Observation of two-dimensional Faraday waves in extremely shallow depth

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A family of two-dimensional Faraday waves in extremely shallow depth (1 mm to 2 mm) of absolute ethanol are observed experimentally using a Hele-Shaw cell that vibrates vertically. The same phenomena are not observed by means of water, ethanol solution, and silicone oil. These Faraday waves are quite different from the traditional ones. These phenomena are helpful to deepen and enrich our understandings about Faraday waves, and besides provide a challenging problem for computational fluid dynamics.

I. INTRODUCTION

The Faraday waves were first observed by Faraday using vertical oscillation [1], which oscillate with a half of the driving frequency [2]. The surprising findings about these waves in the next century would probably be the different patterns, such as stripes, squares, hexagons, stars [3,4], even a steep one with a large symmetric double-peaked crest, which was observed by using a forcing frequency lower than 4 Hz in a tank filled with deep water [5]. A little later, the experiments concerning the existence of three-dimensional patterns near a special resonance point were done by Binks and van de Water [6], who also formulated a remarkably simple explanation of the variation of pattern symmetries with respect to depth [7]. For the Faraday resonance in a rectangular basin filled with water, the dependencies of wave amplitude on excitation frequency for a given wave harmonic were investigated both theoretically and experimentally in the case that the water depth is equal or close to the critical depth (60 mm to 87.6 mm) [8], at which the third-order nonlinear correction to the wave frequency predicted by the linear theory is known to vanish, so they gave a comprehensive description of the fifth-order theory. Das and Hopfinger gave the experimental phase diagram to exhibit the existence of steady state wave motion, and took into account small surface tension and viscosity [9]. The numerical simulation of two-dimensional Faraday waves between two types of fluids was performed by Takagi *et al.* [10]. With a long period of the vibration forcing, they found that the interface turned over and became a multivalued function of the horizontal coordinate. Moreover, the surface elevation changed with time dramatically. In 2011, three researchers in France observed two new types of standing solitary waves in a (nearly two-dimensional) Hele-Shaw cell filled with water [11]. Currently, Li *et al.* [12] observed the highly localized, two-dimensional resonant standing Faraday waves with multiple crests and troughs in a Hele-Shaw cell partly filled with ethanol-water solution, which was vertically oscillated with a single forcing fre-

quency. Especially, they observed an interesting phenomenon that the wave height of these two-dimensional standing solitary waves seems independent of the fluid depth varying from 10 mm to 50 mm [12]. This independence of the wave height on the ethanol-water depth is very interesting. What happens if the liquid depth becomes extremely shallow (such as 1 mm or 2 mm)? This is the motivation of the current work.

In this paper, a family of two-dimensional Faraday waves is observed experimentally in a Hele-Shaw cell filled with absolute ethanol in an extremely shallow depth (1 mm to 2 mm). They are quite different from traditional Faraday waves.

II. EXPERIMENTAL SETUP

A Hele-Shaw cell (made of PMMA) with 300 mm length, 1.7 mm width, and 60 mm depth is filled with absolute ethanol in a rather shallow depth (from 1 mm to 2 mm), fixed onto a shaker. The temperature is nearly 25 °C. The cell is guided vertically with a sinusoidal vibration, and the forcing frequency is fixed on 18 Hz. The acceleration amplitude is measured by an oscilloscope and an amplifier referenced externally to the shaker. A high-speed (400 fps) camera is oriented perpendicular to the cell so as to record the deformation of free surface. Considering the volatilization of absolute ethanol, the cell is sealed. The liquid is replaced every ten minutes and the variance of the depth is measured before and after each experiment.

III. RESULTS

First of all, the experiment is performed by using absolute ethanol in a depth (expressed by D) of 2 mm, with the driving frequency of 18 Hz and the acceleration amplitude of 19.80 m/s^2. The first mode of this family of Faraday waves in the extremely shallow depth is shown in Fig. 1. When these Faraday waves display their maximum (we set this moment as $t = 0$), the crests hold most of liquid, so that the trough becomes a long, almost *horizontal* line segment very close to the bottom of the cell. Then, as the amplitude decreases, more and more liquid flows down to the trough.

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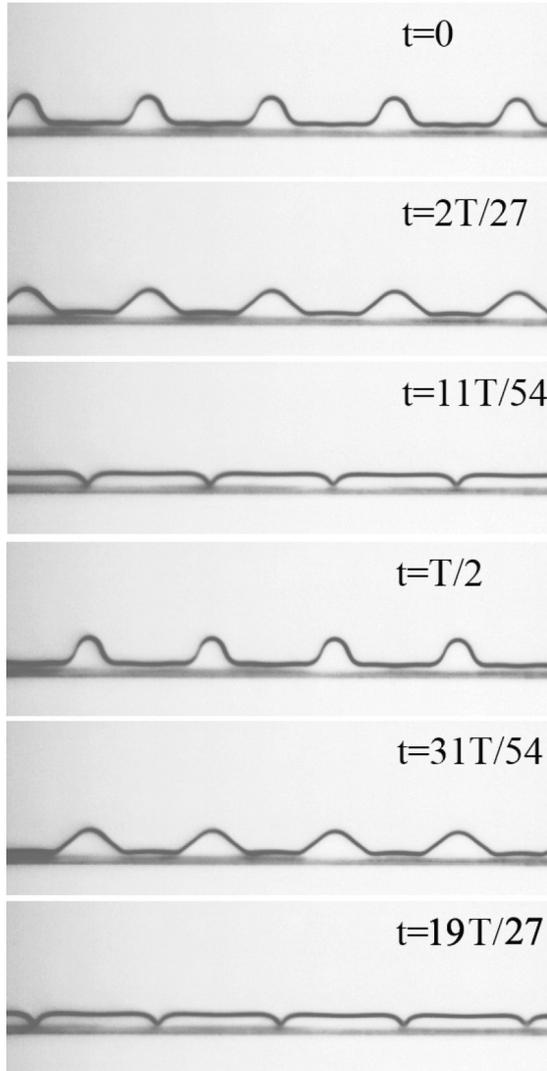[*]sjliao@sjtu.edu.cn; http://numericaltank.sjtu.edu.cn

FIG. 1. Faraday wave profiles in the extreme shallow depth (2 mm) of absolute ethanol by means of the forcing oscillation frequency of 18 Hz and the acceleration amplitude of 19.80 m/s^2 (see Supplemental Material [13]).
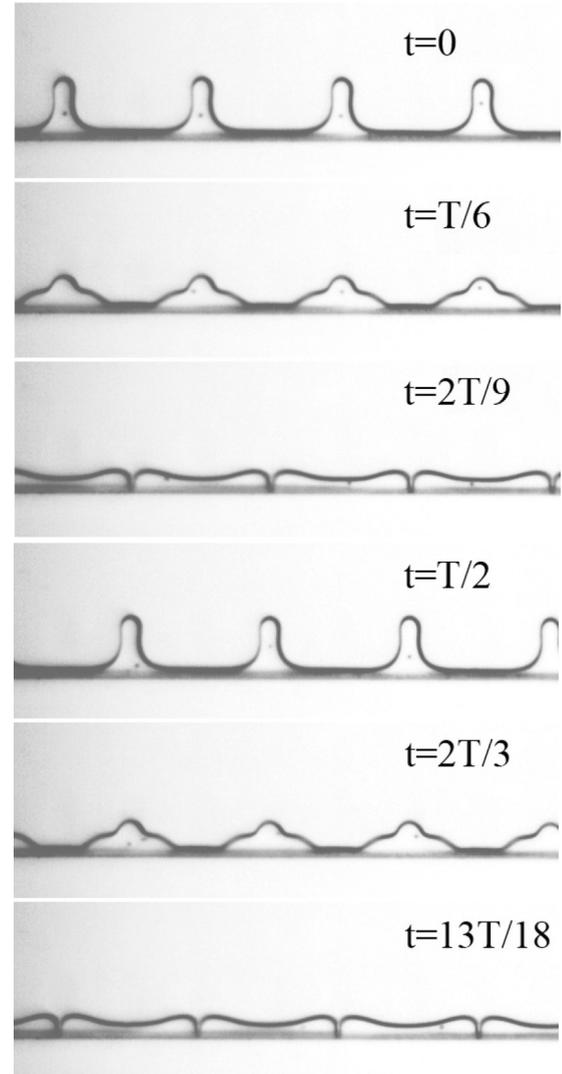
FIG. 2. Faraday wave profiles in the extreme shallow depth (2 mm) of absolute ethanol by means of the forcing oscillation frequency of 18 Hz and the acceleration amplitude of 22.62 m/s^2 (see Supplemental Material [13]).

At $t = 11T/54$, where T denotes the wave period, the crest becomes an almost *horizontal*, long line segment, when the trough becomes rather sharp in a rather small region. As t increases, the fluid continues to move towards the trough until it becomes the crest at $t = T/2$, which again contains most of the fluid. The pattern of the two-dimensional Faraday wave changes periodically.

Secondly, using the same fluid depth and driving frequency as mentioned above, we increase the acceleration amplitude to 22.62 m/s^2. As shown in Fig. 2, the second mode of this family of Faraday waves in the extremely shallow depth is very unique and quite distinct from the traditional ones. When the wave amplitude displays its maximum (at $t = 0$), the crest includes almost all of the fluid, and especially part of the surface of the crest becomes almost vertical, while the trough becomes a *horizontal*, long line segment that almost touches the bottom. As the amplitude decreases, more and more fluid flows back to the trough. At $t = 2T/9$, the trough becomes

very sharp in a rather small region, but the crest in the form of a long line segment is a little bit hollow, *not* horizontal any more, indicating its difference from the first mode of the family of Faraday waves shown in Fig. 1 at $t = 11T/54$. Thereafter, most of the fluid flows towards the trough so that it becomes the crest that contains almost all fluid at its maximum when $t = T/2$. The pattern of this kind of two-dimensional Faraday waves moves periodically as the first one. Besides, near the lateral walls of the cell there are two special waves which only have a half part. Their maximum seems like a very thin, vertical bar which sticks to the walls, not similar to the shape of other waves when their minimum has the same shape with other troughs. This is mainly due to the effect of the viscous boundary layer near the lateral walls, which does not influence the Faraday waves far from the two lateral walls.

Especially, the wave elevations at $T = 11T/54$ and $T = 38T/54$ in Fig. 1 with a *horizontal*, long line-segment crest and a sharp trough are very interesting, so are the elevations at
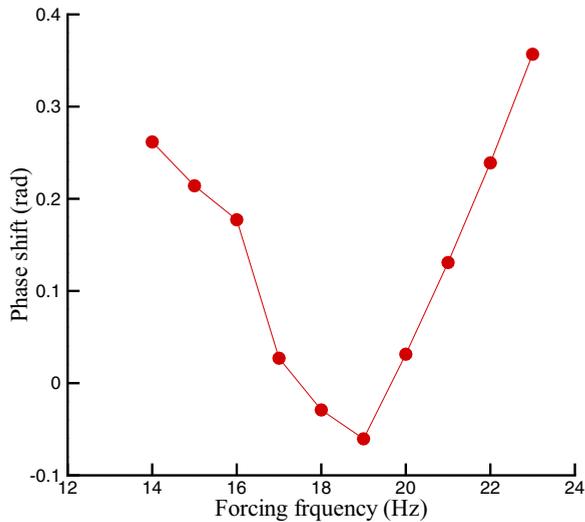
FIG. 3. (Color online) Phase shift between the wave and the forcing signal. Experiments are performed with the driving frequency from 14 Hz to 23 Hz and the acceleration amplitude of 19.80 m/s^2 in the depth of 2 mm.

$T = T/6$ and $T = 2T/9$ in Fig. 2, and especially the highest crest (at $t = 0$) with a vertical steepness and the lowest trough with the longest, horizontal line segment that almost touches the bottom. All of these phenomena are unique.

It is worth noting that, when the oscillation frequency increases from 14 Hz to 23 Hz, there always exist two critical values of the acceleration amplitude, at which these two modes of this family of Faraday waves generates, respectively. It indicates that this family of Faraday waves exists widely. The phase shift also exists between the wave and the forcing signal of the shaker, like the case of the traditional Faraday waves. A bifurcation diagram for it with the frequency in the range of 14 Hz to 23 Hz is shown in Fig. 3.

Obviously, the liquid depth has a profound effect on the first and second modes of this family of Faraday waves (as shown in Figs. 1 and 2). To investigate its effect, the liquid depth is further reduced to 1 mm, with the same driving frequency of 18 Hz but a larger acceleration amplitude of 40.01 m/s^2. The third mode of this family of Faraday waves was observed, as shown in Fig. 4. When the wave amplitude arrives at its maximum (at $t = 0$), the crest looks like that of the second mode shown in Fig. 2, but with a bigger top and a smaller footer, like a drop splashing up, while the long, flat trough almost gets in touch with the bottom in a rather long domain. As the crest decreases, the fluid flows back towards the trough. At $t = T/9$ and $t = 11T/18$, the trough in the form of a long line segment almost touches the bottom, and besides the crest contains so much fluid that the first derivative of the wave elevation seems to be discontinuous at the intersection of the trough and crest. At $t = 11T/54$, the crest becomes long but *not* in a line segment, while the trough is sharp in a rather small domain. The pattern of this kind of two-dimensional Faraday wave oscillates periodically.

To investigate the characteristics of this family of Faraday waves in the extremely shallow depth, we fix the driving frequency on 18 Hz and the acceleration amplitude on
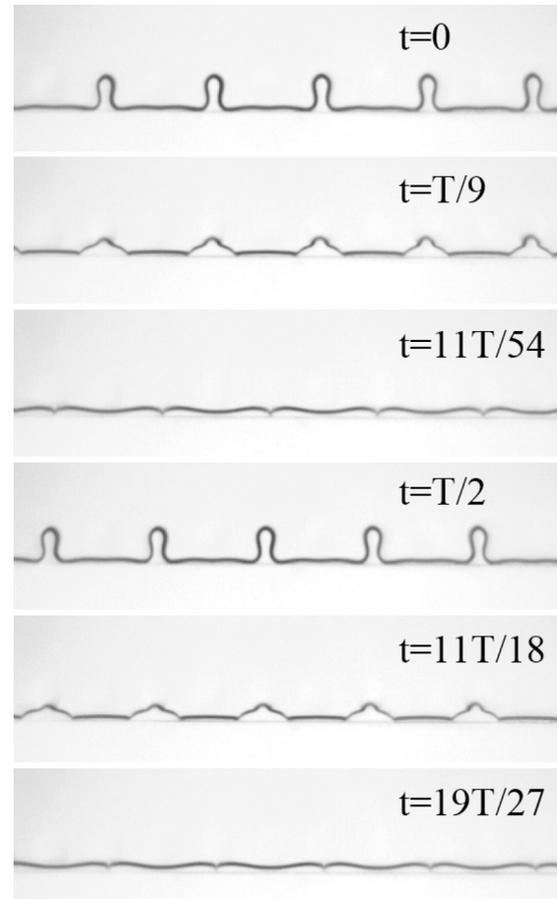

FIG. 4. Faraday wave profiles in the extreme shallow depth (1 mm) of absolute ethanol by means of the forcing oscillation frequency of 18 Hz and the acceleration amplitude of 41.01 m/s^2 (see Supplemental Material [13]).

19.80 m/s^2, but change the depth from 2 mm to 40 mm in steps. The wave heights versus fluid depth are given in Fig. 5. It is found that, when $D \geqslant 10$ mm, the wave heights are nearly the same from 9.5 mm to 10.5 mm (averaged value 9.553 $\pm$ standard deviation 0.103 mm). In other words, the
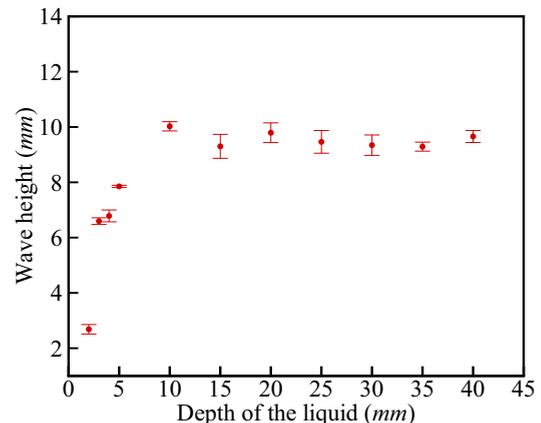

FIG. 5. (Color online) Wave heights vs depths of absolute ethanol. Experiments are performed with the driving frequency of 18 Hz and the acceleration amplitude of 19.80 m/s^2.

wave height is *independent* of the depth in this range. This agrees qualitatively with the experimental results reported by Li *et al.* [12] for the two-dimensional Faraday waves of ethanol solution in deep depth ($D \geqslant 10$ mm). However, when 2 mm $< D < 10$ mm, the wave height decreases linearly with the liquid depth, as shown in Fig. 5. It is very interesting that, when $D \leqslant 2$ mm, the wave height decreases so greatly that even the first derivative of the wave elevation might become discontinuous, as shown in Fig. 4 at $t = T/9$ and $t = 11T/18$. Thus this family of Faraday waves in the extremely shallow depth ($D \leqslant 2$ mm) are essentially different from those in deep depth ($D \geqslant 10$ mm), mainly because there is not enough liquid and especially the viscous effect becomes much larger due to the existence of a very thin boundary layer in the extremely shallow depth, compared with in deep fluid.

To further confirm this, we fix the driving frequency at 18 Hz, and increase the forcing acceleration amplitude in uniform steps (from the value which the waves display with to the threshold of wave breaking) of different depths, respectively, as shown in Fig. 6(a). The interval of forcing acceleration for the existence of stable Faraday waves decreases with depth. It is found that the ratio of wave height to wave depth, i.e., h/D, increases gently with the forcing acceleration in the depth of 5 mm and 10 mm, but more quickly in the depth of 3 mm and 2 mm. Especially, in the depth of 2 mm, the ratio h/D might be two times larger than those in the depth of 5 mm and 10 mm, as shown in Fig. 6(a). In the depth of 2 mm, the ratio h/D first slowly increases as the forcing acceleration amplitude varies from 19 m/s^2 to 22 m/s^2, corresponding to the first mode of the family of Faraday waves as shown in Fig. 1. Then, in the range of 22 m/s^2 to 23 m/s^2, the ratio h/D increases rather quickly, corresponding to the second mode of the family of Faraday waves shown in Fig. 2. However, it is difficult to gain precise values of these two thresholds of forcing acceleration, because they are very sensitive to the environment conditions. As the forcing acceleration amplitude varies from 23 m/s^2 to 24 m/s^2, the ratio h/D grows a little, corresponding to the second mode of the family of Faraday waves as shown in Fig. 2, but near wave breaking.

The wavelengths of this family of Faraday waves are also investigated, as shown in Fig. 6(b). Four forcing acceleration amplitudes are considered in four different depths, respectively, as the driving frequency increases from 14 Hz to 23 Hz. It is found that the average lengths of this family of Faraday waves decrease with the driving frequency but in different slopes for different depth.

Note that the *absolute* ethanol is used in our experiment, for we did *not* observe the same phenomena by means of water, silicone oil, and ethanol solution. Unfortunately, we do not know how to explain this phenomena in theory. In Table I we simply list some properties of these fluids. First of all, the surface tension of distilled water in a PMMA cell is two times larger than that of absolute ethanol, which cannot meet the requirement of the wettability, indicating that the surface tension is an important factor to the family of the Faraday waves in the extremely shallow depth [14]. Secondly, although the surface tension of silicone oil meets the standard, it has a much larger viscosity than that of absolute ethanol, which makes it adhere to the walls. Besides, for the family of Faraday waves in the extremely shallow depth,
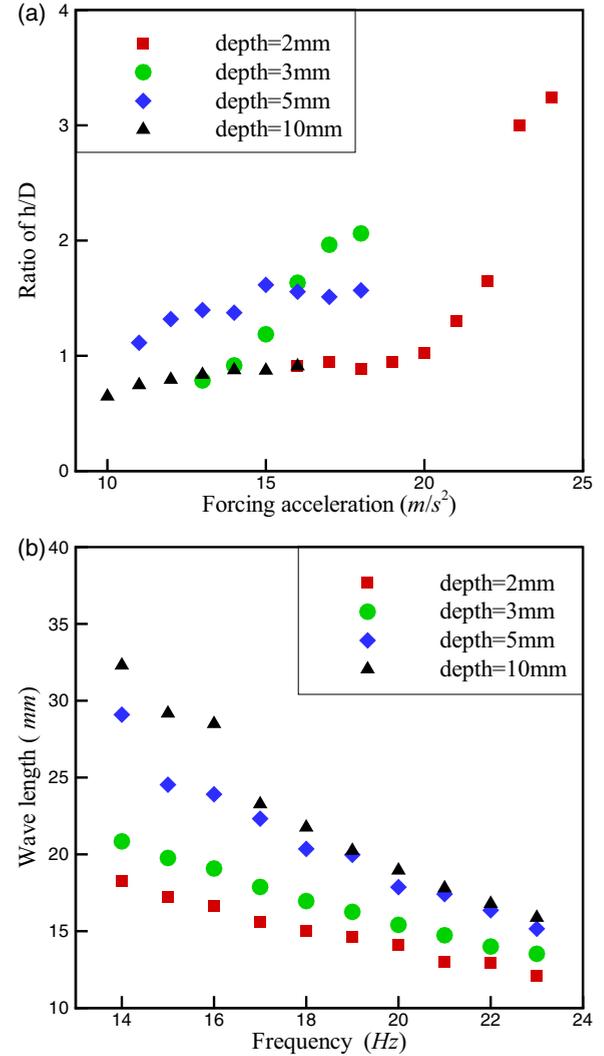

FIG. 6. (Color online) (a) Ratio of h (wave height) to D (depth) vs forcing amplitude for different liquid depth (2 mm, 3 mm, 5 mm, and 10 mm) with the driving frequency of 18 Hz. (b) Wavelength vs wave forcing frequency for given forcing acceleration amplitude in different depth, respectively. (2 mm vs 19.8 m/s^2, 3 mm vs 16 m/s^2, 5 mm vs 14 m/s^2, and 10 mm vs 12 m/s^2).

the horizontal, long trough has such a very thin boundary layer that it almost touches the bottom. Thus the viscous friction must be considered. This suggests that the viscosity is also very important to the family of the Faraday waves in

TABLE I. Relevant properties of some liquids.

Properties (25 °C)	Surface tension (mN/m)	Density (kg/m³)	Viscosity (Cp)
Absolute ethanol	22.51	0.791	1.096
Water	72.01	1	0.8937
Silicone oil	21.0	0.963	100
Ethanol-solution (20%)	37.97	0.969	1.815
Ethanol-solution (50%)	27.96	0.912	2.4

the extremely shallow depth. The density of these fluids are close, suggesting that the density might not be important to the family of Faraday waves. Therefore, the proper values of surface tension and viscosity might be the key points to the family of Faraday waves in the extremely shallow depth. Note that the water-ethanol solution with the concentration between 20% and 50% can display Faraday waves in a little bigger depth (more than 10 mm), but can*not* lead to the family of Faraday waves in extremely shallow depth. In addition, a contrast experiment using a three-dimensional basin with the same parameters as those in the 2D Hele-Shaw cell was done, but similar phenomena were not observed. All of these suggest that the surface tension and viscous effect between the fluid and the sidewall of the cell might play an important role to the family of Faraday waves in the extremely shallow depth.

IV. CONCLUSION

Certainly, more theoretical investigations, numerical simulations, and experiments are needed to further study this family of Faraday waves in the extremely shallow depth. However, it should be emphasized that the trough of these Faraday waves almost gets in touch with the bottom so that it might be a challenge to numerically simulate these experimental phenomena.

In conclusion, we experimentally observe a family of two-dimensional Faraday waves in the extremely shallow depth of absolute ethanol. These phenomena are helpful to deepen and enrich our understandings about Faraday waves, and besides provide a challenging problem for computational fluid dynamics (CFD).

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