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Strings of droplets propelled by coherent waves

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Bouncing walking droplets possess fascinating properties due to their peculiar wave-particle interaction leading to unexpected quantumlike behaviors. We propose a study consisting in droplets walking along annular cavities. We show that, in this geometry, they spontaneously form a string of synchronized bouncing droplets that share a common coherent wave propelling the group at a speed faster than single walkers. The formation of this coherent wave and the collective droplet behaviors are captured by a model. Those are at the opposite of the ones found in two-dimensional geometries. Our results shed light on walking dynamics.

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When a tiny droplet is gently placed on an oscillating liquid interface, which is vibrated with an amplitude A and a frequency f, it is able to bounce without coalescing when the bath maximum acceleration $\Gamma = 4\pi^2 A f^2/g$ is above a threshold Γ_B [1–3]. Actually, an air layer separates the droplet from the vibrated surface preventing coalescence by lubrication. Above another threshold denoted by Γ_F, a pattern of standing waves appears at the liquid surface. This is the Faraday instability [4,5] for which waves are characterized by a wavelength λ_F and a frequency $f_F = f/2$. By approaching the Faraday instability from below ($\Gamma_B < \Gamma < \Gamma_F$), the droplet may start to bounce once every two periods of oscillation. In such a situation, the emitted waves possess the characteristics of the Faraday pattern [6]. Due to the coupling between the droplet and the sum of waves emitted on the liquid surface $\zeta(\vec{r},t)$ by the successive previous impacts, bouncing droplets may start to move horizontally along the liquid surface. They are called walkers [6]. The droplet-wave interaction leads to spectacular quantumlike phenomena at the macroscopic scale. See Ref. [7] for a complete review of quantumlike behaviors in the walking droplets experiment. Processes involving interactions between two or more droplets were investigated. It has been shown that co-orbiting droplets move along circles of quantified radii [8]. In a recent article, the rectilinear motion of two droplets, the so-called promenade mode, was studied. In this article, Borghesi *et al.* [9] show that walkers can move together at quantified average distances with a velocity proportional to their separation. Similar behaviors were already observed in [8]. Here we investigate the walker-walker interactions by placing them into carved rings in the experimental cell. We show that this one-dimensional (1D) system motion leads to diametrically different behaviors than the ones observed in 2D system, due to the coherent wave that couples walkers together.

The experimental setup is the following. Identical droplets of 800 μm diameter are created by an automatic generator [2]. The liquid is silicon oil with a kinematic viscosity $\nu = 20$ cSt, density $\rho = 949$ kg/m^3, and surface tension $\sigma = 20.6$ mN/m. The container is shaken vertically by an electromagnetic system with a controllable amplitude A and a fixed frequency $f = 70$ Hz. The resulting acceleration is measured through an accelerometer fixed to the experimental cell. The dimensionless acceleration Γ has always been kept in the interval $[0.95\Gamma_F, 0.99\Gamma_F]$ with $\Gamma_F = 4.25 \pm 0.04$ for our fluid and frequency. This interval corresponds to a memory $M = \Gamma_F/(\Gamma_F - \Gamma)$ [10] ranging in the interval [20,100]. The wavelength of the Faraday waves is estimated to $\lambda_F = 6$ mm. Since the occurrence of the Faraday instability is highly dependent on the liquid depth in the shallow regime, carved cavities are a straightforward way to control the path of a walking droplet. Indeed, previous experiments [11,12] already use submerged objects in order to create obstacles for walking droplets. Thus, we consider circular rings where droplets walk forever without reaching a boundary. The sketch of an annular cavity is given in Fig. 1(a) with a picture of the experiment taken from above [Fig. 1(b)]. The liquid depth in the center of the cavity is $H = 4$ mm and the width of the annular channel is $D = 7.5$ mm [13]. Elsewhere, the liquid depth H_0 is fixed to 1 mm limiting the propagation of waves. At this depth, a droplet may bounce but cannot walk. A picture of the Faraday instability in the cavity is shown in Fig. 1(c). One observes the limited wave propagation as well as the dependence of Γ_F regarding the fluid depth. Three ring cavities of different radii have been considered with $R = 13.75$, 41.25, and 68.75 mm.

Observations show that typical trajectories in the xy plane of single walkers are mostly circles [14], whatever the size of the cavity, forcing parameters, or memory. Note that there is no central force behind this circular trajectory. The fact that the droplet follows the ring shows that the Faraday waves adopt the symmetry of the cavity [see Fig. 1(c)]. Along the ring, i.e., in the azimuthal direction, waves can be approximated to sinusoidal standing waves, as expected for a 1D system, as illustrated in Fig. 1(b). In the transverse direction, i.e., in the radial direction, the Faraday waves present antinodes in the center of the cavity and evanescent waves are strongly damped outside the ring. The droplet remains in the central part of the cavity. Moreover, the speed v_1 of a single walker is fixed by the forcing parameters of the experiment and the geometry of the cavity. We checked that the speed $v_1 \approx 10$ mm/s of the walker (at memory $M = 20$) is independent of the ring radius R. Furthermore, the memory M of the system does not change qualitatively the observations presented in this Rapid Communication. We will thus work at memory 20 for the whole study. Nevertheless, note that the walker speed strongly depends on the presence of boundaries, especially the fluid depth H and the channel width D.

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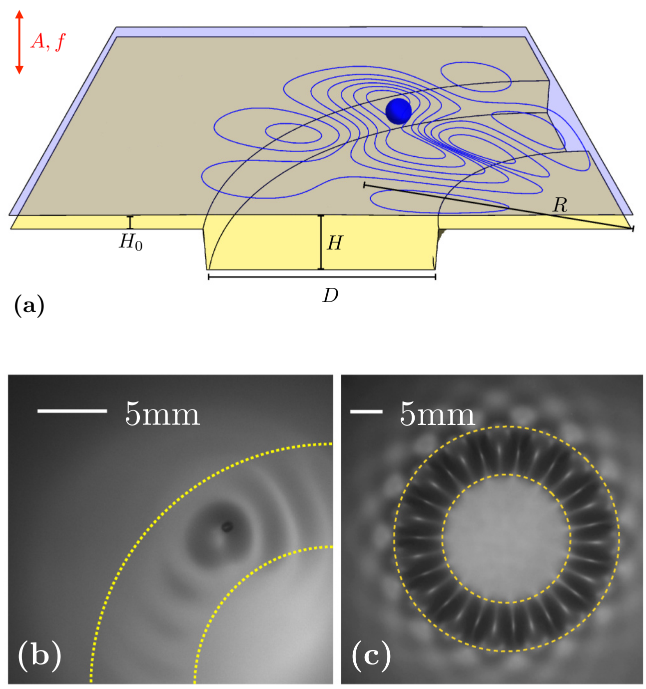

FIG. 1. (Color online) (a) Sketch of a quarter of the annular cavity of width D and radius R. The oil level is adjusted to obtain a depth H in the cavity and a thin layer H_0 elsewhere. A walking droplet tends to remain in the cavity. Contours of the liquid surface $\zeta(\vec{r},t)$ are shown to illustrate that the propagation of waves mostly takes place in the cavity while evanescent waves are observed outside the cavity. (b) Picture of the experiment from above. Yellow dashed lines account for the channel borders. One can observe the waves emitted by the walker at each impact propagating mainly along the channel for a memory $M = 20$. (c) Faraday instability inside the channel above Γ_F. Evanescent waves can be observed outside the channel.

When two or more droplets are placed in the ring, they walk clockwise or counterclockwise depending on their initial conditions. After a while, they start to interact through their waves. The result after long times is the formation of a string of walking droplets moving cooperatively along the ring, sharing a common and coherent wave. The distances between droplets are quantified and correspond roughly to multiples of $\lambda_F/2$ and bounces are (anti)synchronized. Figure 2(a) presents a picture of such a group made of $N = 7$ droplets. In that string, two successive droplets are antisynchronized, meaning that when the first one bounces, the second one is in a free flight and vice versa. In Fig. 2(a) the shadows below each droplet illustrate this ordering. Strings of synchronous droplets can also be created, but their interdistances should be different, as shown below. Figure 2(b) presents a picture of a chain of four droplets. It also offers a visualization of the wave field emitted by the drops, which adapts to the cavity symmetry. Figure 2(c) shows the interdistances between successive droplets in various groups of two droplets that are launched randomly in an annular cavity. In the steady state, their distances are given by the empirical formula

$$s = (n - \epsilon_0)\lambda_F, \tag{1}$$

where ϵ_0 accounts for the dimensional aspect of the waves and their damping due to viscosity. The label n is an integer or a half integer. The fit using Eq. (1) gives $\lambda_F = 6.1 \pm 0.1$ mm and $\epsilon_0 = 0.18 \pm 0.02$. The first observed situation corresponds to $n = 1$, for which the bounces are synchronized. Half-integer values of n correspond to antisynchronized values. The case $n = 1/2$ is particular since the droplets start to orbit around each other and they follow complex trajectories [8]. They

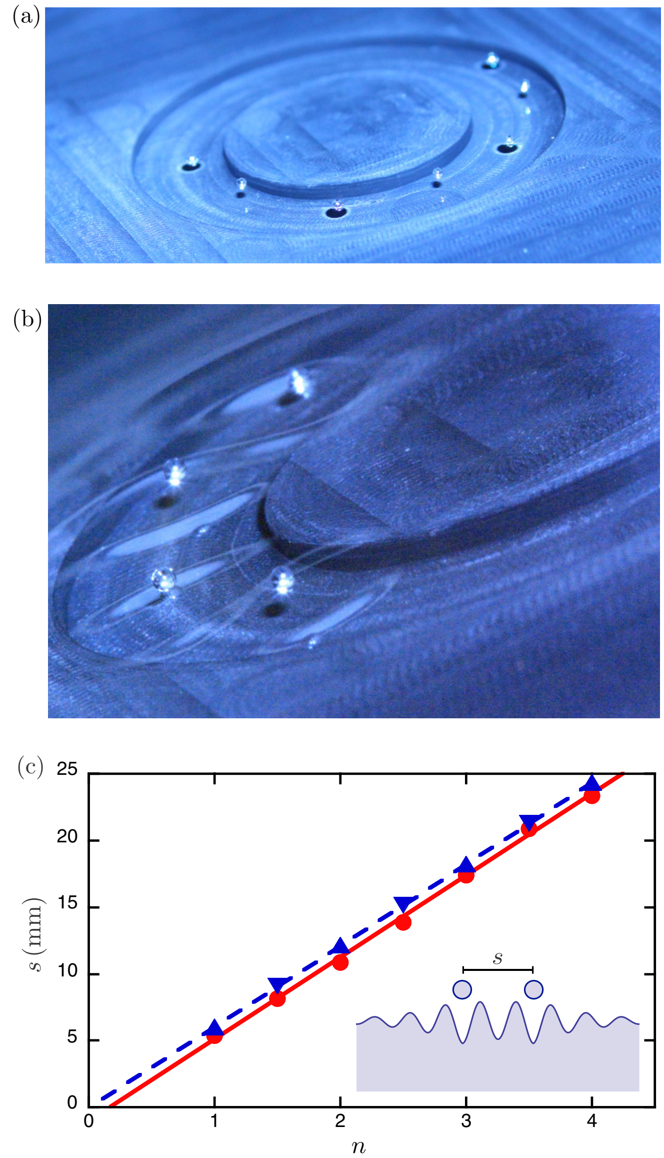

FIG. 2. (Color online) (a) Picture of a group of seven droplets in the small annular cavity. One observes quantified interdistances and antisynchronous bounces for the successive droplets. (b) Four bouncing droplets walking in an annular cavity and creating a coherent wave propelling the group at a speed higher than individual droplets. The coherent wave can be seen to follow the cavity symmetry. One can observe antisynchronous bounces for the successive drops. (c) Red circles represent the droplet interdistances as a function of the label n describing the interaction mode for a memory $M = 20$. The line is a fit using Eq. (1). Error bars are not indicated since they are smaller than the symbol size. Blue triangles are droplet interdistances given by the model for synchronous (up triangles) or antisynchronous bounces (down triangles). The dashed line is a fit using Eq. (1). Note that the quantification does not depend on the value of the memory M.

finally coalesce or vanish in the vibrating bath. The absence of circular motions and promenade modes for other interdistances is assumed to be due to the constraint originating from the boundaries.

The coherence of the wave propelling the string of walkers has been tested [14]. When a string of a few droplets is formed, we used a needle to destroy one of the central droplets. The system appears to be unaffected by the disappearance of this droplet and behaves exactly as before. The distances remain unchanged and the synchronicity is kept. Movies of that experiments are given in Ref. [14]. When the number N of droplets increases, the coherent wave extension increases accordingly. At some point, the wave starts to self-interfere such that the system destabilizes. We checked that it is nearly impossible to form a complete ring of droplets moving cooperatively due to this effect. The only stable groups of droplets are found up to 11 droplets in the smallest ring. In the same spirit, modifying the acceleration will inevitably change the stability of the system. Indeed, at higher memory, e.g., $M = 45$, only seven droplets can be placed.

The collective motion of the string was unexpected because in two dimensions only a few cases lead to a rectilinear motion of the center of mass. Indeed, mainly two behaviors are found: promenade modes and orbiting motions. For both dynamics, the collective velocity is lower than the individual droplets freely moving along the surface [8,9]. We will demonstrate herein a clear violation of this conjecture. Let us analyze the speed of the walkers resulting from the collective effects. Figure 3 shows the speed v_2 of the pair of bouncing droplets as a function of the distance s separating them. The group speed is normalized by the speed v_1 of single droplets in the ring. It is worth noting that this speed v_1 differs from the speed v_1^{2D} of a drop in a cavity without a boundary. The single droplet case v_1 should represent the asymptotic case when the interdistances tends to infinity to neglect interactions. The speeds v_2 obtained for droplet pairs are larger than v_1 in both synchronous and antisynchronous cases. The speed ratio is also seen to decrease exponentially as a function of s. The common coherent wave shared by droplets has therefore a large driving force, as we will explain below.

In order to emphasize the collective effects induced by wider coherent waves, we created groups of N droplets being separated by a single wavelength λ_F. All droplets are therefore bouncing in a synchronized way. The normalized speed v_N/v_1 of the string increases with N, as shown in Fig. 4. The speed of the string seems to saturate for high-N values, meaning that there is a limit for coherence. The origin of this limit is due to the damping of Faraday waves. Indeed, above a droplet number (around $N = 6$ in our experimental conditions) any additional droplet joining the string will not affect the global dynamics. We also created strings with a larger separation between successive walkers, like $3\lambda_F/2$. Similar results (not shown herein) are obtained, but the saturation speed is lower than the previous case.

Actual models [7,10] do not account for 1D geometries where emitted waves strongly depend on the presence of boundaries or on the viscous damping of emitted Faraday waves. As a consequence, we choose to adapt existing model [10] to our experiments. We first assume that the standing waves associated with the successive droplet impacts

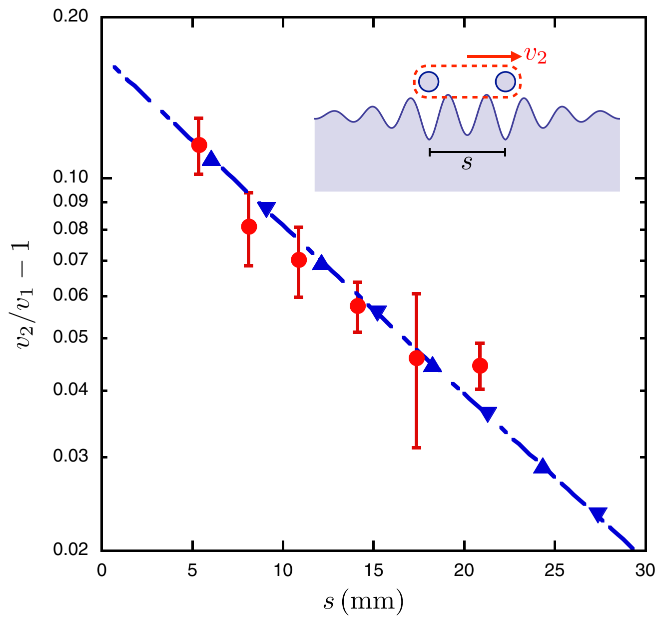

FIG. 3. (Color online) Circles represent the speed of a droplet pair v_2 (on a logarithmic scale) as a function of the distance s between droplets. The speed is normalized by the speed v_1 of a single droplet. Error bars are indicated. The model explained in the text returns quantified interdistances s between droplets as well as specific speeds v_2 for both synchronous (up triangles) and antisynchronous (down triangles) cases. Excellent agreement is found between the model and the experimental data. The dashed curve is an exponential decay fitting the results from the model.

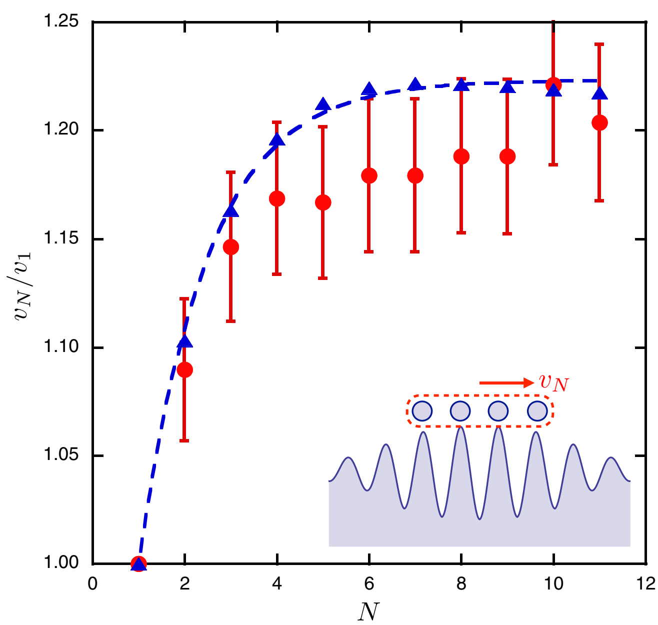

FIG. 4. (Color online) Circles represent the speed v_N of a droplet string as a function of its number N of components. The speed is normalized by the speed v_1 of single droplets. Error bars are indicated. The speed seems to saturate for large systems. The model described in the text captures this effect for both synchronous (up triangles) and antisynchronous (down triangles) cases. Good agreement is found between the model and the experimental data. The dashed curve is a guide for the eye.

are given by damped sine waves along the s coordinate. We reduce the system to a 1D liquid profile $\zeta(s,t)$ along the ring. The driving horizontal force is supposed to be proportional to the slope of the liquid profile at each impact. Since the successive bounces are periodic events, separated by a time $\tau_F = 2/f$, a phenomenological strobed equation of motion is considered for each droplet i. The speed change $u^i_{n+1} - u^i_n$ of droplet i between the nth and $(n+1)$th impacts is given by

$$u^i_{n+1} - u^i_n = -\gamma u^i_n - C_0 \frac{\partial \zeta^{ii}}{\partial s}\bigg|_{n+1} - C_1 \sum_{j\neq i} \frac{\partial \zeta^{ij}}{\partial s}\bigg|_{n+1}. \quad (2)$$

The first term represents some dissipation at bouncing, due to air drag and the shear of the air layer, with a parameter γ. The second term with a coefficient C_0 represents the interaction of the droplet i at the $(n+1)$th impact with the waves produced by the same droplet at previous impacts. The last term with a coefficient C_1 represents the interactions between the droplets in a string. We expect C_1 to be different from C_0 because the system is quasi-one-dimensional rather than purely one dimensional. As a consequence, waves emitted by neighbor droplets are also radiated outside the channel and this effect is taken into account due to C_1 [see Figs. 1(b) and 1(c)]. Since we focus only on droplet motions in the tangential direction, the relevant information for the strobed equation (2) is given by

$$\zeta^{ij} = \zeta_0 \cos\left(\frac{2\pi\left(s^i_{n+1} - s^j_n\right)}{\lambda_F}\right) \exp\left(-\frac{s^i_{n+1} - s^j_n}{\delta}\right), \quad (3)$$

where ζ_0 is an arbitrary parameter containing the wave amplitude and thus on the memory. Assuming $u^1_n = u^2_n = v_2$, only one solution is found with quantified distances, depending on whether successive walkers are in phase or out of phase. The model [15,16] is calibrated as follows. The speed of a lone droplet gives us the ratio C_0/γ and the value of δ is chosen as $2.1\lambda_F$ in order to fit the exponential decrease of v_2; this value is in the range of previous values of δ [9,10]. Finally, C_1 is the only remaining fitting parameter with $C_0/\gamma \approx 0.03$ and $C_0 \approx 20C_1$. Quantification is shown in Fig. 2(c). The agreement with the experimental data is excellent, except for ϵ_0, which remains close to zero in the model. Moreover, it can be shown that the speed of a group is larger than individual speeds. The group speed v_2 is seen to decay with the interdistance between two droplets. This is illustrated in Fig. 3. Extra speed for a pair of droplets is due to the constructive interference of waves emitted from both droplets. Since the wave damping is already taken into account through δ, the fact that C_1 is much lower than C_0 has an origin related to the nontrivial dissipation of waves out of the cavity. Indeed, the self-interaction of a droplet with its wave is a local phenomenon, while cross interactions between droplets involve propagation and reflection of waves along the cavity. Although different interaction coefficients should be considered, the coherent wave dynamics is well captured by our model. By generalizing the calculations to N droplets, it is possible to estimate numerically the speed increase of v_N reported in Fig. 4. Numerical results are shown in this plot (triangles). Qualitative agreement with the experimental data is obtained. Moreover, using different quantified distances for successive droplets, we are able to show that the asymptotic speed limit decreases with the distance between adjacent droplets. In fact, decoherence appears for distant droplets.

In summary, we have investigated bouncing droplets in a geometry confining them to a nearly 1D system. The wave emitted by a walker becomes a damped sine wave. We evidenced a remarkable feature for 1D walkers: They form a group bouncing collectively, leading to a faster motion than independent elements. The fact that coherent waves can be created pushes the analogy between walking droplets and the quantum system further by considering, for example, waveguides. Moreover, 1D systems as considered herein are promising since droplets can be conveyed to collisions and other phenomena that are reminiscent of quantum information or wave fields.

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