense classical as well as quantum plasmas. In the present paper the inverse-bremsstrahlung absorption of an intense x-ray laser field in the dense classical and quantum plasmas is considered in the relativistic-quantum regime considering a wave field exactly, while a scattering potential of a plasma ion as a perturbation. The x-ray radiation power absorbed in plasma is investigated for circularly and linearly polarized waves arising from the second quantized consideration. The Liouville–von Neumann equation for the density matrix is solved analytically for a system initially in thermodynamic equilibrium (grand canonical ensemble). With the help of this solution we investigate the nonlinear inverse-bremsstrahlung absorption rate for Maxwellian as well as for degenerate quantum plasmas. It is shown that depending on the intensity of an incident x ray, one can achieve the quite large absorption coefficients. Hence the considered mechanism may serve as an efficient tool for ultrafast plasma heating.

The organization of the paper is as follows. In Sec. II, the relativistic quantum dynamics of SB is presented with analytical results for density matrix and inverse-bremsstrahlung absorption rate. In Sec. III, we consider the problem numerically. Finally, conclusions are given in Sec. IV.

II. BASIC MODEL AND THEORY

Let us consider the relativistic quantum theory of plasma nonlinear interaction with the arbitrary strong EM wave field by microscopic theory of electrons-ions interaction on the base of the density matrix. The EM wave with the four-wave vector $k \equiv (\omega/c,\mathbf{k})$ is described by the four-vector potential

$$A^{\mu} = (0,\mathbf{A}), \tag{1}$$

where $\mathbf{A}$ is defined as

$$\begin{gathered}\mathbf{A}(\tau) = A_0\{\mathbf{e}_1 \cos\omega\tau + \mathbf{e}_2 g \sin\omega\tau\}, \\ \tau = t - \frac{\boldsymbol{\nu}_0\mathbf{r}}{c}, \quad \boldsymbol{\nu}_0 = \frac{c\mathbf{k}}{\omega}, \quad \mathbf{e}_1\boldsymbol{\nu}_0 = \mathbf{e}_2\boldsymbol{\nu}_0 = \mathbf{e}_1\mathbf{e}_2 = 0,\end{gathered} \tag{2}$$

with the amplitude A_0, unit polarization vectors $\mathbf{e}_{1,2}$, and ellipticity parameter g. The ions are assumed to be at rest and being either randomly or nonrandomly distributed in plasma, the static potential field of which (for nucleus/ion—as a scattering center—the recoil momentum is neglected) is described by the scalar potential

$$A^{(e)}(x) = (\varphi(\mathbf{r}),0), \tag{3}$$

where

$$\varphi(\mathbf{r}) = \sum_{i}^{N_i} \varphi_i(\mathbf{r} - \mathbf{R}_i). \tag{4}$$

Here φ_i is the potential of a single ion situated at the position $\mathbf{R}_i$, and N_i is the number of ions in the interaction region.

To investigate the quantum dynamics of SB we need the quantum kinetic equations for a single particle density matrix, which can be derived arising from the second quantized formalism. As is known, the Dirac equation allows the exact solution in the field of a plane EM wave (Volkov solution). Although the Volkov states are not stationary, as there are no real transitions in the monochromatic EM wave field (due to the violation of energy and momentum conservation laws), the state of an electron with a charge e and mass m in an EM wave field can be characterized by the quasimomentum $\boldsymbol{\Pi}$ and polarization σ, and the particle state in the field (1) is given by the wave function:

$$\begin{aligned}\Psi_{\boldsymbol{\Pi}\sigma} = {} & \left[1 + \frac{e\not{k}\not{A}}{2c(kp)}\right]\frac{u_\sigma(p)}{\sqrt{2E_{\boldsymbol{\Pi}}\mathcal{V}}}\exp\left[-\frac{i}{\hbar}\Pi x\right] \\ & \times \exp\left\{\frac{i}{\hbar}\left[\frac{eA_0}{c(pk)}(\mathbf{e}_1\mathbf{p}\sin\omega\tau - g\mathbf{e}_2\mathbf{p}\cos\omega\tau)\right.\right. \\ & \left.\left. - \frac{e^2A_0^2}{8c^2(pk)}(1-g^2)\sin(2\omega\tau)\right]\right\},\end{aligned} \tag{5}$$

where for any four-vector a: $\not{a} \equiv a_\mu\gamma^\mu$, $\gamma^\mu \equiv \{\gamma^0,\gamma^1,\gamma^2,\gamma^3\}$ are the Dirac matrices, $\mathcal{V}$ is the quantization volume, u_σ is the bispinor amplitude of a free Dirac particle with polarization σ, and $\Pi = (E_{\boldsymbol{\Pi}}/c,\boldsymbol{\Pi})$ is the average four-kinetic momentum or "quasimomentum" of the particle in the periodic field, which is determined via a free particle four-momentum $p = (\mathcal{E}/c,\mathbf{p})$ and relativistic invariant parameter of the wave intensity $\xi_0 = eA_0/mc^2$ by the following equation:

$$\Pi = p + k\frac{m^2c^2}{4kp}(1+g^2)\xi_0^2. \tag{6}$$

From this equation follows that

$$\Pi^2 = m^{*2}c^2, \quad m^* = m\left(1 + \frac{1+g^2}{2}\xi_0^2\right)^{1/2}, \tag{7}$$

where m^* is the effective mass of the particle in the monochromatic wave. The states (5) are normalized by the condition

$$\int \Psi^\dagger_{\boldsymbol{\Pi}'\sigma'}\Psi_{\boldsymbol{\Pi}\sigma}d\mathbf{r} = \frac{(2\pi\hbar)^3}{\mathcal{V}}\delta(\boldsymbol{\Pi} - \boldsymbol{\Pi}')\delta_{\sigma,\sigma'}.$$

Cast in the second quantization formalism, the Hamiltonian is

$$\mathcal{H} = \int \widehat{\Psi}^+\widehat{H}_0\widehat{\Psi}\,d\mathbf{r} + \mathcal{H}_{sb}, \tag{8}$$

where $\widehat{\Psi}$ is the fermionic field operator, $\widehat{H}_0$ is the one-particle Dirac Hamiltonian in the plane EM wave (1), and the interaction Hamiltonian is

$$\mathcal{H}_{sb} = \frac{1}{c}\int \widehat{j}A^{(e)}d\mathbf{r}, \tag{9}$$

with the current density operator

$$\widehat{j} = e\widehat{\Psi}^+\gamma_0\gamma\widehat{\Psi}. \tag{10}$$

Making Fourier transformation

$$A^{(e)}(x) = \frac{1}{(2\pi)^3}\int A^{(e)}(\mathbf{q})e^{-i\mathbf{q}\cdot\mathbf{r}}d\mathbf{q},$$

the expression (9) will have a form

$$\mathcal{H}_{sb} = \frac{1}{c(2\pi)^3}\int \widehat{\Psi}^+V(\mathbf{q})e^{-i\mathbf{q}\cdot\mathbf{r}}\widehat{\Psi}\,d\mathbf{q}\,d\mathbf{r}, \tag{11}$$

where

$$V(\mathbf{q}) = \int\sum_{i}^{N_i} e\varphi_i(\mathbf{r} - \mathbf{R}_i)e^{-i\mathbf{q}\cdot\mathbf{r}}d\mathbf{r}. \tag{12}$$

We pass to the Furry representation and write the Heisenberg field operator of the electron in the form of an expansion in the quasistationary Volkov states (5)

$$\widehat{\Psi}(\mathbf{r},t) = \sum_\sigma\int d\Phi_{\boldsymbol{\Pi}}\widehat{a}_{\boldsymbol{\Pi},\sigma}e^{\frac{i}{\hbar}E_{\boldsymbol{\Pi}}t}\Psi_{\boldsymbol{\Pi}\sigma}(\mathbf{r},t), \tag{13}$$

where $d\Phi_{\boldsymbol{\Pi}} = \mathcal{V}d^3\boldsymbol{\Pi}/(2\pi\hbar)^3$. In Eq. (13) we have excluded the antiparticle operators, since contribution of electron-positron intermediate states will be negligible for considered intensities and photon energies $\varepsilon_\gamma = \hbar\omega \ll mc^2$. The creation and annihilation operators, $\widehat{a}^+_{\mathbf{\Pi},\sigma}(t)$ and $\widehat{a}_{\mathbf{\Pi},\sigma}(t)$, associated with positive energy solutions satisfy the anticommutation rules at equal times,

$$\{\widehat{a}^\dagger_{\mathbf{\Pi},\sigma}(t),\widehat{a}_{\mathbf{\Pi}',\sigma'}(t')\}_{t=t'} = \frac{(2\pi\hbar)^3}{\mathcal{V}}\delta(\mathbf{\Pi}-\mathbf{\Pi}')\delta_{\sigma,\sigma'}\ , \tag{14}$$

$$\{\widehat{a}^\dagger_{\mathbf{\Pi},\sigma}(t),\widehat{a}^\dagger_{\mathbf{\Pi}',\sigma'}(t')\}_{t=t'} = \{\widehat{a}_{\mathbf{\Pi},\sigma}(t),\widehat{a}_{\mathbf{\Pi}',\sigma'}(t')\}_{t=t'} = 0. \tag{15}$$

Taking into account Eqs. (13), (10), (9), and (5), the second quantized Hamiltonian can be expressed in the form

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{sb}(t). \tag{16}$$

The first term in Eq. (16) is the Hamiltonian of Volkov dressed electron field

$$\mathcal{H}_0 = \sum_\sigma \int d\Phi_{\mathbf{\Pi}} E_{\mathbf{\Pi}} \widehat{a}^+_{\mathbf{\Pi},\sigma}\widehat{a}_{\mathbf{\Pi},\sigma}, \tag{17}$$

while the second term

$$\mathcal{H}_{sb}(t) = \sum_{\sigma\sigma'}\int d\Phi_{\mathbf{\Pi}}\int d\Phi_{\mathbf{\Pi}'} M_{\mathbf{\Pi}',\sigma';\mathbf{\Pi},\sigma}(t)\widehat{a}^+_{\mathbf{\Pi}',\sigma'}\widehat{a}_{\mathbf{\Pi},\sigma} \tag{18}$$

is the interaction Hamiltonian describing the SB with amplitudes

$$M_{\mathbf{\Pi}',\sigma';\mathbf{\Pi},\sigma}(t) = \frac{1}{\mathcal{V}}\sum_{s=-\infty}^{\infty} e^{-is\omega t}\mathcal{M}^{(s)}_{\mathbf{\Pi}',\sigma';\mathbf{\Pi},\sigma}, \tag{19}$$

$$\mathcal{M}^{(s)}_{\mathbf{\Pi}',\sigma';\mathbf{\Pi},\sigma} = \frac{V(\mathbf{q}_s)}{2c\sqrt{E_{\mathbf{\Pi}}E_{\mathbf{\Pi}'}}}\overline{u}_{\sigma'}(p') \times\left[\not{\epsilon}_0 B_s + \left(\frac{e\not{B}_{1s}\not{k}\not{\epsilon}_0}{2c(kp')} + \frac{e\not{\epsilon}_0\not{k}\not{B}_{1s}}{2c(kp)}\right) + \frac{e^2(k\epsilon_0)B_{2s}}{2c^2(kp')(kp)}\not{k}\right]u_\sigma(p). \tag{20}$$

In Eq. (20) the vector functions $B^\mu_{1s} = (0,\mathbf{B}_{1s})$ and scalar functions B_s, B_{2s} are expressed via the generalized Bessel functions $G_s(\alpha,\beta,\varphi)$:

$$G_s(\alpha,\beta,\varphi) = \sum_{k=-\infty}^{\infty} J_{2k-s}(\alpha)J_k(\beta)e^{i(s-2k)\varphi}, \tag{21}$$

$$\mathbf{B}_{1s} = \frac{A_0}{2}\{\mathbf{e}_1(G_{s-1}(\alpha,\beta,\varphi)+G_{s+1}(\alpha,\beta,\varphi)) + i\mathbf{e}_2 g(G_{s-1}(\alpha,\beta,\varphi)-G_{s+1}(\alpha,\beta,\varphi))\}, \tag{22}$$

$$B_s = G_s(\alpha,\beta,\varphi), \tag{23}$$

$$B_{2s} = \frac{A_0^2}{2}(1+g^2)G_s + \frac{A_0^2}{2}(1-g^2) \times (G_{s-2}(\alpha,\beta,\varphi)+G_{s+2}(\alpha,\beta,\varphi)), \tag{24}$$

and

$$\hbar\mathbf{q}_s = \mathbf{\Pi}' - \mathbf{\Pi} - s\hbar\mathbf{k} \tag{25}$$

is the recoil momentum. The definition of the arguments α,β,φ are

$$\alpha = \frac{eA_0}{\hbar c}\left[\left(\frac{\mathbf{e}_1\mathbf{p}}{pk}-\frac{\mathbf{e}_1\mathbf{p}'}{p'k}\right)^2 + g^2\left(\frac{\mathbf{e}_2\mathbf{p}}{pk}-\frac{\mathbf{e}_2\mathbf{p}'}{p'k}\right)^2\right]^{1/2}, \tag{26}$$

$$\beta = \frac{e^2A_0^2}{8\hbar c^2}(1-g^2)\left(\frac{1}{pk}-\frac{1}{p'k}\right), \tag{27}$$

$$\tan\varphi = \frac{g\left(\frac{\mathbf{e}_2\mathbf{p}}{pk}-\frac{\mathbf{e}_2\mathbf{p}'}{p'k}\right)}{\left(\frac{\mathbf{e}_1\mathbf{p}}{pk}-\frac{\mathbf{e}_1\mathbf{p}'}{p'k}\right)}. \tag{28}$$

Thus, in order to develop microscopic relativistic quantum theory of the multiphoton inverse-bremsstrahlung absorption of ultrastrong shortwave laser radiation in plasma we need to solve the Liouville–von Neumann equation for the density matrix $\widehat{\rho}$:

$$\frac{\partial\widehat{\rho}}{\partial t} = \frac{i}{\hbar}[\widehat{\rho},\mathcal{H}_0+\mathcal{H}_{sb}(t)], \tag{29}$$

with the initial condition

$$\widehat{\rho}(-\infty) = \widehat{\rho}_G. \tag{30}$$

Here we assume that before the interaction with an EM wave the system was in thermodynamic equilibrium (thermal and chemical) with a reservoir. Thus $\widehat{\rho}_G$ is the density matrix of the grand canonical ensemble:

$$\widehat{\rho}_G = \exp\left[\frac{1}{T_e}\left(\Omega + \sum_\sigma\int d\Phi_{\mathbf{\Pi}}(\mu - E_{\mathbf{\Pi}})\widehat{a}^+_{\mathbf{\Pi},\sigma}\widehat{a}_{\mathbf{\Pi},\sigma}\right)\right]. \tag{31}$$

In Eq. (31) T_e is the electrons temperature in energy units, μ is the chemical potential, and Ω is the grand potential. Note that the initial one-particle density matrix in momentum space is

$$\rho_{\sigma_1\sigma_2}(\mathbf{\Pi}_1,\mathbf{\Pi}_2,-\infty) = \mathrm{Tr}\left(\widehat{\rho}_G\widehat{a}^+_{\mathbf{\Pi}_2,\sigma_2}\widehat{a}_{\mathbf{\Pi}_1,\sigma_1}\right) = n\left(E_{\mathbf{\Pi}_1}\right)\frac{(2\pi\hbar)^3}{\mathcal{V}}\delta(\mathbf{\Pi}_1-\mathbf{\Pi}_2)\delta_{\sigma_1,\sigma_2}, \tag{32}$$

where

$$n\left(E_{\mathbf{\Pi}_1}\right) = \frac{1}{\exp\left[\frac{E_{\Pi_1}-\mu}{T_e}\right]+1}. \tag{33}$$

We consider Volkov dressed SB Hamiltonian $\mathcal{H}_{sb}(t)$ as a perturbation. Accordingly, we expand the density matrix as

$$\widehat{\rho} = \widehat{\rho}_G + \widehat{\rho}^{(1)}.$$

Then taking into account the relations

$$[\widehat{a}^+_{\mathbf{\Pi}',\sigma'}\widehat{a}_{\mathbf{\Pi},\sigma},\widehat{\rho}_G] = \left(1 - e^{\frac{1}{T_e}(E_{\Pi'}-E_{\Pi})}\right)\widehat{\rho}_G\widehat{a}^+_{\mathbf{\Pi}',\sigma'}\widehat{a}'_{\mathbf{\Pi},\sigma}$$

and

$$[\widehat{\rho}_G,\mathcal{H}_0] = 0,$$

for $\widehat{\rho}^{(1)}$ we obtain

$$\widehat{\rho}^{(1)} = \frac{1}{i\hbar}\int_{-\infty}^t dt'\sum_{\sigma\sigma'}\int d\Phi_{\mathbf{\Pi}}\int d\Phi_{\mathbf{\Pi}'} M_{\mathbf{\Pi}',\sigma';\Pi,\sigma}(t') \times e^{\frac{i}{\hbar}(t'-t)(E_{\Pi'}-E_{\Pi})}\left(1-e^{\frac{1}{T_e}(E_{\Pi'}-E_{\Pi})}\right)\widehat{\rho}_G\widehat{a}^+_{\mathbf{\Pi}',\sigma'}\widehat{a}_{\mathbf{\Pi},\sigma}. \tag{34}$$

Now with the help of this solution one can calculate the desired physical characteristics of the SB process. In particular, for the energy absorption rate by the electrons due to the inverse stimulated bremsstrahlung one can write

$$\frac{d\mathcal{E}}{dt} = \mathrm{Tr}\left(\widehat{\rho}^{(1)}\frac{\partial \mathcal{H}_{sb}(t)}{\partial t}\right). \tag{35}$$

It is more convenient to represent the rate of the inverse-bremsstrahlung absorption via the mean number of absorbed photons by per electron, per unit time:

$$\frac{dN_{\gamma e}}{dt} = \frac{1}{\hbar\omega N_e}\frac{d\mathcal{E}}{dt}, \tag{36}$$

where N_e is the number of electrons in the interaction region. Taking into account decomposition

$$\left(1 - e^{\frac{1}{T_e}(E_1 - E_2)}\right)\mathrm{Tr}(\widehat{\rho}_G \widehat{a}_1^{+}\widehat{a}_2\widehat{a}_3^{+}\widehat{a}_4) = \left(1 - e^{\frac{1}{T_e}(E_1 - E_2)}\right)n_1(1-n_2)\delta_{23}\delta_{14},$$

with the help of Eqs. (34), (35), (36), and (20) for large t we obtain

$$\frac{dN_{\gamma e}}{dt} = \sum_{s=1}^{\infty}\frac{dN_{\gamma e}(s)}{dt}, \tag{37}$$

where the partial s-photon absorption rates are given by the formula

$$\frac{dN_{\gamma e}(s)}{dt} = \frac{8\pi s}{\hbar N_e \mathcal{V}^2}\int\int \frac{d\Phi_{\mathbf{\Pi}} d\Phi_{\mathbf{\Pi}'}}{E_{\mathbf{\Pi}} E_{\mathbf{\Pi}'}}|V(\mathbf{q}_s)|^2 \left|\mathcal{B}^{(s)}_{\mathbf{\Pi}';\mathbf{\Pi}}\right|^2 \times \delta(E_{\mathbf{\Pi}'} - E_{\mathbf{\Pi}} + s\hbar\omega)\left(1 - e^{\frac{1}{T_e}(E_{\mathbf{\Pi}'} - E_{\mathbf{\Pi}})}\right) \times n(E_{\mathbf{\Pi}'})(1 - n(E_{\mathbf{\Pi}})), \tag{38}$$

where

$$\left|\mathcal{B}^{(s)}_{\mathbf{\Pi}';\mathbf{\Pi}}\right|^2 = \left\{\left|\mathcal{E}B_s - \frac{e(\mathbf{p}\mathbf{B}_{1s})\omega}{(kp)c} + \frac{e^2\omega B_{2s}}{2c^2(kp)}\right|^2 - \frac{\hbar^2\mathbf{q}_s^2 c^2}{4}|B_s|^2 + \frac{e^2\hbar^2[\mathbf{k}\mathbf{q}_s]^2}{4(kp')(kp)}[|\mathbf{B}_{1s}|^2 - \mathrm{Re}(B_{2s}B_s^*)]\right\}, \tag{39}$$

and $\delta(x)$ is the Dirac δ function that expresses the energy conservation law in the SB process. The obtained expression for the absorption rate is general and applicable to arbitrary polarization, frequency, and intensity of the wave field. This formula is applicable for a grand canonical ensemble and is always positive. With the help of Eqs. (38) and (37) one can calculate the nonlinear inverse-bremsstrahlung absorption rate for Maxwellian, as well as for degenerate quantum plasmas.

III. NUMERICAL RESULTS AND DISCUSSION

For the obtained absorption rate (38) one needs to concretize the ionic potential $V(\mathbf{q}_s)$. In general, one should arise from the Lindhard theory of screening. The latter is applicable for quantum plasmas [46] and in the classical limit coincides with the Debye theory. For our calculations the Thomas-Fermi approximation is sufficient, namely the potential of a single ion φ_i of charge number Z_a varies slowly on the scale of Fermi wavelength $\hbar/p_F$, where p_F is the Fermi momentum. Thus, in this case in Eq. (4) we have screened Coulomb potential

$$\varphi_i(\mathbf{r}) = \frac{eZ_a}{r}\exp(-\varkappa_e r), \tag{40}$$

where

$$\varkappa_e = \left(4\pi e^2\frac{\partial n_e}{\partial \mu}\right)^{1/2} \tag{41}$$

is the Thomas-Fermi wave vector [46] which defines the screening length $\varkappa_e^{-1}$ as a function of the plasma temperature and density of electrons n_e. For Maxwellian plasma $n_e \propto e^{\mu/T_e}$ and $\varkappa_e = (4\pi e^2 n_e/T_e)^{1/2}$. For degenerate plasma $\partial n_e/\partial\mu = 3n_e/(2\varepsilon_F)$ is the density of levels at the Fermi energy ε_F and $\varkappa_e = (6\pi e^2 n_e/\varepsilon_F)^{1/2}$. Thus, taking into account the plasma quasineutrality ($Z_a N_i \simeq N_e$), from Eqs. (4), (40), and (12) for randomly distributed plasma ions we have

$$|V(\mathbf{q}_s)|^2 = N_e\frac{16\pi^2 Z_a e^4}{\left(\mathbf{q}_s^2 + \varkappa_e^2\right)^2}. \tag{42}$$

Integrating in Eq. (38) over $E_{\mathbf{\Pi}'}$ we will obtain the following expression for partial absorption rates:

$$\frac{dN_{\gamma e}(s)}{dt} = \frac{2Z_a e^4 s}{\pi^3\hbar^3 c^4}\int_{m^*c^2+s\hbar\omega} dE_{\mathbf{\Pi}} d\Omega d\Omega' \frac{|\mathbf{\Pi}||\mathbf{\Pi}'|\left|\mathcal{B}^{(s)}_{\mathbf{\Pi}';\mathbf{\Pi}}\right|^2}{\left(\hbar^2\mathbf{q}_s^2 + \hbar^2\varkappa_e^2\right)^2} \times \left(1 - e^{-\frac{s\hbar\omega}{T_e}}\right)n(E_{\Pi} - s\hbar\omega)(1 - n(E_{\mathbf{\Pi}})), \tag{43}$$

where

$$|\mathbf{\Pi}'| = \sqrt{|\mathbf{\Pi}|^2 + \hbar^2 s^2\mathbf{k}^2 - 2\frac{E_{\mathbf{\Pi}} s\hbar\omega}{c^2}}.$$

In general, the analytical integration over solid angles Ω, Ω' and energy is impossible, and one should make numerical integration. The latter for initially nonrelativistic plasma and at the photon energies $\hbar\omega > T_e$ is convenient to make

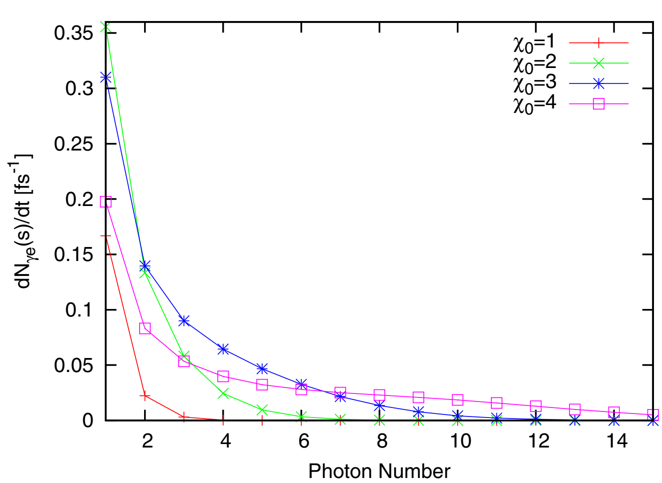

FIG. 1. (Color online) Envelope of partial rate of inverse-bremsstrahlung absorption via the mean number of absorbed photons by per electron, per unit time (in femtosecond^{-1}) for circularly polarized wave in Maxwellian plasma is shown for various wave intensities at $\varepsilon_\gamma = 1$ keV, $Z_a = 13$, $n_e = 10^{23}\mathrm{cm}^{-3}$, and $T_e = 100$ eV.

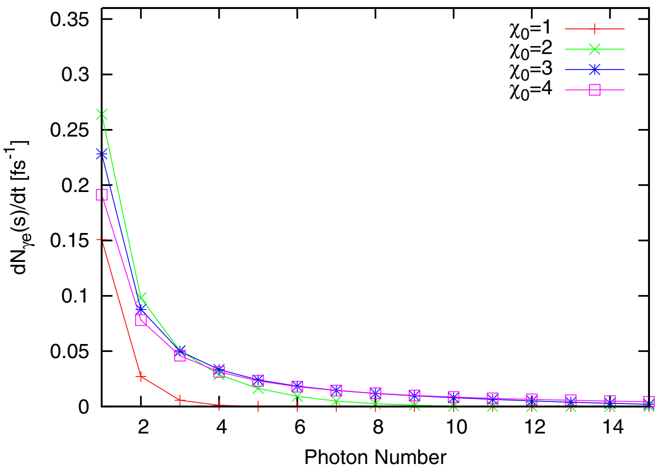

FIG. 2. (Color online) Same as Fig. 1 but for linearly polarized wave.

introducing a dimensionless parameter

$$\chi_0 = \frac{eE_0}{\omega\sqrt{m\hbar\omega}}, \tag{44}$$

which is the ratio of the amplitude of the momentum transferred by the wave field to the momentum at the one-photon absorption. In Eq. (44) the dimensionless parameter $E_0 = \omega A_0\sqrt{1+g^2}/c$ is the amplitude of the electric field strength. Hence the average intensity of the wave expressed via the parameter χ_0 can be estimated as

$$I_{\chi_0} = \chi_0^2 \times 1.74 \times 10^{12}\ \mathrm{W\,cm^{-2}}\left[\frac{\hbar\omega}{\mathrm{eV}}\right]^3.$$

The intensity I_{χ_0} strongly depends on the photon energy $\hbar\omega$. At $\chi_0 \sim 1$, the multiphoton effects become essential. Particularly, for x-ray photons with energies $\varepsilon_\gamma \equiv \hbar\omega = 0.1$–$1$ keV, multiphoton interaction regime can be achieved at the intensities

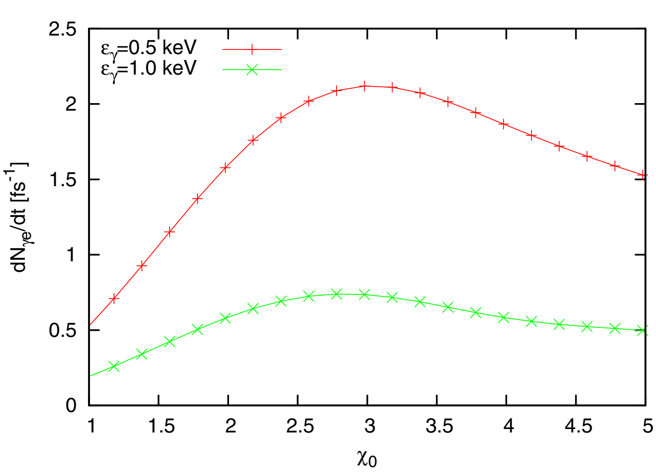

FIG. 3. (Color online) Total rate of inverse-bremsstrahlung absorption for circularly polarized wave in Maxwellian plasma vs the dimensionless parameter χ_0 for various photon energies at $n_e = 10^{23}\mathrm{cm}^{-3}$, and $T_e = 100$ eV.

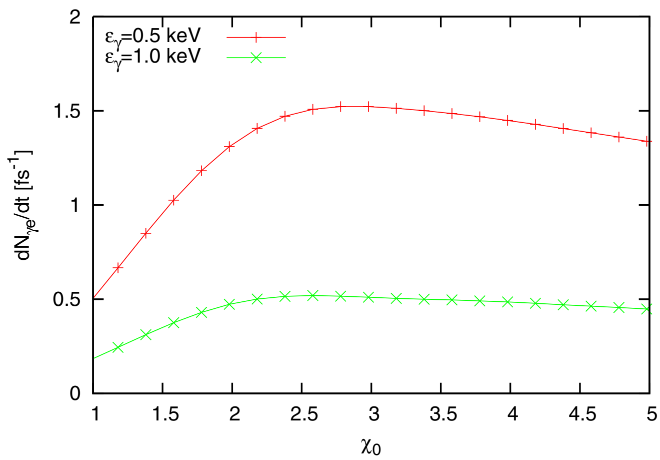

FIG. 4. (Color online) Same as Fig. 3 but for linearly polarized wave.

$I_{\chi_0} \sim 10^{18}$–10^{21} W/cm^2. In the opposite limit $\chi_0 \ll 1$, the multiphoton effects are suppressed.

For all calculations as a reference sample we take ions with $Z_a = 13$ (fully ionized aluminum) and consider plasma of solid densities. In Fig. 1 and Fig. 2 the envelope of the partial rate of inverse-bremsstrahlung absorption for circularly and linearly polarized waves in Maxwellian plasma is shown for various wave intensities at $\varepsilon_\gamma = 1$ keV, $n_e = 10^{23}$ cm^{-3}, and $T_e = 100$ eV. As is seen from these figures, the multiphoton effects become essential with the increase of the wave intensity.

To show the dependence of the inverse-bremsstrahlung absorption rate on the laser radiation intensity, in Fig. 3 and Fig. 4 the total rate (37) with (43) for circularly and linearly polarized waves in Maxwellian plasma versus the parameter χ_0 for various photon energies are shown. As is seen from these figures, the rate strictly depends on the wave polarization, and for the large values of χ_0 it exhibits a tenuous dependence on the wave intensity.

To compare with the linear theory [24], in Fig. 5 we plot scaled absorption rate $\chi_0^{-2}dN_{\gamma e}/dt$ versus χ_0. In the scope of the linear theory the scaled absorption rate does not

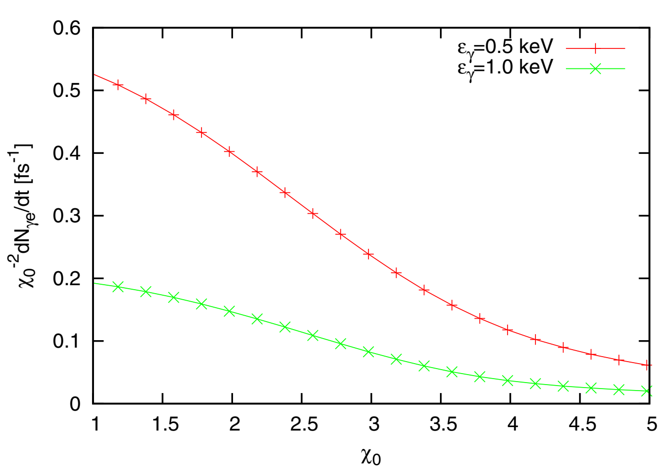

FIG. 5. (Color online) Total rates of the inverse-bremsstrahlung absorption scaled to χ_0^2 vs the parameter χ_0 for setup of Fig. 3.

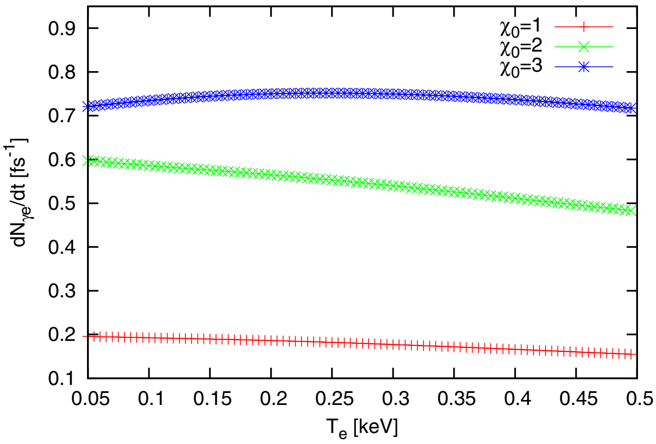

FIG. 6. (Color online) Total rate of inverse-bremsstrahlung absorption of circularly polarized wave in Maxwellian plasma, as a function of the plasma temperature is shown for various wave intensities at $\varepsilon_\gamma = 1$ keV, $Z_a = 13$, and $n_e = 10^{23}$ cm^{-3}.

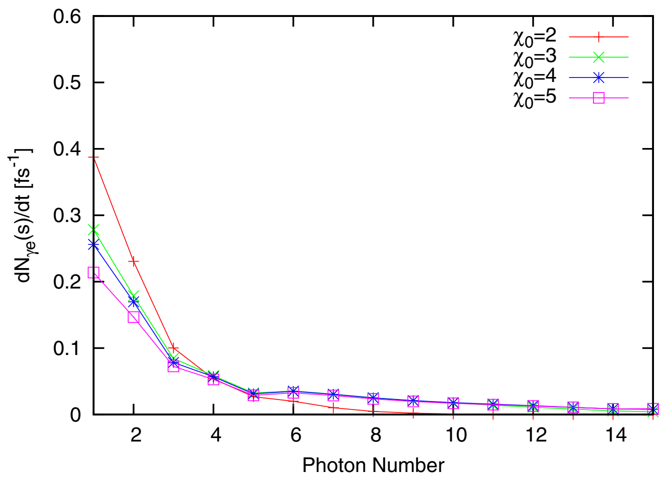

FIG. 8. (Color online) Same as Fig. 7 but for linearly polarized wave.

depend on the wave intensity, while for the large values of χ_0 it is suppressed with the increase of the wave intensity. To show the dependence of the considered process on the plasma temperature, in Fig. 6 we plot the total rate of the inverse-bremsstrahlung absorption of circularly polarized laser radiation in Maxwellian plasma, as a function of the plasma temperature for various wave intensities at $\varepsilon_\gamma = 1$ keV, and $n_e = 10^{23}$ cm^{-3}. The similar picture holds for linearly polarized waves. Here for the large values of χ_0 we have a weak dependence on the temperature, which is a result of the laser modified scattering of electrons irrespective of its' initial state.

We have also made calculations for a degenerate quantum plasma. In Fig. 7 and Fig. 8 the envelope of the partial rate of inverse-bremsstrahlung absorption for circularly and linearly polarized waves in degenerate plasma with Fermi energy $\mu \simeq \varepsilon_F = 11.7$ eV (aluminum) is shown for various wave intensities at $\varepsilon_\gamma = 1$ keV, and $T_e = 0.1\varepsilon_F$. The total rate of inverse-bremsstrahlung absorption via the mean number of absorbed photons by per electron, per unit time in degenerate plasma versus the parameter χ_0 at $\varepsilon_F = 11.7$ eV and $T_e = 0.1\varepsilon_F$ is shown in Fig. 9. As is seen from these figures, the rate strictly depends on the wave polarization, and for the large values of χ_0 it is saturated.

Note that our consideration is valid when the pulse duration τ of an EM wave is restricted by the condition

$$\tau < \nu_{\rm eff}^{-1}, \tag{45}$$

where $\nu_{\rm eff}^{-1}$ is the time scale during which the thermalization of the electrons energy in plasma occurs. In the presence of a laser field the electron-ion binary collisions take place with the effective frequency

$$\nu_{\rm eff} \simeq \frac{2\pi Z_a e^4 n_e}{m^2 \langle {\rm v} \rangle^3} L_{\rm cb}, \tag{46}$$

where $L_{\rm cb}$ is the Coulomb logarithm, and $\langle {\rm v} \rangle$ is the mean values of electrons velocity in the laser field. For moderate

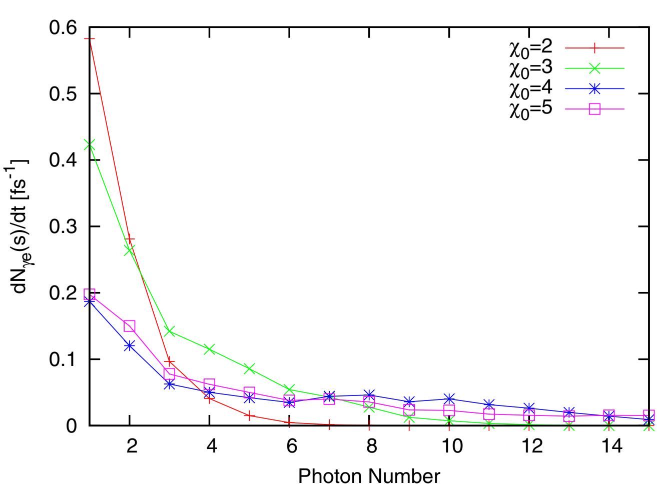

FIG. 7. (Color online) Envelope of partial rate of inverse-bremsstrahlung absorption for circularly polarized wave in degenerate plasma with Fermi energy $\varepsilon_F = 11.7$ eV is shown for various wave intensities at $\varepsilon_\gamma = 1$ keV, $Z_a = 13$, and $T_e = 0.1\varepsilon_F$.

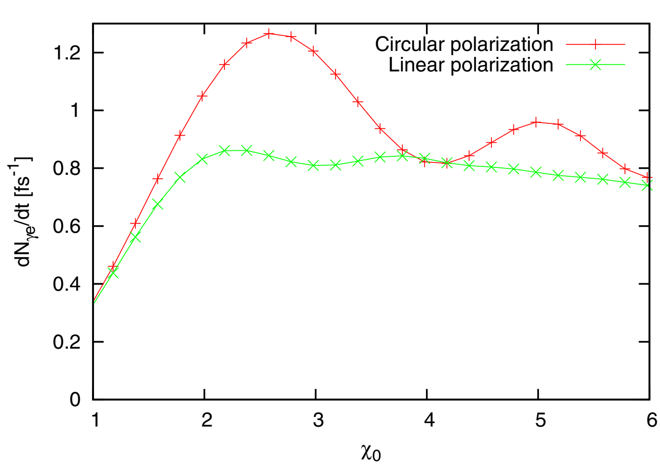

FIG. 9. (Color online) Total rate of inverse-bremsstrahlung absorption for circularly and linearly polarized waves in degenerate plasma vs the parameter χ_0 at $\varepsilon_F = 11.7$ eV, $Z_a = 13$, and $T_e = 0.1\varepsilon_F$.

intensities one can write $\langle \mathrm{v} \rangle \simeq \chi_0 \sqrt{\varepsilon_\gamma / m}$. For the considered parameters we have $\nu_{\mathrm{eff}} \simeq 10^{14}$–$10^{15}\ \mathrm{s}^{-1}$.

In an underdense plasma, there are several types of instabilities [20,21,23], which can be developed on a time scale shorter than the pulse duration. Hence the pulse duration τ of an EM wave should also satisfy the condition

$$\tau < \Gamma^{-1}, \tag{47}$$

where Γ is the maximal increment of the instability of the plasma in the strong wave field. For strong laser fields, the most fast growing instabilities are Raman sidescatter ones [21,23]. The maximal increment of stimulated Raman instability for the range of parameters considered above is given by the formula [21]

$$\Gamma \simeq \frac{\sqrt{3}}{2}\omega\left(\frac{\omega_p^2}{2\omega^2}\xi_0^2\right)^{1/3} = \frac{\sqrt{3}}{2}\omega\left(\frac{\omega_p}{2\omega}\chi_0^2\frac{\hbar\omega_p}{mc^2}\right)^{1/3}, \tag{48}$$

where $\omega_p = (4\pi e^2 n_e/m)^{1/2}$ is the plasma frequency. For the considered parameters we have $\Gamma/\omega \simeq 5 \times 10^{-3}$. Thus the x-ray pulse duration should be $\tau < 0.1$ fs. The latter can be satisfied for x-ray sources. As is seen from Figs. 3, 4, and 9, for the pulse durations $\tau \lesssim 0.1$ fs one can achieve 100 eV absorbed energy by per electron, which means that in plasma of solid densities one can reach the plasma in excess of 10^6 kelvin by x-ray lasers with the intensity parameter $\xi_0 \sim 0.1$.

IV. CONCLUSION

Concluding, we have presented the microscopic relativistic quantum theory of multiphoton inverse-bremsstrahlung absorption of an intense shortwave laser radiation in the classical and quantum plasma. The Liouville–von Neumann equation for the density matrix has been solved analytically considering a wave-field exactly, while a scattering potential of the plasma ions as a perturbation. With the help of this solution we derived a relatively compact expression for the nonlinear inverse-bremsstrahlung absorption rate when electrons are represented by the grand canonical ensemble. Numerical investigation of the obtained results for Maxwellian, as well as for degenerate quantum plasmas at x-ray frequencies and large values of laser fields has been performed. The obtained results demonstrate that for the shortwave radiation the SB rate, being practically independent of the plasma temperature, is saturated with the increase of the wave intensity. The obtained results showed that in the x-ray domain of frequencies one can achieve an efficient absorption of powerful radiation specifically in plasma of solid densities and reach the plasma heating of high temperatures.

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