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Bistable intrinsic charge fluctuations of a dust grain subject to secondary electron emission in a plasma

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A master equation was formulated to study intrinsic charge fluctuations of a grain in a plasma as ions and primary electrons are attached to the grain through collisional collection, and secondary electrons are emitted from the grain. Two different plasmas with Maxwellian and non-Maxwellian distributions were considered. The fluctuations could be bistable in either plasma when the secondary electron emission is present, as two stable macrostates, associated with two stable roots of the charge net current, may exist. Metastablity of fluctuations, manifested by the passage of the grain charge between two macrostates, was shown to be possible.

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I. INTRODUCTION

Various mechanisms, including ion and electron collisional collection and resulting secondary emission of ions or electrons, contribute to the charging of a dust grain in a plasma. Since the collision of plasma particles with the grain occurs at random times, the net electric charge possessed by the grain fluctuates in time even if the plasma parameters such as temperature and number densities are fixed. These kinds of fluctuations, which take place in systems with discrete particles, are known as *intrinsic noise* [1]. This noise cannot be switched off as it is inherent in the actual physical mechanism, e.g., electron or ion electron collision or emission in grain charging mechanism, which is responsible for the evolution of the system. Intrinsic charge fluctuations refer to random variation of the grain charge by this intrinsic noise.

Description of intrinsic charge fluctuations of grains was the subject of a number of studies [2–15]. Cui and Goree [2] studied the fluctuations through a Monte Carlo approach and concluded that they are most important for small grains. The grain charge is correlated with the grain size so this conclusion is consistent with the net elementary charge Z possessed by the grain having fluctuations with $Z_{\rm rms} \propto \sqrt{|\langle Z\rangle|}$ suggested by Morfill *et al.* [16]. Cui and Goree [2] also showed that the fluctuating charge of small grains could experience positive values. It is known that the grain mean charge at equilibrium is negative because a charging grain collects mobile electrons more than ions until it reaches equilibrium. Matsoukas and Russell [3] proposed a one-step process master equation [1] for the grain charge density function, then derived a Fokker-Planck equation for it and showed that if the condition $e^2/4\pi\epsilon_0 R k_B T_e \ll 1$, where R is the radius of the grain and T_e is the electron temperature, is satisfied, the charge distribution at stationary states is Gaussian with average and variance related to the ion and electron currents to the grain. Defining the system size as $\Omega = 4\pi\epsilon_0 R k_B T_e/e^2$, Shotorban [9] derived a Gaussian solution at nonstationary states for the Fokker-Planck equation formulated through the system size expansion of the master equation [1]. In the nonstationary state Gaussian solution, the rate of the mean grain charge correlates with the rate of charge to the net current. This mean equation is the macroscopic equation [1] of the grain charging system, and it is the same equation widely used for negligible fluctuations, which is the charge conservation law for the grain. The rate of the grain charge variance is correlated with the currents and their derivatives evaluated at the charge mean. Shotorban [13] lately extended this model to include multicomponent plasmas where there are various kinds of singly or multiply charged negative or positive ions and showed that the grain charge distribution still follows Gaussianity when Ω is sufficiently large.

In all the above discussed references, collisional collections of electrons and ions were the only mechanism of charging. The secondary electron emission (SEE) of electrons is another mechanism, which is important when sufficiently energetic electrons or ions are present. The electron yield, the ratio of emitted electrons to incident ones, is a function of primary electron energy and the physical and chemical state of the surface. Gordiets and Ferreira [6] obtained an analytical solution for the PDF at stationary states for a master equation that included the effect of the electron detachment, e.g., SEE, assuming that the grain charge does not experience positive values, i.e, a half-infinite range $Z = 0, -1, -2, \dots$. This kind of stationary-state solution is unique for the master equation of a general one-step process with a half-infinite or finite range of the variable whereas it is not unique for a sample space consisting of all integers [1]. Later, Gordiets and Ferreira [8] formulated an improved version of the master equation that they had originally proposed [6], relaxed the half-infinite range assumption, and derived an approximate analytical solution for the PDF at the stationary state. This approximation is not well justified for grains where the PDF varies substantially in a small range of charges. Khrapak *et al.* [7] studied the effects of thermionic emission and UV irradiation, separately, while electron collisional collection was present. They concluded that $Z_{\rm rms} \propto \sqrt{|\langle Z\rangle|}$ is valid for these situations as well. Lately, Mishra and Misra [15] studied the fluctuations in a multicomponent plasma through a population balance equation resembling the master equation. They also included the influence of photoemission from dust through irradiation by laser light in their study. It is noted that all works above but Refs. [10,14] rely on Markov approaches for a description of grain charge fluctuations.

The most well-known effect of SEE on grain charging is perhaps its associated bifurcation phenomena: Two identical

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grains in the same plasma environment may have two different stable charge values, one positive and one negative [17,18]. Thus a small variation in the parameters may cause a rapid change from one equilibrium charge to another. Interestingly, Lai [19] showed that in the spacecraft charging, the bifurcation phenomena caused by SEE could involve three stable equilibrium charge values. The experimental study of Walch *et al.* [20] on charging of grains with energetic electrons that resulted in secondary electron emission, showed the distribution of grain charge could be bimodal. However, they asserted that the lack of a unique value may be due to fluctuations in the plasma parameters or small differences in the grains.

The current study is on the influence of SEE on grain charge intrinsic fluctuations with a focus on bistability. Bistability occurs in stochastic systems with two stable macrostates [1,21]. The bistability of the grain charging system is associated with the bifurcation phenomena described above. The fluctuations in a bistable system may be metastable [1], where the fluctuations are at one macrostate for a while and at a random time, a passage to the other macrostate takes place and at a random time, the system returns to the first macrostate. Whether or no the metastability of grain charge intrinsic fluctuations occurs is investigated in this work. In Sec. II, first, a master equation describing the fluctuations of the grain charge in the presence of SEE mechanism is presented, and then currents of ions, primary electrons and secondary electrons of a Maxwellian plasma and a non-Maxwellian plasma are shown. In Sec. III, results are shown and discussed. Conclusions are made in Sec. IV.

II. MATHEMATICAL FORMULATION

Assuming that the charging of the grain undergoes a Markov process, the following master equation can be formulated for the probability density function of the grain charge $P(Z,t)$:

$$\frac{dP(Z,t)}{dt} = (\mathbb{E}-1)f_0(Z)I_e(Z)P(Z) + \sum_{n=1}^{M-1}(\mathbb{E}^{-n}-1)f_{n+1}(Z)I_e(Z)P(Z) + (\mathbb{E}^{-1}-1)I_i(Z)P(Z), \tag{1}$$

where $\mathbb{E}$ is an operator defined by $\mathbb{E}^k g(Z) = g(Z+k)$ for any integer number k, n indicates the number of secondary electrons emitted from the grain upon the impact of one primary electron, M is the maximum number of secondary electrons that can be emitted, $I_i(Z)$ and $I_e(Z)$ are the currents of ions and primary electrons to the grain, respectively, and $f_n(Z)$ is the probability distribution of emission of n electrons in a single incident of a primary impact, i.e., the fraction of primary electrons that result in the emission of n secondary electrons in one single attachment incident. Hence, the rate of the attachment of the primary electrons that do not cause secondary emission is $f_0(Z)I_e(Z)$, and $f_n(Z)I_e(Z)$ indicate the rate of the attachment of the primary electrons that cause the emission of n secondary electrons in one incident. The jump process associated with the master Eq. (1) is a sudden change that the grain charge experiences once a primary electron is attached and secondary electrons are emitted, or once an ion is attached. In other words, $Z(t)$, the charge of the grain at time t, jumps to $Z(t)+n-1$ once a primary electron is attached and n secondary electrons are emitted, and jumps to $Z(t)+1$ once an ion is attached. The master equation of Gordiets and Ferreira [8] is a special case of Eq. (1) with $M=3$. Also, two following special cases of the master equation (1) regarded as one-step processes are worth noting.

(1) $M=0$, which corresponds to a case where no SEE occurs, i.e., $f_0(Z)=1$ and $f_n(Z)=0$ for $n>0$. In this case, the second term on the right-hand side of Eq. (1) vanishes and the master equation of the grain charing is retrieved [3,9].

(2) $M=1$, which corresponds a case that at most one secondary electron is emitted so $f_0(Z)<1$ and $f_1(Z)=1-f_0(Z)$.

Defining the system size Ω as a reference constant charge number and having changed the variable $Z=\Omega\phi(t)+\Omega^{1/2}\xi$, where Z is modeled by a combination of a deterministic part $\phi(t)$ scaled by Ω, and a random part ξ scaled by $\Omega^{1/2}$, a macroscopic equation associated with Eq. (1) can be derived through the system size expansion method [1,13]:

$$\frac{d\phi}{dt} = a_1(\phi), \tag{2}$$

where $a_1(\phi)=\Omega^{-1}I_n(\Omega\phi)$, $I_n(.)=I_i(.)-I_e(.)+I_s(.)$ is the net current to the grain, and $I_s(.)$ is the SEE current to the grain. A solution of the macroscopic Eq. (2) is a time-dependent macrostate of the grain charging system while the solution of $a_1(\phi)=0$ is a stationary macrostate of the system [1].

Van Kampen [1] classifies the stable, bistable, and unstable stochastic systems through $a_1(\phi)$ in Eq. (2): A stochastic system is stable when $a_1'(\phi)<0$ where $a_1'(\phi)\equiv da_1/d\phi$; it is bistable when there are two stable stationary macrostates, i.e., there are two solutions for $a_1(\phi)=0$ and at the vicinities of them, $a_1'(\phi)<0$ holds; and it is unstable when $a_1'(\phi)>0$. This classification is in harmony with the bifurcation phenomenon reviewed in Sec. I based on the roots of the net current. In other words, having neglected the fluctuations of Z, i.e., $\xi=0$, one obtains $Z=\Omega\phi$ and Eq. (2) is readily simplified to $dZ/dt=I_n(Z)$, which can be used to find the time evolution of Z. If an initial Z is within the domain of attraction of a stable root of $I_n(Z)=0$, then Z approaches it at the stationary state when there are no fluctuations. Each root is associated with one stationary macrostate of the system and the system stability discussed above can be similarly done through the sign of $I_n'(Z)$. When the fluctuations of the grain charge are taken into account, there is a probability for a fluctuation to carry the charge from the domain of attraction of one root to another.

The SEE current is correlated with $I_e(Z)$ and $f_n(Z)$, and this correlation is found through the mean secondary electron yield defined by

$$\overline{n}(Z) = I_s(Z)/I_e(Z). \tag{3}$$

In addition, $\overline{n}(Z)$ is correlated with $f_n(Z)$ through the definition of the mean $\overline{n}(Z)=\sum_{n=1}^{M} nf_n(Z)$. Using these two equations and the normalization condition, i.e.,

$\sum_{n=0}^{M} f_n(Z) = 1$, one obtains

$$I_s(Z) = \left[1 - f_0(Z) + \sum_{n=2}^{M}(n-1)f_n(Z)\right]I_e(Z). \quad (4)$$

Here, a binomial distribution is proposed for $f_n(Z)$:

$$f_n(Z) = \binom{M}{n} p^n (1-p)^{M-n}, \quad (5)$$

where for $M > 0$, $p = \overline{n}(Z)/M$ where $\overline{n}(Z)$ is given in Eq. (3). Binomial distributions are used for the Monte Carlo modeling of SEE in the electron-surface collision [22]. For $M = 1$ in Eq. (5), $f_0(Z) = 1 - \overline{n}(Z)$ and hence, $f_0(Z)I_e(Z)P(Z) = [I_e(Z) - I_s(Z)]P(Z)$. For $M = 1$, the summation term in Eq. (4) is zero. It is noted that $I_e(Z) - I_s(Z)$ is the net electric current to the grain so the charging process for $M = 1$ is modeled as each primary electron impact incident causing no or only one secondary electron emission. In Eq. (5), a sufficient condition for the positivity of $f_n(Z)$ is $1 - p > 0$, which is equivalent to

$$\overline{n}(Z) < M. \quad (6)$$

This inequality sets the requirement for the minimum M.

Following Meyer-Vernet [17], who investigated the bifurcation phenomena associated with SEE, a Maxwellian plasma and non-Maxwellian plasma are considered here. For Maxwellian plasmas, it can be shown [13,23] that

$$I_e(Z) = \Gamma \times \begin{cases} 1 + \frac{Z}{\Omega} & Z \geqslant 0, \\ \exp\left(\frac{Z}{\Omega}\right) & Z < 0, \end{cases} \quad (7)$$

$$I_i(Z) = \Gamma \widehat{n}_i \sqrt{\frac{\widehat{T}_i}{\widehat{m}_i}} \times \begin{cases} 1 - \frac{Z}{\widehat{T}_i\Omega} & Z \leqslant 0, \\ \exp\left(-\frac{Z}{\widehat{T}_i\Omega}\right) & Z > 0, \end{cases} \quad (8)$$

where n_i and T_i are the number density and temperature of ions, respectively. Also, $\widehat{T}_i = T_i/T_e$, $\widehat{m}_i = m_i/m_e$, $\widehat{n}_i = n_i/n_e$,

$$\Omega = \frac{4\pi\epsilon_0 R k_B T_e}{e^2}, \quad (9)$$

$$\Gamma = \pi R^2 n_e \sqrt{\frac{8k_B T_e}{\pi m_e}} = \frac{\Omega \omega_{\rm pe} R}{\sqrt{2\pi}\lambda_{De}}, \quad (10)$$

where $\lambda_{De} = \sqrt{\epsilon_0 k_B T_e / n_e e^2}$ is the electron Debye length and $\omega_{pe} = \sqrt{n_e e^2/\epsilon_0 m_e}$ is the electron plasma frequency.

Using the theory of Sternglass [24], the SEE current is obtained [17]:

$$I_s(Z) = 3.7\delta_M \Gamma \times \begin{cases} \left(1 + \frac{Z}{\Omega \widehat{T}_s}\right)\exp\left(-\frac{Z}{\Omega\widehat{T}_s} + \frac{Z}{\Omega}\right)F_{5,B}\left(\frac{E_M}{4k_B T_e}\right) & Z \geqslant 0, \\ \exp\left(\frac{Z}{\Omega}\right)F_5\left(\frac{E_M}{4k_B T_e}\right) & Z < 0, \end{cases} \quad (11)$$

where

$$F_5(x) = x^2 \int_0^\infty u^5 \exp(-xu^2 - u)du,$$

$$F_{5,B}(x) = x^2 \int_B^\infty u^5 \exp(-xu^2 - u)du,$$

where $B = \sqrt{4k_B T_e Z/\Omega E_M}$ and $\widehat{T}_s = T_s/T_e$ where T_s is the temperature of the emitted secondary electrons. In Eq. (11), δ_M is the maximum yield which is around unity for metals and at the order 2–30 for insulators, and E_M is the peak primary electron energy, a model constant ranging from 300 to 2000 eV. The values of these two parameters for various dust materials can be found in Ref. [17].

Electrons in the non-Maxwellian plasma are assumed to have a bi-Maxwellian distribution that occurs in a number of circumstances. A bi-Maxwellian distribution was derived by Weibel [25] from the Boltzaman equation for electrons with anisotropic distribution of velocities that cause spontaneous growth of transverse electromagnetic waves. This phenomenon, also known as Weibel instability, is observed in astrophysical plasmas ([26], and references therein) and fusion plasmas ([27], and references therein). A bi-Maxwelllian distribution of electrons was also observed in several laboratory setups including Ref. [28]. The experiments of Godyak *et al.* [29] revealed that electrons have a non-Maxwellian distribution in low-pressure capacitively coupled rf plasmas. At a very low pressure, where stochastic electron heating dominates, the distribution is bi-Maxwellian whereas at a moderately low pressure, where collisional electron heating dominates, the distribution is Druyvesteyn. The electron current to the grain in the bi-Maxwellian plasma is obtained by adding an identical term, where n_e and T_e are to be replaced by n_H and T_H, to the right side of Eq. (7) and changing n_e to $n_e - n_H$ in the first term when using this distribution [17].

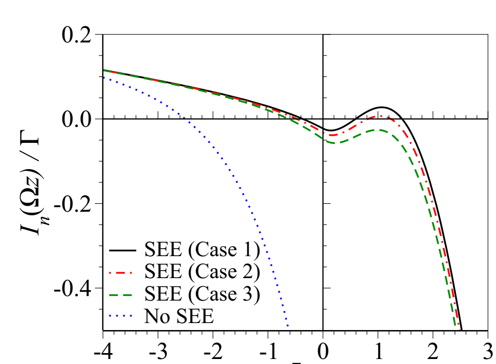

FIG. 1. (Color online) Dimensionless net current versus dimensionless grain charge $z = Z/\Omega$, where $\Omega = 4\pi\epsilon_0 R k_B T_e/e^2$ and $\Gamma = \pi R^2 n_e \sqrt{8k_B T_e/\pi m_e}$, for $T_s/T_e = 1.5$ in a Maxwellian plasma [17]; case 1, $\delta_M = 15$ and $E_M/4kT_e = 45.6$; case 2, $\delta_M = 14.85$ and $E_M/4kT_e = 45.6$; case 3, $\delta_M = 15$ and $E_M/4kT_e = 47$.

III. RESULTS AND DISCUSSIONS

Dimensionless net current is plotted against dimensionless grain potential in Fig. 1 for a Maxwellian plasma [17]. Seen in this figure is that when the SEE mechanism is lacking, the net current curve crosses the horizontal axis only at one point so there is only one root. The system is stable in this case as $I_n'(Z) < 0$ for all values of Z. This negativity is due to attaching electrons, which are more mobile than attaching ions. On the other hand, it is seen in the figure that when the SEE mechanism is present, the net current may have up to three roots, one negative and two positive roots (cases 1 and 2). The root at the middle is unstable whereas two others are stable so the grain charge fluctuations are bistable in cases 1 and 2. In case 3, only one stable root exists and for all values of Z except the domain restricted between local maxima and minima, the system is stable. It is noted that the SEE cases seen in Fig. 1 are different through small changes made in the SEE current parameters δ_M or E_M. The triple root situations in Maxwellian plasmas are conditional on both a high value of δ_M and a somewhat low value of E_M/k_BT_s [17].

The PDF of the grain charge obtained through a numerical solution of the master equation (1) for a Maxwellian plasma (cases 1–3 illustrated in Fig. 1) for two grain sizes $R = 5$ and $R = 30$ nm are shown in Fig. 2. A prominent deviation from Gaussian distribution is observed for most of the cases seen in this figure. However, the PDF was verified to be very close to Gaussian for $M = 1$ in Figs. 2(d)–2(f) as compared to a Gaussian solution obtained by the system size expansion method [9,13]. For $R = 5$ nm and $M = 1$, seen in Fig. 2(c), the PDF is bimodal, i.e., with two distinct local maxima, and for two other values of M, it is not. It is born in mind that for all cases in Figs. 2(a)–2(d), there are two stable roots of the net current so they all are considered bistable according to the

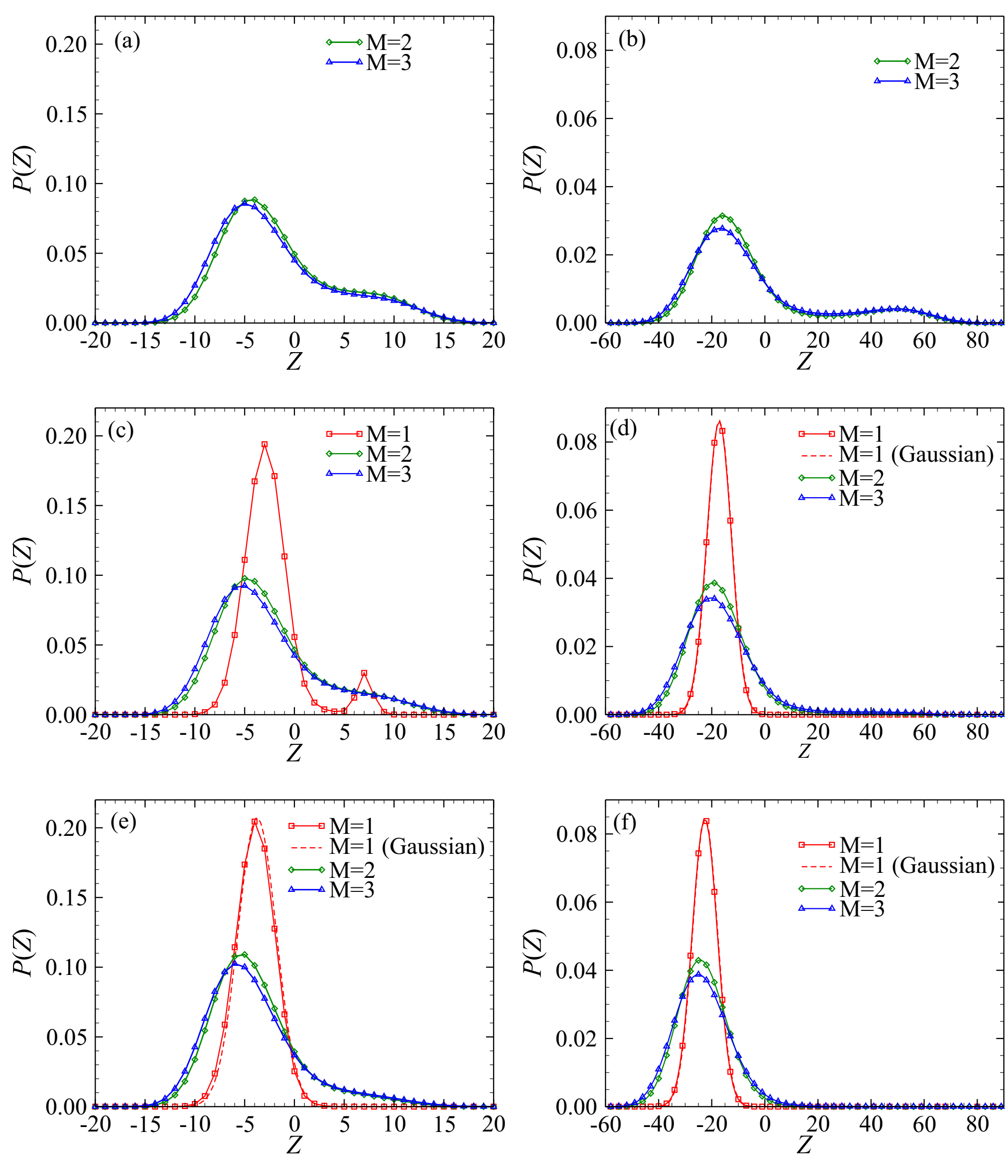

FIG. 2. (Color online) Probability density function of grain charge in a Maxwellian plasma; (a) case 1 with $R = 5$ nm; (b) case 1 with $R = 30$ nm; (c) case 2 with $R = 5$ nm; (d) case 2 with $R = 30$ nm; (e) case 3 with $R = 5$ nm; (f) case 3 with $R = 30$ nm. Gaussian solutions are obtained by the system size expansion method [9,13]. See the caption of Fig. 1 for parameters associated with cases 1–3.

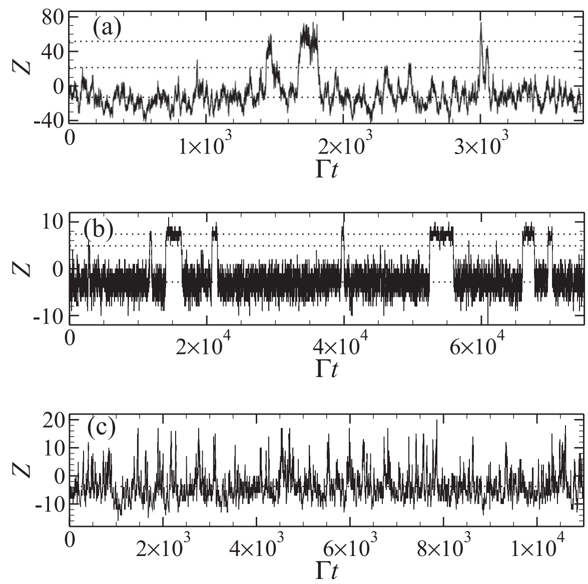

FIG. 3. Grain charge variation in time in a Maxwellian plasma with SEE; (a) case 1 with $R = 30$ nm and $M = 3$; (b) case 2 with $R = 5$ nm and $M = 1$; (c) case 3 with $R = 5$ nm and $M = 3$. The dotted lines show the roots of the net current.

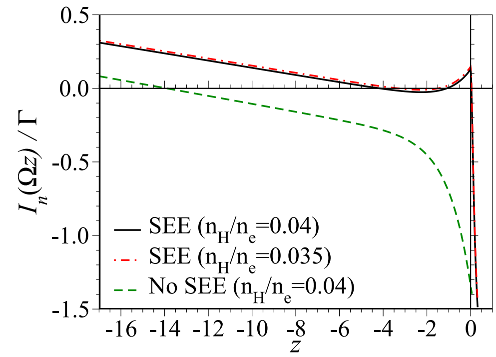

FIG. 4. (Color online) Dimensionless net current versus dimensionless grain charge $z = Z/\Omega$ in a bi-Maxwellian plasma; $T_e = T_i = 25$ eV, $\delta_M = 3$, $E_M/k_B T_e = 16$, $T_s/T_e = 1$, $T_H/T_e = 100$, and $M = 3$ [17].

classification at the beginning of this section. A bimodal PDF is also seen in Fig. 2(b) for a larger grain with $R = 30$ nm. However, when the same SEE parameters are used for smaller grain $R = 5$ nm, no bimodal distribution is observed [see Fig. 2(a)]. No bimodal distribution is observed in Figs. 2(e) and 2(f) which is for the SEE cases with only one root of the net current. Although, in these two subfigures, the deviation of the distribution from Gaussianity is substantial for $M = 2$ and 3.

Figure 3 displays time histories of grain charges. The discrete stochastic method [13], adapted from Gillespie's algorithm [30,31]), is utilized to simulate the grain charge variation governed by the master equation (1). Time histories seen in Figs. 3(a) and 3(b) are for the bistable cases shown in Fig. 2(b) for $M = 3$ and Fig. 2(c) for $M = 1$, respectively. The fluctuations in these two cases are characterized by two distinct time scales: one associated with fluctuations around two stable charges and the other associated with the spontaneous switches between them. A system with this behavior is called metastable [1]. A switch from the negative stable charge to the positive one is attributed to a sequence of incidents most of which increase the grain charge by one or two elementary charges. These incidents could be the attachment of an ion or the attachment of a primary electron that results in the emission of two or more of secondary electrons. On the other hand, the switch from the positive stable charge to the negative one is attributed to a sequence of incidents most of which are the attachments of a primary electron without emitting a secondary electron. Figure 3(c) which corresponds to the PDF shown in Fig. 2(e) with $M = 3$ is not bistable as the net current in this case has only one root.

Shown in Fig. 4 is the net current variation against the grain charge in a bi-Maxwellian plasma. For the shown SEE cases, there are two negative and one positive roots for the net current so the system is bistable in both SEE cases. The positive root is very close to the origin of the coordinates and the net current has a very sharp variation around this root. Although the curves of the SEE cases shown in this figure seem very similar, they are different as the negative roots in the case with $n_H/n_e = 0.035$ are slightly closer to each other than the case with $n_H/n_e = 0.04$. The root of the net current in the "No SEE" case is substantially far from the roots of the SEE cases. Moreover, it is found that in the "No SEE" case, the net charge (the root of the net current) in the bi-Maxwellian plasma, where $n_H/n_e = 0.04$, is with a factor of five larger than that in a Maxwellian plasma where $n_H/n_e = 0$. The reason for this large difference is that a small fraction (slightly over 4%) of the electrons in the bi-Maxwellian plasma have much higher temperature, $T_H/T_e = 100$, whereas all electrons in the Maxwellian plasma have the cold temperature T_e. An increase in the electron temperature results in a more negative net charge on the grain, as an electron has more kinetic energy in the average sense to overcome the repulsion between itself and the grain [3]. It is also noted that a triple root situation is expected to occur in a bi-Maxwellian plasma more than a Maxwellian plasma. According to Meyer-Vernet [17], a T_e higher than a few 10 eV and a T_H higher than several 100 eV, values typical in the solar wind and possible in planetary

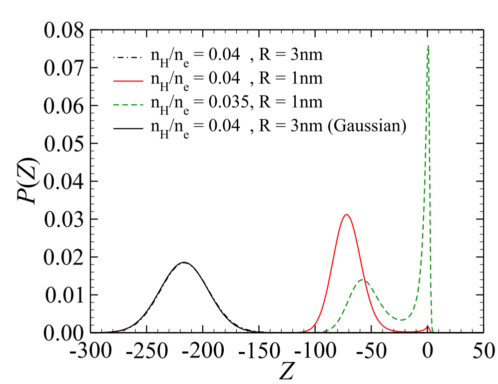

FIG. 5. (Color online) Probability density function of grain charge in a bi-Maxwellian plasma. See the caption of Fig. 4 for parameters.

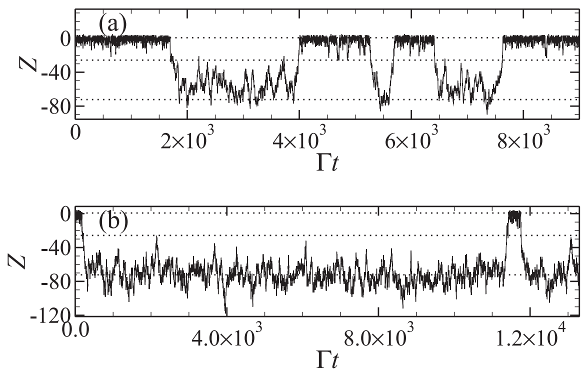

FIG. 6. Grain charge fluctuations in a bi-Maxwellian plasma with SEE, $M = 3$, and initial charge $Z(0) = 0$; (a) $n_H/n_e = 0.035$ and $R = 1$ nm; (b) $n_H/n_e = 0.04$ and $R = 1$ nm.

magnetospheres, should result in triple roots for grains with nonmetallic surfaces.

Figure 5 displays the grain charge PDF in the studied bi-Mawellian plasma for two grain sizes. All three cases shown in this figure are associated with the SEE cases in Fig. 4 and the system is classified bistable in all. However, the bimodal distribution is observed for smaller grain with $R = 1$ nm at both $n_H/n_e = 0.035$ and 0.04. Although the difference between these two values is around 13%, the bimodal forms of their associated PDFs are very different. The peak value of the PDF seen at around $Z = 0$ for $n_H/n_e = 0.035$ is at least an order of magnitude larger than that for $n_H/n_e = 0.04$. For this case, the value of the left peak is an order of magnitude larger than the right peak. For the grain with a larger radius $R = 3$ nm, no bimodal behavior is observed. For this case, also, a Gaussian solution is obtained by the system size expansion with an initial condition $\langle Z(0)\rangle/\Omega = -3$. An excellent agreement between the Gaussian solution and the master equation solution is observed. When $\langle Z(0)\rangle = 0$ is used, the solution at the stationary state is a sharp Gaussian function at around $Z = 0$. Time history of the grain charge is shown in Figs. 6(a) and 6(b) associated with solid- and dashed-line PDFs, respectively, in Fig. 5. An obvious metastability is observed for these two cases.

IV. SUMMARY AND CONCLUSIONS

A master equation was formulated to include the effect of secondary electron emission in addition to collisional attachment of ions and electrons on the intrinsic charge fluctuations of a grain. Grain charging in both Maxwellian and non-Maxwellian plasmas was considered. In both plasmas, the fluctuations could be bistable, as the system could have two stable macrostates. In the absence of the SEE mechanism, the bistabillity is not possible as the system always has a single macrostate. It was shown that if the system is bistable, the grain charge can be metastable. That is a situation where the fluctuations are characterized by two distinct time scales—one associated with fluctuations around two stable charges and the other associated with the spontaneous switches between them. A switch from the negative stable charge to the positive one is attributed to a sequence of incidents almost all of which increase the grain charge by one or two elementary charges. On the other hand, the switch from the positive stable charge to the negative one is attributed to a sequence of incidents most of which are the attachments of a primary electron without resulting in the emission of a secondary electron.

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