Dephasing-assisted selective incoherent quantum transport

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Selective energy transport throughout a quantum network connected to more than one reaction center can play an important role in many natural and technological considerations in photosystems. In this work, we propose a method in which an excitation can be transported from the original site of the network to one of the reaction centers arbitrarily using independent sources of dephasing noises. We demonstrate that in the absence of dephasing noises, the coherent evolution of the system does not have any role in energy transport in the network. Therefore, incoherent evolution via application of dephasing noises throughout a selected path of the network leads to complete transferring of the excitation to a desired reaction center.

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I. INTRODUCTION

The efficient transport of optical excitation energy through a network of coupled many-body quantum systems has recently become the subject of extensive study in both natural and artificial systems [1–3]. A particular example is energy transport in the molecular structure of biological systems ranging in scale from a few atoms to large macromolecular structures, such as light harvesting (photosynthetic) complexes [4–6]. In general, the overall effect of environment on the quantum transport process in a system is expected to be negative. However, in a large variety of quantum systems, such as chromophoric light-harvesting complexes, the interaction with the environment can result in increased quantum transport efficiency. In fact, interplaying between coherent and incoherent dynamics provides the optimal way for quantum transport in many noisy systems [7–10]. Many efforts have been devoted to study the ways in which quantum transport is optimally affected by the interplay of coherent and incoherent dynamics arising from environmental noises [11–23], a phenomenon called environment-assisted quantum transport (ENAQT) or dephasing-assisted quantum transport (DAQT).

On the other hand, there are quantum systems in which the coherent evolution is completely suppressed due to destructive interferences [7–9], and therefore the optimal dynamics of the system is purely incoherent, which is the issue of interactions between the system and its fluctuating environments. Destructive interferences can be removed locally or globally in the quantum system when it is affected by its fluctuating environment [10]. Indeed, energy transport through pure incoherent evolution in a quantum system can be regarded as direct evidence for the remarkable existence of long-lived quantum coherence and wavelike behavior, which plays an important role in this way [4].

In this paper, we investigate selective quantum transport of excitation energy throughout a regular network, such as a two-dimensional hexagonal-like network of interacting two-level chromophores or sites. As illustrated in the text, there are a number of sinks considered as reaction centers irreversibly attached to the network. The network structure is considered in such way that the coherent part of the dynamics of an excitation created in one of the sites, such as an initial site, is completely suppressed. Therefore, the selective nature of the transport in the network is related to the selective applications of local independent dephasing noises along a path, such as a one-dimensional quantum transport prototype, which connect the initial site to one of the sinks. In fact, application of dephasing noises along a particular path removes the destructive interferences throughout that path in the network. In this way, the excitation energy is incoherently transported from the initial site to the aforementioned sink. This process can also be regarded to implement for the other reaction centers. On the other hand, to evaluate the optimality of transport in the network, we investigate the optimal effect of dephasing noises on the efficiency of transport along the one-dimensional prototype. Also, in this way, the effect of energy mismatch between sites on the efficiency of transport is highlighted. It is observed that the optimal transport is robust with respect to the dephasing noises and energy mismatches. Therefore, it is concluded that the optimal conditions for the transport in a two-dimensional network lie within the optimal conditions of the one-dimensional case.

This paper is organized as follows: In Sec. II, we demonstrate one-dimensional incoherent quantum transport along with an investigation of its optimality conditions, which, in turn, make up the basic ingredients for quantum transport in a two-dimensional hexagonal-like network. Section III is devoted to explaining the transport of excitation energy in the two-dimensional network incoherently, using local independent dephasing noises. Finally, a brief conclusion is presented in Sec. IV.

II. ONE-DIMENSIONAL QUANTUM TRANSPORT PROTOTYPE

We consider a network such as that depicted in Fig. 1(a), in which the vertices or sites are as two-level chromophoric systems interacting with each other, corresponding to edges of the network. The Hamiltonian for this system is considered as

$$H = \sum_{j=1}^{3N+1} \hbar\omega_j \sigma_j^+ \sigma_j^- + \sum_{\{j,l\}\in E} \hbar\nu_{j,l}(\sigma_j^- \sigma_l^+ + \sigma_j^+ \sigma_l^-), \quad (1)$$

where $\sigma_j^+ = |j\rangle\langle 0|$ and $\sigma_j^- = |0\rangle\langle j|$ are the raising and lowering operators for a two-level system positioned at the

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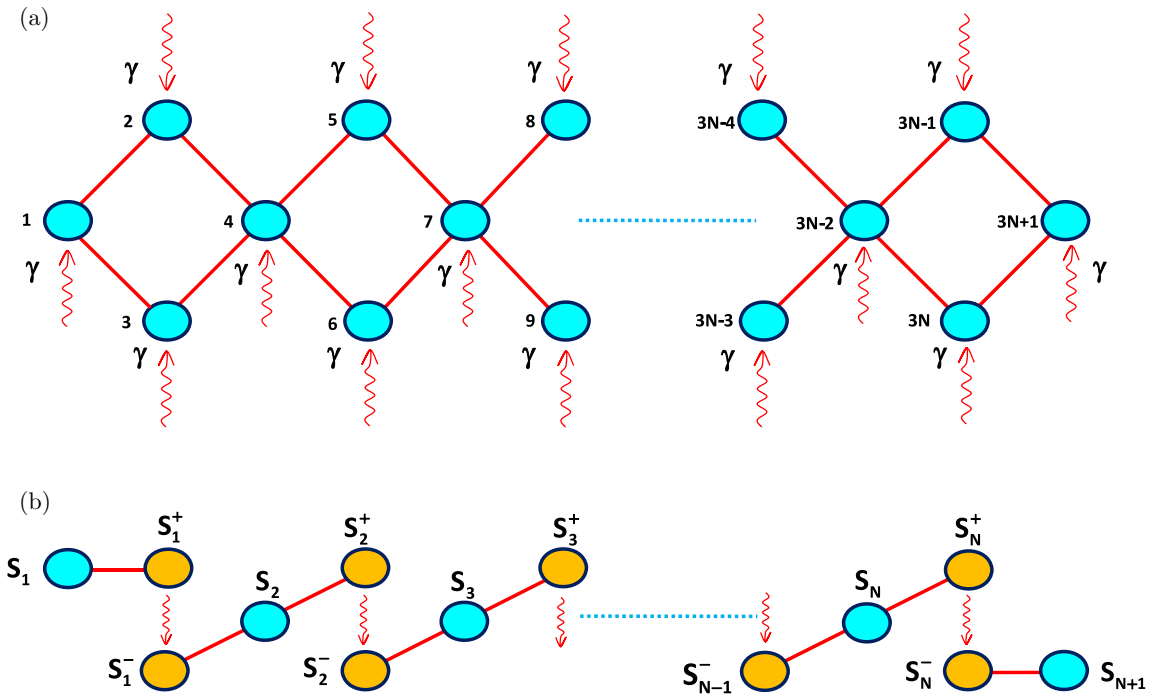

FIG. 1. (Color online) (a) A network of two-level systems coupled to each other as $\nu_{3j-2,3j-1} = \nu_{3j-2,3j} = \nu_{3j-1,3j+1} = -\nu_{3j,3j+1} = J$ and attached to independent dephasing noises. (b) The same network under the change of basis. Invariant subspaces are incoherently connected by the dephasing noises.

jth vertex or site; the state $|j\rangle$ denotes an excitation in site j, and $|0\rangle$ indicates no excitation in that site. The energy of a typical site j is $\hbar\omega_j$, $\nu_{j,l}$ is the strength of coupling between the jth and lth sites, denoting the hopping rate of an excitation between them, and E is the set of edges of the network depicted in Fig. 1(a), corresponding to the coupling between the sites. In general, we have considered $\nu_{3j-2,3j-1} = \nu_{3j-2,3j}$ and $\nu_{3j-1,3j+1} = -\nu_{3j,3j+1}$ for the coupling strength, and $\omega_{3j-1} = \omega_{3j}$ for site energies with $j = 1,2,3,\ldots,N$. We introduce another set of basis in the single excitation subspace in terms of a standard basis as

$$\begin{aligned} |s_j\rangle &:= |3j-2\rangle, \quad |s_{j+1}\rangle := |3j+1\rangle, \\ |s_j^{\pm}\rangle &:= \frac{1}{\sqrt{2}}(|3j-1\rangle \pm |3j\rangle), \end{aligned} \tag{2}$$

where $j = 1,2,3,\ldots,N$. The transformation that gives the basis set (2) from the standard ones is as follows:

$$\begin{aligned} U = &\sum_{j=1}^{N+1} |3j-2\rangle\langle 3j-2| + \frac{1}{\sqrt{2}}\sum_{j=1}^{N}(|3j-1\rangle\langle 3j-1| \\ &+ |3j-1\rangle\langle 3j| + |3j\rangle\langle 3j-1| - |3j\rangle\langle 3j|). \end{aligned} \tag{3}$$

U is clearly unitary, i.e., $UU^\dagger = U^\dagger U = 1$. Hence, by choosing the basis set in (2), the Hamiltonian (1) is left with a direct sum structure as [see Fig. 1(b)]

$$H = \bigoplus_{j=1}^{N+1} H_j, \tag{4}$$

where

$$\begin{aligned} H_1 &= \omega_1|s_1\rangle\langle s_1| + \omega_2|s_1^+\rangle\langle s_1^+| + \sqrt{2}\nu_{1,2}(|s_1\rangle\langle s_1^+| + |s_1^+\rangle\langle s_1|), \\ H_{j+1} &= \omega_{3j-1}|s_j^-\rangle\langle s_j^-| + \omega_{3j+1}|s_{j+1}\rangle\langle s_{j+1}| + \omega_{3j+2}|s_{j+1}^+\rangle \\ &\quad \times \langle s_{j+1}^+| + \sqrt{2}\nu_{3j-1,3j+1}(|s_j^-\rangle\langle s_{j+1}| + |s_{j+1}\rangle\langle s_j^-|) \\ &\quad + \sqrt{2}\nu_{3j+1,3j+2}(|s_{j+1}\rangle\langle s_{j+1}^+| + |s_{j+1}^+\rangle\langle s_{j+1}|), \\ &\quad j = 1,2,3,\ldots,N-1, \\ H_{N+1} &= \omega_{3N-1}|s_N^-\rangle\langle s_N^-| + \omega_{3N+1}|s_{N+1}\rangle\langle s_{3N+1}| \\ &\quad + \sqrt{2}\nu_{3N-1,3N+1}(|s_N^-\rangle\langle s_{N+1}| + |s_{N+1}\rangle\langle s_N^-|), \end{aligned} \tag{5}$$

and their respective invariant subspaces can be regarded as

$$\begin{aligned} \mathcal{H}_1 &= \text{span}\{|s_1\rangle, |s_1^+\rangle\}, \\ \mathcal{H}_{j+1} &= \text{span}\{|s_j^-\rangle, |s_{j+1}\rangle, |s_{j+1}^+\rangle\}, \quad j = 1,2,3,\ldots,N-1, \\ \mathcal{H}_{N+1} &= \text{span}\{|s_N^-\rangle, |s_{N+1}\rangle\}. \end{aligned} \tag{6}$$

To give a clear physical picture for the basis introduced in (2), we assume, without loss of generality, that $\omega_j = \omega$ and $\nu_{3j-2,3j-1} = \nu_{3j-2,3j} = \nu_{3j-1,3j+1} = -\nu_{3j,3j+1} = J$. By this consideration, the Hamiltonian (1) becomes

$$\begin{aligned} H = &\sum_{j=1}^{N} \hbar J[\sigma_{3j-2}^-(\sigma_{3j-1}^+ + \sigma_{3j}^+) + \sigma_{3j-2}^+(\sigma_{3j-1}^- + \sigma_{3j}^-) \\ &+ (\sigma_{3j-1}^- - \sigma_{3j}^-)\sigma_{3j+1}^+ + (\sigma_{3j-1}^+ - \sigma_{3j}^+)\sigma_{3j+1}^-], \end{aligned} \tag{7}$$

where we have neglected the term responsible for the trivial dynamics. It is assumed that, at $t = 0$, we have an excitation in the initial site of the network ($|S_1\rangle := |1\rangle$). After time t, it

is evolved by the Hamiltonian (7) as

$$e^{-iHt}|S_1\rangle = \cos(\sqrt{2}Jt)|S_1\rangle - i\sin(\sqrt{2}Jt)|S_1^+\rangle. \quad (8)$$

As is observed, the excitation prepared at the initial site of the network evolves to share itself between sites 2 and 3, and therefore it constructs the entangled state $|S_1^+\rangle$ after time $t = \frac{\pi}{2\sqrt{2}J}$ and ultimately oscillates between $|S_1\rangle$ and $|S_1^+\rangle$. In other words, as is clear from (8), the evolution of the excitation prepared at the initial site is restricted to the subspace $\mathcal{H}_1$ and is governed by a (sub)Hamiltonian $H_1 = \sqrt{2}J(|s_1\rangle\langle s_1^+| + |s_1^+\rangle\langle s_1|)$ [24]. Now consider the case in which an excitation is shared between two sites $3j-1$ and $3j$, such that we have the entangled state $|S_j^-\rangle$ ($j = 1,2,\ldots,N-1$). Then the evolution of this state is given by

$$e^{-iHt}|S_j^-\rangle = \cos^2(Jt)|S_j^-\rangle - i\sqrt{2}\sin(Jt)\cos(Jt)|S_{j+1}\rangle - \sin^2(Jt)|S_{j+1}^+\rangle. \quad (9)$$

The state $|S_j^-\rangle$ evolves to $|S_{j+1}^+\rangle$ after time $t = \frac{\pi}{2J}$ and vice versa. This evolution relies in the subspace $\mathcal{H}_{j+1}$ and is governed by the (sub)Hamiltonian $H_{j+1} = \sqrt{2}J(|s_j^-\rangle\langle s_{j+1}| + |s_{j+1}\rangle\langle s_j^-| + |s_{j+1}\rangle\langle s_{j+1}^+| + |s_{j+1}^+\rangle\langle s_{j+1}|)$ [24]. And finally, in a similar way as (8), the evolution of excitation from the entangled state $|S_N^-\rangle$ under Hamiltonian H in (7) only leads to the state $|S_{N+1}\rangle$ and vice versa. Therefore, the Hamiltonian H only acts on the subspace $\mathcal{H}_{N+1}$, and its action is equivalent to the action of (sub)Hamiltonian $H_{N+1} = \sqrt{2}J(|s_N^-\rangle\langle s_{N+1}| + |s_{N+1}\rangle\langle s_N^-|)$ on that subspace. Therefore, we see that the basis introduced in (2) is obtained fundamentally from time evolution processes of the system.

These arguments show that Hamiltonian (7) [and generally Hamiltonian (1)] has a direct sum structure on the basis set (2), which is suitable for our purposes. Therefore, in this way, if we have an excitation localized in one of the subspaces (6), its coherent evolution is also restricted to the respective subspace. By this consideration, the evolution of excitation cannot be performed in the whole network. In particular, if we have an excitation in the initial site, it cannot be received in the site $3N+1$ under only pure coherent evolution. This partial evolution of the system indeed returns to the destructive interferences in the network, which is a pure quantum effect. Now we consider a case in which the site $3N+1$ of the network is attached to a reaction center denoted as a sink in such way that the dynamical evolution from the system to the sink is irreversible. Therefore, the population of the sink is always zero when the excitation at time $t = 0$ is in the initial site of the network ($|1\rangle$).

For the aim of this paper, we consider that $\nu_{3j-2,3j-1} = \nu_{3j-2,3j} = \nu_{3j-1,3j+1} = -\nu_{3j,3j+1} = J$ for $j = 1,2,3,\ldots,N$. Now consider a situation in which each of the two-level systems of the network is in contact with its fluctuating environment. These interactions affect the dynamics of the network in the form of dephasing noise (Fig. 1). Under these considerations, the Lindblad-type master equation for the density matrix of the system is written as

$$\dot{\rho} = \frac{i}{\hbar}[\rho, H] + L_{\text{deph}}(\rho) + L_{\text{sink}}(\rho), \quad (10)$$

where $L_{\text{deph}}(\rho)$ is the Lindblad operator corresponding to the action of the dephasing noises on the ρ given by

$$L_{\text{deph}}(\rho) = \sum_{j=1}^{3N+1}[\gamma_j(2\sigma_j^+\sigma_j^-\rho\sigma_j^+\sigma_j^- - \{\sigma_j^+\sigma_j^-, \rho\}], \quad (11)$$

where $\gamma_1 = \gamma_2 = \cdots = \gamma_{3N+1} = \gamma$ are the rates of dephasing processes that randomize the corresponding phases of the local excitations and excitations in the form $|S_j^+\rangle \in \mathcal{H}_j$, with $j = 1,2,\ldots,N$, produced by the time-evolution processes. Therefore, each of the randomized $|S_j^+\rangle$ has a nonzero overlap along the respective $|S_j^-\rangle \in \mathcal{H}_{j+1}$, with $j = 1,2,\ldots,N$. Consequently, the evolution of an excitation from a particular subspace to its neighbor subspace will be possible incoherently [Fig. 1(b)]. On the other hand, the additional sink site is populated by an irreversible decay process from a chosen site (for this case site $3N+1$) as described by the Lindblad operator

$$L_{\text{sink}}(\rho) = \Gamma(2\sigma_{3N+2}^+\sigma_{3N+1}^-\rho\sigma_{3N+1}^+\sigma_{3N+2}^- - \{\sigma_{3N+1}^+\sigma_{3N+2}^-\sigma_{3N+2}^+\sigma_{3N+1}^-, \rho\}), \quad (12)$$

where $\Gamma = 2\gamma$ is the rate of dissipative process that reduces the number of excitations in the system and traps it in the sink. The sink population or efficiency of transport is given by

$$P_{\text{sink}}(t) = 2\Gamma_{3N+2}\int_0^t \rho_{3N+1,3N+1}(t')dt'. \quad (13)$$

It should be noted that the dynamics preserves the total excitation number in the system, and for each N the coherent part of the evolution of the system is completely suppressed. The optimality of incoherent dynamics of the system is defined by the best way to couple the system to its independent environments, such that the sink site is populated in the shortest possible time. To achieve the optimal conditions for the transport, we consider three separate cases. For the first case, it is assumed that all sites of the network with $N = 4$ interact, in an optimal way, with their respective dephasing environments. Figure 2(a) shows the populations of site 1 and the sink versus time. The transfer time for the sink to be completely populated is $t = 505.89$ in this case. The inset in Fig. 2(a) gives the population of the sink, P_{sink}, at a fixed time of $t = 505.89$, as a function of γ.

In the second case, only the sites $3j-1$ and $3j$, with $j = 1,2,3,4$, are subjected to dephasing noises. In an optimal way, the efficiency of transport is improved with respect to the previous case (with transfer time $t = 391.27$), as shown in Fig. 2(b). This shows that the effects of dephasing noises on the sites $3j-2$ and $3j+1$ ($j = 1,2,3,4$) reduce the efficiency of transport. In other words, the removal rate of destructive interferences should not be smaller than the decay rate of phase of the related wave function. For the third case, it is interesting to note that the effect of noises on the only sites $3j-1$ (or $3j$), with $j = 1,2,3,4$, improves the optimal transport better than the previous cases (with transfer time $t = 379.1$) as depicted in Fig. 2(c). In the next section, we show that when the transport of excitation in a two-dimensional network through a particular one-dimensional path is demanded, the conditions in the second case are more effective than the others.

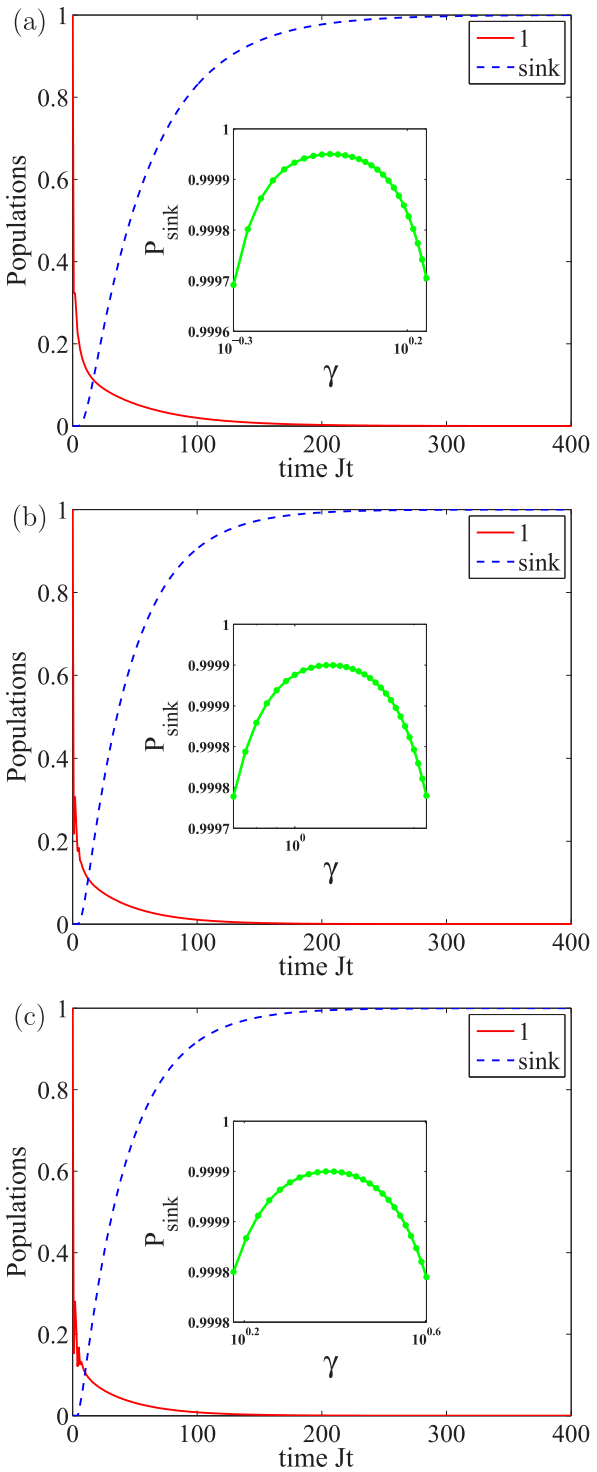

FIG. 2. (Color online) Populations of site 1 (solid red line) and the sink (dashed blue line) for $N = 4$ corresponding to Fig. 1, with $\omega_1 = \omega_2 = \cdots = \omega_{13} = 50$. The network is affected by independent dephasing noises in three different ways: (a) all sites are attached to the noise with optimal rate $\gamma_{\text{opt}} = 0.95$, (b) the sites $3j-1$ and $3j$ ($j = 1,2,3,4$) are attached with $\gamma_{\text{opt}} = 1.22$, and (c) the sites $3j-1$ ($j = 1,2,3,4$) are attached, with $\gamma_{\text{opt}} = 2.44$. Each inset shows the dependence of P_{sink} at a fixed time (the respective transfer time) as a function of γ. The initially sharp rise is due to the increasing rapidity of destruction of invariant subspaces, while the decreasing rate is due to the quantum Zeno effects.

Before analyzing the transport process in the two-dimensional case, let us consider the robustness of optimal transport versus energy mismatch of sites for the second case. First, we consider the energy of sites $3j-2$ with $j = 1,3,5$ as $\hbar(\omega-\delta)$ and with $j = 2,4$ as $\hbar\omega$. As observed from Fig. 3(a), the optimality of transport with respect to γ is robust due to the mentioned energy disorders. Now, let us consider another case in which for $j = 1,3,5$ the energy of sites is $\hbar(\omega-\delta)$ and for $j = 2,4$ it is $\hbar(\omega+\delta)$, which is more disordered than the first case. As is observed from Fig. 3(b), the robustness of optimal transport with respect to this type of disorder is less than the previous one.

III. TWO-DIMENSIONAL CASE

In this stage, we develop the process of excitation transport across a two-dimensional multisink network, such that the transport can take place in a completely selective way to each reaction center. Since the evolution is purely incoherent in the network, the transport of an excitation to a particular sink needs to establish artificial couplings between the network and independent fluctuating environments throughout a path connecting the initial site to that sink. Toward that end, let us introduce the following Hamiltonian as a building block for constructing the network, corresponding to Fig. 4, as follows:

$$\begin{aligned} H_\mu = &\sum_{j=0}^{3}\hbar\omega_{\mu_j}\sigma_{\mu_j}^{+}\sigma_{\mu_j}^{-} + \sum_{j=1}^{6}\hbar\omega_{\mu_{0j}}\sigma_{\mu_{0j}}^{+}\sigma_{\mu_{0j}}^{-} \\ &+ \sum_{j=1}^{6}\hbar\nu_{\mu_0,\mu_{0j}}\left(\sigma_{\mu_0}^{+}\sigma_{\mu_{0j}}^{-} + \sigma_{\mu_0}^{-}\sigma_{\mu_{0j}}^{+}\right) \\ &+ \sum_{j=1}^{2}\hbar\Big[\nu_{\mu_1,\mu_{0j}}\left(\sigma_{\mu_1}^{+}\sigma_{\mu_{0j}}^{-} + \sigma_{\mu_1}^{-}\sigma_{\mu_{0j}}^{+}\right) \\ &+ \nu_{\mu_2,\mu_{0j+2}}\left(\sigma_{\mu_2}^{+}\sigma_{\mu_{0j+2}}^{-} + \sigma_{\mu_2}^{-}\sigma_{\mu_{0j+2}}^{+}\right) \\ &+ \nu_{\mu_3,\mu_{0j+4}}\left(\sigma_{\mu_3}^{+}\sigma_{\mu_{0j+4}}^{-} + \sigma_{\mu_3}^{-}\sigma_{\mu_{0j+4}}^{+}\right)\Big]. \end{aligned} \tag{14}$$

In general, we assume that $\nu_{\mu_0,\mu_{01}} = \nu_{\mu_0,\mu_{02}}$, $\nu_{\mu_0,\mu_{03}} = \nu_{\mu_0,\mu_{04}}$, $\nu_{\mu_0,\mu_{05}} = \nu_{\mu_0,\mu_{06}}$, $\nu_{\mu_1,\mu_{01}} = -\nu_{\mu_1,\mu_{02}}$, $\nu_{\mu_2,\mu_{03}} = -\nu_{\mu_2,\mu_{04}}$, and $\nu_{\mu_3,\mu_{05}} = -\nu_{\mu_3,\mu_{06}}$, as well as $\omega_{\mu_{01}} = \omega_{\mu_{02}}$, $\omega_{\mu_{03}} = \omega_{\mu_{04}}$, and $\omega_{\mu_{05}} = \omega_{\mu_{06}}$. However, for the purpose of this paper, it is enough to rewrite the assumptions as $\nu_{\mu_0,\mu_{0j}} = J$, $\omega_{\mu_i} = \omega_{\mu_{0j}} = \omega$ $(i = 0,1,2,4$ and $j = 1,2,\ldots,6)$, and $\nu_{\mu_1,\mu_{01}} = -\nu_{\mu_1,\mu_{02}} = \nu_{\mu_2,\mu_{03}} = -\nu_{\mu_2,\mu_{04}} = \nu_{\mu_3,\mu_{05}} = -\nu_{\mu_3,\mu_{06}} = J$. Now, we introduce a new basis set as follows:

$$\begin{aligned} &|\mu_j\rangle, \quad j = 0,1,2,3, \\ &|\mu_1^{\pm}\rangle = \frac{1}{\sqrt{2}}(|\mu_{01}\rangle \pm |\mu_{02}\rangle), \quad |\mu_2^{\pm}\rangle = \frac{1}{\sqrt{2}}(|\mu_{03}\rangle \pm |\mu_{04}\rangle), \\ &|\mu_3^{\pm}\rangle = \frac{1}{\sqrt{2}}(|\mu_{05}\rangle \pm |\mu_{06}\rangle). \end{aligned} \tag{15}$$

Under these considerations, we find that the Hamiltonian H_μ, in the new basis, takes a direct sum structure as

$$H_\mu = \bigoplus_{j=0}^{3} H_{\mu_j}, \tag{16}$$

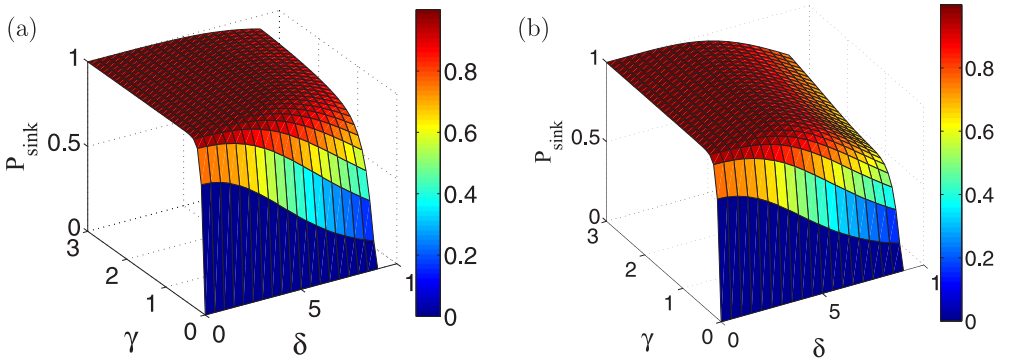

FIG. 3. (Color online) Robustness of P_{sink} around the optimal value of dephasing noise due to energy mismatch between sites. (a) The energies of sites $3j-2$ with $j=1,3,5$ are $\hbar(\omega-\delta)$ and those with $j=2,4$ are $\hbar\omega$. (b) The energies of sites $3j-2$ with $j=1,3,5$ are $\hbar(\omega-\delta)$ and those with $j=2,4$ are $\hbar(\omega+\delta)$. For both cases, the sites $3j-1$ and $3j$ are attached to dephasing noises.

where their corresponding representative subspaces are

$$\begin{aligned}\mathcal{H}_{\mu_0} &= \text{span}\{|\mu_0\rangle,|\mu_1^+\rangle,|\mu_2^+\rangle,|\mu_3^+\rangle\},\\ \mathcal{H}_{\mu_j} &= \text{span}\{|\mu_j\rangle,|\mu_j^-\rangle\}, \quad j=1,2,3.\end{aligned} \tag{17}$$

As the one-dimensional quantum transport prototype, the evolution of the closed system described by the Hamiltonian H_μ is governed by one of the (sub)Hamiltonians of Eq. (16) and contained in one of the respective invariant subspaces of Eq. (17) (depending on the position of the initial state). If excitation is prepared at site μ_1 as an initial state, then it cannot be transferred either to site μ_2 or to μ_3, because they lie in different invariant subspaces. However, if some

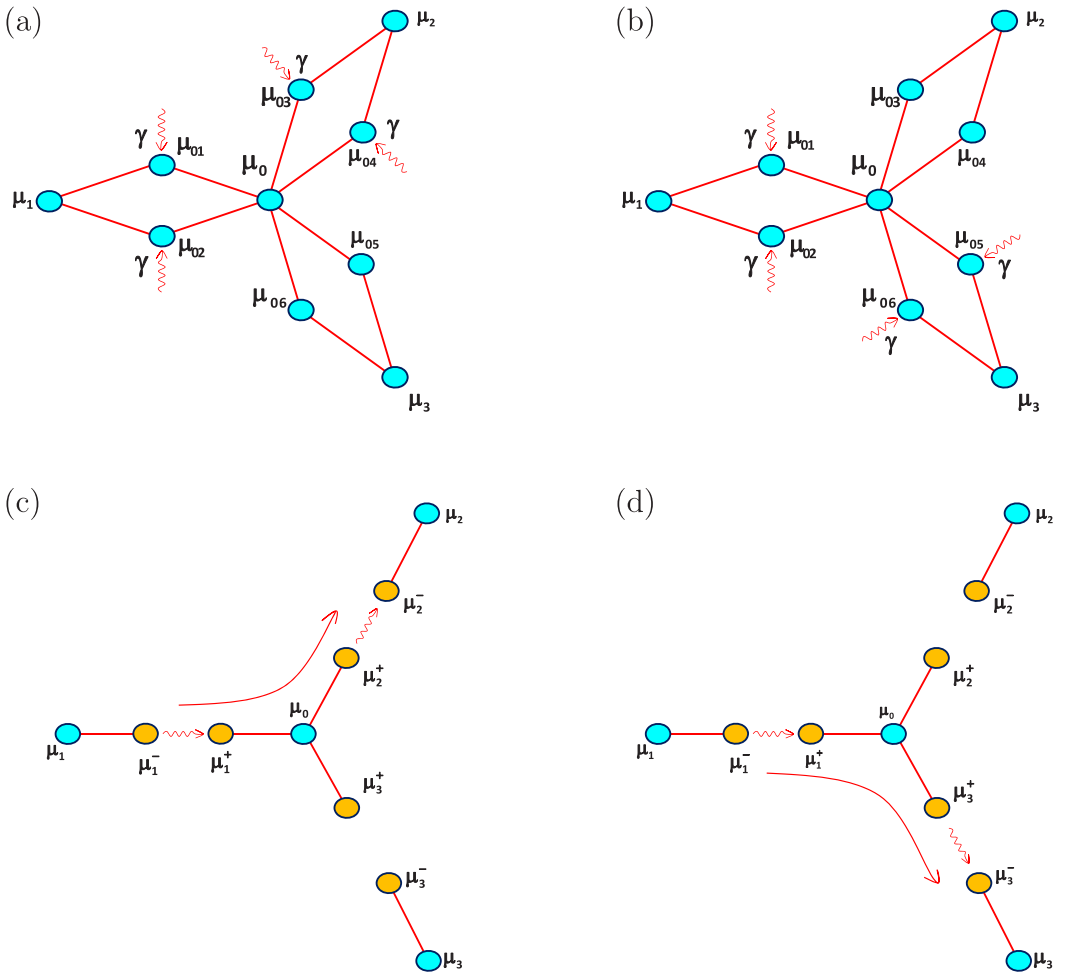

FIG. 4. (Color online) Selective incoherent quantum transport building block in two dimensions. (a) Incoherent transport from site μ_1 to site μ_2 and (b) incoherent transport from site μ_1 to site μ_3. Parts (c) and (d) show invariant subspaces structure of (a) and (b), respectively, and incoherent connection between them through the noises.

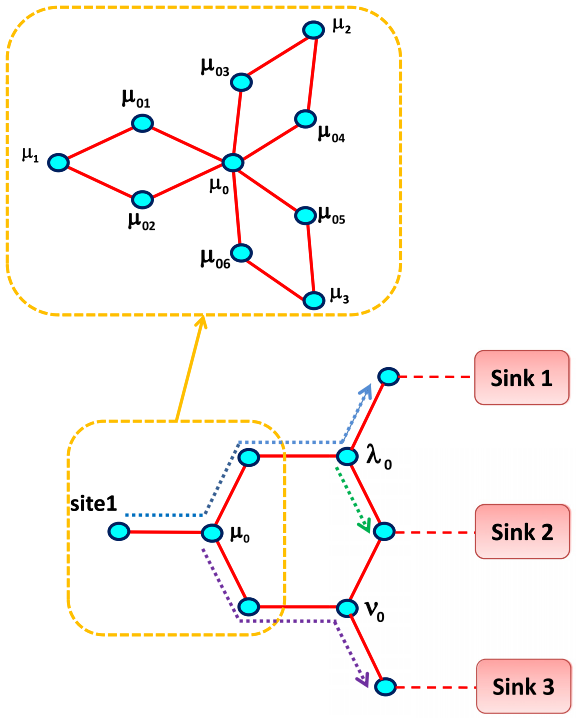

FIG. 5. (Color online) A network with three sinks. Energy excitation can be transferred from site 1 to one of the sinks arbitrarily.

of these invariant subspaces are tailored to each other by some processes, such as interacting the network at sites, for example, μ_{0j} with $j = 1,2,3,4$ ($j = 1,2,5,6$) with fluctuating environments, then the quantum transport of excitation can be possible from μ_1 to μ_2 (μ_3), as seen in Fig. 4.

Now, using the previous discussion and Fig. 4 as a building block, a two-dimensional quantum network can be constructed, whereby the energy of excitation can be transferred from an initial site to one of the reaction centers attached to the network. Consider, for example, a network with three identical sinks as reaction centers corresponding to Fig. 5, in which the complete transport of excitation prepared in site 1 arbitrarily to one of the reaction centers is required. The Hamiltonian of the system is given as

$$H = \sum_{\mu} H_{\mu}, \tag{18}$$

where H_μ is as in Eq. (14). It is clear that the Hamiltonian of the network has also a direct sum structure, and therefore the evolution of the system is restricted to one of the related invariant subspaces. The direct sum structure of H is as

$$\begin{aligned} H = {} & H_{\mu_1} \oplus H_{\lambda_2} \oplus H_{\nu_3}, \\ & \oplus H_{\mu_0} \oplus H_{\lambda_0} \oplus H_{\nu_0}, \\ & \oplus H_{\mu,\lambda} \oplus H_{\mu,\nu} \oplus H_{\lambda,\nu}, \end{aligned} \tag{19}$$

and their corresponding invariant subspaces are

$$\begin{aligned} \mathcal{H}_a &= \text{span}\{|a\rangle, |a^-\rangle\}, \quad a = \mu_1, \lambda_2, \nu_3, \\ \mathcal{H}_{b0} &= \text{span}\{|b_0\rangle, |b_1^+\rangle, |b_2^+\rangle, |b_3^+\rangle\}, \quad b = \mu, \lambda, \nu, \\ \mathcal{H}_{\mu,\lambda} &= \text{span}\{|\mu_2^-\rangle, |\mu_2(\lambda_1)\rangle, |\lambda_1^-\rangle\}, \\ \mathcal{H}_{\mu,\nu} &= \text{span}\{|\mu_3^-\rangle, |\mu_3(\nu_1)\rangle, |\nu_1^-\rangle\}, \\ \mathcal{H}_{\lambda,\nu} &= \text{span}\{|\lambda_3^-\rangle, |\lambda_3(\nu_2)\rangle, |\nu_2^-\rangle\}. \end{aligned} \tag{20}$$

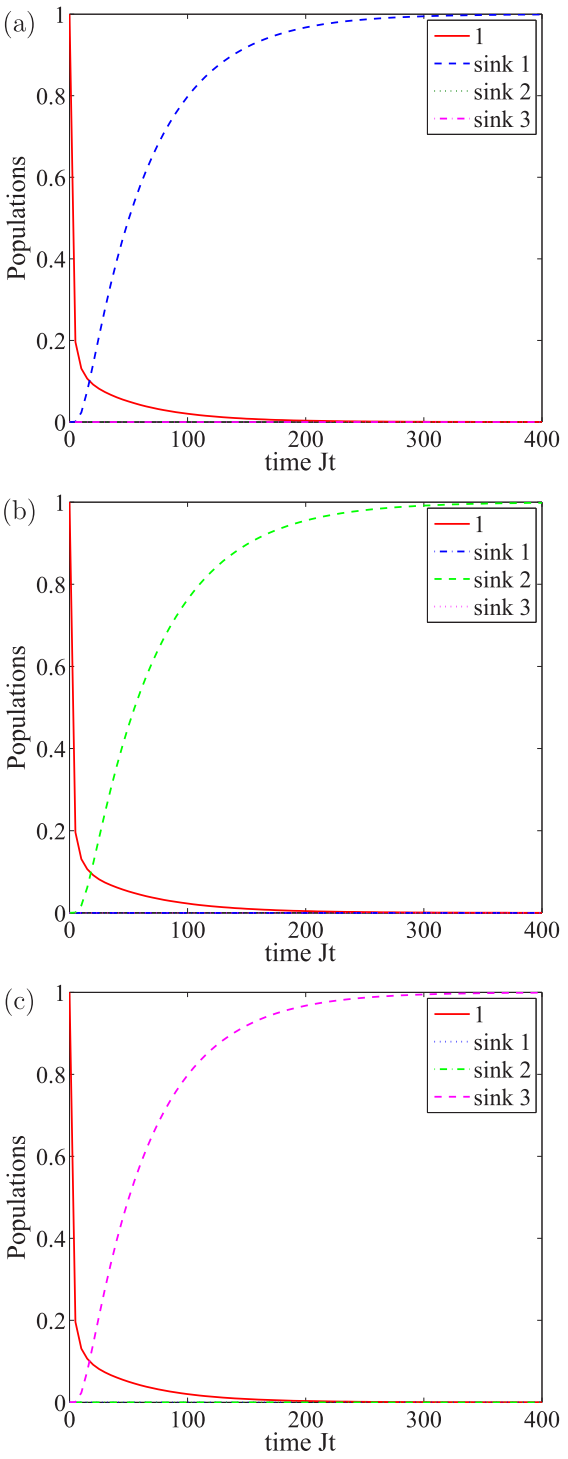

FIG. 6. (Color online) Complete transfer of excitation from site 1 (solid red line) selectively to (a) sink 1 (dashed blue line), (b) sink 2 (dashed green line), and (c) sink 3 (dashed pink line).

As discussed previously, the evolution of an excitation prepared at site 1 is only restricted within the invariant subspace H_{μ_1} (Fig. 5). Consider that the sinks are attached to the sites λ_2, λ_3 (or ν_2), and ν_3 with equal strength of coupling as $\Gamma = 2\gamma$,

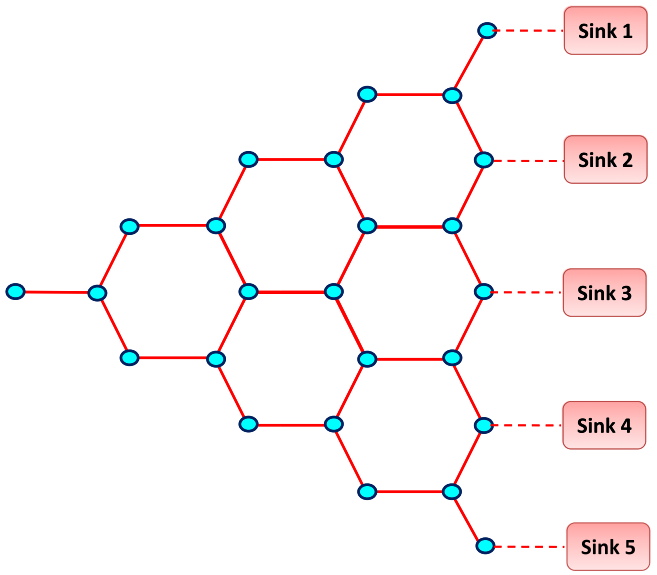

FIG. 7. (Color online) Extended network with more than three sinks.

where γ is the rate of dephasing noise on a typical site. The populations of the sinks are

$$P_{\text{sink}\,1}(t) = 2\Gamma \int_0^t \rho_{\lambda_2,\lambda_2}(t')dt',$$
$$P_{\text{sink}\,2}(t) = 2\Gamma \int_0^t \rho_{\lambda_3,\lambda_3}(t')dt', \qquad (21)$$
$$P_{\text{sink}\,3}(t) = 2\Gamma \int_0^t \rho_{\nu_3,\nu_3}(t')dt'.$$

Now consider the effect of dephasing noises with equal rate γ on the sites μ_{01}, μ_{02}, μ_{03}, μ_{04}, λ_{01}, λ_{02}, λ_{03}, and λ_{04}. We see that the excitation is optimally transferred from site 1 (or μ_1) to the sink 1, completely without any penetration to the other sinks, as shown in Fig. 6(a). On the other hand, the excitation is completely transferred to the sink 2 if the dephasing noises affect the sites μ_{01}, μ_{02}, μ_{03}, μ_{04}, λ_{01}, λ_{02}, λ_{05}, and λ_{06}, as is obvious from Fig. 6(b). And in a similar way, if the sites μ_{01}, μ_{02}, μ_{05}, μ_{06}, ν_{01}, ν_{02}, ν_{05}, and ν_{06} are affected by dephasing noises, only sink 3 is populated [Fig. 6(c)]. Hence, transport of excitation to one of the reaction centers is only possible through incoherent coupling of the network to dephasing environments, which in turn tailor a number of invariant subspaces throughout the path connecting the site 1 to the desired reaction center. The transport process discussed here for each sink through a particular one-dimensional path is optimal and similar to the conditions of the second case of a one-dimensional prototype, as discussed in the previous section.

The discussed scheme for selective transport of energy can be easily extended to other larger networks (Fig. 7). In general, the conditions for the respective optimal transport will be different.

IV. CONCLUSIONS

In this work, we presented a method for energy transport in a two-dimensional network in a selective way. In this approach, the coherent part of the evolution is an unwanted process for selective transfer of excitation energy. Therefore, the network is designed in such a way that the coherent evolution is completely suppressed by itself. So the evolution of the system, whose existence depends on the existence of interactions between the system and independent environmental fluctuating noises, is completely incoherent. If the interactions are established throughout a particular path of the network, the evolution takes place along that path incoherently. In particular, the path can be considered as a connection link between the site 1 (the excitation was prepared initially in this site) and one of the sinks or reaction centers, so in this way the excitation can be completely transferred to the respective reaction center. On the other hand, it was observed that the optimal transport throughout a particular path in the two-dimensional network is not similar to the optimality of quantum transport for a one-dimensional prototype. It is interesting to note that since the evolution takes place in a particular path rather than in the whole network, from the dissipation and losing point of view (which can occur in chromophores or sites), quantum transport can be performed in a more efficient way. This may lead to further investigations in the future.

An additional point of view is the effect of reorganization energy shift for each site, which interacts with the related environment. Since all of the interacting sites with the environments are identical (as are the environments), they all experience an identical energy shift. The optimality of transport under this kind of energy shift, for Markovian and non-Markovian environments, can also be investigated in the future.

[1] D. Ferrari, G. L. Celardo, G. P. Berman, R. T. Sayre, and F. Borgonovi, J. Phys. Chem. C **118**, 20 (2014).
[2] P. K. Ghosh, A. Yu. Smirnov, and F. Nori, J. Chem. Phys. **134**, 244103 (2011).
[3] P. K. Ghosh, A. Yu. Smirnov, and F. Nori, Phys. Rev. E **84**, 061138 (2011).
[4] G. S. Engel, T. R. Calhoun, E. L. Read, T.-K. Ahn, T. Mancal, Y.-C. Cheng, R. E. Blankenship, and G. R. Fleming, Nature (London) **446**, 782 (2007).
[5] E. Collini, C. Y. Wong, K. E. Wilk, P. M. G. Curmi, P. Brumer, and G. D. Scholes, Nature (London) **463**, 644 (2010).
[6] R. Hildner, D. Brinks, J. B. Nieder, R. J. Cogdell, and N. F. van Hulst, Science **340**, 1448 (2013).
[7] F. Caruso, A. W. Chin, A. Datta, S. F. Huelga, and M. B. Plenio, J. Chem. Phys. **131**, 105106 (2009).
[8] A. W. Chin, A. Datta, F. Caruso, S. F. Huelga, and M. B. Plenio, New J. Phys. **12**, 065002 (2010).
[9] M. B. Plenio and S. F. Huelga, New J. Phys. **10**, 113019 (2008).

[10] P. Rebentrost, M. Mohseni, I. Kassal, S. Lloyd, and A. Aspuru-Guzik, New. J. Phys. **11**, 033003 (2009).
[11] R. de J. León-Montiel, I. Kassal, and J. P. Torres, J. Phys. Chem. B **118**, 10588 (2014).
[12] M. Mohseni, P. Rebentrost, S. Lloyd, and A. Aspuru-Guzik, J. Chem. Phys. **129**, 174106 (2008).
[13] J. Cao and R. J. Silbey, J. Phys. Chem. A **113**, 13825 (2009).
[14] A. Perdomo, L. Vogt, A. Najmaie, and A. Aspuru-Guzik, Appl. Phys. Lett. **96**, 093114 (2010).
[15] S. M. Vlaming, V. A. Malyshev, and J. Knoester, J. Chem. Phys. **127**, 154719 (2007).
[16] M. Mohseni, A. Shabani, S. Lloyd, and H. Rabitz, J. Chem. Phys. **140**, 035102 (2014).
[17] L. Campos Venuti and P. Zanardi, Phys. Rev. B **84**, 134206 (2011).
[18] S. Hoyer, M. Sarovar, and K. B. Whaley, New J. Phys. **12**, 065041 (2010).
[19] P. Nalbach, J. Eckel, and M. Thorwart, New J. Phys. **12**, 065043 (2010).
[20] P. Nalbach, D. Braun, and M. Thorwart, Phys. Rev. E **84**, 041926 (2011).
[21] T. Scholak, F. de Melo, T. Wellens, F. Mintert, and A. Buchleitner, Phys. Rev. E **83**, 021912 (2011).
[22] T. Scholak, T. Wellens, and A. Buchleitner, J. Phys. B **44**, 184012 (2011).
[23] A. Shabani, M. Mohseni, H. Rabitz, and S. Lloyd, Phys. Rev. E **89**, 042706 (2014).
[24] M. Christandl, N. Datta, T. C. Dorlas, A. Ekert, A. Kay, and A. J. Landahl, Phys. Rev. A **71**, 032312 (2005).