# Modulation of a compressional electromagnetic wave in a magnetized electron-positron quantum plasma

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Amplitude modulation of a compressional electromagnetic wave in a strongly magnetized electron-positron pair plasma is considered in the quantum magnetohydrodynamic regime. The important ingredients of this study are the inclusion of the external strong magnetic field, Fermi quantum degeneracy pressure, particle exchange potential, quantum diffraction effects via the Bohm potential, and dissipative effect due to collision of the charged carriers. A modified-nonlinear Schödinger equation is developed for the compressional magnetic field of the electromagnetic wave by employing the standard reductive perturbation technique. The linear and nonlinear dispersions of the electromagnetic wave are discussed in detail. For some parameter ranges, relevant to dense astrophysical objects such as the outer layers of white dwarfs, neutron stars, and magnetars, etc., it is found that the compressional electromagnetic wave is modulationally unstable and propagates as a dissipated electromagnetic wave. It is also found that the quantum effects due to the particle exchange potential and the Bohm potential are negligibly small in comparison to the effects of the Fermi quantum degeneracy pressure. The numerical results on the growth rate of the modulation instability is also presented.

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## I. INTRODUCTION

Electron-positron ($e$-$p$) plasmas, also known as pair plasmas, are found in the early universe [1,2], in astrophysical objects, e.g., pulsars [3], supernova remnants, and active galactic nuclei [4,5] in $\gamma$-ray bursts [6], at the center of the Milky Way galaxy [7], and even in the ultrashort laser pulse-matter interactions [8]. In the astrophysical systems, the $e$-$p$ pairs can be created by collisions between particles that are accelerated by electromagnetic and electrostatic waves and/or by gravitational forces. In the pulsar environments, there is also a possibility of pair creation through the high-energy curvature radiation photons that are triggered by charged particles streaming along the curved magnetic field [9,10] with a resulting collection of positrons at the polar caps of the pulsar [11–13]. Recent experiments have opened up the possibility of creating $e$-$p$ plasmas in the laboratory. High energy laser-plasma interactions and fusion devices are also the sources of $e$-$p$ plasmas. Intense laser-plasma interaction experiments have recently reported the production of MeV electrons and also conclusive evidence of positron production [14–16] through electron collisions [17]. Positrons have also been created in large tokamaks [18] via collisions between MeV electrons and the thermal particles. The developments of the above experiments makes it possible to consider performing laboratory experiments on the $e$-$p$ plasmas [19–21].

The pair plasmas are composed of electrons and positrons of equal mass and opposite charges. Due to the equal mass of the pairs, there are fewer spatial and temporal scales on which collective effects arise. Some of the unique features of the neutral $e$-$p$ plasma may be stated as follows. (i) Same dynamical properties for electrons and positrons: Owing to the same masses and electric charge magnitudes for the electrons and positrons, their dynamical behavior is the same. This is to be contrasted with the electron-ion ($e$-$i$) or electron-hole ($e$-$h$) plasma. The dynamical time scales are different from those in $e$-$i$ and $e$-$h$ plasmas. In the case of the $e$-$i$ plasma, for example, the relation among the electron-electron ($ee$), ion-ion ($ii$), and electron-ion ($ei$) relaxation time scales $\tau_{ee}$, $\tau_{ii}$, and $\tau_{ei}$ is $\tau_{ee} : \tau_{ii} : \tau_{ei} \sim 1 : (m_i/m_e)^{1/2} : m_i/m_e$, where $m_i$ and $m_e$ are the masses of the ion and electron, respectively. Due to this hierarchy of the time scales the $e$-$i$ plasma may exist as a two temperature plasma. For the $e$-$p$ plasma, on the other hand, the electron-electron ($--$), positron-positron ($++$), and electron-positron ($-+$) relaxation time scales are comparable: $\tau_{--} : \tau_{++} : \tau_{-+} = 1 : 1 : 1/2$. Thus, it is not possible to produce an $e$-$p$ plasma with each component in thermal equilibrium and either with $T_- \gg T_+$ or $T_- \ll T_+$. Here, $T_\mu$ is the temperature of the species $\mu$ ($-$ for electrons and $+$ for positrons). (ii) Coupling to electromagnetic waves in the presence of a magnetic field: In the presence of a magnetic field, the electron and positron perform gyromotion at the same frequency in opposite directions. This is to be contrasted to the case with $e$-$i$ plasma. For a charge neutral $e$-$p$ plasma, the plasma couples to the left- and right-circularly polarized waves equally, which is in contrast to the $e$-$i$ plasma.

In most of the astrophysical plasmas, the electrons and the positrons exist in relativistic regimes and therefore most of the research works have been directed toward relativistic $e$-$p$ plasmas [22,23] (and references therein). The classical study of collective behavior of $e$-$p$ plasmas is important to understand some aspects of astrophysical plasma situations because $e$-$p$ plasma radiates effectively by emission of cyclotron radiations [24].

The theory of nonlinear wave propagation in a relativistic electron-positron plasma with a strong external magnetic field has been studied by Lominadze *et al.* [25], for possible application to explain the radiation mechanism in pulsar microstructure. In this study, a nonlinear wave theory is presented for the excitation of low-frequency Alfven waves by waves resonantly interacting with the electrons and the positrons of the ultrarelativistic pair plasma that can explain the high effective temperatures of pulsar radio emissions. Later on, Stenflo *et al.* [26] investigated the nonlinear propagation of electromagnetic waves in magnetized electron-positron plasmas. The nonlinear interaction of the external magnetic field-aligned circularly polarized electromagnetic radiation with a cold electron-positron plasma has been considered in their investigation. Modulational instabilities and wave localization have been discussed after deriving a set of nonlinear equations describing the coupling of the electromagnetic radiation with the cold electrostatic oscillations.

Quantum plasma have attracted a renewed attention in recent years. The inclusion of the quantum terms in the plasma fluid equations—such as quantum diffraction effects, modified equations of state, and the spin degrees of freedom—leads to a variety of new physical phenomena. The study of quantum plasma becomes important when the de Broglie wavelength associated with the charged particles becomes of the order or greater than the interparticle distance of the system, and the plasma behaves like a Fermi gas and quantum mechanical effects play a significant role in the dynamics [27–30]. The quantum hydrodynamic (QHD) model is a useful approximation to study the short-scale collective phenomena, such as waves, instabilities, and nonlinear structures, etc., in dense plasmas [27,28,31]. The QHD model generalizes the usual fluid model with the inclusion of the statistical degenerate pressure and quantum diffraction (also known as the Bohm potential) terms. The validity of the QHD model is limited to those systems that are large compared to the Fermi lengths of the species in the system. In most of the earlier studies [27,29,30,32–35], it was shown that the presence of positrons plays a significant role in the formation of topological solitons, obtained from the solution of the Korteweg–de Vries (KdV) equation in dense plasmas. The low-frequency waves, such as ion-acoustic waves, drift waves, etc., have been studied in quantum *e-p-i* plasmas with their application to neutron stars and pulsars [36]. Furthermore, low and arbitrary amplitude nonlinear structures have also been studied in dense *e-p-i* plasmas and/or *e-i* plasmas [37–40]. However, in most of the recent investigations, quantum effects are considered mostly involving electron-ion or dusty plasmas without external magnetic field [41–50].

Recently, Shukla *et al.* [51] have carried out a study on the electromagnetic solitary pulses in a magnetized *e-p* plasma. They have considered a compressional electromagnetic wave propagating perpendicular to the external magnetic field and have developed a full set of nonlinear equations for the propagation of the wave. They have found that solitary electromagnetic pulses can exist within a definite velocity range. It is also found in that investigation that a strong localization of pulses occurs in the cold pair plasma. In their study, they have not considered the Fermi degeneracy pressure and the quantum diffraction effect. On the other hand, Mahmood *et al.* [39] have recently investigated electrostatic solitary waves in a nonrelativistic quantum *e-p-i* plasma with Fermi degenerate electrons and positrons. They have derived the Kadomtsev-Petviashvili (KP) equation and studied the properties of localized acoustic solitary waves (topological soliton) for parameters relevant to dense astrophysical objects, e.g., the outer layers of white dwarfs, neutron stars, and magnetars, etc. However, the effect of magnetic field is missing in their study. Furthermore, their study excludes possible electromagnetic wave perturbations. It is our understanding that, for the consideration of parameter range of astrophysical objects, the effect of external magnetic field should be included in these investigations for the propagation of electrostatic and electromagnetic waves.

In the present paper, we consider the model of Shukla *et al.* [51] for the perpendicular propagation of a compressional electromagnetic wave pulse in a strong external magnetic field, with the inclusion of the degenerate quantum pressure law [52,53] with external magnetic field, particle exchange potential, Bohm quantum effects, and also dissipative effect due to particle collisions. Thus, our model is to study the propagation of an electromagnetic wave pulse (packet) in a Fermi degenerate *e-p* collisional quantum plasma in the presence of a strong external magnetic field. It should be mentioned here that the study of quantum *e-p* plasma remains in the nonrelativistic regime if the Fermi energies of electrons and positrons are much smaller than their rest energies, and this holds in our present study.

We study the linear properties of the compressional electromagnetic pulse propagating in the degenerate *e-p* quantum plasma in a strong external magnetic field. We also study the amplitude modulation of the slow evolution of the electromagnetic wave pulse. We apply the standard reductive perturbation technique [54] to derive a modified-nonlinear Schrödinger equation (modified-NLSE) for the propagation of the compressional electromagnetic wave in the *e-p* pair plasma. We also discuss the possible modulational instability of the wave amplitude and carry out some parameter studies on the dissipative nonlinear compressional electromagnetic wave.

We have become interested to study modulational instability because it is one of the most ubiquitous types of instabilities in nature and plays an important role in the formation of plasma turbulence. It affects the spectrum of the turbulent oscillations, the acceleration and heating of charged particles, the emission of electromagnetic waves, and so on.

The paper is organized as follows. Section II describes the hydrodynamical model of the quantum *e-p* plasma and presents the set of highly nonlinear equations for the evolution of the large amplitude compressional electromagnetic wave pulse. In this section, a modified-NLSE is derived by applying the reductive perturbation technique. A nonlinear dispersion relation and hence an expression for the growth rate of the modulational instability of the dissipative compressional electromagnetic wave is derived in this section. Results are discussed in Sec. III along with graphical representation. Finally, Sec. IV concludes the paper.

## II. MATHEMATICAL MODEL

We consider the propagation of a large amplitude dissipative compressional electromagnetic wave, $\mathbf{B}' = \hat{z}\,\delta B\, \exp{[i(kx - \omega t)]}$, along the $x$ direction of a three-dimensional rectangular Cartesian geometry, in a degenerate quantum *e-p* pair plasma in the presence of an external magnetic field $\mathbf{B}_0 = B_0\hat{z}$: where $B_0$ is the strength of the external magnetic field, $\delta B$ is the slowly varying modulated compressional magnetic field of the electromagnetic perturbation $(k,\omega)$, $k$ and $\omega$ are respectively the wave number and angular frequency of the carrier wave, and $\hat{z}$ is the unit vector along the $z$ axis. Thus the total magnetic field along the $z$ axis is $\mathbf{B}_T = \mathbf{B}_0 + \delta\mathbf{B}\, \exp{[i(kx - \omega t)]}$. The nonlinear propagation of the compressional electromagnetic wave is then governed by the following quantum magnetohydrodynamics (QMHD) set of equations:

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{v}_\alpha) = 0. \tag{1}$$

$$m\left(\frac{\partial}{\partial t} + \mathbf{v}_\alpha \cdot \nabla\right)\mathbf{v}_\alpha = q_\alpha \mathbf{E} + \frac{q_\alpha}{c}\mathbf{v}_\alpha \times \mathbf{B}_T - \frac{1}{n_\alpha}\nabla P_\alpha + \nabla V_{xc\alpha} + \nabla V_{B\alpha} - m\nu_0 \mathbf{v}_\alpha, \tag{2}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}_T}{\partial t}, \tag{3}$$

$$\nabla \times \mathbf{B}_T = \frac{4\pi}{c}\sum_\alpha q_\alpha \mathbf{v}_\alpha, \tag{4}$$

where $\alpha = e$ (for electrons) and $\alpha = p$ (for positrons), $q_\alpha = ee_\alpha$ is the charge, $m$ is the mass of an electron or a positron, $\mathbf{v}_\alpha$ is the velocity of species $\alpha$; $\mathbf{E}$ is the self-consistent electric field of the compressional electromagnetic wave; $e_\alpha = -1$ for electrons, $e_\alpha = +1$ for positrons, $e$ is the magnitude of the electronic charge, $c$ is the speed of light in a vacuum, and the quantity $\nu_0$ is the average effective collision frequency of electrons (positrons).

In our model, we consider strong magnetic field with Landau quantization effect and therefore the following appropriate quantum pressure law [53] is adopted:

$$P_\alpha = \frac{4|q_\alpha|B_0(2m)^{1/2}E_{F\alpha}^{3/2}}{3(2\pi)^2\hbar^2 c}\left[1 + 2\sum_{l=1}^{l_m}\left(1 - \frac{l\hbar\omega_{c\alpha}}{k_B T_{F\alpha}}\right)^{3/2}\right],$$

where the value of $l_m$ is fixed by the largest integer $l$ that satisfies $k_B T_{F\alpha} - l\hbar\omega_{c\alpha} \leqslant 0$, $\omega_{c\alpha} = |q_\alpha|B_0/mc$, $k_B$ is the Boltzmann constant, $\hbar = h/2\pi$, and $h$ is Planck's constant. $E_{F\alpha}$ and $T_{F\alpha}$ are respectively the Fermi energy and Fermi temperature and are related to the particle number density $n_\alpha$ by the relation $k_B T_{F\alpha} \equiv E_{F\alpha} = \hbar^2(3\pi^2 n_\alpha)^{2/3}/2m$. The above expression for the quantum pressure with strong external magnetic field was derived in detail earlier by Eiezer *et al.* [52]. After simplification, the expression for the quantum pressure can be cast into the following form:

$$P_\alpha = \frac{1}{2}\hbar\omega_{c\alpha}n_\alpha\left[1 + 2\sum_{l=1}^{l_m}\left(1 - \frac{l\hbar\omega_{c\alpha}}{k_B T_{F\alpha 0}}\right)^{3/2}\right], \tag{5}$$

$T_{F\alpha 0}$ is given by $k_B T_{F\alpha 0} = E_{F\alpha 0} = \hbar^2(3\pi^2 n_{\alpha 0})^{(2/3)}/2m$, $n_{\alpha 0}$ is the equilibrium particle number density of species $\alpha$. We also consider the exchange correlation potential of the electrons and the positrons [55]:

$$V_{xc\alpha} = -0.985e^2 n_\alpha^{1/3}\left[1 + \frac{0.034}{a_B n_\alpha^{1/3}}\ln\left(1 + 18.37 a_B n_\alpha^{1/3}\right)\right], \tag{6}$$

where $a_B = \hbar^2/me^2$. The origin of this potential is the following: The interaction of the plasma species can be separated into a Hartree term due to the electrostatic potential of the plasma particle number density and a particle exchange potential [56]. The exchange-correlation potential energy can be described as an unknown functional of the particle number density, and was introduced by Kohn and Sham [57] in the case of electrons. In ordinary circumstances, because of the weak nature, the particle exchange-correlation effect is usually neglected. However, the role of the particle exchange-correlation effects become important when the particle number density is high and the particle temperature is low, such as in some astrophysical plasmas and plasmas involving modern technologies [58–60].

We also consider the Bohm potential due to the tunneling effect of the plasma particles [27]:

$$V_{B\alpha} = \frac{\hbar^2}{2m}\left(\frac{1}{\sqrt{n_\alpha}}\frac{\partial^2\sqrt{n_\alpha}}{\partial x^2}\right). \tag{7}$$

The above Bohm potential arises directly from the Schödinger equation, and is responsible for typical quantumlike behavior involving tunneling and wave-packet spreading and comes from the nonlinear coupling between the scalar potential associated with the space charge electric field and the particle (electron or positron) wave function [28]. The importance of this term increases for comparatively larger values of the propagation wave number and the particle number density.

In a quasineutral electron-positron plasma, we assume $n_e = n_p = n$, where $n_e$ and $n_p$ are the number densities of the electrons and positrons in the plasma and thus we can set $n_{\alpha 0} = n_0$ and $P_\alpha = P$. The continuity equations for the electrons and positrons predict that the $x$ components of the electron and positron fluid velocities must be equal: $v_{ex} = v_{px} \equiv v_x$. Hence, the continuity equation, Eq. (1), for both the electrons and positrons can be written as

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv_x) = 0. \tag{8}$$

From Eq. (2), the $x$ components of the electron and positron momentum equations become

$$m\left(\frac{\partial}{\partial t} + v_x\frac{\partial}{\partial x}\right)v_x = -eE_x - \frac{e}{c}v_{ey}B_T - \frac{1}{n}\frac{\partial P}{\partial x} + \frac{\partial V_{xc}}{\partial x} + \frac{\partial V_B}{\partial x} - m\nu_0 v_x, \tag{9}$$

and

$$m\left(\frac{\partial}{\partial t} + v_x\frac{\partial}{\partial x}\right)v_x = eE_x + \frac{e}{c}v_{py}B_T - \frac{1}{n}\frac{\partial P}{\partial x} + \frac{\partial V_{xc}}{\partial x} + \frac{\partial V_B}{\partial x} - m\nu_0 v_x. \tag{10}$$

Eliminating $E_x$ from Eq. (9) by using Eq. (10), we have

$$m\left(\frac{\partial}{\partial t} + v_x\frac{\partial}{\partial x}\right)v_x = \frac{e}{2c}(v_{py} - v_{ey})B_T - \frac{1}{n}\frac{\partial P}{\partial x} + \frac{\partial V_{xc}}{\partial x} + \frac{\partial V_B}{\partial x} - m\nu_0 v_x. \tag{11}$$

By using Ampere's law, Eq. (4), Eq. (11) yields

$$\left(\frac{\partial}{\partial t} + v_x\frac{\partial}{\partial x}\right)v_x = -\frac{B_T}{8\pi mn}\frac{\partial B_T}{\partial x} - \frac{1}{mn}\frac{\partial P}{\partial x} + \frac{1}{m}\frac{\partial V_{xc}}{\partial x} + \frac{1}{m}\frac{\partial V_B}{\partial x} - \nu_0 v_x. \tag{12}$$

Again from Eq. (2), the $y$ components of the electron and positron momentum equations are

$$m\left(\frac{\partial}{\partial t}+v_x\frac{\partial}{\partial x}\right)v_{ey}=-eE_y+\frac{e}{c}v_xB_T-m\nu_0v_{ey}, \tag{13}$$

and

$$m\left(\frac{\partial}{\partial t}+v_x\frac{\partial}{\partial x}\right)v_{py}=eE_y-\frac{e}{c}v_xB_T-m\nu_0v_{py}. \tag{14}$$

Subtracting Eq. (13) from Eq. (14), we have

$$m\left(\frac{\partial}{\partial t}+v_x\frac{\partial}{\partial x}\right)(v_{py}-v_{ey}) = 2eE_y-2\frac{e}{c}v_xB_T-m\nu_0(v_{py}-v_{ey}). \tag{15}$$

The above equation can be rewritten using Ampere's law, Eq. (4), as

$$E_y=\frac{1}{c}v_xB_T-\frac{mc\nu_0}{8\pi e^2}\frac{1}{n}\frac{\partial B_T}{\partial x}-\frac{mc}{8\pi e^2}\times\left[\left(\frac{\partial}{\partial t}+v_x\frac{\partial}{\partial x}\right)\left(\frac{1}{n}\frac{\partial B_T}{\partial x}\right)\right], \tag{16}$$

which after using Faraday's law, Eq. (3), takes the following form:

$$\frac{\partial B_T}{\partial t}+\frac{\partial}{\partial x}(v_xB_T)=\frac{mc^2\nu_0}{8\pi e^2}\frac{\partial}{\partial x}\left(\frac{1}{n}\frac{\partial B_T}{\partial x}\right)+\frac{mc^2}{8\pi e^2}\times\frac{\partial}{\partial x}\left[\left(\frac{\partial}{\partial t}+v_x\frac{\partial}{\partial x}\right)\left(\frac{1}{n}\frac{\partial B_T}{\partial x}\right)\right]. \tag{17}$$

Equations (8), (12), and (17) are the governing nonlinear equations for the compressional electromagnetic wave in a hot magnetized $e$-$p$ plasma. We normalize quantities according to the following prescription: $N=n/n_0$, $V=v_x/C_A$, where $C_A=B_0/\sqrt{8\pi n_0 m}$ is the Alfven speed, $B=B_T/B_0$; $t$ and $x$ are normalized by the cyclotron frequency, $\omega_c=eB_0/mc$, and the Alfven wavelength $\lambda_A=C_A/\omega_c$, respectively, i.e., $T=t\omega_c$ and $X=x/\lambda_A$. The plasma $\beta$ of the electron-positron plasma is defined by $\beta_p=8\pi n_0k_BT_F/B_0^2$ ($T_F\equiv T_{F\alpha}$, $\alpha=e,p$). With these normalizations, the set of equations, given by Eqs. (8), (12), and (17), along with Eqs. (5)–(7), becomes

$$\frac{\partial N}{\partial T}+\frac{\partial}{\partial X}(NV)=0, \tag{18}$$

$$\frac{\partial V}{\partial T}+V\frac{\partial V}{\partial X}=-\frac{B}{N}\frac{\partial B}{\partial X}-\beta N^{-1}\frac{\partial N}{\partial X}-\sigma N^{-2/3}\frac{\partial N}{\partial X}+\frac{H^2}{2}\frac{\partial}{\partial X}\left(\frac{1}{\sqrt{N}}\frac{\partial^2}{\partial X^2}\sqrt{N}\right)-\mu V, \tag{19}$$

$$\frac{\partial B}{\partial T}+\frac{\partial}{\partial X}(VB)=\mu\frac{\partial}{\partial X}\left(\frac{1}{N}\frac{\partial B}{\partial X}\right)+\frac{\partial}{\partial X}\left(\frac{\partial}{\partial T}+V\frac{\partial}{\partial X}\right)\left(\frac{1}{N}\frac{\partial B}{\partial X}\right), \tag{20}$$

where $\mu=\nu_0/\omega_c$,

$$\beta=\frac{1}{2}\frac{\hbar\omega_c}{mC_A^2}\left[1+2\sum_{l=1}^{l_m}\left(1-\frac{l\hbar\omega_c}{k_BT_{F0}}\right)^{3/2}\right],$$

$$\sigma=\frac{0.328n_0^{1/3}e^2}{mC_A^2}\left[1+\frac{0.625}{1+18.37a_Bn_0^{1/3}}\right],$$

and $H=\hbar\omega_c/2mC_A^2$.

### A. Linear dispersion relation for the compressional electromagnetic wave

For a small perturbation $(\widetilde{\omega},k)$, we consider $N=1+\delta N$, $V=\delta V$, and $B=1+\delta B$ and letting $\delta N$, $\delta V$, and $\delta B$ vary as $\sim\exp[i(kX-\widetilde{\omega}T)]$; the linearized parts of Eqs. (18)–(20) give the following dispersion equation:

$$(1+k^2)\widetilde{\omega}^3+i\mu(1+k^2+k^2)\widetilde{\omega}^2-k^2\times[1+\mu^2+(1+k^2)f]\widetilde{\omega}-i\mu k^4f=0, \tag{21}$$

where $f=\beta+\sigma+H^2k^2$. First, let us consider the collisionless case by setting $\mu\equiv\nu_0/\omega_c=0$, then we get the following dispersion relation from Eq. (21) with $\widetilde{\omega}\equiv\omega$:

$$\omega^2=k^2f+\frac{k^2}{1+k^2}. \tag{22}$$

The dispersion relation, Eq. (22), can be rewritten as

$$\omega^2=k^2(\beta+\sigma+k^2H^2)+\frac{k^2}{1+k^2}. \tag{23}$$

We can identify the importance of different terms of the right hand side of Eq. (23). The combination of the first three terms of the right hand side of Eq. (23) gives the electrostatic contribution and the last term gives the electromagnetic contribution to the linear dispersion of the compressional electromagnetic wave. Let us denote the first, second, third, and fourth terms of the right hand side of Eq. (23) by $T_{DP}=k^2\beta$, $T_{XP}=k^2\sigma$, $T_{BP}=k^4H^2$, and $T_{EM}=k^2/(1+k^2)$. We also define $T_{ES}=T_{DP}+T_{XP}+T_{BP}$.

Next, we solve Eq. (21) numerically for a possible dispersion relation with nonzero linear damping ($\mu=\nu_0/\omega_c\neq0$) of the wave due to collisions. From the appropriate solution, the real part of $\widetilde{\omega}$, $\omega=\mathrm{Re}[\widetilde{\omega}]$ and the imaginary part of $\widetilde{\omega}:\gamma=\mathrm{Im}[\widehat{\omega}]$ can easily be obtained. We plot $\omega$ and $|\gamma|$ as functions of the carrier wave number $k$ in Figs. 3 and 4.

### B. Derivation of the modified nonlinear Schrödinger equation

To derive the modified-NLSE, we apply the standard reductive perturbation technique [61] with the following stretched coordinates: $\xi=\epsilon(X-v_0T)$ and $\tau=\epsilon^2T$, where $v_0$ is a constant and will be determined later from the compatibility condition of the reductive perturbation analysis and $\epsilon$ is a small positive quantity less than 1, and is known as the perturbation parameter. Expanding the dependent variables $N$, $V$, and $B$ as

$$N=1+\sum_{n=1}^{\infty}\epsilon^n\sum_{l=-\infty}^{\infty}N_l^{(n)}(\xi,\tau)\,e^{il(kX-\omega T)}, \tag{24}$$

$$V = \sum_{n=1}^{\infty} \epsilon^n \sum_{l=-\infty}^{\infty} V_l^{(n)}(\xi,\tau)\, e^{il(kX-\omega T)}, \tag{25}$$

$$B = 1 + \sum_{n=1}^{\infty} \epsilon^n \sum_{l=-\infty}^{\infty} B_l^{(n)}(\xi,\tau)\, e^{il(kX-\omega T)}, \tag{26}$$

$\omega$ and $k$ are the angular frequency and the wave number of the carrier electromagnetic wave. The quantities $N_l^{(n)}(\xi,\tau)$, $V_l^{(n)}(\xi,\tau)$, and $B_l^{(n)}(\xi,\tau)$ are the $l$th harmonic of the $n$th order slowly varying field variables $N$, $V$, and $B$ respectively, and these satisfy the reality condition $A_{-l}^{(n)} \equiv A_l^{(n)*}$ and the asterisk denotes complex conjugation. We note that according to the stretched variables and the expanded variables in Eqs. (24)–(26), we have two coordinate systems: the $(X,T)$ coordinate system, in which there is a rapid variation of the field quantities of the carrier wave with wave number $k$ and angular frequency $\omega$; and the $(\xi,\tau)$ coordinate system, in which there is a slowly modulated wave packet. In our derivation of the modified-NLSE, the value of the normalized collision frequency $\mu$ is very small compared to the electron cyclotron frequency $\omega_c$; in this situation we can set [62]

$$\mu = \eta\epsilon^2, \tag{27}$$

where $\eta$ is a finite quantity of the order of unity. If we now substitute the above expansions, given by Eqs. (24)–(26) for the field variables with Eq. (27) in the set of equations, Eqs. (18)–(20), we can easily obtain the $l$th harmonic of the $n$th order reduced equations. We omit listing the reduced equations here as they are very lengthy and will not be much use to most of the readers.

If we consider the first harmonic of the first order quantities ($n = 1$ and $l = 1$) in the reduced equations, the following linear dispersion relation of the compressional electromagnetic wave is easily obtained:

$$\omega^2 = k^2 f + \frac{k^2}{1+k^2}, \tag{28}$$

where $f$ is defined below Eq. (21) and the quantities $\beta$, $\sigma$, and $H$ are defined earlier. Equation (28) coincides with the linear dispersion relation, Eq. (22), obtained earlier when the value of the normalized collision frequency $\mu$ is set to zero. Similarly, from the first harmonic of the second order quantities ($n = 2$ and $l = 1$), and using Eq. (28), we obtain the following compatibility condition:

$$(1+k^2)(kff_1 + \omega f_2) + kf_3 = 0,$$

where the expressions for $f_1$, $f_2$, and $f_3$ are given in the Appendix. This compatibility condition eventually gives the following expression for the group velocity $v_0$ of the compressional electromagnetic wave:

$$\begin{aligned} v_0 = {} & [\omega(1+k^2)\{(1+k^2)(f+f_0)+2\} - 2k^2\omega] \\ & \times[k(1+k^2)\{1+f(1+k^2)+\omega^2(1+k^2)/k^2\}]^{-1}, \end{aligned} \tag{29}$$

where $f_0 = \beta + \sigma + 3H^2k^2$.

It should be mentioned here that we are particularly interested in the low-frequency ($\omega < \omega_c$) limit for the compressional electromagnetic wave mode for long wavelength perturbation $(\omega,k)$.

Here, we do not go into the detail derivation of the modified-NLSE, but only state briefly the procedure. From the $l = 2$ part of the second order ($n = 2$) reduced equations, the second order harmonic mode $B_2^{(2)}$ of the carrier wave is obtained in terms of $B_1^{(1)}B_1^{(1)}$, where $B_1^{(1)}$ is the first order mode, obtained in deriving the linear dispersion relation, Eq. (28). The mode $B_2^{(2)}$ comes from the nonlinear self-interaction. Similarly, from the $l = 0$ part of the third order ($n = 3$) reduced equations, the zeroth harmonic mode $B_0^{(2)}$ is obtained in terms of $|B_1^{(1)}|^2$. Its expression cannot be obtained from the second order equations. Finally, substituting the expressions for $B_2^{(2)}$ and $B_0^{(2)}$ into the $l = 1$ component of the third order ($n = 3$) part of the reduced equations, we finally obtain the following modified-NLSE for the nonlinear propagation of the compressional electromagnetic wave:

$$i\frac{\partial a}{\partial\tau} + P\frac{\partial^2 a}{\partial\xi^2} + Q|a|^2 a = -i\Upsilon a, \tag{30}$$

where $a \equiv B_1^{(1)}$ and

$$\begin{aligned} P = {} & \left[\frac{\omega}{k}(1+k^2)\left\{\frac{f_0}{\omega}(f_3-f_1) + 3H^2k(1+k^2) - v_0\frac{f_3}{k}\right\}\right. \\ & \left. + f(1+k^2)\left\{\frac{f_3}{k} - \frac{v_0}{\omega}(f_3-f_1)\right\} - \left(\omega + 2kv_0 - \frac{f_3}{k}\right)\right] \\ & \times\left[(1+k^2)\left\{1 + f(1+k^2) + \frac{\omega^2}{k^2}(1+k^2)\right\}\right]^{-1}, \end{aligned} \tag{31}$$

$$\begin{aligned} Q = {} & -\left[L_1 + fF_1(1+k^2) + \frac{\omega}{k}G_1(1+k^2)\right] \\ & \times\left[(1+k^2)\left\{1 + f(1+k^2) + \frac{\omega^2}{k^2}(1+k^2)\right\}\right]^{-1}, \end{aligned} \tag{32}$$

and

$$\begin{aligned} \Upsilon = {} & \eta\left[k^2 + \frac{\omega^2}{k^2}(1+k^2)^2\right] \\ & \times\left[(1+k^2)\left\{1 + f(1+k^2) + \frac{\omega^2}{k^2}(1+k^2)\right\}\right]^{-1}. \end{aligned} \tag{33}$$

The quantities $f_1$, $f_2$, $f_3$, $L_1$, $F_1$, and $G_1$ appearing in Eqs. (31) and (32) are given in the Appendix.

The coefficients $P$ and $Q$ appearing in the modified-NLSE, given by Eq. (30), are known as the dispersion coefficient and nonlinear coefficient respectively. The signs of $P$ and $Q$ determine whether or not the slowly varying wave amplitude is modulationally stable. If the signs of $P$ and $Q$ are such that $PQ < 0$, the wave amplitude is modulationally stable and the corresponding solution of the modified-NLSE is called a dark soliton [63] in the absence of collisions. On the other hand, if $PQ > 0$, then the wave amplitude may be modulationally unstable and the solution of the modified-NLSE in this case is called a bright soliton [63] in the absence of collisions. The question of dark soliton or bright soliton appears only when the right hand side of Eq. (30) is zero, i.e., in the collisionless situation. On the other hand, in the collisional case, i.e., if there is dissipation in the system, the modified-NLSE does not admit any stationary envelope solitonic solution. However, we are particularly interested about modulation instability of the nonlinear dissipative compressional electromagnetic wave in the long wavelength limit, and this is discussed next.

### C. Modulation instability of the dissipative compressional electromagnetic wave

Let us analyze the modulational instability of the dissipative compressional electromagnetic wave described by the modified-NLSE, Eq. (30), in the case of $PQ > 0$, where $P$ and $Q$ are respectively the coefficients of the dispersive and the nonlinear terms. We write the amplitude $a(\xi,\tau)$ as [62]

$$a(\xi,\tau) = [a_0 + \delta a(\xi,\tau)]\ \exp\left[-i\int_0^\tau \Delta(\tau)\, d\tau - \Upsilon\tau\right], \quad (34)$$

where $\Delta$ is a possible nonlinear frequency shift, and $a_0$ (real constant) is the amplitude of the pump carrier wave. By expanding $\delta a(\xi,\tau)$ as

$$\delta a(\xi,\tau) = (U + iW)\ \exp\left[i\left(K\xi - \int_0^\tau \Omega(\tau)\, d\tau\right)\right],$$

where $U$ and $W$ are respectively the real and imaginary parts of $\delta a(\xi,\tau)$, the following nonlinear dispersion relation for the modulation of the compressional electromagnetic wave is obtained:

$$\Omega^2 = (PK^2)^2\left(1 - \frac{K_c^2(\tau)}{K^2}\right), \quad (35)$$

where $K_c^2(\tau) = (2Q|a_0|^2/P)\ \exp(-2\Upsilon\tau)$. From Eq. (35), the local-instability growth rate is given by

$$\Gamma \equiv \mathrm{Im}[\Omega(\tau)] = |PK^2|\left(\frac{K_c^2(\tau)}{K^2} - 1\right). \quad (36)$$

We observe from the nonlinear dispersion relation, Eq. (35), that if $K_c^2(\tau) \leqslant K^2$, then $\Omega^2 \geqslant 0$, there will be no growth of the wave. However, when $K_c^2(\tau) > K^2$, then $\Omega^2$ becomes negative, and there will be absolute growth of the wave. Thus for absolute growth,

$$\frac{2Q|a_0|^2}{P}\ \exp(-2\Upsilon\tau) > K^2.$$

We observe that as $\tau$ increases, the left hand side of the above inequality decreases and this will continue until $\tau = \tau_{\max}$, when $\Omega$ becomes zero, and the growth of the wave ceases. After $\tau > \tau_{\max}$, $\Omega^2 \geqslant 0$, and the wave becomes modulationally stable. Thus, by equating $K_c^2(\tau)$ with $K^2$, we have

$$\frac{2Q|a_0|^2}{P}\ \exp(-2\Upsilon\tau_{\max}) = K^2,$$

from which we obtain

$$\tau_{\max} = \frac{1}{2\Upsilon}\ln\left(\frac{2Q|a_0|^2}{PK^2}\right).$$

The quantity $\tau_{\max}$ is the modulation instability period in the collisional plasma [62]. This instability period is absent in plasmas without collision. Thus, the instability growth will cease when

$$\tau \geqslant \tau_{\max} = \frac{1}{2\Upsilon}\ln\left(\frac{2Q|a_0|^2}{PK^2}\right). \quad (37)$$

We observe from the above expression for $\tau_{\max}$, that by increasing the damping $\Upsilon$, the instability period decreases, and we can say that by introducing some damping to the system, the instability of the wave can be suppressed.

### D. Solution of the modified-NLSE

Here we present the dissipative nonlinear compressional electromagnetic rogue wave solution of the modified-NLSE, Eq. (30), within the instability region, where the signs of the dispersion coefficient $P$ and the nonlinear coefficient $Q$ of the modified-NLSE are the same, i.e., $PQ > 0$. The required solution can be easily obtained by substituting $\Phi(\xi,\tau) = a(\xi,\tau)\exp(\Upsilon\tau)$ into Eq. (30) giving [64]

$$i\frac{\partial\Phi}{\partial\tau} + P\frac{\partial^2\Phi}{\partial\xi^2} + e^{-2\Upsilon\tau}Q|\Phi|^2\Phi = 0. \quad (38)$$

Since the damping (dissipative) coefficient $\Upsilon$ is very small, we have $e^{-2\Upsilon\tau} \approx 1/(1+2\Upsilon\tau)$ by the Taylor expansion. Using this approximation, Eq. (38) can be written as

$$i\frac{\partial\Phi}{\partial\tau} + P\frac{\partial^2\Phi}{\partial\xi^2} + Q\sigma_D|\Phi|^2\Phi = 0, \quad (39)$$

where $\sigma_D = e^{-2\Upsilon\tau} \approx 1/(1+2\Upsilon\tau)$. By substituting $\zeta(\xi,\tau) = \sigma_D(\tau)\xi$, $\chi = \sigma_D(\tau)\tau$, and $\widehat{\Phi} = \Phi\ \exp[i(\Upsilon\sigma_D(\tau)\xi^2/2P)]$, we can transform Eq. (39) into the following standard NLSE:

$$i\frac{\partial\widehat{\Phi}}{\partial\chi} + P\frac{\partial^2\widehat{\Phi}}{\partial\zeta^2} + Q|\widehat{\Phi}|^2\widehat{\Phi} = 0. \quad (40)$$

The above standard NLSE has a rational solution that is localized on a nonzero background and localized both in $\zeta$ and $\chi$ directions, that is, in $\xi$ and $\tau$ directions and the ultimate rogue wave solution for the dissipative NLSE, Eq. (30), is obtained as [65]

$$a(\xi,\tau) = \sqrt{\frac{2P}{Q}}\left[\frac{4 + 16iP\sigma_D(\tau)\tau}{1 + 16\{P\sigma_D(\tau)\tau\}^2 + 4\{\sigma_D(\tau)\xi\}^2} - 1\right] \times \exp\left[-i\left\{\frac{\Upsilon\sigma_D(\tau)\xi^2}{2P}\right\}\right]\sqrt{\sigma_D(\tau)}. \quad (41)$$

The above solution (41) predicts the concentration of the compressional electromagnetic wave in a small region due to the nonlinear properties of the electron-positron pair plasma. This solution is able to concentrate a significant amount of the compressional electromagnetic wave energy into a relatively small area in space [65]. In the next section, the dissipative solution, Eq. (41), of the modified-NLSE, Eq. (30), is plotted against the position coordinate $\xi$ for a fixed value of $\tau$, taking $\Upsilon$ (responsible for the dissipative effect) as a parameter.

## III. RESULTS AND DISCUSSIONS

In this section, we analyze the linear as well as nonlinear compressional electromagnetic wave dispersion relations. For the numerical appreciation of our results, we consider the following parameters of the outside layers of white dwarf or neutron star (a compact star) as an $e$-$p$ pair plasma [39]: The external magnetic field, $B_0 \sim 10^{11}$–$10^{12}$ G; the equilibrium particle (electron or positron) number density, $n_0 \sim 5 \times 10^{27}$ to $2 \times 10^{28}$ cm$^{-3}$, the Fermi temperature $T_{F0}$ is

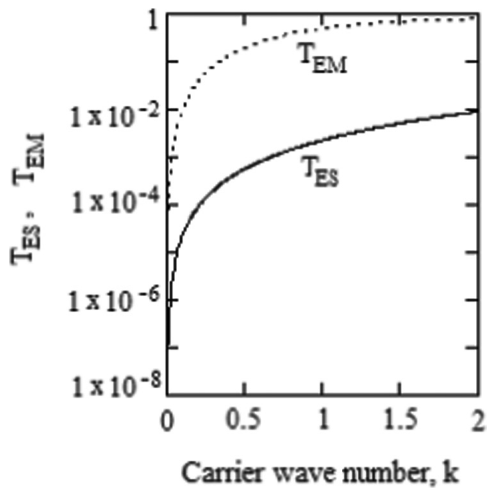


FIG. 1. Variation of the terms $T_{ES}$ and $T_{EM}$ of the right hand side of the linear dispersion, Eq. (23), vs $k$. The solid curve is for $T_{ES}$ and the dotted curve is for $T_{EM}$. The parameters of the electron-positron plasma are as follows: $n_0 = 1 \times 10^{28}$ cm$^{-3}$, $B_0 = 1.2 \times 10^{12}$ G, and $\mu = \nu_0/\omega_c = 0$.

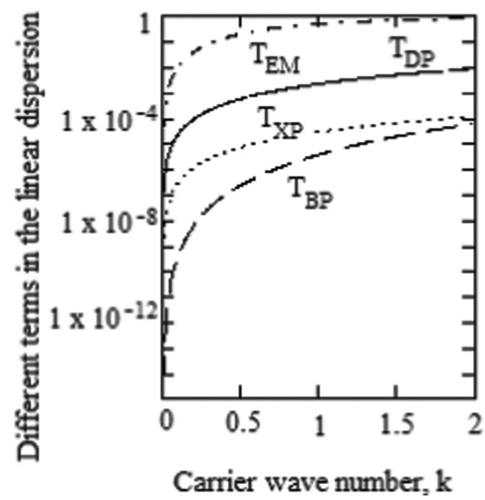


FIG. 2. Variation of the terms $T_{DP}$, $T_{XP}$, $T_{BP}$, and $T_{EM}$ vs $k$. The solid, dotted, dashed, and the dot-dashed curves are for the terms $T_{DP}$, $T_{XP}$, $T_{BP}$, and $T_{EM}$ respectively. The parameters of the electron-positron plasma are as follows: $n_0 = 1 \times 10^{28}$ cm$^{-3}$, $B_0 = 1.2 \times 10^{12}$ G, and $\mu = \nu_0/\omega_c = 0$.

obtained from the relation $E_{F0} = k_B T_{F0} = \hbar^2 (3\pi^2 n_0)^{2/3}/2m$, which in the present case, ranges $T_{F0} = 1.24 \times 10^8$ K to $T_{F0} = 3.12 \times 10^8$ K. The ratio of the Fermi energy and the rest energy of an electron or positron, $E_{F0}/mc^2$, for the particle number density under consideration, ranges 0.021–0.053, which is much smaller than unity. Thus, our model correctly describes the dense plasma in the nonrelativistic regime occurring in outer layers of the white dwarf or compact stars and even in the future laboratory experiments with high power laser interactions. It is to be mentioned here that in the astrophysical $e$-$p$ environment where particle number density is around $10^{28}$ cm$^{-3}$, the pair annihilation can be ignored [39].

Figure 1 shows the variation of the terms $T_{ES}$ and $T_{EM}$ with respect to the carrier wave number $k$. The solid curve is for $T_{ES}$, which is the sum of the first three terms of the right hand side of Eq. (23), that is, $T_{ES} = k^2\beta + k^2\sigma + k^4 H^2$, and the dotted curve is for $T_{EM}$, which is the last term, $T_{EM} = k^2/(1+k^2)$, of Eq. (23). As mentioned earlier, $T_{ES}$ accounts for the electrostatic nature, whereas $T_{EM}$ accounts for the electromagnetic nature of the compressional electromagnetic wave. The curves are plotted for plasma $\beta$, $\beta_p = 4 \times 10^{-3}$. We observe from Fig. 1 that the wave preserves the electromagnetic character for the values of the carrier wave number $k$ ranging 0–2.

Figure 2 shows the variation of the individual terms of the right hand side of Eq. (23) with respect to the carrier wave number $k$. The solid curve is for the term $T_{DP}$, which accounts for the degenerate quantum pressure, the dotted curve is for the exchange correlation term $T_{XP}$, the dashed curve is for the Bohm potential term $T_{BP}$, while the dot-dashed curve is for the electromagnetic term $T_{EM}$. From this figure, we notice that the electromagnetic term $T_{EM}$ and the degenerate quantum pressure term $T_{DP}$ are the most important terms in contributing to the linear dispersion while the effects of the exchange correlation potential and the Bohm potential are negligible for the compressional electromagnetic wave.

Figures 3 and 4 show the variation of the angular frequency $\omega$ and the magnitude of the linear damping rate $\gamma$ due to collisions with respect to the carrier wave number $k$ taking external magnetic field $B_0$ as a parameter. The solid curve is for $B_0 = 1.4 \times 10^{12}$ G, the dashed curve is for $B_0 = 0.5 \times 10^{12}$ G, while the dot-dashed curve is for $B_0 = 0.3 \times 10^{12}$ G for both figures. The plasma $\beta$, $\beta_p$ in these cases, are $3.49 \times 10^{-3}$, 0.027, and 0.076 respectively. We observe that as we decrease the external magnetic field, the plasma $\beta$ increases, however the wave is still electromagnetic in character. We also notice from Fig. 4 that as the external magnetic field is decreased the linear damping slowly decreases for comparatively higher values of the carrier wave number $k$.

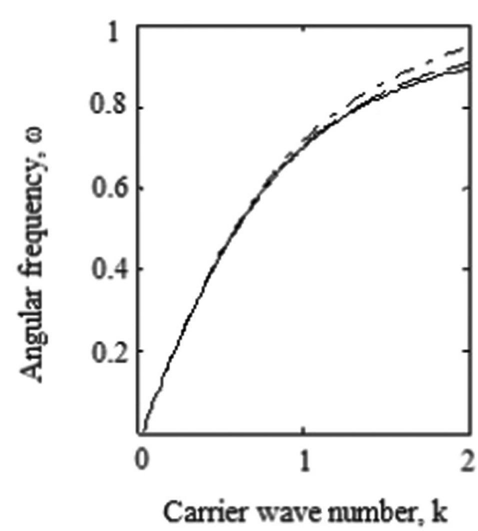


FIG. 3. The linear dispersion relation $\omega$ vs $k$ taking external magnetic field $B_0$ as a parameter. The solid, dashed, and dot-dashed curves are for the external magnetic field $B_0 = 1.4 \times 10^{12}$ G, $B_0 = 0.5 \times 10^{12}$ G, and $B_0 = 0.3 \times 10^{12}$ G respectively. Other parameters are as follows: $n_0 = 1 \times 10^{28}$ cm$^{-3}$ and $\mu = \nu_0/\omega_c = 0.04$.

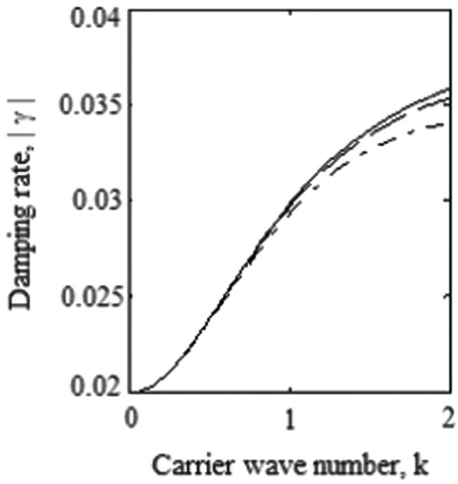


FIG. 4. The linear damping rate $|\gamma|$ vs $k$ taking external magnetic field $B_0$ as a parameter. The solid, dashed, and dot-dashed curves are for the external magnetic field $B_0 = 1.4 \times 10^{12}$ G, $B_0 = 0.5 \times 10^{12}$ G, and $B_0 = 0.3 \times 10^{12}$ G respectively. Other parameters are as follows: $n_0 = 1 \times 10^{28}$ cm$^{-3}$ and $\mu = \nu_0/\omega_c = 0.04$.

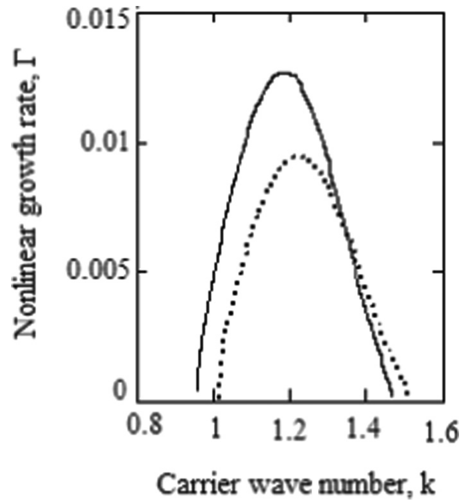


FIG. 6. The modulation growth rate $\Gamma$ with respect to the carrier wave number $k$. The solid and dotted curves are for $B_0 = 1.4 \times 10^{12}$ G and $B_0 = 0.3 \times 10^{12}$ G respectively. Other parameters are as follows: $n_0 = 1 \times 10^{28}$ cm$^{-3}$, modulation wave number $K = 0.01$, $a_0 = 0.01$.

Figure 5 shows the variation of the ratio $P/Q$, of the dispersive coefficient $P$ and the nonlinear coefficient $Q$ of the modified-NLSE, given by Eq. (30), with respect to the carrier wave number $k$, keeping all the nonlinear terms, viz., the terms due to the degenerate quantum particle pressure, the exchange-correlation potential, and the Bohm potential in the particle momentum balance equation, Eq. (19), with $n_0 = 1 \times 10^{28}$ cm$^{-3}$ and $B_0 = 1 \times 10^{12}$ G. We see from this figure that the ratio $P/Q$ becomes positive after the carrier wave number assumes the value around 0.92 which is called the critical wave number, indicating that the compressional electromagnetic wave mode is modulationally unstable at and above this value of the carrier wave number $k$. It could be shown that by changing the value of the external magnetic field $B_0$, the value of the critical wave number is slightly different than the value shown in Fig. 5.

Figure 6 shows the variation of the modulational instability growth rate $\Gamma$ with respect to the carrier wave number $k$ keeping all the nonlinear terms in the momentum balance equation of the carriers with $n_0 = 1 \times 10^{28}$ cm$^{-3}$ for two different values of the external magnetic field $B_0$. The solid and the dotted curves are for the external magnetic field $B_0 = 1.4 \times 10^{12}$ G and $B_0 = 0.3 \times 10^{12}$ G respectively. We observe

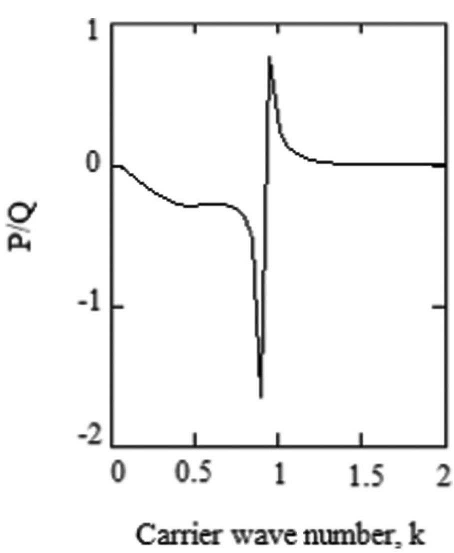


FIG. 5. The variation of the ratio $P/Q$ of the coefficients of the dispersive and the nonlinear terms of the NLSE, Eq. (30), vs the carrier wave number $k$. The parameters of the electron-positron plasma are as follows: $n_0 = 1 \times 10^{28}$ cm$^{-3}$ and $B_0 = 1 \times 10^{12}$ G.

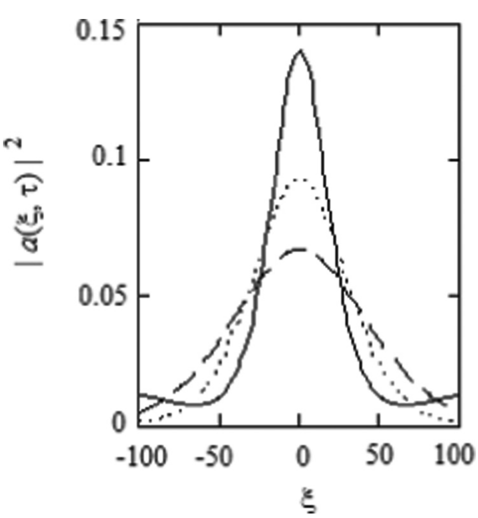


FIG. 7. Variation of the modulus-square $|a(\xi,\tau)|^2$ of the amplitude of the nonlinear dissipative compressional electromagnetic wave with respect to the position coordinate $\xi$ taking $\tau = 40$ with damping $\Upsilon$ as a parameter. The solid, dotted, and dashed curves are for the normalized damping parameters $\Upsilon = 0.74$, 1.47, and 2.21 respectively. The other parameters of the electron-positron plasma are the following: $B_0 = 1.4 \times 10^{12}$ G, $n_0 = 1 \times 10^{28}$ cm$^{-3}$, and carrier wave number $k = 0.95$.

from this figure that the modulation growth rate is noticeable for a domain of the values of the carrier wave number $k$. It is observed from this figure that as the external magnetic field $B_0$ decreases, the growth rate drastically decreases and the curve is shifted to the right in the $\Gamma$ vs $k$ diagram.

Figure 7 shows the variation of the modulus square of the amplitude of the dissipative compressional electromagnetic wave with respect to the position coordinate $\xi$. The solid, dotted, and dashed curves are for the dissipative parameter $\Upsilon = 0.736$, 1.473, and 2.209 respectively for a particular time $\tau = 40$, with $n_0 = 1 \times 10^{28}$ cm$^{-3}$ and $B_0 = 1.4 \times 10^{12}$ G. We observe from this figure that the amplitude of the modulated wave decreases with increasing the damping.

## IV. CONCLUSION

We have investigated the linear and nonlinear wave dispersions of a compressional electromagnetic wave in a dense quantum $e$-$p$ pair plasma in the presence of a strong external magnetic field, usually found in some astrophysical objects. A modified-NLSE is derived for the slow evolution of the compressional electromagnetic wave amplitude. We have discussed both the linear and nonlinear dispersions of the wave propagating in the quantum electron-positron pair plasma along with the dissipation effects due to collisions of the charged particles. For the considered parameter set, the compressional wave amplitude is unstable after a critical value of the carrier wave number $k$. For the case of the modulationally unstable wave, we have studied the modulation instability including the effects of the particle degenerate quantum pressure, exchange-correlation potential and the Bohm potential on the amplitude modulation of the compressional electromagnetic wave. There is a finite modulation period of the unstable compressional wave because of the collisional effect of the charge carriers. From the numerical results, it is found that the carrier wave number and the external magnetic field have profound effects on the dispersion properties of the electromagnetic wave. Important ingredients introduced in this investigation are the inclusion of the quantum statistical degeneracy pressure, exchange-correlation force, and the quantum diffraction effect via the Bohm potential. It is also found that the quantum effects due to the particle exchange potential and the Bohm potential are negligibly small in comparison with the effects of the Fermi quantum degeneracy pressure. The numerical results on the growth rate of the modulation instability are also presented. In conclusion, we stress that the dissipative compressional electromagnetic wave and its stability investigated in this paper can be associated with low-frequency electromagnetic disturbances that can occur in nonrelativistic strongly magnetized pair plasmas, such as those in the outer layers of white dwarfs, neutron stars, magnetars, active galactic nuclei, and even in the future laser-plasma laboratory experiments. Finally, new aspects of nonlinear plasma waves and their role are to be explored with regard to electron and positron acceleration in relativistic situations with strong magnetic field as considered in this investigation.

## APPENDIX

The parameters $F_1$, $G_1$, $L_1$, and $f_1$–$f_{17}$ appearing in Eqs. (31) and (32) during the derivation of the modified-NLSE, Eq. (30), are listed below:

$$F_1 = k(1+k^2)\left[f_{10} + f_{16} + \frac{\omega}{k}(f_{11}+f_{17})\right], \tag{A1}$$

$$G_1 = k\left[f_9 + f_{15} - f_{17} + (1+k^2)\left\{1 - f_9 - f_{17} + \frac{\omega}{k}(f_{10}+f_{16}) - 7H^2k^2 f_{11} - \left(\beta + \frac{2\sigma}{3}\right)(f_{11}+f_{17})\right\}\right], \tag{A2}$$

$$L_1 = k\left[\frac{\omega}{k}(1+k^2)(f_9+f_{15}) + f_{10} + f_{16} + k\omega\{2(1+k^2)f_9 - f_{11} + f_{17}\} + k^2\left\{\frac{2\omega}{k}(1+k^2)f_9 - f_{10} + f_{16}\right\} + k\omega(1+k^2)^2\right], \tag{A3}$$

$$f_1 = -\frac{1}{k}(1+k^2)(\omega - kv_0), \tag{A4}$$

$$f_2 = \frac{v_0\omega}{k}(1+k^2) - (1+k^2)f_0 - 1, \tag{A5}$$

$$f_3 = v_0 + 2k\omega + k^2 v_0 - \frac{\omega}{k}(1+k^2), \tag{A6}$$

$$f_4 = -\omega(1+k^2)^2, \tag{A7}$$

$$f_5 = \beta + \sigma + 4H^2k^2, \tag{A8}$$

$$f_6 = -\frac{k}{2}\left[\frac{\omega^2}{k^2}(1+k^2)^2 - k^2 - (1+k^2)^2\left(\beta + \frac{2\sigma}{3} + H^2k^2\right)\right], \tag{A9}$$

$$f_7 = 1 + 4k^2, \tag{A10}$$

$$f_8 = -\omega(1+k^2)^2, \tag{A11}$$

$$f_9 = -\frac{1}{\omega}\left(f_7 - \frac{k^2}{\omega^2 - k^2 f_5}\right)^{-1}\left[f_8 + \frac{k(\omega f_6 + k f_4 f_5)}{\omega^2 - k^2 f_5}\right], \tag{A12}$$

$$f_{10} = \frac{k\omega f_9 - \omega f_6 - k f_4 f_5}{\omega^2 - k^2 f_5}, \tag{A13}$$

$$f_{11} = \frac{1}{\omega}(k f_{10} - f_4), \tag{A14}$$

$$f_{12} = -\frac{2\omega}{k}(1+k^2)^2, \tag{A15}$$

$$f_{13} = -\frac{\omega^2}{k^2}(1+k^2)^2 + k^2 - \frac{k}{\omega}(f_3 - f_1) + (1+k^2)^2\left(\beta + \frac{2\sigma}{3} + 3H^2k^2\right), \tag{A16}$$

$$f_{14} = -(1+k^2)\left(\frac{2\omega}{k} + 5k\omega + k^2 v_0\right) + k^2(2f_3 - f_1), \tag{A17}$$

$$f_{15} = \frac{1}{v_0\left(1+\beta+\sigma-v_0^2\right)}\left[(\beta+\sigma)f_{12} + v_0 f_{13} - \left(\beta+\sigma-v_0^2\right)f_{14}\right], \tag{A18}$$

$$f_{16} = \left(\frac{1}{\beta+\sigma-v_0^2}\right)\left[(\beta+\sigma)f_{12} + v_0 f_{13} - v_0 f_{15}\right], \tag{A19}$$

$$f_{17} = \frac{1}{v_0}(f_{16} - f_{12}). \tag{A20}$$

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